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State	Finished
Completed on	Wednesday, April 26, 2017, 12:00 AM
Time taken	1 day 22 hours
Points	11.00/11.00
Grade	**10.00** out of 10.00 (**100**%)

Question 1

Correct

1.00 points out of 1.00

Which of these is logically equivalent to the negation of "If wishes were fishes, the sea would be full."

Select one:

- a. Wishes are fishes, and the sea is not full. ✓
- b. If the sea were full, then wishes would be fishes.
- c. If wishes were not fishes, the sea would not be full.
- d. If the sea were not full, then wishes would not be fishes.

The correct answer is: Wishes are fishes, and the sea is not full.

Question 2

Correct

1.00 points out of 1.00

Claim: $\sqrt{2} + \sqrt{6} < \sqrt{15}$.

How might our proof by contradiction start?

Select one:

- a. Suppose not. That is, suppose $\sqrt{2} + \sqrt{6} \geq \sqrt{15}$.
- b. Suppose not. Since $\sqrt{2} + \sqrt{6} \geq \sqrt{15}$, $\sqrt{2} \geq \sqrt{15} - \sqrt{6}$
- c. Suppose not. That is, suppose $\sqrt{2} + \sqrt{6} > \sqrt{15}$.

The correct answer is: Suppose not. That is, suppose $\sqrt{2} + \sqrt{6} \geq \sqrt{15}$.

Question 3

Correct

1.00 points out of 1.00

In a lottery, 5 balls are selected from a bin of twenty balls numbered 1 through 20. (The order doesn't matter.) Professor Luckless always guesses a 5-ball combination that includes her lucky number 7, and does not include her unlucky number 13. How many different days can she play the lottery without repeating a 5-ball guess?

Select one:

- a. $\binom{20}{4}$
- b. $\binom{19}{3}$
- c. $\binom{18}{4}$ ✓
- d. $\binom{19}{4}$
- e. $\binom{20}{5}$
- f. $\binom{18}{3}$
- g. $\binom{18}{5}$

The correct answer is: $\binom{18}{4}$

Question 4

Correct

1.00 points out of 1.00

$$\sum_{k=1}^{n} k \cdot \binom{n}{k} = ?$$

[Hint: You may use the fact that $\sum_{i=0}^{N} \binom{N}{i} = 2^N$.]

Select one:

- a. $2^n - 1$
- b. 2^n
- c. $n \cdot 2^{n-1}$ ✓
- d. 2^{n-1}
- e. $k \cdot 2^n$
- f. $2^n - n$

Your answer is correct.

The correct answer is: $n \cdot 2^{n-1}$

Question 5

Correct

1.00 points out of 1.00

How many ways are there to select 5 scoops of ice cream to put into a bowl, chosen from Baskin Robbins 31 flavors? A flavor may be selected multiple times.

Select one:

a. $\binom{35}{5}$

b. $\binom{34}{5}$

c. $\binom{32}{5}$

d. $\binom{36}{5}$

e. $\binom{31}{5}$

f. $\binom{33}{5}$

The correct answer is: $\binom{35}{5}$

Question **6**

Correct

1.00 points out of 1.00

We will say that a positive integer n is *cube-free* if there is no integer $k > 1$ such that $k^3 \mid n$. Let C be the set of all cube-free integers.

We will say that a positive integer n is *modest* if every prime factor of n is less than 10 (i.e., 2, 3, 5 or 7). Let M be the set of all modest integers.

What is $|C \cap M|$ (i.e., how many cube-free modest integers are there)?

(Hint: remember that every positive integer greater than 1 has a unique representation as a product of primes.)

Answer: 81 ✓

The correct answer is: 81

Question **7**

Correct

1.00 points out of 1.00

Suppose that A and B are sets. Then $\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$

Select one:

- a. never ✓
- b. always
- c. sometimes

The correct answer is: never

Question **8**

Correct

1.00 points out of 1.00

Suppose that k is a fixed positive integer. Let's define a function $f : \mathbb{Z} \to \mathbb{P}(\mathbb{Z})$ by

$$f(x) = \{y \in \mathbb{Z} \ : \ \exists n \in \mathbb{Z}, x = y + kn\}$$

Now suppose that a and b are both integers, where a < b and $a \equiv b \pmod{k}$. Which of these correctly describes the relationship between f(a) and f(b)?

Select one:

- a. f(a) is a proper subset of f(b)
- b. f(a) and f(b) are disjoint
- c. f(a) = f(b)

The correct answer is: f(a) = f(b)

Question 9

Correct

1.00 points out of 1.00

Let $A = \{a, b, c, d, e, f, g, h\}$.

Suppose that P is a partition of A, which already contains the following elements

$\{b, d, f\}$
$\{c, h\}$
$\{e\}$

Then the other element(s) of P could be

Select one or more:

- [x] a. $\{a\}$ and $\{g\}$ ✓
- [ ] b. $\{a, d, g\}$
- [ ] c. a and g
- [x] d. $\{a, g\}$ ✓
- [ ] e. $\{g\}$ (i.e. as the only other element of the partition)
- [ ] f. $\{a, g\}$ and Ø

The correct answers are: $\{a, g\}$, $\{a\}$ and $\{g\}$

Question **10**

Correct

1.00 points out of 1.00

For any integer n, define

$$F(n) = \{p \in \mathbb{Z} \mid n^4 + 1 = p^4 + 1\}$$

Then let P be the partition of the integers that contains all the sets F(n).   That is:

$$P = \{F(n) \mid n \in \mathbb{Z}\}$$

Then the cardinality of P (i.e. the number of elements in P) is

Select one:

- a. not defined
- b. 4
- c. 0
- d. 1
- e. infinite ✓

The correct answer is: infinite

Question **11**

Correct

1.00 points out of 1.00

Let $A = \mathcal{P}(X)$ denote the power set of the set X, and let $X = \{1, 2, 3, 4, 5\}$. Consider the relation R on A such that pRq if and only if $p \cap q \neq \emptyset$. Select those statements that are true for the relation R.

Select one or more:

- [ ] a. R is reflexive
- [x] b. R is an equivalence relation ✗
- [x] c. R is finite ✓
- [ ] d. R is transitive
- [x] e. R is symmetric ✓

Your answer is correct.

The correct answers are: R is symmetric
, R is finite