CS/ECE 374 B Fall 2019 Jiashuo Tong (jtong8)

Homework 0 Problem 1

Solution:

(a) Let x and y be arbitrary strings.
Assume for any string m where $|m| < |x|$ that $digsum(m\dot{y}) = digsum(m) + digsum(y)$. There are two cases to consider:

1. If $x = \epsilon$, then

$$\begin{aligned} digsum(x \cdot y) &= digsum(\epsilon \cdot y) && (x = \epsilon) \\ &= digsum(\epsilon) + digsum(y) && \text{(definition of digsum)} \\ &= digsum(x) + digsum(y) && (x = \epsilon) \end{aligned}$$

2. If $x = a \cdot m$ for some symbol a and some string m, then

$$\begin{aligned} digsum(x \cdot y) &= digsum\big((a \cdot m) \cdot y\big) && (x = \epsilon) \\ &= digsum\big(a \cdot (m \cdot y)\big) && \text{(definition of } \cdot\text{)} \\ &= a + digsum(m \cdot y) && \text{(definition of digsum)} \\ &= a + digsum(m) + digsum(y) && \text{(inductive hypothesis)} \\ &= digsum(a \cdot m) + digsum(y) && \text{(definition of digsum)} \\ &= digsum(x) + digsum(y) && (x = am) \end{aligned}$$

In both cases, we conclude that $digsum(xy) = digsum(x) + digsum(y)$.

(b) Let x be an arbitrary string.
Assume for any sring m where $|m| < |x|$ that $digsum(m^R) = digsum(m)$. There are two cases to consider:

1. If $x = \epsilon$, then

$$\begin{aligned} digsum(x^R) &= digsum(\epsilon^R) && (x = \epsilon) \\ &= digsum(\epsilon) && \text{(definition of }^{\mathrm{R}}\text{)} \\ &= digsum(x) && (x = \epsilon) \end{aligned}$$

2. If $x = a \cdot m$, then

$$\begin{aligned} digsum(x^R) &= digsum\big((a \cdot m)^R\big) && (x = a \cdot m) \\ &= digsum(m^R \cdot a^R) && \text{(definition of }^{\mathrm{R}}\text{)} \\ &= digsum(m^R \cdot a) && \text{(definition of }^{\mathrm{R}}\text{)} \\ &= digsum(m^R) + digsum(a) && \text{(conclusion from (a))} \\ &= digsum(m) + digsum(a) && \text{(inductive hypothesis)} \\ &= digsum(am) && \text{(definition of digsum)} \\ &= digsum(x) && (x = a \cdot m) \end{aligned}$$

In both cases, we conclude that $digsum(x^R) = digsum(x)$.

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Homework 0 Problem 2

Solution:

(a) Prove by contradiction: Assume that 374 IS in L_{odd}

1. 374 complies with the 1st form:
   This is not possible because 374 has three digits, while the 1st form has one digit.
2. 374 complies with the 2nd form:
   This is not possible because the 2nd form starts with an even digit, while 374 starts with 3 which is odd.
3. 374 complies with the 3rd form:
   This is not possible because the 3rd form ends with an odd digit, while 374 ends with 4 which is even.

374 is not fit for all three cases. Therefore, 374 is not in L_{odd}.

(b) Let x and y be arbitrary strings.
Assume for any string m where $|m| < |x|$ that $digsum(m)$ is odd. There are three cases to consider:

1. If $x \in L$ for $x \in \{1, 3, 5, 7, 9\}$, then $digsum(x)$ will be odd in this case, since any element of $\{1, 3, 5, 7, 9\}$ is odd.
2. If $x = a \cdot m$, then

$$\begin{aligned}
digsum(x) &= digsum(a \cdot m) && (x = am) \\
&= a + digsum(m) && \text{(definition of } L_{odd} \text{ digsum)} \\
&= even + odd && \text{(a is even, m is odd by inductive hypothesis)} \\
&= odd && \text{(even plus odd is odd)}
\end{aligned}$$

3. If $x = a \cdot m \cdot b$, then

$$\begin{aligned}
digsum(x) &= digsum(a \cdot m \cdot b) && (x = amb) \\
&= digsum(a \cdot (m \cdot b)) && \text{(definition of } \cdot\text{)} \\
&= digsum(a) + digsum(m \cdot b) && \text{(definition of digsum)} \\
&= digsum(a) + digsum(m) + digsum(b) && \text{(definition of digsum)} \\
&= odd + odd + odd && \text{(a and b are odd, m is odd by inductive hypothesis)} \\
&= odd && \text{(odd plus odd plus odd is odd)}
\end{aligned}$$

In all three cases, we conclude that $digsum(x)$ is odd.

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Homework 0 Problem 3

Solution: Let n be an arbitrary non-negative integer. There are four cases to consider:

- $a \in L_{bad}^1$ for $a \in \{\epsilon, 0, 00\}$
- $a \in L_{bad}^2$ for $a \in \{\epsilon, 1, 11\}$
- $a \in L_{bad}^1$ for $a \in \{1, 11\}$ and $x \in L_{bad}^1$
- $a \in L_{bad}^2$ for $a \in \{0, 00\}$ and $x \in L_{bad}^2$
- $x \in L_{bad}$ if $x \in L_{bad}^1$ or $x \in L_{bad}^2$

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