The D Function

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We will develop a function D which maps square matrices to numbers.
D must satisfy the following conditions:

1. $D(I_n) = 1$. (Identity property)

2. If A has a zero row, $D(A) = 0$. (Zero row property)

3. If A' is formed from adding a multiple of a row in A to another row in A, then $D(A) = D(A')$. (Reduction property)

4. If A' is formed from multiplying a row in A by a constant c, then $D(A') = cD(A)$. (Scaling property)

Comment on notation: $D(A)$ can also be written in terms of its row vectors: $D(r_1^\intercal, r_2^\intercal, \ldots r_n^\intercal)$.

From these basic axioms we will develop the following theorems:

1. If any row is a multiple of another, then $D(A) = 0$.

2. If the rows of A are linearly dependent, then $D(A) = 0$

3. D gives 0 if any column of A is all zeroes.

4. If any two rows of A are swapped, then $D(A) = -D(A)$. (Swap property)

5. $D(A) = 0$ iff A is singular, i.e. not invertible (Singular property).

6. If A is upper or lower triangular, then $D(A)$ is the product of the diagonal elements. (Triangular property)

7. If A,B, and C are identical expect that row k of A is $a^\intercal$, row k of B is $b^\intercal$, and row k of C is $s^\intercal$ where $s = a + b$, then $D(C) = D(A) + D(B)$. (Multilinearity)

8. Define M_{ij}, the minor of A at position (i, j) to be the matrix obtained from A by knocking out both row i and column j of A.
   If a_{ij} is the only nonzero entry in either row i or column j, then $D(A) = a_{ij}(-1)^{i+j}D(M_{ij})$. (Block property)