Regular Star Polygons : Angles and Lengths

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A regular star polygon is defined by the radius of the circumscribing circle and by its Schlafli symbol {n/m} where n is the number of vertices and m is the step used to connect the vertices. The vertices are numbered from 0 to n-1 and regularly placed on the circle. Starting from the vertex 0, the star is built step by step by connecting the vertex i to the vertex (i+m) % n until the initial vertex 0 is reached (% is the modulo operator). If n and m are coprime (d=gcd(m,n)=1), all vertex are reached and the star is done. if n and m are not coprime (d>1) the construction is done via stellation: the figure is composed of d star polygons {(n/d) / (m/d)}. Examples: an hexagram {6/2} is composed of 2 triangles, i.e. {6/2} ⇒ 2×{3}; {12/3} ⇒ 3×{4} and {30/12} ⇒ 6×{5/2}.

Input data 1

r: radius of circumscribing circle (drawn in blue)
n: number of vertices (numbered from 0 to n-1)
m: step to connect edges, vertex *i* is linked to vertex *(i+m) % n*
    1 corresponds to a convex polygon (drawn in orange)
    n/2 is the "narrowest" concave polygon (drawn in black+red)

Case 1: the star is drawn with internal segments (e.g. as done by hand we tracing a 5-points star, i.e. pentagram)

Case 2: the star is drawn without internal segments.

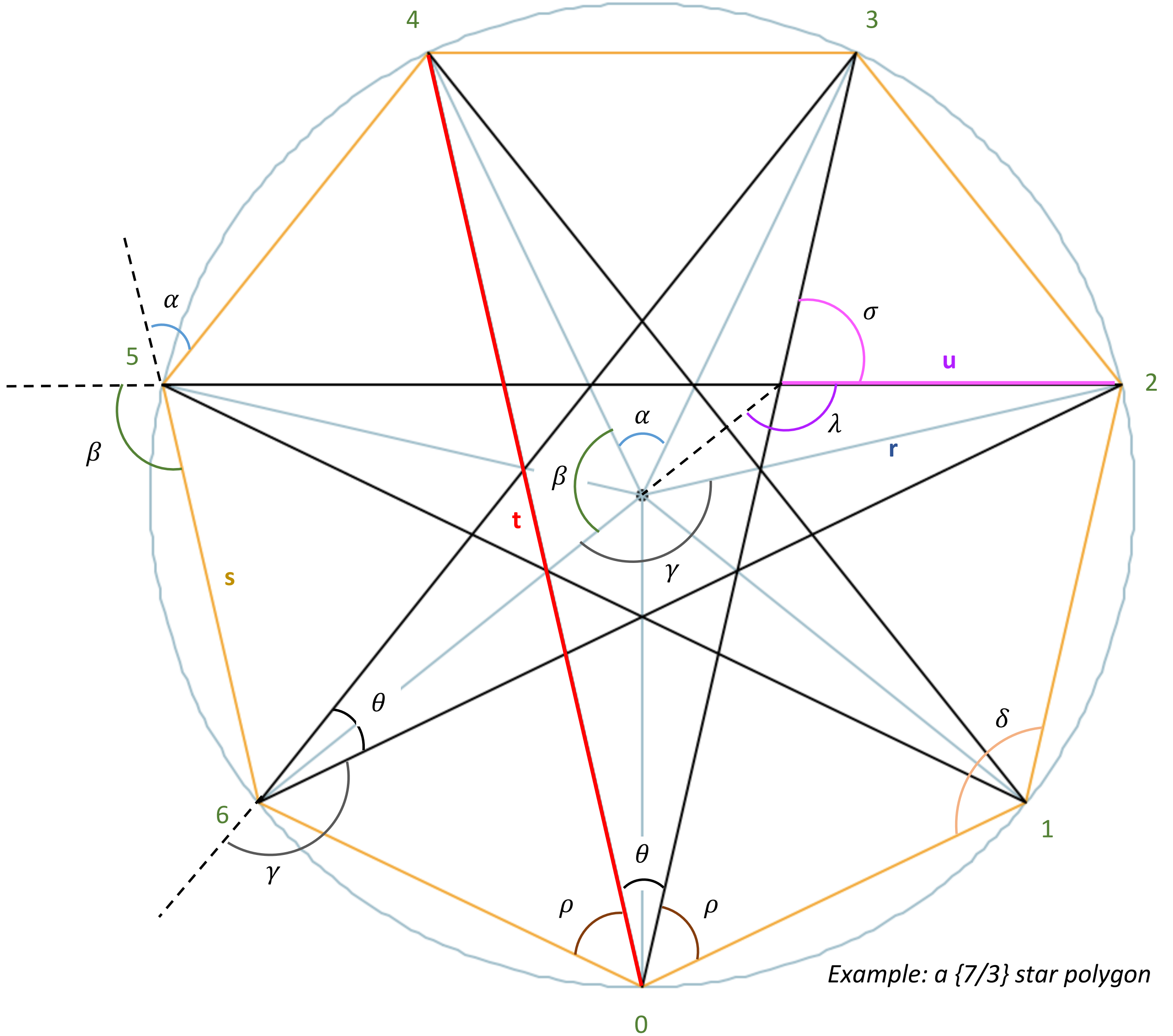
Input data 2

r: radius of circumscribing circle
n: number of vertices
u: length of the edge

Case 3: from u compute angles θ and ρ and then as Case 2

Angles (degrees) and lengths
(font colors corresponds to values needed in the above cases)

$\alpha = 360 / n$
$\beta = \alpha \times (m + 1) / 2$ or $\beta = \gamma - \rho$
$\gamma = \alpha \times m$
$\delta = (n - 2) / n * 180$
$\theta = 180 - \gamma$ or $\theta = \delta - 2 \times \rho$
$\rho = (\delta - \theta) / 2$ or $\rho = \mathrm{acos}(s / (2 \times u))$
$\lambda = 180 - (\alpha + \theta) / 2$
$\sigma = 180 - 2 * \rho$
$s = 2 \times r \times \sin(\alpha / 2)$
$t = 2 \times r \times \sin(\gamma / 2)$
$u = \sin(\alpha / 2) \times r / \sin(\lambda)$



Example: a {7/3} star polygon