Ordinary Differential Equations (ODEs), a short recap

Oscar Eriksson
oerikss@kth.se

ODEs, what are they good for?

- A powerful tool to model dynamical system.
- A concrete and simple example is the Lotka–Volterra equations, which models the dynamics between the population of predators and prey over time.
- Other applications for ODEs include modeling electrical, mechanical, and chemical systems.

The components of an ODE and what we are after

- Independent variable (usually named t and interpreted as time).
- Dependent variable, e.g $x(t)$, a function of the independent variable.
- Differential equation, e.g. $x'(t) = -x(t)$, where $x'(t) = \frac{dx(t)}{dt}$.
- The typical problem is to find $x : \mathrm{R} \to \mathrm{R}$, so that $x'(t) = -x(t)$ holds for all t in some interval.

Explicit First-Order ODEs

Explicit $x'(t)$ is written as an explicit function of $x(t)$.

First-Order Only first-order derivatives ($\frac{dx(t)}{dt}$ but not $\frac{d^2x(t)}{dt^2}$, $\frac{d^3x(t)}{dt^3}$, and so on).

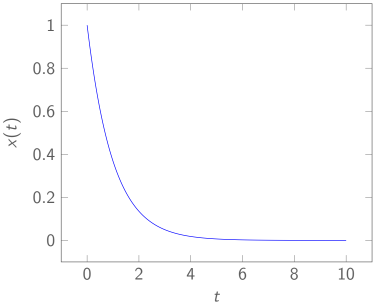
Ordinary We only have one independent variable t, as opposed to partial differential equations.

Solutions to an ODE

- $x(t) = Ce^{-t}$ is a solution to $x'(t) = -x(t)$ for all constants C.
- I.e. the differential equation has an infinite number of solutions.
- We can find a unique solution by giving initial values for x, e.g. $x(0) = 1$.
- This gives one solution $x(t) = e^{-t}$.
- An ODE and initial values forms an Initial Value Problem (IVP).

Numerical solutions to IVPs

- We can only find closed form solutions to IVPs in special cases.
- We therefore resort to approximate numerical solvers.
- The input to such solvers are typically a callback function encoding the ODE, inital values, and an interval where we seek a solution.
- The output is a solution trace for the dependent variables over the input interval.



Systems of ODEs and a concrete example

A system of ODEs is just a system of coupled differential equations, e.g. the Lotka–Volterra equations, which models the dynamics between the population of predators $y(t)$ and prey $x(t)$ over time.

$$x'(t) = \alpha x(t) - \beta x(t)y(t)$$
$$y'(t) = \gamma x(t)y(t) - \delta y(t),$$

where α, β, γ, and δ are parameters that determine how the species interact.