<u>MAAI past paper 2023</u>

3)

b)

i.

Person has good health : H

Person has good sleep : S

Person has balanced diet : D

Person does exercise : E

"A person has good health if and only if the person has a good sleep and has a balanced diet."

$H \leftrightarrow (S \land D)$

"The person does not have a balanced diet, but he has good sleep."

$\neg D \land S$

"If a person does exercises, then it appears as having a balanced diet."

$E \rightarrow D$

"The person does exercise."

E

"Therefore, that person has good health."

H

ii.

$[H \leftrightarrow (S \land D)] \land [\neg D \land S] \land [E \rightarrow D] \land [E] \rightarrow H$

Methods that can check whether the argument is valid:

1. Truth Table
2. Inference rules
3. Equivalence
4. Conjunctive normal forms

iii.

③ (b)

~~ii)~~

iii) CNF means we can only have $\wedge, \vee, \neg$.
(Not $\rightarrow$, $\leftrightarrow$).

* After converting to CNF, we break the expression formed by the connective "$\wedge$".

Converting to CNF ⇒

1) $H \leftrightarrow S \wedge D$

$= [H \rightarrow (S \wedge D)] \wedge [(S \wedge D) \rightarrow H]$

$= [\neg H \vee (S \wedge D)] \wedge [\neg(S \wedge D) \vee H]$

$= [(\neg H \vee S) \wedge (\neg H \vee D)] \wedge [(\neg S \vee \neg D) \vee H]$

$= [\neg H \vee S] \wedge [\neg H \vee D] \wedge [\neg S \vee \neg D \vee H]$

Now, We have CNF's ⇒

① $(\neg H \vee S)$ —— ①

② $(\neg H \vee D)$ —— ②

③ $(\neg S \vee \neg D \vee H)$ —— ③

2) $\neg D \wedge S$

④ $\neg D$ —— ④

⑤ S —— ⑤

3) $E \rightarrow D$

$= \neg E \vee D$

⑥ $\neg E \vee D$ —— ⑥

4) ⑦ E —— ⑦

Assume that the conclusion, H is false.

$\therefore$, $\neg H$ is true

$\neg H$ $\qquad$ $\neg S \vee \neg D \vee H$

$\neg S \vee \neg D$

Assume that the conclusion, H is false.

$\therefore$, $\neg H$ is true

$\neg H$ $\qquad$ $\neg S \vee \neg D \vee H$

$\neg S \vee \neg D$ $\qquad$ S

$\neg D$ $\qquad$ $\neg E \vee D$

$\neg E$ $\qquad$ E

\# (Contradiction)

This contradicts, since E and $\neg E$ can't be true at the same time.

$\therefore$, Assumption is false.

$\therefore$, Conclusion is true

iv.

1. **Standardization of Logic Representation:**

CNF provides a uniform way to represent logical expressions, enabling algorithms to process information systematically and efficiently. This standardization is critical for tasks such as automated theorem proving and logic programming.

2. **Efficient Problem Solving:**

CNF simplifies logical formulas into a series of parts, making them easy to solve using proof systems. This makes it great for solving problems like constraint satisfaction and knowledge representation