

Task 2.6

Hand calculate Task 2.2:

$$\begin{aligned} P(S_1) &= \mu_{f \rightarrow S_1}(S_1) \mu_{g \rightarrow S_1}(S_1) \\ &= f(S_1) \cdot \left(\sum_{E_1} g(S_1, E_1) \cdot 1 \right) \end{aligned}$$

Thus,

$$\begin{aligned} P(S_1 = 0) &= f(S_1 = 0) \cdot \left(\sum_{E_1} g(S_1, E_1) \right) \\ &= f(S_1 = 0) \cdot (g(S_1 = 0, E_1 = 0) + g(S_1 = 0, E_1 = 1)) \\ &= 0.85 \cdot (0.1 + 0.2) \\ &= 0.255 \end{aligned}$$

$$\begin{aligned} P(S_1 = 1) &= f(S_1 = 1) \cdot \left(\sum_{E_1} g(S_1, E_1) \right) \\ &= f(S_1 = 1) \cdot (g(S_1 = 1, E_1 = 0) + g(S_1 = 1, E_1 = 1)) \\ &= 0.15 \cdot (0 + 0.5) \\ &= 0.075 \end{aligned}$$

Normalizing the probabilities, we obtain

$$\begin{aligned} P(S_1 = 0) &= \frac{0.255}{0.255 + 0.075} = 0.7727 \\ P(S_1 = 1) &= \frac{0.075}{0.255 + 0.075} = 0.2273 \end{aligned}$$

Hand calculate Task 2.4:

$$\begin{aligned} P(S_1 = 0) &= f(S_1 = 0) \cdot g(S_1 = 0, E_1 = 1) \\ &= 0.85 \cdot 0.2 \\ &= 0.17 \\ P(S_1 = 1) &= f(S_1 = 1) \cdot g(S_1 = 1, E_1 = 1) \\ &= 0.15 \cdot 0.5 \\ &= 0.075 \end{aligned}$$

Normalizing the probabilities, we obtain

$$\begin{aligned} P(S_1 = 0) &= \frac{0.17}{0.17 + 0.075} = 0.6939 \\ P(S_1 = 1) &= \frac{0.075}{0.17 + 0.075} = 0.3061 \end{aligned}$$