Interval Selection in Data Streams: Weighted Intervals and the Insertion-deletion Setting

Jacques Dark, Adithya Diddapur[1], Christian Konrad[1]

[1]University of Bristol

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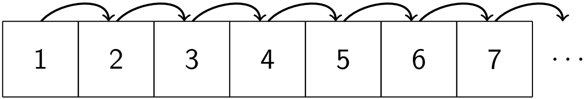
Goal

Given the input elements one at a time, compute an (approximate) solution using as **little space** as possible.



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- **Key Question:** What can we achieve across this regime?

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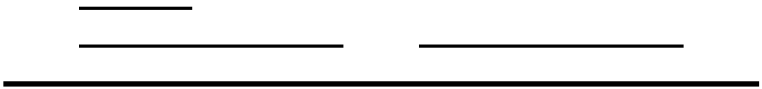

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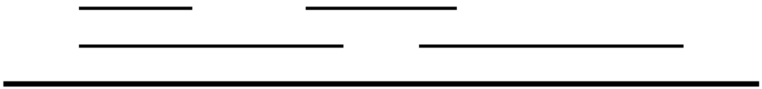

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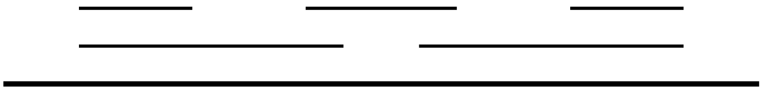

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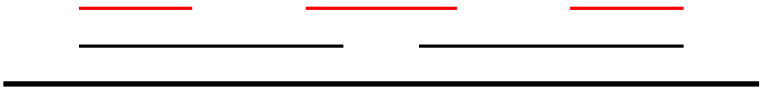

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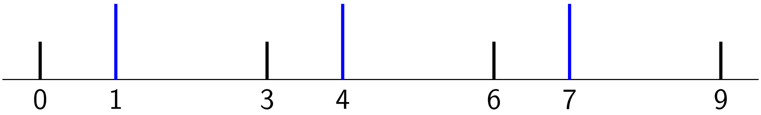

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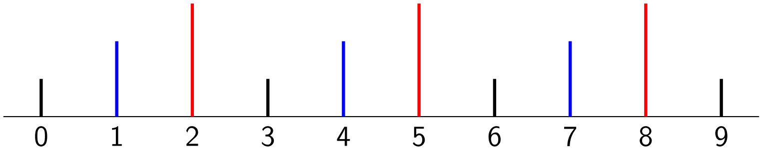

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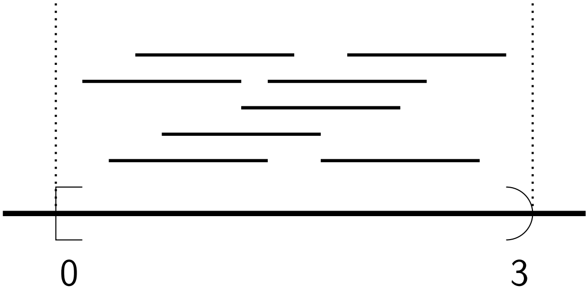

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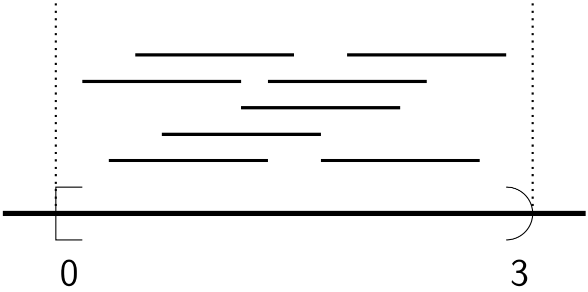

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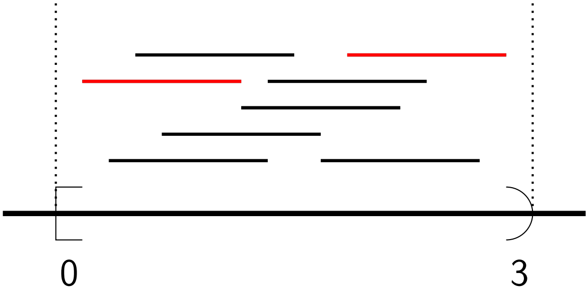

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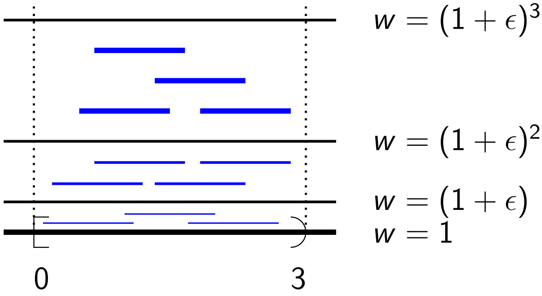

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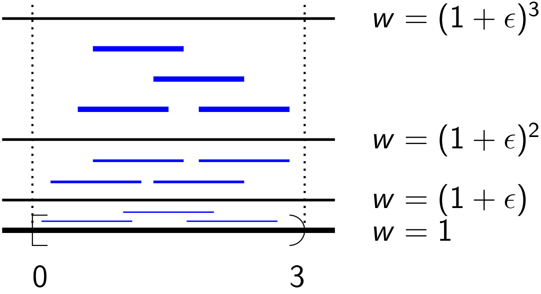

- This gives a $(1+\epsilon)$-approximation within the heaviest $\log(1/\epsilon)/\epsilon$ weight classes.

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Chained Index Problem - $\mathrm{CHAIN_p(k)}$ [CDK19]

$$
\begin{array}{ccccccccc}
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X^1 & & X^2 & & & & X^{p-1} & & \\
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\end{array}
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Theorem

[FNSZ20] Any (randomised) protocol for $\mathrm{CHAIN}_p(k)$ with success probability at least $2/3$ must send a message of size $\Omega(k/p^2)$.

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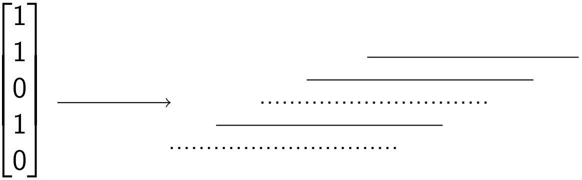

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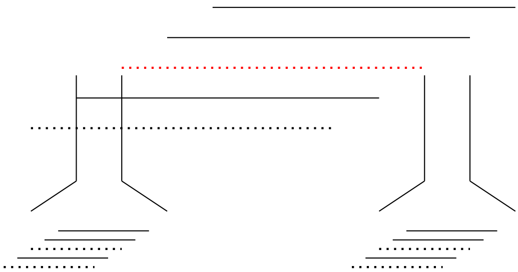

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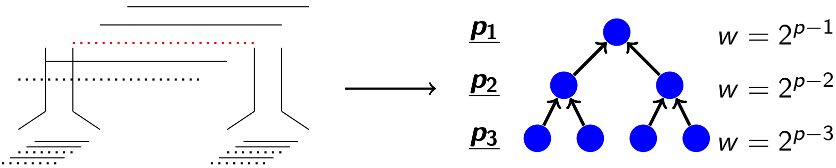

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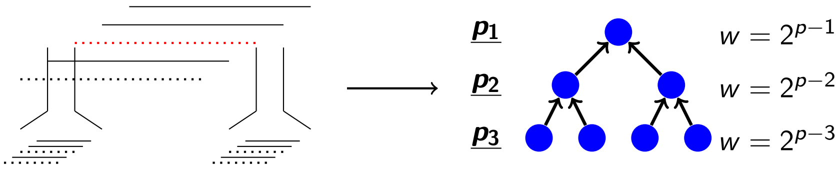

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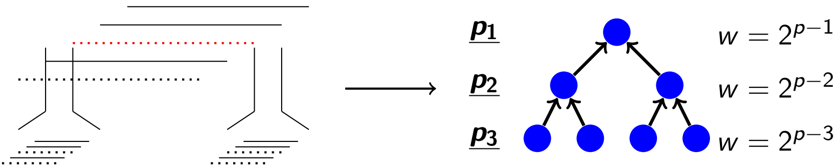

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Theorem

A $(p - \epsilon)$-approximate alg. solves the $CHAIN_p(k)$ instance, and so requires space $\Omega(k/p) = \Omega(n)$ for constant p.

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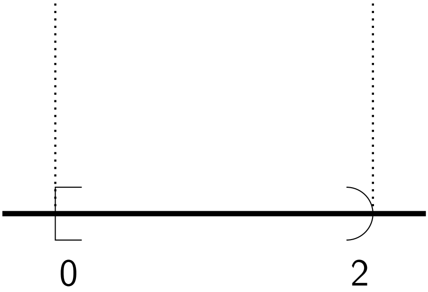

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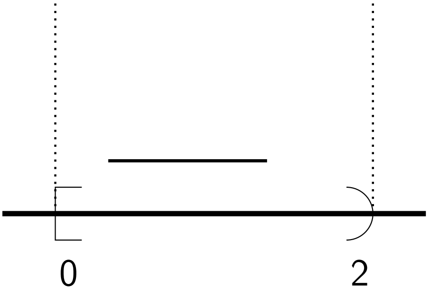

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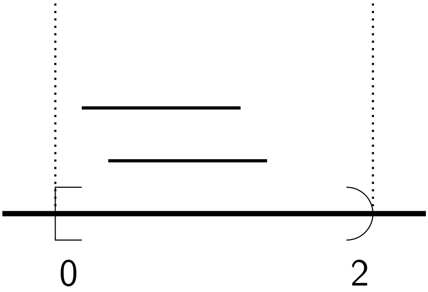

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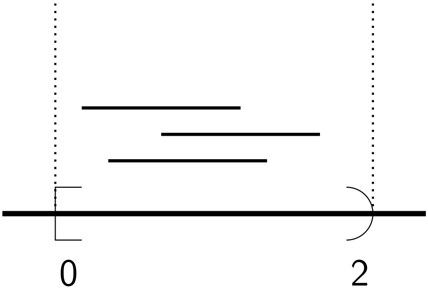

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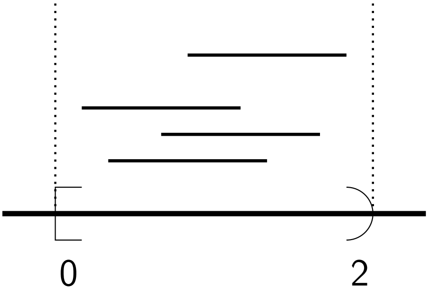

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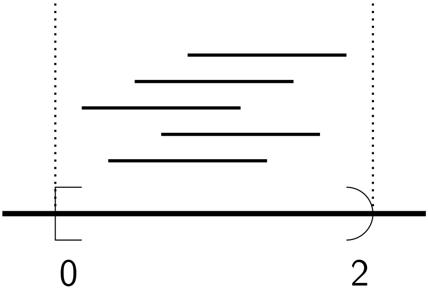

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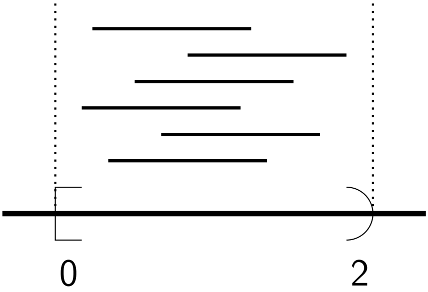

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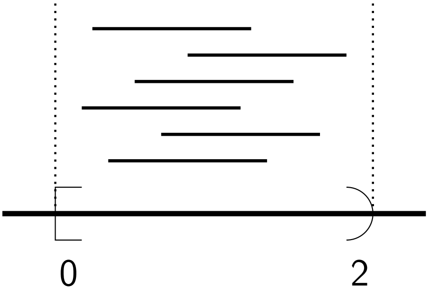

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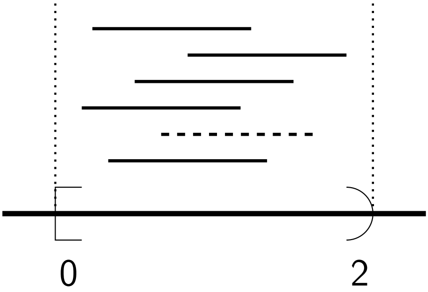

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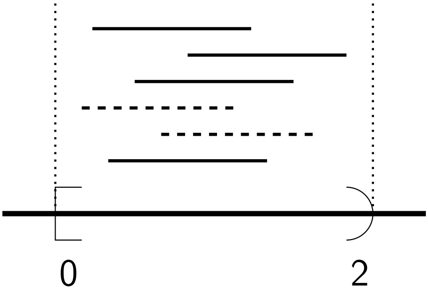

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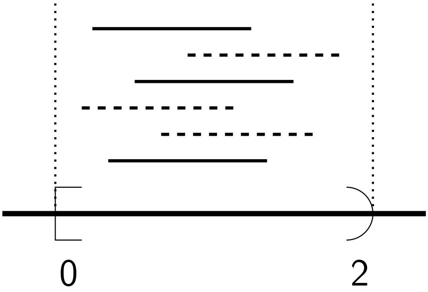

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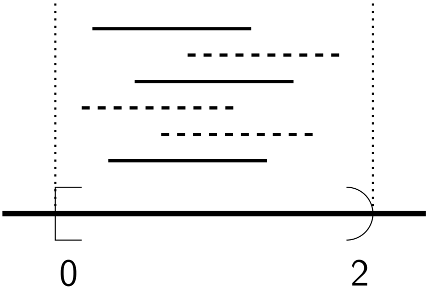

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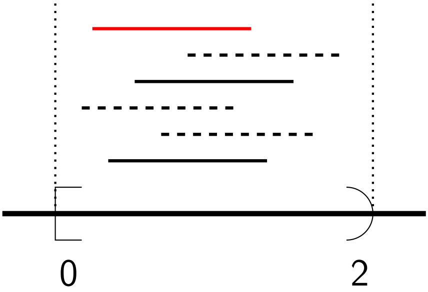

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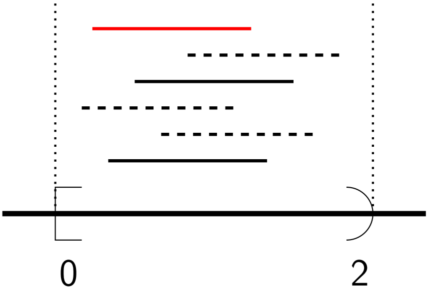

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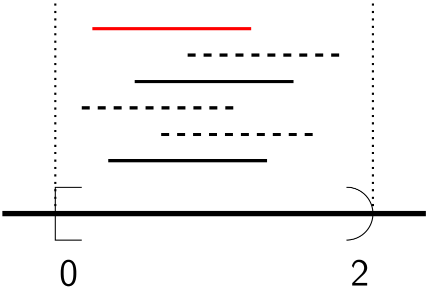

Problem

ℓ_0-Sampling Sample a uniform random surviving element from an insertion-deletion stream.

Theorem

[JST11] There is an algorithm for ℓ_0-Sampling which succeeds w.h.p. in space $O(\log^3 n)$.

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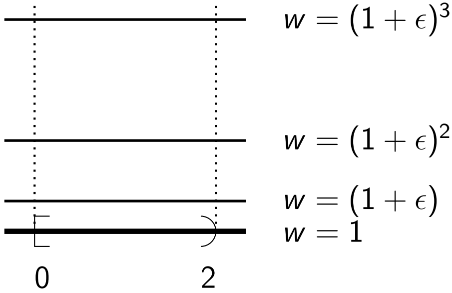

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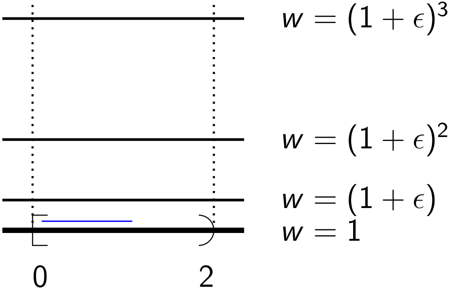

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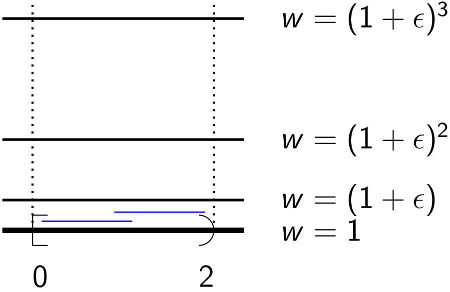

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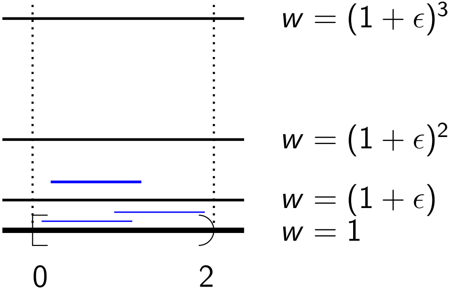

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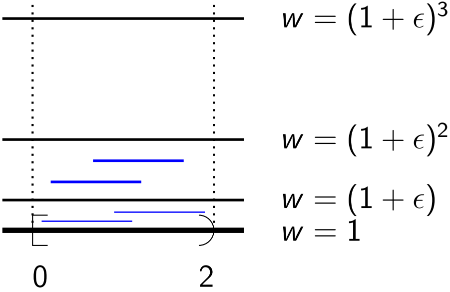

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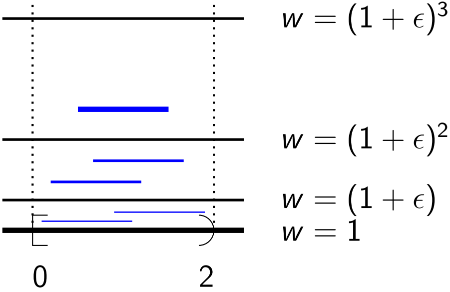

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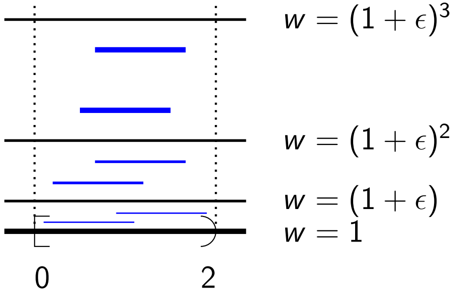

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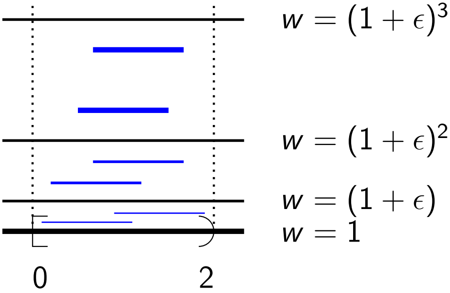

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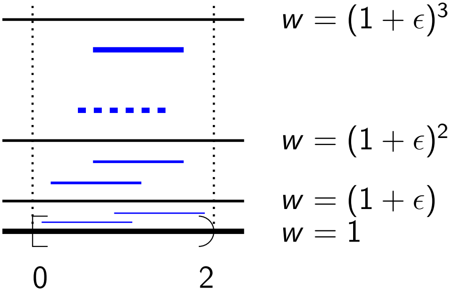

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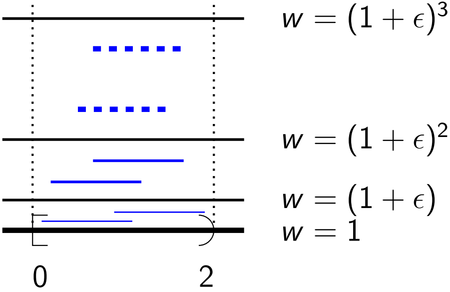

Insertion-Deletion: Unit-Length Intervals

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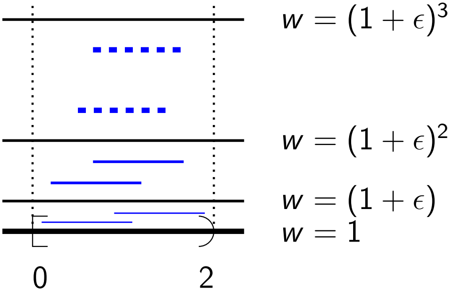
- Need all $\log(W)/\epsilon$ weight classes this time.



State of the Art

		Insertion-Only		Insertion-Deletion	
		UB - $O_\epsilon(\|OPT\|)$	LB - $\Omega(n)$	UB - $\tilde{O}_\epsilon(\|OPT^*\|)$	LB - $\Omega(n)$
Unweighted	Unit-Length	$3/2$ [EHR12]	$3/2 - \epsilon$ [EHR12]	$\mathbf{2}$	$2 - \epsilon$ [BCW20]
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Weighted	Unit-Length	$\mathbf{3/2} + \epsilon$	$3/2 - \epsilon$ [EHR12]	$\mathbf{2} + \epsilon$	$2 - \epsilon$ [BCW20]
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Insertion-Deletion: Variable-Length Intervals

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Augmented Chained Index Problem - AUG-CHAIN$_p$(k)

$$\begin{array}{ccccccccc}
\underline{P_1} & \xrightarrow{M^1} & \underline{P_2} & \longrightarrow & \cdots & \longrightarrow & \underline{P_{p-1}} & \xrightarrow{M^{p-1}} & \underline{P_p} \\
X^1 & & X^2 & & & & X^{p-1} & & \\
 & & \sigma^1 & & & & \sigma^{p-2} & & \sigma^{p-1} \\
 & & X^1_+ & & & & X^{p-2}_+ & & X^{p-2}_+
\end{array}$$

- $X^i \in \{0,1\}^k$, $\sigma^i \in [k]$, $X^i_+ = X^i[0 : \sigma^i - 1]$.
- **Promise:** $X^i[\sigma^i] = z \in \{0,1\}$ for each i.
- **Goal:** Recover z using small messages.

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$$\begin{tikzcd}
\underline{P_1} \arrow[r, bend left, "M^1"] & \underline{P_2} \arrow[r, bend left] & \cdots \arrow[r, bend left] & \underline{P_{p-1}} \arrow[r, bend left, "M^{p-1}"] & \underline{P_p} \\
X^1 & X^2 & & X^{p-1} & \\
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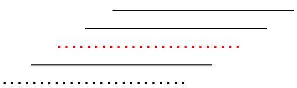
Theorem

Any (randomised) protocol for AUG-CHAIN$_p$(k) with success probability at least $2/3$ must send a message of size $\Omega(k/p^2)$.

Insertion-Deletion: Variable-Length Intervals

- **Key Idea:** Prefixes allow players to introduce valid deletions in the stream.



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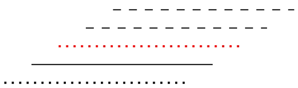

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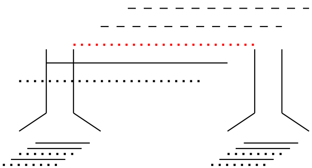

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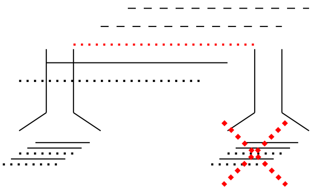

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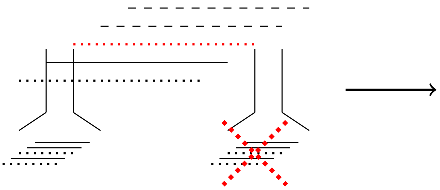

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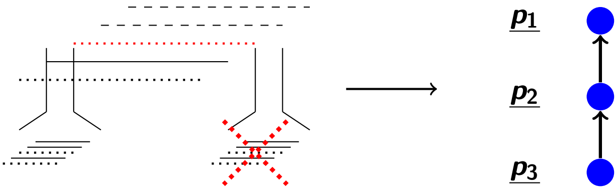

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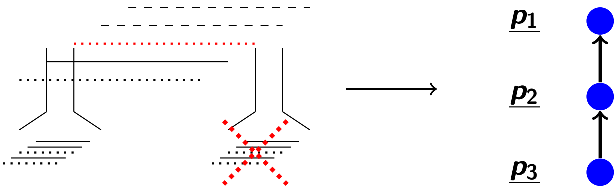

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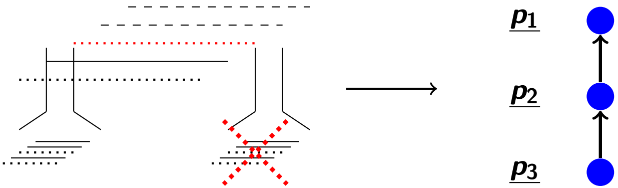

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Theorem

Any $(p - \epsilon)$-approximate alg. solves the AUG-CHAIN$_p(k)$ instance, and so requires space $\Omega(k/p) = \Omega(n)$ for constant p.

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- [CDK18] ask about a gap for unit squares:
  - **UB:** 3-approx.
  - **LB:** 5/2-approx.

References I

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