Interval Selection in Insertion-Only Streams

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The Streaming Setting

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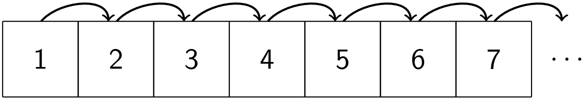
Goal

Given the input elements one at a time, compute an (approximate) solution using as **little space** as possible.



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- **Key Question:** What can we achieve across this regime?

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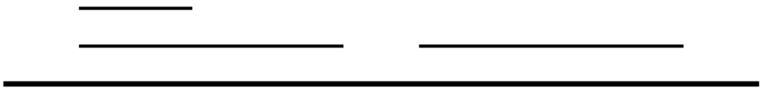

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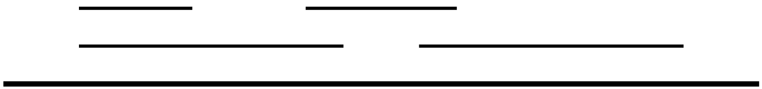

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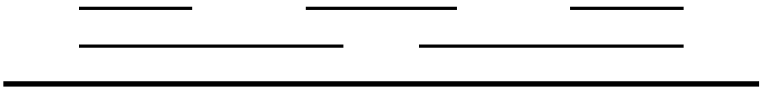

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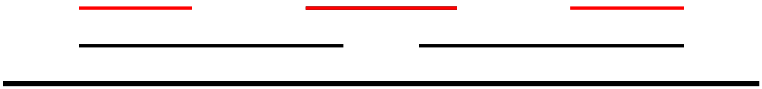

State of the Art

		Insertion-Only		Insertion-Deletion	
		UB - $O_\epsilon(\lvert OPT \rvert)$	LB - $\Omega(n)$	UB - $\tilde{O}_\epsilon(\lvert OPT^* \rvert)$	LB - $\Omega(n)$
Unweighted	Unit-Length				
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Weighted	Unit-Length				
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	Variable-Length	2 [EHR12]	$2-\epsilon$ [EHR12]		
Weighted	Unit-Length		$3/2-\epsilon$ [EHR12]		$2-\epsilon$ [BCW20]
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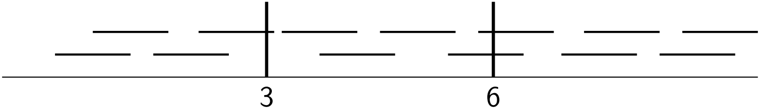

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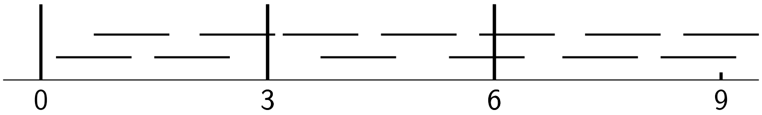

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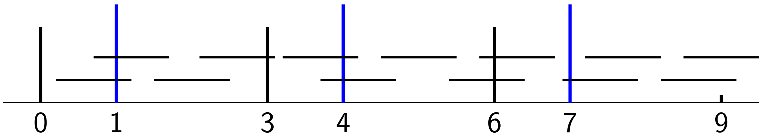

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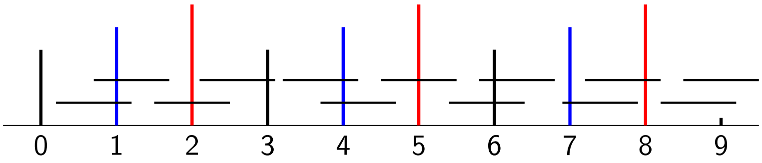

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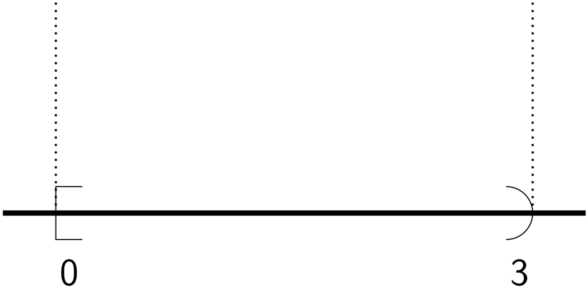

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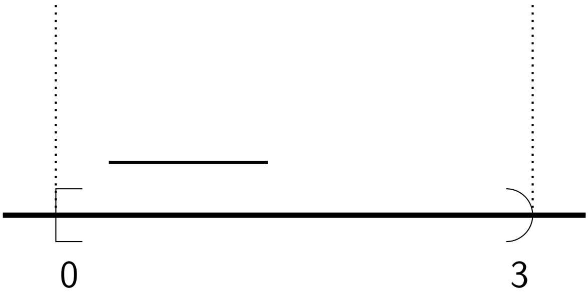

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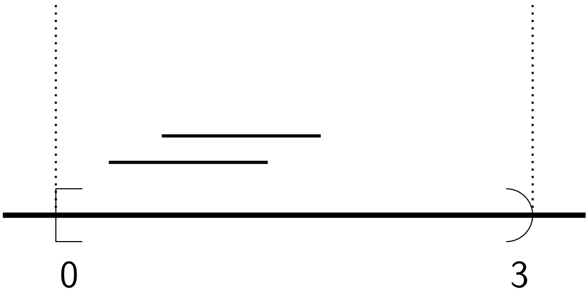

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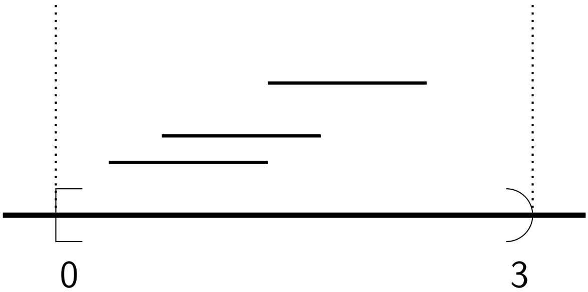

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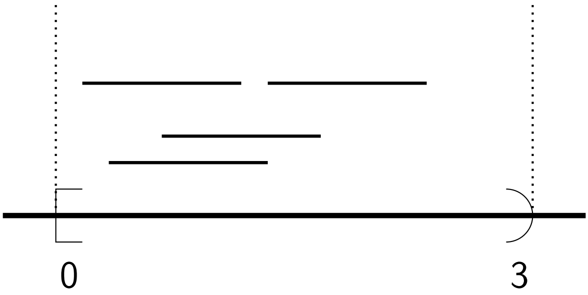

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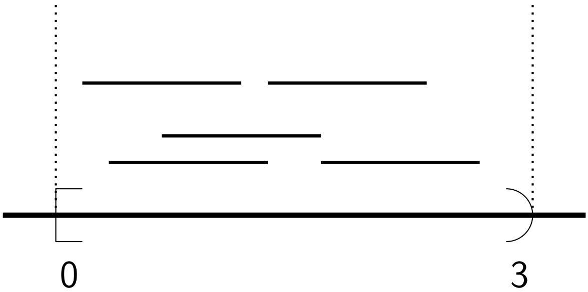

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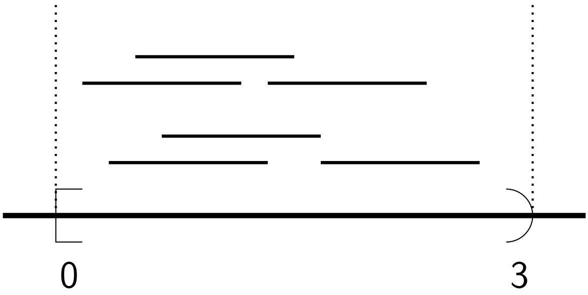

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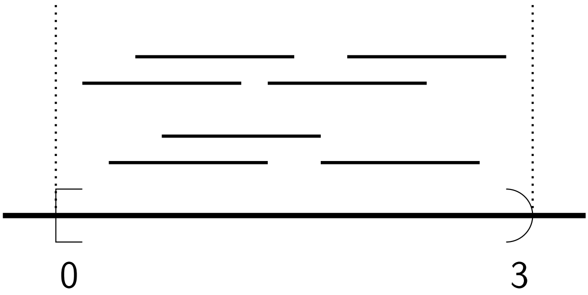

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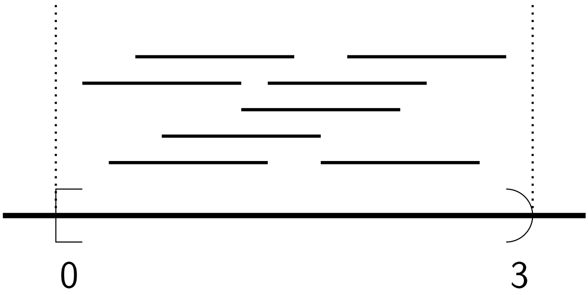

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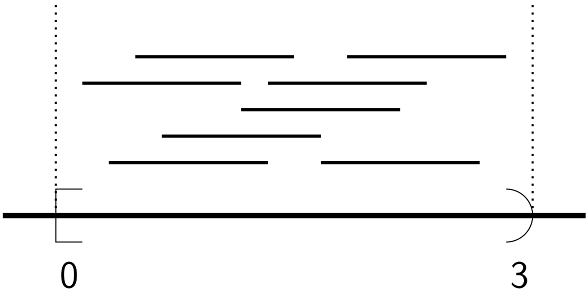

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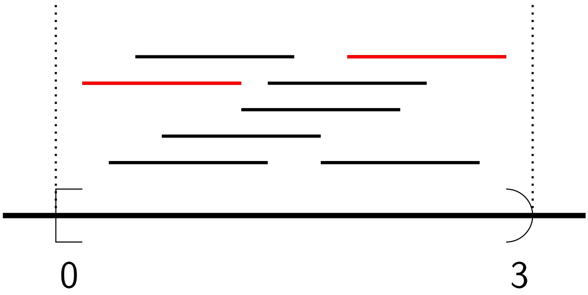

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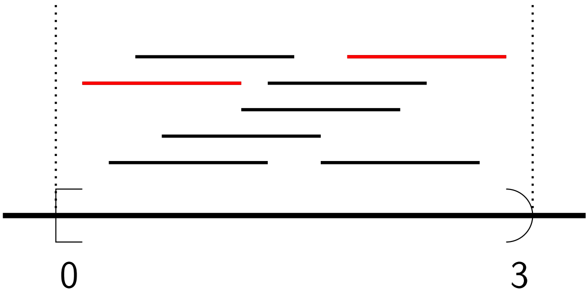

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- This gives a 3/2-approximation by our earlier argument.

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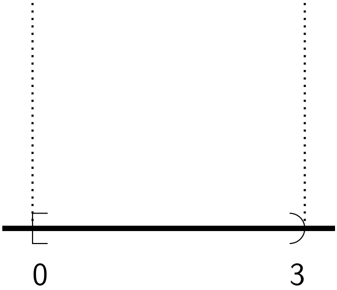

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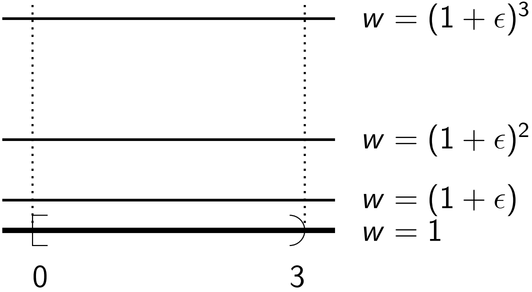

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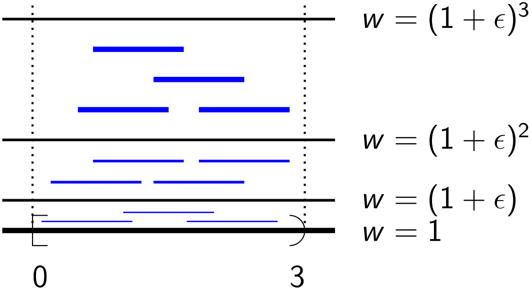

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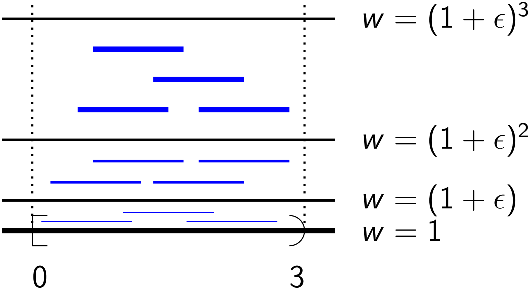

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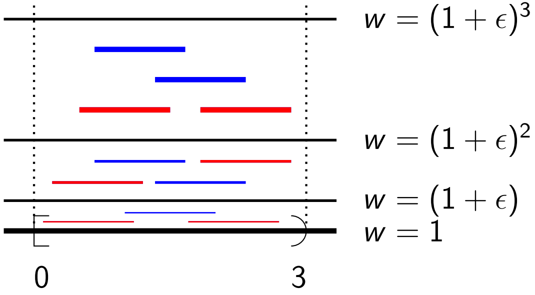

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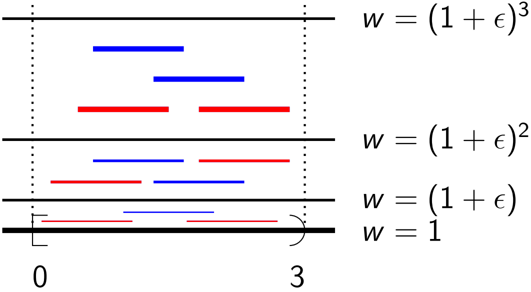

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- This gives a $(1+\epsilon)$-approximation within the heaviest $\log(1/\epsilon)/\epsilon$ weight classes.

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Index Problem - INDEX(k)

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Index Problem - INDEX(k)

Alice

Unweighted Unit-Length Intervals

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Index Problem - INDEX(k)

Alice | Bob

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Index Problem - INDEX(k)

Alice	Bob	Out

Unweighted Unit-Length Intervals

- LB via communication complexity.

Index Problem - INDEX(k)

Alice	Bob	Out
X		

Unweighted Unit-Length Intervals

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Index Problem - INDEX(k)

Alice	Bob	Out
X	σ	

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Index Problem - INDEX(k)

Alice	Bob	Out
X	σ	Z

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Index Problem - INDEX(k)

Alice	$\xrightarrow{M}$	Bob	Out
X		σ	Z

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Index Problem - INDEX(k)

$$\frac{\text{Alice}}{X} \xrightarrow{M} \frac{\text{Bob}}{\sigma} \longrightarrow \frac{\text{Out}}{Z}$$

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Index Problem - INDEX(k)

$$\begin{tikzcd} \frac{\text{Alice}}{X} \arrow[r, bend left, "M"] & \frac{\text{Bob}}{\sigma} \arrow[r, bend left] & \frac{\text{Out}}{Z} \end{tikzcd}$$

- $X \in \{0,1\}^k$, $\sigma \in [k]$.

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Theorem

[KNR99] Any (randomised) protocol for INDEX(k) with success probability at least $2/3$ must send a message of size $\Omega(k)$.

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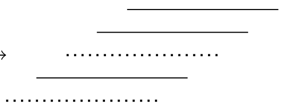
$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow$$



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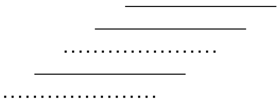

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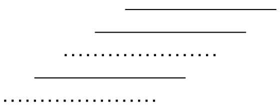

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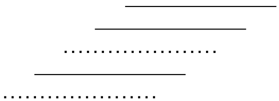
$J = 4$



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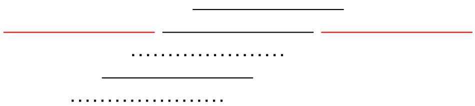

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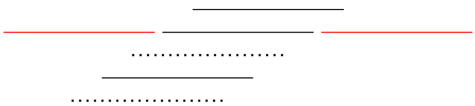

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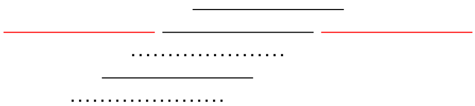
- **Key Idea:** Use J to create two wing intervals.
- This constructs a set of intervals which has an independent set of size 3 if $Z = 1$ and size 2 if $Z = 0$.



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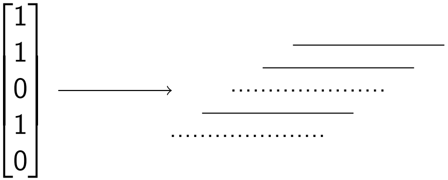
- **Alice:**
  1. Use X to create the clique gadget.



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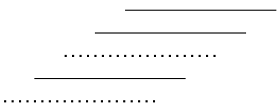
- **Alice:**
  1. Use X to create the clique gadget.
  2. Run the streaming algorithm on this set of intervals and set the state of the algorithm to Bob.



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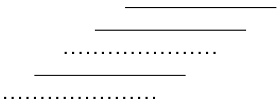

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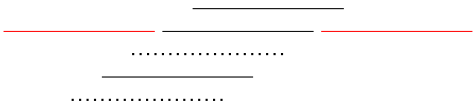

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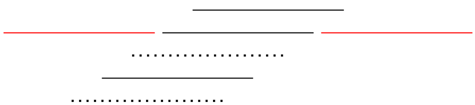
- **Bob:**
  1. Use J to create the wing intervals.
  2. Using the state sent by Alice, continue running the algorithm on the new intervals and output the result.



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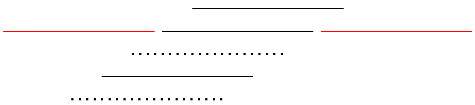

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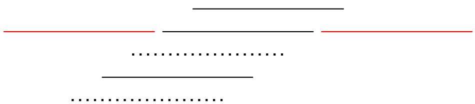
How does this apply to streaming?

- **Key Idea:** The message size is exactly the memory required for the algorithm.



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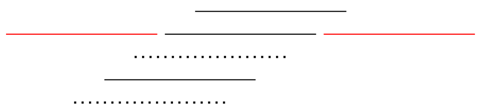
- **Key Idea:** The message size is exactly the memory required for the algorithm.
- An algorithm to compute a better-than 3/2-approximation can be used to solve INDEX(k).



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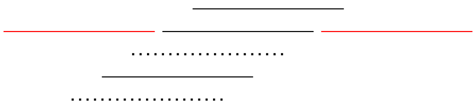
How does this apply to streaming?

- **Key Idea:** The message size is exactly the memory required for the algorithm.
- An algorithm to compute a better-than 3/2-approximation can be used to solve INDEX(k).
- By the INDEX(k) lower bound, such an algorithm must use space $\Omega(k) = \Omega(n)$.



State of the Art

		Insertion-Only		Insertion-Deletion	
		UB - $O_\epsilon(\lvert OPT\rvert)$	LB - $\Omega(n)$	UB - $\tilde{O}_\epsilon(\lvert OPT^*\rvert)$	LB - $\Omega(n)$
Unweighted	Unit-Length	$3/2$ [EHR12]	$\mathbf{3/2} - \epsilon$ [EHR12]	$\mathbf{2}$	$2 - \epsilon$ [BCW20]
	Variable-Length	2 [EHR12]	$2 - \epsilon$ [EHR12]	$*$	$\mathbf{\Theta(1)}$
Weighted	Unit-Length	$\mathbf{3/2} + \epsilon$	$3/2 - \epsilon$ [EHR12]	$\mathbf{2} + \epsilon$	$2 - \epsilon$ [BCW20]
	Variable-Length	$*$	$\mathbf{\Theta(1)}$	$*$	$\mathbf{\Theta(1)}$

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Unweighted	Unit-Length	$3/2$ [EHR12]	$3/2-\epsilon$ [EHR12]	$\mathbf{2}$	$2-\epsilon$ [BCW20]
	Variable-Length	2 [EHR12]	$2-\epsilon$ [EHR12]	$*$	$\boldsymbol{\Theta(1)}$
Weighted	Unit-Length	$\mathbf{3/2}+\epsilon$	$3/2-\epsilon$ [EHR12]	$\mathbf{2}+\epsilon$	$2-\epsilon$ [BCW20]
	Variable-Length	$*$	$\boldsymbol{\Theta(1)}$	$*$	$\boldsymbol{\Theta(1)}$

A Closing Thought

Communication Complexity can be used to achieve non-trivial lower bounds for streaming algorithms!

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