

## MU Test Plan — Test 11 ( $\beta$ - $\Delta Q$ Phase Diagram)

### Goal

Map **survival regimes** as a function of:

- $\beta$  (Truth strength)
- $\Delta Q = Q_{\text{fast}} - Q_{\text{slow}}$  (action cost gap)

We'll visualize where the system shows:

- **Coexistence (near-degenerate)**: both branches sizable
- **Hierarchical coexistence**: slow dominates, fast suppressed but nonzero
- **Near-collapse of fast**: fast effectively negligible

### Parameters

- $\hbar = 0.1$
- $\gamma = 1.0$
- $\beta$  sweep:  $8.0 \rightarrow 10.0$  (step 0.25)
- $\Delta Q$  sweep: generated by varying fast-path slope (we'll produce a range that spans small to large  $\Delta Q$ )
- Slow path:  $r_{\text{slow}}(t) = 0.25 \cdot t$
- Fast paths:  $r_{\text{fast}}(t) = \text{offset} + \text{slope} \cdot t$ , with  $\text{offset} = 0.6$  and slopes in a list to span  $\Delta Q$
- Selector:  $T(r) = 1/(1 + |r - 0.25|)$
- Window: 1.0

### Predictions

- For **small  $\Delta Q$**  and **moderate  $\beta$** , **coexistence**:  $w_{\text{fast}}/w_{\text{slow}} \sim O(1)$ .
- As  **$\Delta Q$  increases**, the **ratio drops exponentially**:  $\log(w_f/w_s) \approx -\Delta Q/\hbar \log(w_f/w_s) \approx -\Delta Q/\hbar$  (Hierarchy Law).
- As  **$\beta$  increases**, absolute weights rise, but **ratios follow  $\Delta Q/\hbar$** , so **phase boundaries mostly vertical** in the  $(\beta, \Delta Q)$  plane (weak  $\beta$ -dependence of ratio).

### What to look for

- Heatmap of  **$\log_{10}(w_{\text{fast}} / w_{\text{slow}})$**  with contour lines to separate regimes:
  - $\geq -1$  (ratio  $\geq 0.1$ ): **coexistence**
  - $(-1, -4]$ : **hierarchical coexistence**
  - $\leq -6$ : **fast effectively collapsed**

