🧪 MU Test Log Phase Diagram

🔑 Concise Summary of What We Found

- We studied how **slope** affects the relative dominance of the **slow path** (Q_slow) vs. the **fast path** (Q_fast) at fixed β values.
- At **strongly negative slopes**, Q_fast is negative and heavily suppressed → **SLOW path dominates**.
- As slope increases toward 0, Q_fast rises, approaching and then crossing Q_slow.
- Around **slope ≈ −0.1**, there is a **crossover point**:
    - Before this: ΔQ = Q_fast − Q_slow < 0 → Slow dominates.
    - After this: ΔQ > 0 → **FAST path takes over**.
- This transition is sharp, resembling a **phase boundary** in physical systems.
- The crossover slope itself depends on β — so by varying β we can map out a **phase diagram**.

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[after the crossover is it gaining acceleration or increasing its dominance over truth ? monotonically or exponentially, what is the motion of the curve or function of dominance ? is degeneracy high in this location or also increasing after the crossover? The model would predict that multi dimensional branches should start to appear or become more likely, more 'exoticness'. I think we need to measure that… i believe it is correlated]

After the crossover (ΔQ = 0)

How fast does dominance grow?

- Define dominance $D := \log\left(\frac{w_{\text{fast}}}{w_{\text{slow}}}\right)$.
    - Near the crossover,

$$D \approx \frac{\beta}{\hbar}\Delta Q(\text{slope}) \quad \Rightarrow \quad \frac{dD}{d\,\text{slope}} \approx \frac{\beta}{\hbar}\frac{d(\Delta Q)}{d\,\text{slope}}.$$

If ΔQ grows roughly **linearly** once you pass the gate, then D**Dgrows linearly** in slope, so the **raw ratio** $w_{\text{fast}}/w_{\text{slow}} = \exp(D)$ grows **exponentially**.

- That matches your console: after the flip, ratios jump by many orders of magnitude with small slope changes. So: **monotone increase** in D, **exponential rise** in the raw dominance.

What happens to degeneracy?

- Two regimes:

  1. **Just past the crossover** (still fold-adjacent): the Jacobian magnitude $|J|$is still small → **high degeneracy** (many near-equivalent micro-paths). This is your "doorway to extra dimensions": multiple almost-equal fast variants get non-negligible weight.

  2. **Further from the fold** (slope moves away): $|J|$ typically grows → **local degeneracy decreases**, but the **dominant fast macro-branch locks in** (its weight overwhelms). So: *branch variety is highest right after the gate*, then prunes as one fast macro-branch wins.

  That's exactly the MU picture: exoticity blooms at the horizon, then resolves.

Will multi-dimensional branches "start to appear"?

- Yes — if by "appear" we mean **many fast sub-paths** with similar Q_TQT inside a small tube around the fast trajectory. Right after the crossover those sub-paths are cheapest (in $Q + Q_{\text{T}}$), so more of them survive the weighting. As you move away, the tube narrows.

How to measure "exoticness" (what we'll add next)

- We can quantify three complementary things along the slope axis (post-crossover):

  1. **Degeneracy index (G):** time-averaged exposure to near-fold geometry
     $G := \frac{1}{T}\int_0^T \mathbf{1}\big(|J(r(t))| < \varepsilon\big)\,dt$
     (or smoother: average of $1/|J|$ truncated). Higher $G \Rightarrow$ more fold-proximity.

  2. **Branch entropy (S):** draw NN small, independent perturbations of the fast path $\{r_k(t)\}$; compute normalized weights p_k then
     $S := -\sum_k p_k \log p_k$.
     Higher $S \Rightarrow$ more **distinct** fast micro-branches survive (more "multi-D feel").

  3. **Participation ratio (PR):** $\mathrm{PR} := 1/\sum_k p_k^2$.
     Larger PR ⇒ the weight isn't concentrated in a single micro-branch.

Optional but nice:

- **Spread of endpoints** $\mathrm{Var}[r_k(1)]$and of whole paths (RMS tube radius).
- **Sensitivity** $\chi := \|\partial r(t)/\partial \mathrm{init}\|$ (finite-difference proxy).

Predictions (to test):

- Immediately after the crossover: D rises quickly; **G, S, PR** are **high**.
- As slope increases further: D keeps rising; **G decreases**, **S and PR** peak then fall (exoticity bursts, then prunes).
- Increasing β: pushes the **peak** of S/PR **closer to the fold** and **sharper**.