📁 Test 14: Phase Diagram with Critical Marking

Goals

- To generate a **phase diagram** over a grid of (β, slope) values.
- Track **ΔQ = Q_fast – Q_slow**, the **weight ratio** $R = \frac{w_{\text{fast}}}{w_{\text{fast}} + w_{\text{slow}}}$, and mark where transitions occur.
- **Console log each line** for your copy-paste workflow.
- Write results into a **CSV file** for structured analysis.
- Produce **plots**:
  1. ΔQ vs slope (for each β).
  2. Weight ratio vs slope (for each β).
- Add a **vertical line** on each plot showing where ΔQ ≈ 0, marking the **critical slope** for transition.

Interpretation Strategy

- **ΔQ = 0** is the *geometric degeneracy point* → transition between slow- and fast-dominated dynamics.
- **R (ratio)** indicates which path actually dominates dynamically.
- Together, these form the **critical manifold** in parameter space.
- Plotting across β (8–10) shows whether the critical slope shifts with β (suggesting dimensionality effects).

📁 Updated Test 14 (extended)

Goals

- ◦ Keep all the previous outputs (console, CSV, separate ΔQ and Ratio plots).
  - ◦ Add a **dual-axis plot**:
    - **Left y-axis** → ΔQ (difference between Q_fast and Q_slow).
    - **Right y-axis** → Ratio (fast / total).
  - ◦ This will allow us to **see directly whether the critical ΔQ=0 point aligns with the Ratio=0.5 threshold** across different β values.
  - ◦ Critical for identifying whether the **geometric and dynamic transitions coincide or separate** — which could reveal hidden dimensional effects.