Final-Part A

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The forward kinematic equations of the robot are as follows:

$$\begin{cases} P_x = a_3C_1C_2C_3 - a_3C_1S_2S_3 + a_2C_1C_2 \\ P_y = a_3S_1C_2C_3 - a_3S_1S_2S_3 + a_2S_1C_2 \\ P_z = a_3S_2C_3 + a_3C_2S_3 + a_2S_2 + d_1 \end{cases}$$

The inverse kinematic equations of the robot are as follows:

$$\theta_1 = \tan^{-1}(p_y / p_x)$$

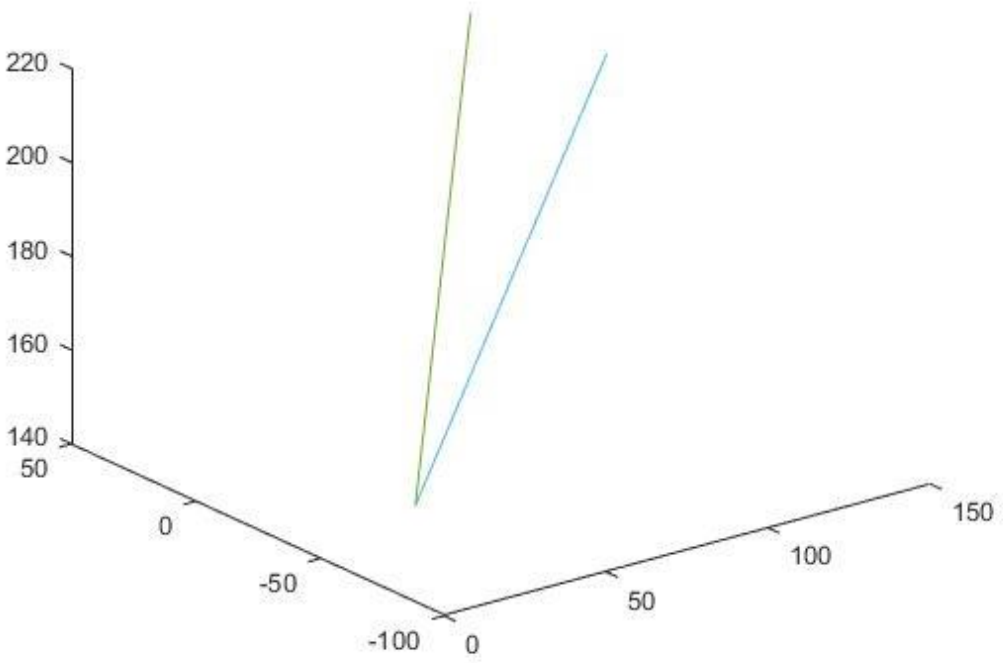
$$\theta_3 = \cos^{-1}\left[\frac{P_x^2+P_y^2+(P_z-d_1)^2-(a_2^2+a_3^2)}{2a_2a_3}\right]$$

$$\theta_2 = Sin^{-1}\left(\frac{(P_z-d_1)}{\sqrt{m_1^2+m_2^2}}\right) - \tan^{-1}(\frac{m_2}{m_1})$$

1.

```
A=[101 21 220]; B=[21 -61 150]; C=[150,30,200];
X=[A;B];
R1=[];
n=size(X,1);
for i=1:n-1
    y=linspace3d(X(i,:),X(i+1,:),21);
    if (i~=1)
        y=y(2:end,:);
    end
    R1=[R1;y];
end
R1=[R1 zeros(size(R1,1),3)];
x=R1(:,1); y=R1(:,2); z=R1(:,3);
[theta1,theta2,theta3]=inv3arm(x,y,z);
R1(:,4)=theta1; R1(:,5)=theta2(:,1); R1(:,6)=theta3;
%fprintf('R=\n');
%disp(R1);
```

```
plot3(x,y,z);
hold on;

Y=[B;C];
R2=[];
n=size(Y,1);
for i=1:n-1
    y=linspace3d(Y(i,:),Y(i+1,:),21);
    if (i~=1)
        y=y(2:end,:);
    end
    R2=[R2;y];
end
R2=[R2 zeros(size(R2,1),3)];
x=R2(:,1); y=R2(:,2); z=R2(:,3);
[theta1,theta2,theta3]=inv3arm(x,y,z);
R2(:,4)=theta1; R2(:,5)=theta2(:,21); R2(:,6)=theta3;
%fprintf('R2=\n');
%disp(R2);
plot3(x,y,z);
R=[];
R=[R1;R2];
fprintf('R=\n');
disp(R);
xlswrite('testdata.xlsx',R);
```



R=

101.0000	21.0000	220.0000	11.7456	-85.1596	67.5021
97.0000	16.9000	216.5000	9.8833	-88.2314	74.2015
93.0000	12.8000	213.0000	7.8366	-90.7673	80.3595
89.0000	8.7000	209.5000	5.5831	-92.8895	86.0848
85.0000	4.6000	206.0000	3.0977	-94.6777	91.4498
81.0000	0.5000	202.5000	0.3537	-96.1868	96.5043
77.0000	-3.6000	199.0000	-2.6768	-97.4566	101.2827
73.0000	-7.7000	195.5000	-6.0213	-98.5180	105.8081
69.0000	-11.8000	192.0000	-9.7045	-99.3963	110.0951
65.0000	-15.9000	188.5000	-13.7455	-100.1145	114.1507
61.0000	-20.0000	185.0000	-18.1527	-100.6952	117.9754
57.0000	-24.1000	181.5000	-22.9190	-101.1622	121.5634
53.0000	-28.2000	178.0000	-28.0163	-101.5427	124.9028
49.0000	-32.3000	174.5000	-33.3922	-101.8681	127.9757
45.0000	-36.4000	171.0000	-38.9690	-102.1753	130.7578
41.0000	-40.5000	167.5000	-44.6485	-102.5069	133.2195
37.0000	-44.6000	164.0000	-50.3210	-102.9102	135.3263
33.0000	-48.7000	160.5000	-55.8777	-103.4354	137.0413
29.0000	-52.8000	157.0000	-61.2225	-104.1314	138.3280
25.0000	-56.9000	153.5000	-66.2809	-105.0409	139.1545

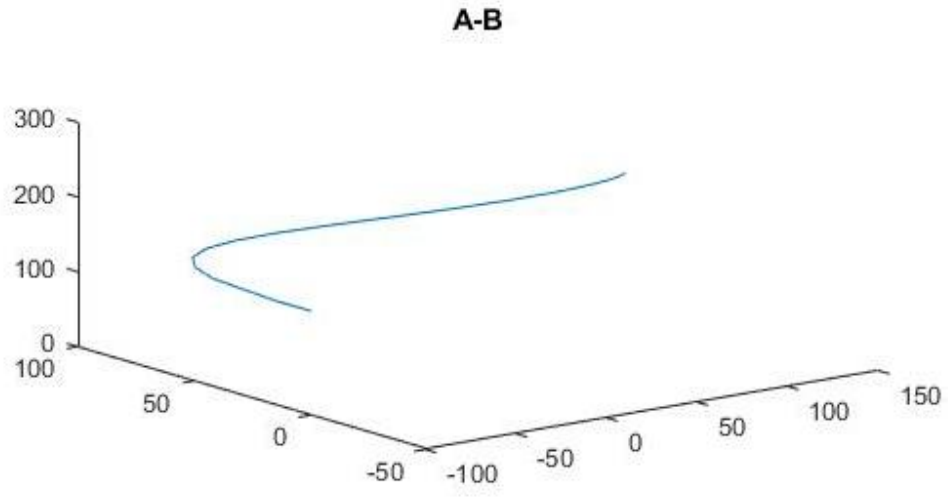
21.0000	-61.0000	150.0000	-71.0033	-106.1947	139.4982
21.0000	-61.0000	150.0000	-71.0033	-98.0927	139.4982
27.4500	-56.4500	152.5000	-64.0677	-97.3577	139.3101
33.9000	-51.9000	155.0000	-56.8483	-96.9355	138.3817
40.3500	-47.3500	157.5000	-49.5635	-96.7903	136.7521
46.8000	-42.8000	160.0000	-42.4438	-96.8663	134.4815
53.2500	-38.2500	162.5000	-35.6901	-97.0971	131.6390
59.7000	-33.7000	165.0000	-29.4443	-97.4150	128.2917
66.1500	-29.1500	167.5000	-23.7814	-97.7563	124.4974
72.6000	-24.6000	170.0000	-18.7186	-98.0643	120.3011
79.0500	-20.0500	172.5000	-14.2322	-98.2891	115.7340
85.5000	-15.5000	175.0000	-10.2754	-98.3865	110.8129
91.9500	-10.9500	177.5000	-6.7912	-98.3156	105.5401
98.4000	-6.4000	180.0000	-3.7213	-98.0350	99.9031
104.8500	-1.8500	182.5000	-1.0108	-97.4988	93.8719
111.3000	2.7000	185.0000	1.3897	-96.6503	87.3945
117.7500	7.2500	187.5000	3.5233	-95.4123	80.3883
124.2000	11.8000	190.0000	5.4273	-93.6697	72.7220
130.6500	16.3500	192.5000	7.1331	-91.2355	64.1783
137.1000	20.9000	195.0000	8.6676	-87.7700	54.3619
143.5500	25.4500	197.5000	10.0535	-82.5428	42.4148
150.0000	30.0000	200.0000	11.3099	-73.2598	25.5213

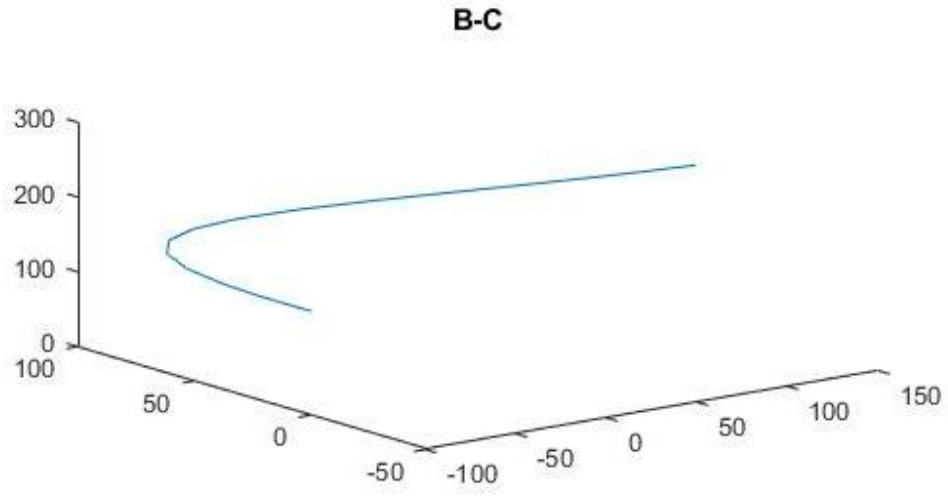
2.

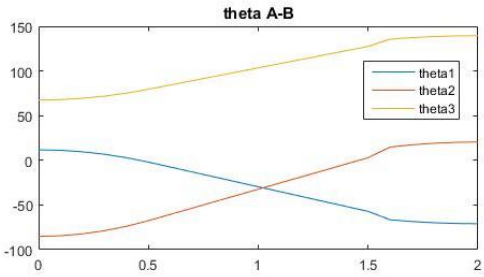
```
A=[101 21 220]; B=[21 -61 150];
[theta1i,theta2i,theta3i]=inv3arm(A(1),A(2),A(3));
thetai=[theta1i,theta2i,theta3i];
[theta1f,theta2f,theta3f]=inv3arm(B(1),B(2),B(3));
thetaf=[theta1f,theta2f,theta3f];

ti=0; tf=2; tb=0.5;
w=(thetaf-thetai)/(tf-tb);
t=ti:0.1:tf;

for i=1:length(t)
    if (t(i)>=0 & t(i)<=tb)
        thetaA(i,:)=thetai+0.5*w/tb*t(i)^2;
        velA(i,:)=w*t(i)/tb;
        accA(i,:)=w/tb;
    elseif (t(i)>tb & t(i)<=tf-tb)
        thetaA(i,:)=thetai+0.5*w*tb+w*(t(i)-tb);
        velA(i,:)=w;
        accA(i,:)=0;
    else
        thetaA(i,:)=thetaf-0.5*w*(tf-t(i))^2;
        velA(i,:)=w/tb*(tf-t(i));
        accA(i,:)=-w/tb;
    end
end

Px=[]; Py=[]; Pz=[];
for i=1:size(thetaA,1)
    [x,y,z]=for3arm(thetaA(i,1),thetaA(i,2),thetaA(i,3));
    Px=[Px; x]; Py=[Py; y]; Pz=[Pz; z];
end

subplot(3,3,1),plot(t,thetaA); title('theta A-B');
legend('theta1','theta2','theta3');
subplot(3,3,4),plot(t,velA); title('velocity A-B');
legend('theta1','theta2','theta3');
subplot(3,3,7),plot(t,accA); title('acceleration A-B');
```

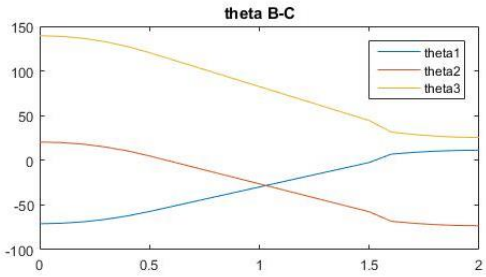
```
legend('theta1','theta2','theta3');
subplot(3,3,3),plot3(Px,Py,Pz); title('A-B');


B=[21 -61 150]; C=[150 30 200];
[theta1i,theta2i,theta3i]=inv3arm(B(1),B(2),B(3));
thetai=[theta1i,theta2i,theta3i];
[theta1f,theta2f,theta3f]=inv3arm(C(1),C(2),C(3));
thetaf=[theta1f,theta2f,theta3f];

ti=0; tf=2; tb=0.5;
w=(thetaf-thetai)/(tf-tb);
t=ti:0.1:tf;

for i=1:length(t)
    if (t(i)>=0 & t(i)<=tb)
        thetaB(i,:)=thetai+0.5*w/tb*t(i)^2;
        velB(i,:)=w*t(i)/tb;
        accB(i,:)=w/tb;
    elseif (t(i)>tb & t(i)<=tf-tb)
        thetaB(i,:)=thetai+0.5*w*tb+w*(t(i)-tb);
        velB(i,:)=w;
        accB(i,:)=0;
    else
        thetaB(i,:)=thetaf-0.5*w*(tf-t(i))^2;
        velB(i,:)=w/tb*(tf-t(i));
        accB(i,:)=-w/tb;
    end
end

Px=[]; Py=[]; Pz=[];
for i=1:size(thetaB,1)
    [x,y,z]=for3arm(thetaB(i,1),thetaB(i,2),thetaB(i,3));
    Px=[Px; x]; Py=[Py; y]; Pz=[Pz; z];
end
subplot(3,3,2),plot(t,thetaB); title('theta B-C');
legend('theta1','theta2','theta3');
subplot(3,3,5),plot(t,velB); title('velocity B-C');
```

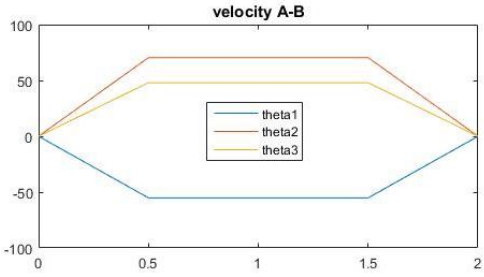
```
legend('theta1','theta2','theta3');
subplot(3,3,8),plot(t,accB); title('acceleration B-C');
legend('theta1','theta2','theta3');
subplot(3,3,6),plot3(Px,Py,Pz); title('B-C');

fprintf('thetaA=\n');
disp(thetaA);
fprintf('thetaB=\n');
disp(thetaB);
```

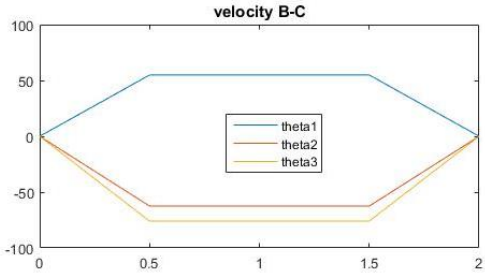


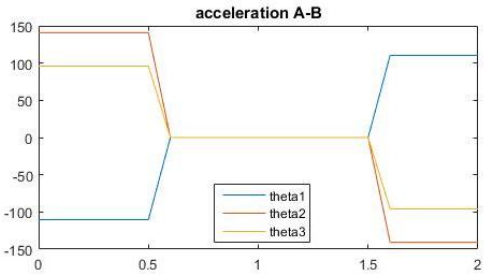


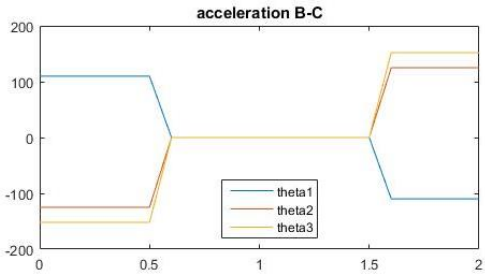

theta A-B
theta1
theta2
theta3


theta B-C
theta1
theta2
theta3


velocity A-B
theta1
theta2
theta3


velocity B-C
theta1
theta2
theta3


acceleration A-B
theta1
theta2
theta3


acceleration B-C
theta1
theta2
theta3

thetaA=

11.7456	-85.1596	67.5021
11.1940	-84.4552	67.9821
9.5390	-82.3421	69.4220
6.7807	-78.8202	71.8219
2.9191	-73.8897	75.1817
-2.0459	-67.5503	79.5015
-7.5625	-60.5066	84.3012
-13.0791	-53.4630	89.1009
-18.5957	-46.4193	93.9007
-24.1123	-39.3756	98.7004
-29.6289	-32.3319	103.5002
-35.1455	-25.2882	108.2999
-40.6621	-18.2445	113.0996
-46.1787	-11.2008	117.8994
-51.6953	-4.1572	122.6991
-57.2118	2.8865	127.4989
-66.5901	14.8608	135.6584
-68.5209	17.3261	137.3383
-69.9000	19.0870	138.5383
-70.7275	20.1436	139.2582
-71.0033	20.4957	139.4982

thetaB=

-71.0033	20.4957	139.4982
-70.4546	19.8707	138.7384
-68.8083	17.9956	136.4588
-66.0645	14.8704	132.6596
-62.2233	10.4952	127.3407
-57.2845	4.8698	120.5020
-51.7969	-1.3805	112.9036
-46.3094	-7.6309	105.3051
-40.8218	-13.8813	97.7067
-35.3343	-20.1317	90.1082
-29.8467	-26.3820	82.5097
-24.3592	-32.6324	74.9113
-18.8716	-38.8828	67.3128
-13.3841	-45.1331	59.7144

```
   -7.8965   -51.3835    52.1159
   -2.4089   -57.6339    44.5174
    6.9199   -68.2595    31.6000
    8.8405   -70.4471    28.9406
   10.2124   -72.0097    27.0410
   11.0356   -72.9473    25.9012
   11.3099   -73.2598    25.5213
```

```
>> disp(velA)
         0         0         0
  -11.0332   14.0874    9.5995
  -22.0664   28.1747   19.1990
  -33.0996   42.2621   28.7984
  -44.1328   56.3495   38.3979
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -55.1660   70.4369   47.9974
  -44.1328   56.3495   38.3979
  -33.0996   42.2621   28.7984
  -22.0664   28.1747   19.1990
  -11.0332   14.0874    9.5995
         0         0         0
```

```
>> disp(velB)
         0         0         0
   10.9751  -12.5007  -15.1969
   21.9502  -25.0015  -30.3938
   32.9253  -37.5022  -45.5908
   43.9004  -50.0030  -60.7877
   54.8755  -62.5037  -75.9846
```

```
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    54.8755   -62.5037   -75.9846
    43.9004   -50.0030   -60.7877
    32.9253   -37.5022   -45.5908
    21.9502   -25.0015   -30.3938
    10.9751   -12.5007   -15.1969
           0         0         0

>> disp(accA)
  -110.3320   140.8737    95.9948
  -110.3320   140.8737    95.9948
  -110.3320   140.8737    95.9948
  -110.3320   140.8737    95.9948
  -110.3320   140.8737    95.9948
  -110.3320   140.8737    95.9948
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
           0         0         0
    110.3320 -140.8737   -95.9948
    110.3320 -140.8737   -95.9948
    110.3320 -140.8737   -95.9948
    110.3320 -140.8737   -95.9948
    110.3320 -140.8737   -95.9948
```

```
>> disp(accB)
   109.7510 -125.0074 -151.9692
   109.7510 -125.0074 -151.9692
   109.7510 -125.0074 -151.9692
   109.7510 -125.0074 -151.9692
   109.7510 -125.0074 -151.9692
   109.7510 -125.0074 -151.9692
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
            0         0         0
  -109.7510  125.0074  151.9692
  -109.7510  125.0074  151.9692
  -109.7510  125.0074  151.9692
  -109.7510  125.0074  151.9692
  -109.7510  125.0074  151.9692
```