Esercizio 1)

$$p(y|\mu) = \mathcal{N}(y|\mu, \sigma^2)$$

$$\mu_{MLE} = \arg\max_{\mu} p(y|\mu)$$

$$= \arg\max_{\mu} \log\left(p(y|\mu)\right)$$

$$= \arg\max_{\mu} \log\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}\right)$$

$$= \arg\max_{\mu} \log\left(e^{-\frac{(y-\mu)^2}{2\sigma^2}}\right)$$

$$= \arg\max_{\mu} -\frac{(y-\mu)^2}{2\sigma^2}$$

$$\frac{\partial -\frac{(y-\mu)^2}{2\sigma^2}}{\partial \mu} = \frac{y-\mu}{\sigma^2}$$

$$\frac{y-\mu}{\sigma^2} = 0 \iff y = \mu$$

$$\frac{\partial -\frac{(y-\mu)^2}{2\sigma^2}}{(\partial \mu)^2} = -\frac{1}{\sigma^2} < 0$$

$$\Longrightarrow \quad MLE_{\mu} = y$$