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## 1. Template Java

```
import java.util.*;
import java.io.*;
import java.lang.*;
import java.math.BigInteger;

public class TEMPLATE {
    public static void main(String[] args) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        Task solver = new Task();
        solver.solve(1, in, out);
        out.close();
    }
}

class Task {
    public void solve(int testNumber, InputReader in, PrintWriter out) {

    }
}

class InputReader {
    public BufferedReader reader;
    public StringTokenizer tokenizer;

    public InputReader(InputStream stream) {
        reader = new BufferedReader(new InputStreamReader(stream),
32768);
        tokenizer = null;
    }

    public String next() {
        while (tokenizer == null || !tokenizer.hasMoreTokens()) {
            try {
                tokenizer = new StringTokenizer(reader.readLine());
            } catch (IOException e) {
                throw new RuntimeException(e);
            }
        }
    }
}
```

```
    }
    return tokenizer.nextToken();
}
public int nextInt() {
    return Integer.parseInt(next());
}
public long nextLong() {
    return Long.parseLong(next());
}
public double nextDouble() {
    return Double.parseDouble(next());
}
}
```

## 2. Knuth-Morris-pratt (Precompute & Checking)

Precompute :

```
Arrays.fill(a, 0);
for(int i = 1; i < n; i++) {
    int j = a[i - 1];
    while(j > 0 && s[i] != s[j]) j = a[j - 1];
    if(s[i] == s[j]) a[i] = j + 1;
}
```

Checking :

```
int[] b = computeKMP(pattern);
int j = 0;
for(int i = 0; i < text.length(); i++) {
    if(pattern.charAt(j) == text.charAt(i)) {
        i++; j++;
    } else if(j > 0) {
        j = b[j - 1];
    } else {
        i++;
    }
    if(j == pattern.length()) {
        // found a match
    }
}
```

```

        return i - pattern.length();
    }
}
return NOT_FOUND;

```

### 3. Const Big Prime Number

```

1e9 + 9, 1e9 + 87, 1e9 + 4207, 2e9 + 89, 2e9 + 143, 2e9 + 11, 2e9 + 1851, 2e9
+ 2153,
252097800623, 1e15 - 11, 1e15 + 37,

```

### 4. Miller Rabin Big Primality test

```

vector<long long> A({2, 3, 5, 7, 11, 13, 17, 19, 23});
// if n < 3,825,123,056,546,413,051, it is enough to test a = 2, 3, 5,
7, 11, 13, 17, 19, and 23.
long long fastmul(long long a, long long b, long long n) {
    long long ret = 0;
    while (b) {
        if (b & 1)
            ret = (ret + a) % n;
        a = (a + a) % n;
        b >>= 1;
    }
    return ret;
}

long long fastexp(long long a, long long b, long long n) { //compute
(a^b) mod n
    long long ret = 1;
    while (b) {
        if (b & 1)
            ret = fastmul(ret, a, n);
        a = fastmul(a, a, n);
        b >>= 1;
    }
    return ret;
}

bool mrtest(long long n)
{
    if(n == 1) return false;
    long long d = n-1;
    long long s = 0;
    while(d % 2 == 0)

```

```

{
    s++;
    d /= 2;
}
for(long long j=0;j<(long long)A.size();j++)
{
    if(A[j] > n-1) continue;
    long long ad = fastexp(A[j], d, n);
    if(ad % n == 1) continue;
    bool notcomp = false;
    for(long long r=0;r<=max(0LL,s-1);r++)
    {
        long long rr = fastexp(2,r,n);
        long long ard = fastexp(ad, rr, n);
        if(ard % n == n-1) {notcomp = true; break;}
    }
    if(!notcomp)
    {
        return false;
    }
}
return true;
}

```

### 5. Extended Euclidean Algorithm

```

long long x, y, d; // ax + by = d
void extendedEuclidean(long long a, long long b) {
    if(b == 0) { x = 1; y = 0; d = a; return; }
    extendedEuclidean(b, a % b);
    long long xx, yy;
    xx = y;
    yy = x - (a/b)*y;
    x = xx; y = yy;
}

```

## 6. FFT biasa & FFT versi modular arithmetic (perkalian polinom)

```

/***** FFT dengan complex *****/
typedef complex<double> cd;
typedef vector< cd > vcd;

// asumsi ukuran as = 2^k, dengan k bilangan bulat positif
vcd fft(const vcd &as) {
    int n = (int)as.size();
    int k = 0;
    while((1<<k) < n) k++;
    vector< int > r(n);
    r[0] = 0;
    int h = -1;
    for(int i = 1; i<n; i++) {
        if((i & (i-1)) == 0)
            h++;
        r[i] = r[i ^ (1 << h)];
        r[i] |= (1<<(k-h-1));
    }
    vcd root(n);
    for(int i = 0; i<n; i++) {
        double ang = 2.0*M_PI*i/n;
        root[i] = cd(cos(ang), sin(ang));
    }

    vcd cur(n);
    for(int i = 0; i<n; i++)
        cur[i] = as[r[i]];

    for(int len = 1; len < n; len <= 1) {
        vcd ncur(n);
        int step = n/(len <= 1);
        for(int pdest = 0; pdest < n; pdest++) {
            for(int i = 0; i<len; i++) {
                cd val = root[i*step]*cur[pdest + len];
                ncur[pdest] = cur[pdest] + val;
                ncur[pdest + len] = cur[pdest] - val;
                pdest++;
            }
            pdest += len;
        }
    }
}

```

```

        cur.swap(ncur);
    }
    return cur;
}

vcd inv_fft(const vcd& fa) {
    vcd res = fft(fa);
    for(int i = 0; i<n; i++) {
        res[i] /= n;
    }
    reverse(res.begin() + 1, res.end());
    return res;
}

/***** FFT dengan Modular Arithmetic *****/
const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;

void fft (vector<int> &a, bool invert) {
    int n = (int) a.size();

    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }

    for (int len=2; len<=n; len<=1) {
        int wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = int (wlen * 111 * wlen % mod);
        for (int i=0; i<n; i+=len) {
            int w = 1;
            for (int j=0; j<len/2; ++j) {
                int u = a[i+j], v = int (a[i+j+len/2] *
111 * w % mod);
                a[i+j] = u+v < mod ? u+v : u+v-mod;
                a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
            }
            w = w * wlen % mod;
        }
    }
}

```

```

        w = int (w * 111 * wlen % mod);
    }
}
if (invert) {
    int nrev = reverse (n, mod);
    for (int i=0; i<n; ++i)
        a[i] = int (a[i] * 111 * nrev % mod);
}
}

```

## 7. Gaussian Elimination

```

vector<double> gauss(vector< vector<double> >& A) {
    int n = A.size();

    for (int i=0; i<n; i++) {
        double maxEl = abs(A[i][i]);
        double maxRow = i;
        for (int k=i+1; k<n; k++) {
            if (abs(A[k][i]) > maxEl) {
                maxEl = abs(A[k][i]);
                maxRow = k;
            }
        }
        for (int k=i; k<n+1; k++) {
            double tmp = A[maxRow][k];
            A[maxRow][k] = A[i][k];
            A[i][k] = tmp;
        }
        for (int k=i+1; k<n; k++) {
            double c = -A[k][i]/A[i][i];
            for (int j=i; j<n+1; j++) {
                if (i==j) {
                    A[k][j] = 0;
                } else {
                    A[k][j] += c * A[i][j];
                }
            }
        }
    }
    vector<double> x(n);
}

```

```

for (int i=n-1; i>=0; i--) {
    x[i] = A[i][n]/A[i][i];
    for (int k=i-1; k>=0; k--) {
        A[k][n] -= A[k][i] * x[i];
    }
}
return x;
}

```

## 8. Maxflow Dinic

```

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct Dinic {
    int N;
    vector<vector<Edge> > G;
    vector<Edge *> dad;
    vector<int> Q;

    Dinic(int N) : N(N), G(N), dad(N), Q(N) {}

    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }

    long long BlockingFlow(int s, int t) {
        fill(dad.begin(), dad.end(), (Edge *) NULL);
        dad[s] = &G[0][0] - 1;

        int head = 0, tail = 0;
        Q[tail++] = s;
        while (head < tail) {
            int x = Q[head++];
            for (int i = 0; i < G[x].size(); i++) {
                Edge &e = G[x][i];
                if (!dad[e.to] && e.cap - e.flow > 0) {

```

```

                dad[e.to] = &G[x][i];
                Q[tail++] = e.to;
            }
        }
    }
    if (!dad[t]) return 0;

    long long totflow = 0;
    for (int i = 0; i < G[t].size(); i++) {
        Edge *start = &G[G[t][i].to][G[t][i].index];
        int amt = INF;
        for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
            if (!e) { amt = 0; break; }
            amt = min(amt, e->cap - e->flow);
        }
        if (amt == 0) continue;
        for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
            e->flow += amt;
            G[e->to][e->index].flow -= amt;
        }
        totflow += amt;
    }
    return totflow;
}

long long GetMaxFlow(int s, int t) {
    long long totflow = 0;
    while (long long flow = BlockingFlow(s, t))
        totflow += flow;
    return totflow;
}
};

```

## 9. Minimum Cost Max Flow

```

const int inf = 1e8;

struct Edge {
    int from, to, cap, flow, cost;
    Edge(int from, int to, int cap, int flow, int cost) :
        from(from), to(to), cap(cap), flow(flow), cost(cost) {}
};

```

```

};

struct MCMF {
    int n, s, t;
    vector< vector< int > > adj;
    vector< int > par;
    vector< Edge > vEdge;
    vector< long long > dist;
    MCMF(int _n, int _s, int _t) : n(_n), adj(n), s(_s), t(_t) {}
    void addEdge(int from, int to, int cap, int cost) {
        adj[from].push_back(vEdge.size());
        adj[to].push_back(vEdge.size());
        vEdge.push_back(Edge(from, to, cap, 0, cost));
    }
    long long augment(int v, int minflow = inf) {
        if (v == s) {
            return minflow;
        }
        if (par[v] < 0) {
            return 0;
        }
        long long flow;
        Edge &e = vEdge[par[v]];
        if (v == e.from) {
            flow = augment(e.to, min(minflow, e.flow));
            e.flow -= flow;
        }
        else {
            flow = augment(e.from, min(minflow, e.cap - e.flow));
            e.flow += flow;
        }
        return flow;
    }
    long long findingPath() {
        //dijkstra
        set< pair< long long, int > > st;
        dist.assign(n, inf);
        par.assign(n, -1);
        dist[s] = 0;
        st.insert(make_pair(dist[s], s));
        while (!st.empty()) {
            set< pair< long long, int > >::iterator begin = st.begin();

```

```

int v = begin->second;
st.erase(begin);
for(int i = 0; i<adj[v].size(); i++) {
    Edge &e = vEdge[adj[v][i]];
    if(e.from == v) {
        if(e.cap > e.flow && dist[e.to] > dist[v] + e.cost) {
            st.erase(make_pair(dist[e.to], e.to));
            dist[e.to] = dist[v] + e.cost;
            st.insert(make_pair(dist[e.to], e.to));
            par[e.to] = adj[v][i];
        }
    }
    else {
        if(e.flow > 0 && dist[e.from] > dist[v] - e.cost) {
            st.erase(make_pair(dist[e.from], e.from));
            dist[e.from] = dist[v] - e.cost;
            st.insert(make_pair(dist[e.from], e.from));
            par[e.from] = adj[v][i];
        }
    }
}
return augment(t, inf);
}
pair< long long, long long > EdmondKarp() {
    long long maxflow = 0, mincost = 0;
    long long flow;
    while(flow = findingPath()) {
        maxflow += flow;
        mincost += flow * dist[t];
    }
    return make_pair(mincost, maxflow);
}
};

```

## 10. MinCost MaxFlow with Potential

```

#include <bits/stdc++.h>

using namespace std;

#define INF 1000000000

```

```

#define MAXN 500

struct Edge
{
    int t, c, w, r;
    Edge(int _t, int _c, int _w, int _r): t(_t), c(_c), w(_w), r(_r)
    {};
};

int pot[MAXN+5], prv[MAXN+5], dist[MAXN+5], vis[MAXN+5];
vector<Edge> edge[MAXN+5];

pair<int, int> mcmf(int n, int s, int t)
{
    fill(pot, pot+n, 0);
    int mf = 0, mc = 0;
    while (true) {
        priority_queue<pair<int, int> > pq;
        fill(dist, dist+n, INF);
        fill(vis, vis+n, 0);
        pq.push(make_pair(0, s));
        dist[s] = 0;
        while (!pq.empty()) {
            pair<int, int> top = pq.top();
            pq.pop();
            int v = top.second, c = -top.first;
            if (vis[v]) continue;
            vis[v] = 1;
            for (int i = 0; i < edge[v].size(); ++i) {
                Edge &e = edge[v][i];
                int u = e.t;
                if (e.c == 0) continue;
                int ndist = dist[v] + e.w + pot[v]-pot[u];
                if (ndist < dist[u]) {
                    dist[u] = ndist;
                    prv[u] = e.r;
                    pq.push(make_pair(-ndist, u));
                }
            }
        }
        int v = t;
        if (dist[t] == INF) break;
        int flow = INF;
    }
}

```



```

    for (int i = 0; i < n; ++i) pot[i] += dist[i];
    while (v != s) {
        Edge &r = edge[v][prv[v]], &e = edge[r.t][r.r];
        flow = min(flow, e.c);
        v = r.t;
    }
    mf += flow;
    v = t;
    while (v != s) {
        Edge &r = edge[v][prv[v]], &e = edge[r.t][r.r];
        e.c -= flow;
        r.c += flow;
        mc += e.w * flow;
        v = r.t;
    }
}
return make_pair(mf, mc);
}

int main()
{
    int n, m, s, t;
    scanf("%d%d%d%d", &n, &m, &s, &t);
    for (int i = 0; i < m; ++i) {
        int u, v, c, w;
        scanf("%d%d%d%d", &u, &v, &c, &w);
        Edge a(v,c,w,edge[v].size()), b(u,0,-w,edge[u].size());
        edge[u].push_back(a);
        edge[v].push_back(b);
    }
    pair<int, int> ret = mcmf(n, s, t);
    printf("%d %d\n", ret.first, ret.second);
    return 0;
}

```

## 11. Mincost MaxFlow with Negative Cost

```

/** Max Flow Min Cost */
/* complexity: O(min(E^2 V log V, E log V F)) */
const int maxnodes = 2010;

int nodes = maxnodes;
int prio[maxnodes], curflow[maxnodes], prevedge[maxnodes],

```

```

prevnode[maxnodes], q[maxnodes], pot[maxnodes];
bool inqueue[maxnodes];

struct Edge {
    int to, f, cap, cost, rev;
};

vector<Edge> graph[maxnodes];

void addEdge(int s,int t,int cap,int cost){
    Edge a ={t,0, cap, cost, graph[t].size()};
    Edge b ={s,0,0,-cost, graph[s].size()};
    graph[s].push_back(a);
    graph[t].push_back(b);
}

void bellmanFord(int s,int dist[]){
    fill(dist, dist + nodes,1000000000);
    dist[s]=0;
    int qt =0;
    q[qt++] = s;
    for(int qh =0;(qh - qt)% nodes !=0; qh++){
        int u = q[qh % nodes];
        inqueue[u]=false;
        for(int i =0; i <(int) graph[u].size(); i++){
            Edge &e = graph[u][i];
            if(e.cap <= e.f)continue;
            int v = e.to;
            int ndist = dist[u]+ e.cost;
            if(dist[v]> ndist){
                dist[v]= ndist;
                if(!inqueue[v]){
                    inqueue[v]=true;
                    q[qt++% nodes]= v;
                }
            }
        }
    }
}
}
}
}

```

```

pair<int, int> minCostFlow(int s, int t, int maxf){
    // bellmanFord can be safely commented if edges costs are non-
    negative
    bellmanFord(s, pot);
    int flow = 0;
    int flowCost = 0;
    while(flow < maxf){
        priority_queue<ll, vector<ll>, greater<ll>> q;
        q.push(s);
        fill(prio, prio + nodes, 1000000000);
        prio[s] = 0;
        curflow[s] = 1000000000;
        while(!q.empty()){
            ll cur = q.top();
            int d = cur >> 32;
            int u = cur;
            q.pop();
            if(d != prio[u]) continue;
            for(int i = 0; i < (int) graph[u].size(); i++){
                Edge &e = graph[u][i];
                int v = e.to;
                if(e.cap <= e.f) continue;
                int nprio = prio[u] + e.cost + pot[u] - pot[v];
                if(prio[v] > nprio){
                    prio[v] = nprio;
                    q.push(((ll) nprio << 32) + v);
                    prevnode[v] = u;
                    prevedge[v] = i;
                    curflow[v] = min(curflow[u], e.cap - e.f);
                }
            }
        }
        if(prio[t] == 1000000000) break;
        for(int i = 0; i < nodes; i++) pot[i] += prio[i];
        int df = min(curflow[t], maxf - flow);
        flow += df;
        for(int v = t; v != s; v = prevnode[v]){
            Edge &e = graph[prevnode[v]][prevedge[v]];
            e.f += df;
            graph[v][e.rev].f -= df;
            flowCost += df * e.cost;
        }
    }
}

```

```

    return make_pair(flow, flowCost);
}

/* usage example:
* addEdge (source, target, capacity, cost)
* minCostFlow(source, target, INF) -> flow, flowCost
*/

```

## 12. Maximum Cardinality Bipartite Matching

The code below finds a augmenting path:

```

bool dfs(int v){ // v is in X, it returns true if and only if there is
    an augmenting path starting from v
    if(mark[v])
        return false;
    mark[v] = true;
    for(auto &u : adj[v])
        if(match[u] == -1 || dfs(match[u])) // match[i] = the
            vertex i is matched with in the current matching, initially -1
            return match[v] = u, match[u] = v, true;
    return false;
}

```

An easy way to solve the problem is:

```

for(int i = 0; i < n; i++) if(match[i] == -1){
    memset(mark, false, sizeof mark);
    dfs(i);
}

```

But there is a faster way:

```

while(true){
    memset(mark, false, sizeof mark);
    bool fnd = false;
    for(int i = 0; i < n; i++) if(match[i] == -1 && !mark[i])
        fnd |= dfs(i);
    if(!fnd)
        break;
}

```

### 13. Finding Cut Vertices & Cut Edges

```
// Tarjan version again
void dfs(int v) {
    low[v]= num[v] = ++cntr;
    for(auto u : adj[v]) {
        if(num[u] == -1) {
            par[u] = v;
            if(v == Root) rootChild++;

            dfs(u);

            if(low[u] >= num[v])
                articulation_vertex[v] = true;
            if(low[u] > num[v])
                printf("Edge (%d %d) is a bridge\n", v, u);

            low[v] = min(low[v], low[u]);
        }
        else if(u != parent[v])
            low[v] = min(low[v], num[u]);
    }
}

// Inside Main
cntr = 0;
num.assign(n, -1);
low.assign(n, 0);
par.assign(n, -1);
articulation_vertex.assign(n, 0);
for(int i = 0; i < n; i++) if(num[i] == -1) {
    Root = i;
    rootChild = 0;
    dfs(i);
    articulation_vertex[i] = (rootChild > 1);
}
```

### 14. Rumus-rumus kombin

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$

$$\sum_{k=0}^n k^2 \binom{n}{k} = (n + n^2)2^{n-2}$$

$$\sum_{j=0}^n \binom{m}{j} \binom{n-m}{k-j} = \binom{n}{k}$$

$$\sum_{m=0}^n \binom{m}{j} \binom{n-m}{k-j} = \binom{n+1}{k+1},$$

$$\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}.$$

$$\sum_{j=0}^{\lfloor \frac{n}{2} \rfloor} \binom{m}{j}^2 = \binom{2m}{m}.$$

$$\sum_{k=0}^n \binom{n-k}{k} = F(n+1).$$

$$\sum_{i=0}^n i \binom{n}{i}^2 = \frac{n}{2} \binom{2n}{n}$$

$$\sum_{i=0}^n i^2 \binom{n}{i}^2 = n^2 \binom{2n-2}{n-1}.$$

$$\sum_{k=q}^n \binom{n}{k} \binom{k}{q} = 2^{n-q} \binom{n}{q}$$

Dixon Identity:

$$\sum_{k=-a}^a (-1)^k \binom{2a}{k+a}^3 = \frac{(3a)!}{(a!)^3}$$

$$\sum_{k=-a}^a (-1)^k \binom{a+b}{a+k} \binom{b+c}{b+k} \binom{c+a}{c+k} = \frac{(a+b+c)!}{a! b! c!}, \text{ where}$$

$a, b, \text{ and } c$  are non-negative integers

$$\binom{n}{k_1, k_2, \dots, k_r} = \frac{n!}{k_1! k_2! \dots k_r!}$$

$$\binom{z}{m} \binom{z}{n} = \sum_{k=0}^m \binom{m+n-k}{k, m-k, n-k} \binom{z}{m+n-k}$$

### Lucas' Theorem :

For non-negative integers  $m$  and  $n$  and a prime  $p$ , the following [congruence relation](#) holds:

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base  $p$  expansions of  $m$  and  $n$  respectively. This uses the

convention that  $\binom{m}{n} = 0$  if  $m < n$ .

Example : (combinatorics in small mod when  $\text{mod} < n$  &&  $\text{mod} < k$ )

```
int comb[mod][mod];
int c(int n, int k) {
    return n == 0? 1 : comb[n%mod][k%mod] * c(n/mod, k/mod) % mod;
}
```

### Faulhaber's Formula

$$(n+1)^{k+1} - 1 = \sum_{m=1}^n ((m+1)^{k+1} - m^{k+1}) = \sum_{p=0}^k \binom{k+1}{p} (1^p + 2^p + \dots + n^p)$$

Examples:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2} \text{ (the [triangular numbers](#))}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + 3n^2 + n}{6}$$

(the [square pyramidal numbers](#))

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2 = \frac{n^4 + 2n^3 + n^2}{4}$$

(the [squared triangular numbers](#))

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$= \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$$

$$1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

$$= \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$$

$$1^6 + 2^6 + 3^6 + \dots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$$

$$= \frac{6n^7 + 21n^6 + 21n^5 - 7n^3 + n}{42}$$

## 15. Template Geometri

```
// rotate p by theta degrees CCW w.r.t origin(0, 0)
point rotate(point p, double tetha) {
    // rotate matrix R(theta) = [cos(theta) -sin(theta)]
    //                               [sin(theta)  cos(theta)]
    double rad = tetha * PI / 180.0;
    return point(p.x*cos(rad) - p.y*sin(rad), p.x*sin(rad) +
p.y*cos(rad));
}

// (LINE)
struct line { double a,b,c; };
void pointToLine(point p1, point p2, line *l) {
    if(p1.x == p2.x) {
        l->a = 1.0; l->b = 0.0; l->c = -p1.x;
    }
    else {
        l->a = -(double)(p1.y-p2.y)/(p1.x-p2.x)l
```

```
        l->b = 1.0;
        l->c = -(double)(l->a * p1.x) - (l->b * p1.y);
    }
}
bool areIntersect(line l1, line l2, point * p) {
    if(areSame(l1, l2)) return false;
    if(areParallel(l1, l2)) return false;

    p->x = (l2.b*l1.c - l1.b*l2.c)/(l2.a*l1.b - l1.a*l2.b);
    if(fabs(l1.b) > EPS)
        p->y = (l1.a*p->x + l1.c)/l1.b;
    else
        p->y = (l2.a*p->x + l2.c)/l2.b;
    return true;
}
double area(const vector< point > & P) {
    double result = 0.0;
    for(int i = 0; i< (int)P.size()-1; i++) {
        result += (P[i].x * P[i+1].y - P[i].y*P[i+1].x);
    }
    return fabs(result)/2.0;
}
// calculate angle between BA and BC
double angle(point a, point b, point c) {
    double ux = a.x - b.x, uy = a.y - b.y;
    double vx = c.x - b.x, vu = c.y - b.y;
    return acos(ux*vx + uy*vy)/sqrt((ux*ux + uy*uy)*(vx*vx + vy*vy)); }
// check if point p inside (CONVEX/CONCAVE) polygon vp
int inPolygon(point p, const vector< point > & vp) {
    int wn = 0, n = (int)vp.size() - 1;
    for(int i = 0; i<n; i++) {
        long long cs = cross(vp[i+1], vp[i], p);
        if(cs == 0 && 1LL * (vp[i].x - p.x) * (vp[i+1].x - p.x ) <= 0 &&
1LL * (vp[i].y - p.y) * (vp[i+1].y - p.y ) <= 0)
            return 1;
        if(vp[i].y <= p.y) {
            if(vp[i+1].y > p.y && cs > 0)
                wn++;
        }
        else {
            if(vp[i+1].y <= p.y && cs < 0)
                wn--;
        }
    }
}
```

```

}
return wn;
}

```

## 16. Find two center of same size circle from its intersection and their radius

```

bool circle2PtsRad(point p1, point p2, double r, point *c) { // answer
at *c
    double d2 = (p1.x - p2.x) * (p1.x - p2.x) + (p1.y - p2.y)*(p1.y
- p2.y);
    double det = r * r / d2 - 0.25;
    if(det<0.0) return false;
    double h = sqrt(det);
    c->x = (p1.x + p2.x) * 0.5 + (p1.y-p2.y)*h;
    c->y = (p1.y + p2.y) * 0.5 + (p2.x-p1.x)*h;
    return true;
}

```

## 17. The Great-Circle Distance (SPHERES)

```

double gcDistance(double plat, double plong, double qlat, double,
qlong ,double radius) {
    plat *= PI/180; plong *= PI/180;
    qlat *= PI/180; qlong *= PI/180;
    return radius * acos(cos(plat)*cos(plong)*cos(qlat)*cos(qlong)
+
    cos(plat)*sin(plong)*cos(qlat)*sin(qlong) +
    sin(plat)*sin(qlat));
}

```

## 18. Cutting Polygon with a Straight Line

```

const long double EPS = 1e-7;

struct point {
    double x, y;
}

```

```

point(double x, double y) : x(x), y(y) {}
};

double cross(point p, point q, point r) {
    return (p.x - q.x) * (r.y - q.y) - (p.y - q.y) * (r.x - q.x);
}

// line segment p-q intersect with line A-B
point lineIntersectSeg(point p, point q, point A, point B) {
    double a = B.y - A.y;
    double b = A.x - B.x;
    double c = B.x * A.y - A.x * B.y;
    double u = fabs(a * p.x + b * p.y + c);
    double v = fabs(a * q.x + b * q.y + c);
    return point((p.x*v + q.x*u)/(u+v), (p.y*v + q.y*u)/(u+v));
}

// cuts polygon Q along the line formed by point a-> point b
// (note: the last point must be the same as the first point)
vector<point> cutPolygon(point a, point b, vector<point> Q) {
    vector<point> P;
    for(int i = 0; i<(int)Q.size(); i++) {
        double left1 = cross(a, b, Q[i]), left2 = 0.0;
        if(i != (int)Q.size()-1) left2 = cross(a, b, Q[i+1]);
        if(left1 > -EPS) P.push_back(Q[i]);
        if(left1 * left2 < -EPS)
            P.push_back(lineIntersectSeg(Q[i], Q[i+1], a,
b));
    }

    if(P.empty()) return P;
    if(fabs(P.back().x - P.front().x) > EPS ||
    fabs(P.back().y - P.front().y) > EPS)
        P.push_back(P.front());
    return P;
}

```

## 19. Convex hull (Graham's Scan & Andrew's Monotone Chain)

```

typedef pair<long long,long long> point;

```

```

#define x first
#define y second
// (p-q) x (r-q)
long long cross(point p, point q, point r) {
    return (p.x - q.x) * (r.y - q.y) - (p.y - q.y) * (r.x - q.x);
}

bool collinear(point a, point o, point b) {
    return cross(a, o, b) == 0;
}

// true if point r is on the left side of line pq
bool ccw(point p, point q, point r) {
    return cross(p, q, r) > 0;
}

point pivot;

long long dist2(point a, point b) {
    return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
}

bool angle_cmp(point a, point b) {
    if(collinear(pivot, a, b)) {
        return dist2(a, pivot) < dist2(b, pivot);
    }
    return ccw(pivot, a, b);
}

bool cmp(point a, point b) {
    return a.y < b.y || (a.y == b.y && a.x < b.x);
}

// P tidak siklik, P[0] tidak mengulang di P.back()
// return convex hull siklik, P[0] mengulang di P.back()
vector<point> ConvexHull(vector<point> P) {
    int i, j, n = (int) P.size();
    if(n < 3)
        return P;
    int P0 = 0;
    for(i = 1; i < n; i++) {
        if(cmp(P[P0], P[P[i]])) {
            P0 = i;
        }
    }

```

```

    }
    swap(P[0], P[P0]);
    pivot = P[0];
    if(collinear(P.back(), P[0], P[1])) {
        vector<point> S;
        S.push_back(P[0]);
        S.push_back(P.back());
        return S;
    }
    sort(++P.begin(), P.end(), angle_cmp);
    int k = P.size() - 1;
    while(k && collinear(P[0], P[k-1], P[k])) k--;
    reverse(P.begin() + k, P.end());
    vector<point> S;
    S.push_back(P[n-1]);
    S.push_back(P[0]);
    S.push_back(P[1]);
    i = 2;
    while(i < n) {
        j = (int) S.size() - 1;
        if(ccw(S[j-1], S[j], P[i])) S.push_back(P[i++]);
        else S.pop_back();
    }
    S.pop_back();
    return S;
}

int main(void)
{
    int n;
    scanf("%d", &n);
    vector<point> p;
    for(int i = 0; i < n; i++) {
        int a, b;
        scanf("%d %d", &a, &b);
        p.push_back(point(a, b));
    }
    vector<point> ch = ConvexHull(p);
    cout << ch.size() << endl;
    for(auto it : ch) {
        printf("%I64d %I64d\n", it.x, it.y);
    }
}

```

```

    return 0;
}

```

```

// Andrew's Monotone Chain
struct Point {
    int x, y;
    Point() {}
    Point(int x, int y): x(x), y(y) {}

    Point operator + (const Point& a) const {
        return Point(x+a.x, y+a.y);
    }
    Point operator - (const Point& a) const {
        return Point(x-a.x, y-a.y);
    }
    int operator % (const Point& a) const {
        return x*a.y - y*a.x;
    }
    bool operator<(const Point &rhs) const { return make_pair(y,x) <
make_pair(rhs.y,rhs.x); }
    bool operator==(const Point &rhs) const { return make_pair(y,x) ==
make_pair(rhs.y,rhs.x); }
};

int ccw(Point a, Point b, Point c) {
    int t = (b - a) % (c - a);
    if (t == 0) return 0;
    if (t < 0) return -1;
    return 1;
}

typedef vector< Point > Polygon;
int area2(Point a, Point b, Point c) { return a%b + b%c + c%a; }
bool between(const Point &a, const Point &b, const Point &c) {
    return (area2(a,b,c) == 0 && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-
b.y)*(c.y-b.y) <= 0);
}

void ConvexHull(vector<Point> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()), pts.end());
    vector<Point> up, dn;
    for (int i = 0; i < pts.size(); i++) {

```

```

        while (up.size() > 1 && area2(up[up.size()-2], up.back(),
pts[i]) >= 0) up.pop_back();
        while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(),
pts[i]) <= 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--)
pts.push_back(up[i]);

    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i]))
dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
}

```

## 20. Pick's Theorem

Given a [simple polygon](#) constructed on a grid of equal-distanced points (i.e., points with [integer](#) coordinates) such that all the polygon's vertices are grid points, **Pick's theorem** provides a simple [formula](#) for calculating the [area](#)  $A$  of this polygon in terms of the number  $i$  of *lattice points in the interior* located in the polygon and the number  $b$  of *lattice points on the boundary* placed on the polygon's perimeter.<sup>[1]</sup>

$$A = i + \frac{b}{2} - 1.$$



## 21. Strongly Connected Component

```
/****** Tarjan's SCC *****/
vector< int > num, low, S, vis;
int cntr, numCC;

void tarjanSCC(int v) {
    low[v] = num[v] = ++cntr;
    vis[v] = 1;
    S.push_back(v);
    for(auto u : adj[v]) {
        if(num[u] == -1)
            tarjanSCC(u);
        if(vis[u])
            low[v] = min(low[v], low[u]);
    }
    if(low[v] == num[v]) {
        printf("SCC %d :", ++numCC);
        while(1) {
            int u = S.back(); S.pop_back(); vis[u] = 0;
            printf(" %d", u);
            if(u == v)
                break;
        }
    }
}

// In MAIN();
num.assign(n, -1);
low.assign(n, 0);
vis.assign(n, 0);
cntr = numCC = 0;
for(int i = 0; i<n; i++)
    if(num[i] == -1)
        tarjanSCC(i);
```

## 22. Suffix Array + LCP

```
// suffix array
const int N = 1e5 + 5;
```

```
string s;
int sa[N], pos[N], lcp[N], tmp[N], gap, n;

bool cmp_sa(int a, int b) {
    if(pos[a] - pos[b])
        return pos[a] < pos[b];
    a += gap; b += gap;
    return (a < n && b < n) ? pos[a] < pos[b] : a > b;
}

void build_sa() {
    n = s.size();
    for(int i = 0; i<n; i++)
        sa[i] = i, pos[i] = s[i];
    for(gap = 1;; gap <= 1) {
        sort(sa, sa + n, cmp_sa);
        for(int i = 1; i<n; i++) tmp[i] = tmp[i-1] + cmp_sa(sa[i-1], sa[i]);
        for(int i = 0; i<n; i++) pos[sa[i]] = tmp[i];
        if(tmp[n-1] == n-1) break;
    }
}

void build_lcp() {
    for(int i = 0, k = 0; i<n; i++) if(pos[i] - n + 1) {
        for(int j = sa[pos[i] + 1]; s[j + k] == s[i + k]; k++);
        lcp[pos[i]] = k;
        if(k) k--;
    }
}
```

## 23. Manacher Algorithm (Palindrom)

Sumber : [http://e-maxx.ru/algo/palindromes\\_count](http://e-maxx.ru/algo/palindromes_count)

```
vector<int> d1 (n);
int l=0, r=-1;
for (int i=0; i<n; ++i) {
    int k = (i>r ? 0 : min (d1[l+r-i], r-i)) + 1;
    while (i+k < n && i-k >= 0 && s[i+k] == s[i-k]) ++k;
    d1[i] = k--;
    if (i+k > r)
```

```

        l = i-k, r = i+k;
    }
    vector<int> d2 (n);
    l=0, r=-1;
    for (int i=0; i<n; ++i) {
        int k = (i>r ? 0 : min (d2[l+r-i+1], r-i+1)) + 1;
        while (i+k-1 < n && i-k >= 0 && s[i+k-1] == s[i-k]) ++k;
        d2[i] = --k;
        if (i+k-1 > r)
            l = i-k, r = i+k-1;
    }

    Sumber : http://codeforces.com/blog/entry/12143

    vector< vector<int> > p(2, vector<int>(n,0)); //p[1][i] even, p[0][i] odd palindrom
    center i
    for (int z=0, l=0, r=0; z < 2; z++, l=0, r=0)
        for (int i = 0; i < n; i++) {
            if (i < r) p[z][i] = min(r-i+!z, p[z][l+r-i+!z]);
            int L = i-p[z][i], R = i+p[z][i]-!z;
            while (L-1 >= 0 && R+1 < n && s[L-1] == s[R+1]) p[z][i]++, L--, R++;
            if (R > r) l = L, r = R;
        }

```

## 24. Implicit Treap

```

/**
 * Treap uses implicit key
 * This Implementation : maintain array, can insert and delete in any
 * position, can reverse interval
 */

#include <bits/stdc++.h>
using namespace std;

typedef struct item * pitem;

struct item

```

```

{
    int cnt, value, prior;
    bool rev;
    pitem l, r;
    item(int prior, int value) : cnt(1), rev(false), prior(prior),
    value(value), l(NULL), r(NULL) {}
};

int cnt(pitem t) {
    return t ? t->cnt : 0;
}

void upd_cnt(pitem it) {
    if (it)
        it->cnt = cnt(it->l) + cnt(it->r) + 1;
}

void push(pitem it) {
    if (it && it->rev) {
        it->rev = false;
        swap(it->l, it->r);
        if (it->l) it->l->rev ^= true;
        if (it->r) it->r->rev ^= true;
    }
}

void merge(pitem & t, pitem l, pitem r) {
    push(l);
    push(r);
    if (!l || !r)
        t = l ? l : r;
    else if (l->prior > r->prior)
        merge(l->r, l->r, r), t = l;
    else
        merge(r->l, l, r->l), t = r;
    upd_cnt(t);
}

void split(pitem t, pitem & l, pitem & r, int key, int add = 0) {
    if (!t)
        return void (l = r = 0);
    int cur_key = cnt(t->l) + add;
    if (key <= cur_key)
        split(t->l, l, t->l, key, add), r = t;
}

```

```

else
    split(t->r, t->r, r, key, add + cnt(t->l) + 1), l = t;
upd_cnt(t);
}

void reverse(pitem t, int l, int r) {
    pitem t1, t2, t3;
    split(t, t1, t2, l);
    split(t2, t2, t3, r-l+1);
    t2->rev ^= true;
    merge(t, t1, t2);
    merge(t, t, t3);
}

void output (pitem t) {
    if (!t) return;
    push (t);
    output (t->l);
    printf ("%d ", t->value);
    output (t->r);
}

int main() {
    int n;
    scanf("%d", &n);
    srand(time(NULL));
    pitem root = NULL;
    for (int i = 0; i < n; i++) {
        int a;
        scanf("%d", &a);
        pitem cur = new item(rand(), a);
        if (root)
            merge(root, root, cur);
        else
            root = cur;
    }
    int m;
    scanf("%d", &m);
    for (int i = 0; i < m; i++) {
        int l, r;
        scanf("%d %d", &l, &r);
        reverse(root, l, r);
        output(root);
        printf("\n");
    }
}

```

```

}
return 0;
}

```

## 25. Convex Hull Trick

```

#include <bits/stdc++.h>

using namespace std;
struct line {
    long long a, b, get(long long x) {
        return a*x + b;
    }
    long double getd(long double x) {
        return x * a + b;
    }
};

struct convex_hull_trick {
    line * hull;
    int size;
    convex_hull_trick(int sz) : size(0) {
        hull = new line[sz+1];
    }
    bool isbad(line prev, line cur, line next) {
        return (prev.b - cur.b) * (next.a - cur.a) >= (cur.b - next.b) *
        (cur.a - prev.a);
    }
    void add(line nl) {
        hull[size++] = nl;
        while(size > 2 && isbad(hull[size-3], hull[size-2], hull[size-1]))
            hull[size-2] = nl, size--;
    }

    long long query(long long x) {
        int l, r;
        l = 0; r = size-1;
        while(l < r) {
            int m = (l + r) >> 1;
            if(hull[m].get(x) <= hull[m+1].get(x))
                l = m+1;
            else
                r = m;
        }
    }
}

```

```

    }
    return hull[l].get(x);
}
};
const int N = 2e5 + 5;
long long sum[N];
int a[N];

int main() {
    int n;
    scanf("%d", &n);
    long long ans = 0, add = 0;
    sum[0] = 0;
    for(int i = 1; i <= n; i++) {
        scanf("%d", &a[i]);
        sum[i] = sum[i-1] + a[i];
        ans += a[i] * (long long)i;
    }
    convex_hull_trick hull(n);
    hull.size = 0;
    for(int i = 1; i <= n; i++) {
        hull.add((line){i, -sum[i-1]});
        add = max(add, hull.query(a[i]) + sum[i-1] - a[i]*(long long)i);
    }
    hull.size = 0;
    for(int i = n; i > 0; i--) {
        hull.add((line){-i, -sum[i]});
        add = max(add, hull.query(-a[i]) + sum[i] - a[i]*(long long)i);
    }
    cout << ans + add << endl;
    return 0;
}

```

## 26. Z Algorithm

```

string s;
cin >> s;
int L = 0, R = 0;
int n = s.size();
for (int i = 1; i < n; ++i) {
    if (i > R) {
        L = R = i;
        while (R < n && s[R] == s[R-L]) ++R;
        Z[i] = R-L; --R;
    }
}

```

```

else {
    int k = i-L;
    if (Z[k] < R-i+1) Z[i] = Z[k];
    else {
        L = i;
        while (R < n && s[R] == s[R-L]) ++R;
        Z[i] = R-L; --R;
    }
}
}
}

```

## 27. Aho Corassick

```

/** Aho-Corasick Dictionary Matching */
const int NALPHABET = 26;

struct Node {
    Node** children, go;
    bool leaf;
    char charToParent;
    Node* parent, suffLink, dictSuffLink;
    int count, value;

    Node(){
        children = new Node*[NALPHABET];
        go = new Node*[NALPHABET];
        for(int i = 0; i < NALPHABET; ++i){
            children[i] = go[i] = NULL;
        }
        parent = suffLink = dictSuffLink = NULL;
        leaf = false;
        count = 0;
    }
};

Node* createRoot(){
    Node* node = new Node();
    node->suffLink = node;
    return node;
}

void addString(Node* node, const string& s, int value = -1){
    for(int i = 0; i < s.length(); ++i){
        int c = s[i] - 'a';
        if(node->children[c] == NULL){

```

```

Node* n =new Node();
n->parent = node;
n->charToParent = s[i];
node->children[c]= n;
}
node = node->children[c];
}
node->leaf =true;
node->count++;
node->value = value;
}

Node* suffLink(Node* node);
Node* dictSuffLink(Node* node);
Node* go(Node* node,char ch);
int calc(Node* node);

Node* suffLink(Node* node){
    if(node->suffLink ==NULL){
        if(node->parent->parent ==NULL){
            node->suffLink = node->parent;
        }else{
            node->suffLink = go(suffLink(node->parent),node->charToParent);
        }
    }
    return node->suffLink;
}

Node* dictSuffLink(Node* node){
    if(node->dictSuffLink ==NULL){
        Node* n = suffLink(node);
        if(node == n){
            node->dictSuffLink = node;
        }else{
            while(!n->leaf && n->parent !=NULL){
                n = dictSuffLink(n);
            }
            node->dictSuffLink = n;
        }
    }
    return node->dictSuffLink;
}

```

```

Node* go(Node* node,char ch){
    int c = ch - 'a';
    if(node->go[c]==NULL){
        if(node->children[c]!=NULL){
            node->go[c]= node->children[c];
        }else{
            node->go[c]= node->parent ==NULL? node : go(suffLink(node), ch);
        }
    }
    return node->go[c];
}

int calc(Node* node){
    if(node->parent ==NULL){
        return 0;
    }else{
        return node->count + calc(dictSuffLink(node));
    }
}

int main(){
    Node* root = createRoot();
    addString(root,"a",0);
    addString(root,"aa",1);
    addString(root,"abc",2);

    string s("abcaadc");
    Node* node = root;
    for(int i =0; i < s.length();++i){
        node = go(node, s[i]);
        Node* temp = node;
        while(temp != root){
            if(temp->leaf){
                printf("string (%d) occurs at position %d\n", temp->value, i);
            }
            temp = dictSuffLink(temp);
        }
    }
    return 0;
}

```

## 28. Blossom

```

/** Maximum Matching on General Graph */
/* Blossom |  $O(V^3)$  */

int lca(vector<int>&match, vector<int>&base, vector<int>&p, int a, int b){
    vector<bool> used(SZ(match));
    while(true){
        a = base[a];
        used[a]=true;
        if(match[a]==-1)break;
        a = p[match[a]];
    }
    while(true){
        b = base[b];
        if(used[b])return b;
        b = p[match[b]];
    }
    return -1;
}

void markPath(vector<int>&match, vector<int>&base,
vector<bool>&blossom, vector<int>&p, int v, int b, int children){
    for(; base[v] != b; v = p[match[v]]){
        blossom[base[v]] = blossom[base[match[v]]] = true;
        p[v] = children;
        children = match[v];
    }
}

int findPath(vector<vector<int>>&graph, vector<int>&match,
vector<int>&p, int root){
    int n = SZ(graph);
    vector<bool> used(n);
    FORIT(it, p)*it = -1;
    vector<int> base(n);
    for(int i = 0; i < n; ++i) base[i] = i;

    used[root]=true;
    int qh = 0;
    int qt = 0;
    vector<int> q(n);
    q[qt++] = root;

```

```

while(qh < qt){
    int v = q[qh++];
    FORIT(it, graph[v]){
        int to = *it;
        if(base[v] == base[to] || match[v] == to)continue;
        if(to == root || match[to] != -1 && p[match[to]] != -1){
            int curbase = lca(match, base, p, v, to);
            vector<bool> blossom(n);
            markPath(match, base, blossom, p, v, curbase, to);
            markPath(match, base, blossom, p, to, curbase, v);
            for(int i = 0; i < n; ++i){
                if(blossom[base[i]]){
                    base[i] = curbase;
                    if(!used[i]){
                        used[i] = true;
                        q[qt++] = i;
                    }
                }
            }
        }
        elseif(p[to] == -1){
            p[to] = v;
            if(match[to] == -1)return to;
            to = match[to];
            used[to] = true;
            q[qt++] = to;
        }
    }
}
return -1;
}

int maxMatching(vector<vector<int>> graph){
    int n = SZ(graph);
    vector<int> match(n, -1);
    vector<int> p(n);
    for(int i = 0; i < n; ++i){
        if(match[i] == -1){
            int v = findPath(graph, match, p, i);
            while(v != -1){
                int pv = p[v];
                int ppv = match[pv];
                match[v] = pv;
                match[ppv] = v;
            }
        }
    }
}

```

```

        v = ppv;
    }
}

int matches = 0;
for(int i = 0; i < n; ++i){
    if(match[i] != -1){
        ++matches;
    }
}
return matches / 2;
}

```

## 29. Minimum Cut Stoer - Wagner

```

// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//  $O(|V|^3)$ 
//
// INPUT:
// - graph, constructed using AddEdge()
//
// OUTPUT:
// - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights){
    int N = weights.size();
    VI used(N), cut, best_cut;
    int best_weight = -1;

```

```

for(int phase = N-1; phase >= 0; phase--){
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for(int i = 0; i < phase; i++){
        prev = last;
        last = -1;
        for(int j = 1; j < N; j++){
            if(!added[j] && (last == -1 || w[j] > w[last])) last = j;
        }
        if(i == phase-1){
            for(int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
            for(int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
            used[last] = true;
            cut.push_back(last);
            if(best_weight == -1 || w[last] < best_weight){
                best_cut = cut;
                best_weight = w[last];
            }
        }
        else{
            for(int j = 0; j < N; j++){
                w[j] += weights[last][j];
                added[last] = true;
            }
        }
    }
}
return make_pair(best_weight, best_cut);
}

```

## 30. Chinese Remainder Theorem

```

// Chinese remainder theorem (special case): find z such that
//  $z \% x = a$ ,  $z \% y = b$ . Here, z is unique modulo  $M = \text{lcm}(x, y)$ .
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b){
    int s, t;
    int d = extended_euclid(x, y, s, t);
    if(a % d != b % d) return make_pair(0, -1);
    return make_pair(mod(s * b * x + t * a * y, x * y) / d, x * y / d);
}

// Chinese remainder theorem: find z such that
//  $z \% x[i] = a[i]$  for all i. Note that the solution is

```

```

// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a){
    PII ret = make_pair(a[0], x[0]);
    for(int i = 1; i < x.size(); i++){
        ret = chinese_remainder_theorem(ret.second, ret.first, x[i],
a[i]);
        if(ret.second == -1) break;
    }
    return ret;
}

```

## 31. Simplex

```

// Two-phase simplex algorithm for solving linear programs of the form
//
// maximize c^T x
// subject to Ax <= b
//            x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//        above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c
// as arguments. Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

```

```

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c):
        m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, VD(n+2)){
        for(int i = 0; i < m; i++)
            for(int j = 0; j < n; j++) D[i][j] = A[i][j];
        for(int i = 0; i < m; i++){
            B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];
        }
        for(int j = 0; j < n; j++){ N[j] = j; D[m][j] = -c[j];}
        N[n] = -1; D[m+1][n] = 1;
    }

    void Pivot(int r, int s){
        for(int i = 0; i < m+2; i++) if(i != r)
            for(int j = 0; j < n+2; j++) if(j != s)
                D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for(int j = 0; j < n+2; j++) if(j != s) D[r][j] /= D[r][s];
        for(int i = 0; i < m+2; i++) if(i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }

    bool Simplex(int phase){
        int x = phase == 1 ? m+1 : m;
        while(true){
            int s = -1;
            for(int j = 0; j <= n; j++){
                if(phase == 2 && N[j] == -1) continue;
                if(s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s])
                    s = j;
            }
            if(D[x][s] >= -EPS) return true;
            int r = -1;
            for(int i = 0; i < m; i++){
                if(D[i][s] <= 0) continue;
                if(r == -1 || D[i][n+1] / D[i][s] < D[r][n+1] / D[r][s])

```



```

        D[i][n+1]/ D[i][s]== D[r][n+1]/ D[r][s]&& B[i]< B[r])
        r = i;
    }
    if (r == -1) return false;
    Pivot(r, s);
}
}

DOUBLE Solve(VD &x){
    int r = 0;
    for(int i = 1; i < m; i++) if(D[i][n+1] < D[r][n+1]) r = i;
    if(D[r][n+1] <= -EPS){
        Pivot(r, n);
        if(!Simplex(1) || D[m+1][n+1] <= -EPS)
            return numeric_limits<DOUBLE>::infinity();
        for(int i = 0; i < m; i++) if(B[i] == -1){
            int s = -1;
            for(int j = 0; j <= n; j++)
                if(s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] <
N[s])
                    s = j;
            Pivot(i, s);
        }
        if(!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        x = VD(n);
        for(int i = 0; i < m; i++) if(B[i] < n) x[B[i]] = D[i][n+1];
        return D[m][n+1];
    }
};

int main(){
    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        {6, -1, 0},
        {-1, -5, 0},
        {1, 5, 1},
        {-1, -5, -1}
    };
    DOUBLE _b[m] = {10, -4, 5, -5};
    DOUBLE _c[n] = {1, -1, 0};

```

```

VVD A(m);
VD b(_b, _b + m);
VD c(_c, _c + n);
for(int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

LPSolver solver(A, b, c);
VD x;
DOUBLE value = solver.Solve(x);
cerr << "VALUE: " << value << endl;
cerr << "SOLUTION:";
for(size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
cerr << endl;
return 0;
}

```