Robust Communication Complexity of Approximate Maximum Matching

Adithya Diddapur[1], Pavel Dvorák[2], Christian Konrad[1]

[1]University of Bristol, [2]Charles University

The Communication Complexity Setting

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- Two-Party One-Way Communication Complexity.

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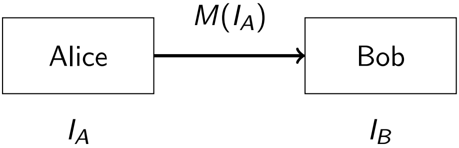

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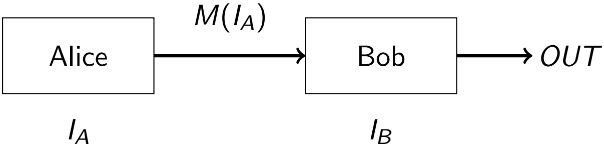

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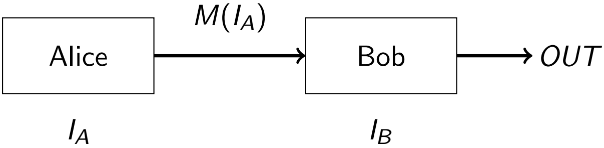

- **Aim:** Compute some function $f(I_A, I_B)$ with a message which is as small as possible.

Approximate Maximum Matching

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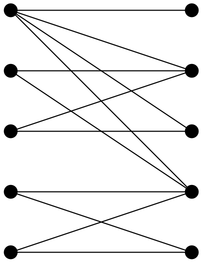

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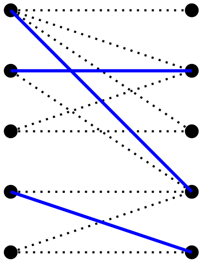

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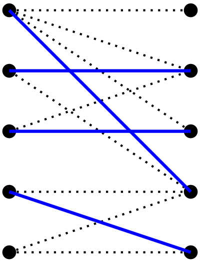

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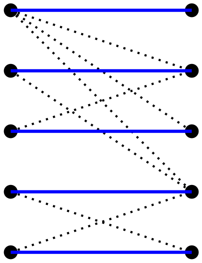

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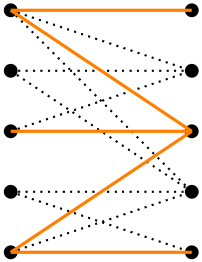

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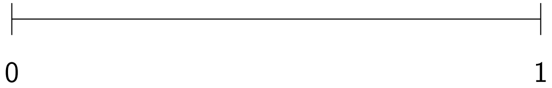

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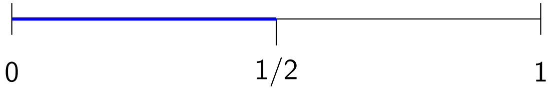

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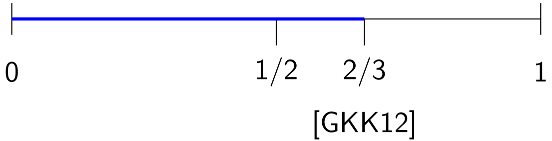

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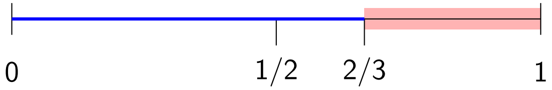

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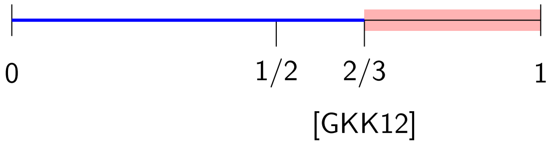

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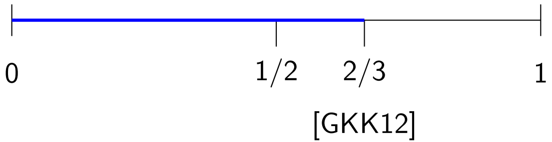

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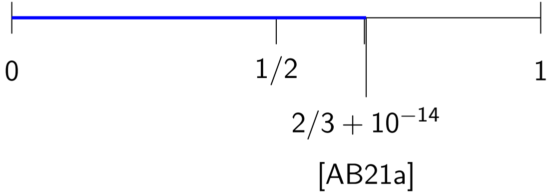

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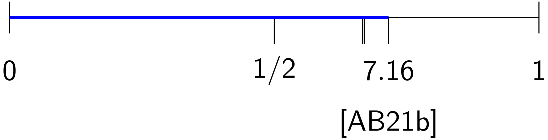

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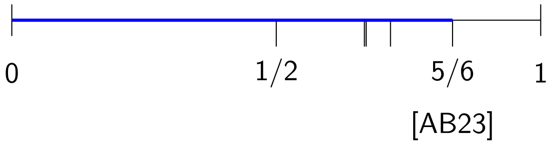

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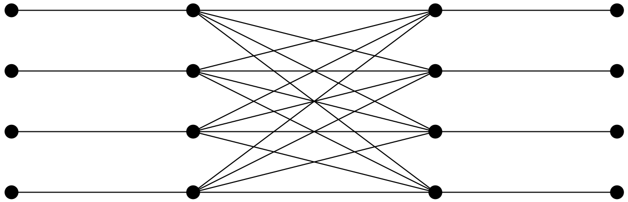

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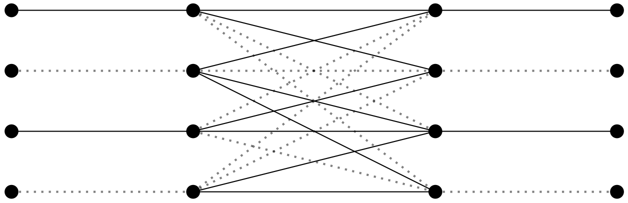

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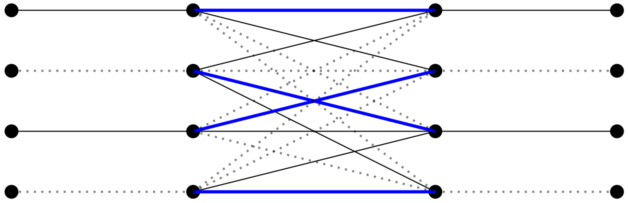

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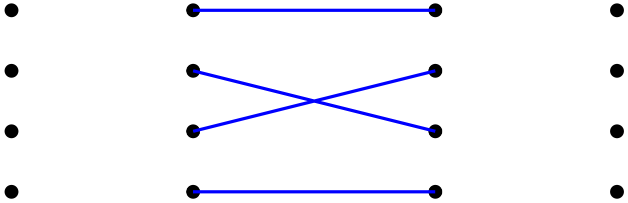

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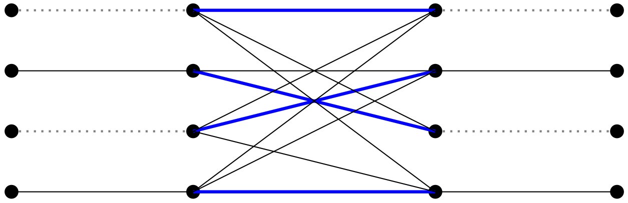

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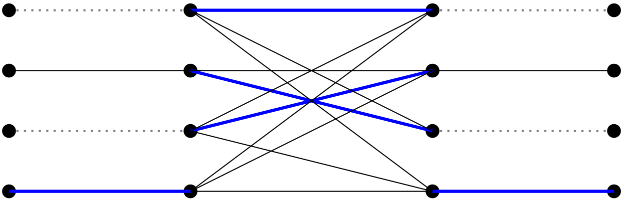

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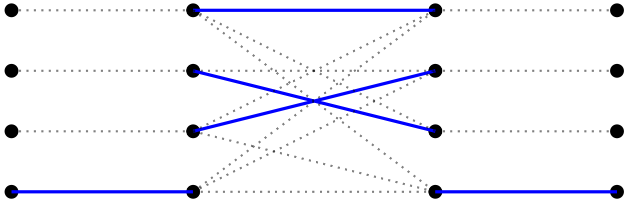

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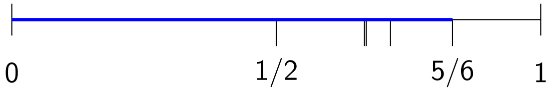

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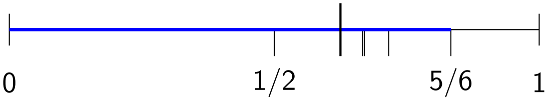

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A protocol which sends a maximum matching cannot exceed a $5/6$*-approximation in expectation.*

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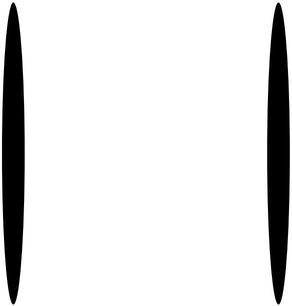

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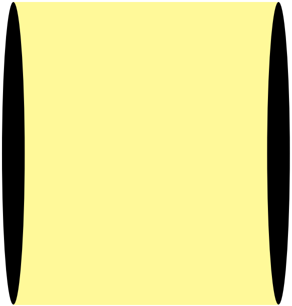

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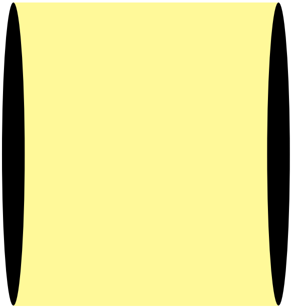

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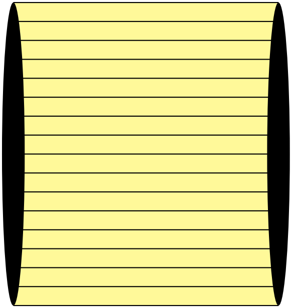

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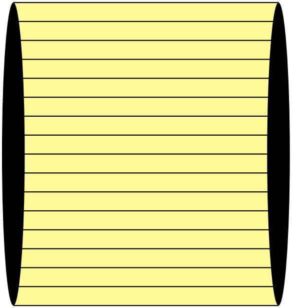

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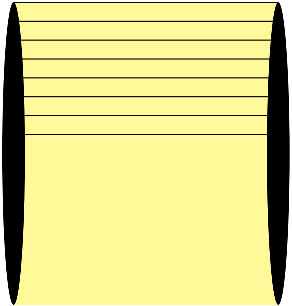

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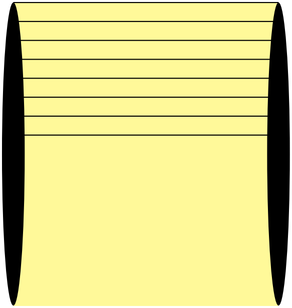

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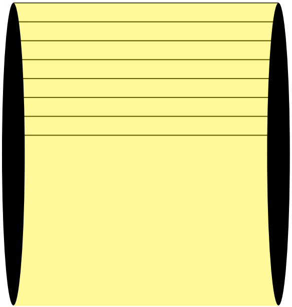

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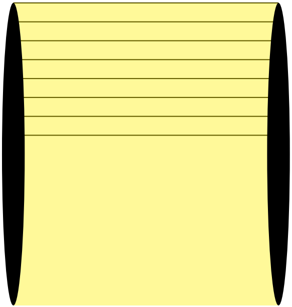

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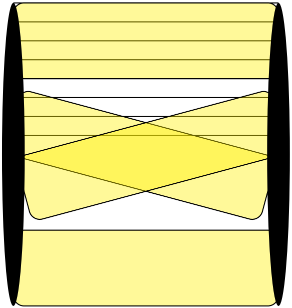

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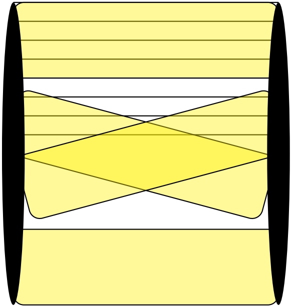

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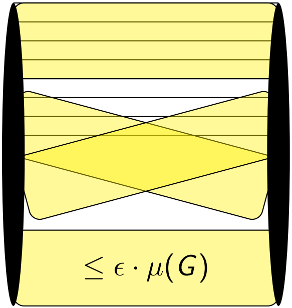

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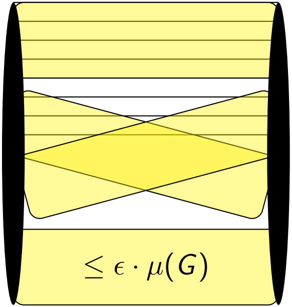

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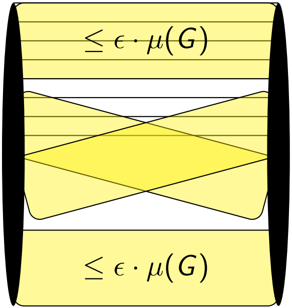

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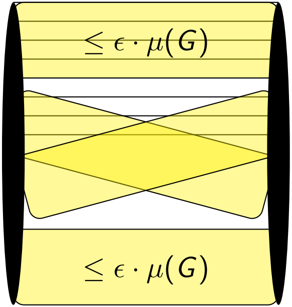

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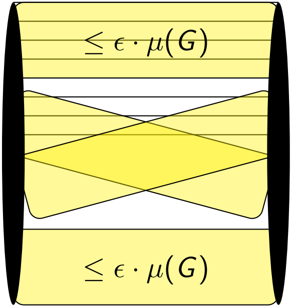

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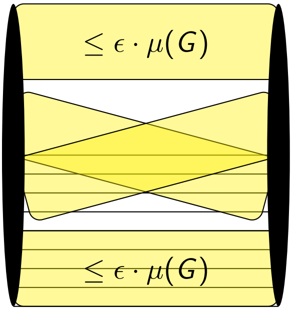

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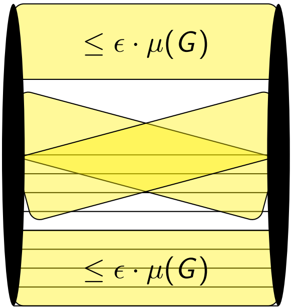

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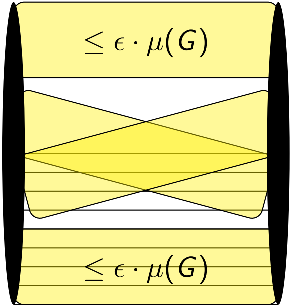
9. Solve $(1 - 2\epsilon) = (\frac{1}{2} + \epsilon)$.



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Do these techniques help us go beyond $5/6$?

References I

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