The D Function

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I'm going to develop a function D which maps square matrices to numbers. D must satisfy the following conditions:

1. $D(I_n) = 1$. (Identity property)

2. If A has a zero row, $D(A) = 0$. (Zero row property)

3. If A' is formed from adding a multiple of a row in A to another row in A, then $D(A) = D(A')$. (Reduction property)

4. If A' is formed from multiplying a row in A by a constant c, then $D(A') = cD(A)$. (Scaling property)

Comment on notation: $D(A)$ can also be written in terms of its row vectors: $D(r_1^\mathsf{T}, r_2^\mathsf{T}, \ldots r_n^\mathsf{T})$.

From these basic axioms we will develop the following theorems:

1. If any row is a multiple of another, then D(A)=0.

    Proof. If $r_i^\mathsf{T} = cr_j^\mathsf{T}$, where c is a constant, then $D(\ldots, r_i^\mathsf{T}, \ldots r_j^\mathsf{T} \ldots) = D(\ldots, cr_j^\mathsf{T}, \ldots r_j^\mathsf{T} \ldots)$ $\square$