SDR notes

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Chapter 1

Demodulation of 802.15.4

1.1 Demodulation equation

The receiver uses the following equation to demodulate the signal.

$$d(t) = R_Q(t) \times R_I(t-\tau) - R_Q(t-\tau) \times R_I(t) \tag{1.1}$$

where the $R_I(t)/R_Q(t)$ are received In-phase/Quad-phase signals, τ is the sampling period. Bit decoding depends on the sign of $d(t)$. If $d(t) > 0$, the decoded bit equals 1 and vice versa.

1.2 Decoding principle

The 802.15.4 standard uses OQPSK modulation. The In-phase and Quad-phase are shifted by 1/2 symbol time. The symbol time is 1/2 period of sine wave. Namely, the In-phase and Quad-phase are shifted by 1/4 period of sine wave. Thus, the In-phase and Quad-phase can be expressed as $\pm sin(t+\theta)$ and $\pm cos(t+\theta)$. Where $\theta \in \{0, \frac{\pi}{2}\}$.

For example, one combination of In-phase/Quad-phase is $cos(t)$ and $sin(t), 0 < t < \frac{pi}{2}$. By applying the demodulation equation,

$$d(t) = sin(t) \times cos(t-\tau) - sin(t-\tau) \times cos(t) \tag{1.2}$$

$$d(t) = sin(t)\left[cos(t)cos(\tau) + sin(t)sin(\tau)\right] - cos(t)\left[sin(t)cos(\tau) - cos(t)sin(\tau)\right] \tag{1.3}$$

$$d(t) = sin^2(t)sin(\tau) + cos^2(t)sin(\tau) = sin(\tau) \tag{1.4}$$

Therefore, $d(t)$ depends on the sampleing period only. In this example, $d(t)$ is greater than 0 and remains constant during the half-symbol period. The following table lists the total combination of I/Q signals.

In-phase	Quad-phase	decoded equation
$cos(t)$	$sin(t)$	$sin(\tau)$
$cos(t)$	$-sin(t)$	$-sin(\tau)$
$-cos(t)$	$sin(t)$	$-sin(\tau)$
$-cos(t)$	$-sin(t)$	$sin(\tau)$
$sin(t)$	$cos(t)$	$-sin(\tau)$
$sin(t)$	$-cos(t)$	$sin(\tau)$
$-sin(t)$	$cos(t)$	$sin(\tau)$
$-sin(t)$	$-cos(t)$	$-sin(\tau)$

Table 1.1

Therefore, the receiver simply determines the incoming bit from the sign of $d(t)$. Note that the I/Q signals coming at the symbol rate of 1μs, and the I/Q shift by 0.5μs. Every 0.5μs generates a bit. Bit rate is 2MHz and half-byte is mapped to 32 bits chip sequence, which is 16μs.

Chapter 2

Frequency Mismatch

2.1 Carrier frequency mismatch in single TX/RX

Considering the transmitted In-phase signal $S_I(t)$, Quad-phase signal $S_Q(t)$ and the carrier frequency of transmitter f_{c1}. The transmitted signal can be writted as $S(t)$

$$S(t) = S_I(t) \times cos(2\pi \times f_{c1} \times t) + S_Q(t) \times sin(2\pi \times f_{c1} \times f) \tag{2.1}$$

Ignoring the channel characteristic and absense of noice, the received signal $R(t) = S(t)$. If there is a mismatch in carrier frequency, the receiver carrier frequency $f_{c2} \neq f_{c1}$. The received In-phase signal can be expressed as
$r_I(t) = R(t) \times cos(2\pi \times f_{c2} \times t)$.

$$r_I(t) = [S_I(t) \times cos(\omega_{c1}t) + S_Q(t) \times sin(\omega_{c1}t)] \times cos(\omega_{c2}t) \tag{2.2}$$
$$r_I(t) = S_I(t) \times cos(\omega_{c1}t) \times cos(\omega_{c2}t) + S_Q(t) \times sin(\omega_{c1}t) \times cos(\omega_{c2}t) \tag{2.3}$$

Where $\omega_{c1,2} = 2\pi \times f_{c1,2}$. Simplifying the equation by product to sum identities.

$$r_I(t) = \frac{1}{2}S_I(t)\left[cos((\omega_{c1} - \omega_{c2})t) + cos((\omega_{c1} + \omega_{c2})t)\right] + \frac{1}{2}S_Q(t)\left[sin((\omega_{c1} - \omega_{c2})t) + sin((\omega_{c1} + \omega_{c2})t)\right] \tag{2.4}$$

By removing high frequency components $(\omega_{c1} + \omega_{c2})$. In-phase baseband signal $r_{IB}(t)$ can be expressed

$$r_{IB}(t) = \frac{1}{2}S_I(t) \times cos(\Delta\omega t) - \frac{1}{2}S_Q(t) \times sin(\Delta\omega t) \tag{2.5}$$

Similarly, Quad-phase baseband signal $r_{QB}(t)$

$$r_{QB}(t) = \frac{1}{2}S_I(t) \times sin(\Delta\omega t) + \frac{1}{2}S_Q(t) \times cos(\Delta\omega t) \tag{2.6}$$

where $\Delta\omega = \omega_{c2} - \omega_{c1}$. By Rearranging these queations, we can express the following.

$$\begin{bmatrix} r_{IB}(t) \\ r_{QB}(t) \end{bmatrix} = \begin{bmatrix} cos(\Delta\omega t) & -sin(\Delta\omega t) \\ sin(\Delta\omega t) & cos(\Delta\omega t) \end{bmatrix} \times \begin{bmatrix} {}^1/_2 \cdot S_I(t) \\ {}^1/_2 \cdot S_Q(t) \end{bmatrix}$$

We can clearly see that the frequency mismatch is equivalent to the rotation of complex coordinate with angular velocity $\Delta\omega$. Hense, by measuring the rotation speed and direction, we can back calculate the frequency offset between receiver and transmitter. In the O-QPSK modulation, I/Q signal at any given time map to a unit circle on constellation. Measuring both clockwise and counter clockwise rotation angle at fixed interval tells the receiver's carrier frequency is leading or legging. With this information, radio is able to self-compensate the frequency mismatch and we called it *Automatic Frequency Compensation* or AFC.

From table 2.1, data shows that the ARR decrease in SFD-Latch type AGC when AFC is enabled. Since the AFC tries to sompensate the frequency difference between TX and RX, the duration of local minimum becomes larger whereas no significant change on Continuous AGC mode. Continuous AGC with AFC could potentially increase the reception rate in longer identical packet collision.

2.1.1 Automatic Frequency Compensation

The 802.15.4 uses O-QOSK modulation. At any given of time, the in-phase and quad-phase have only 8 combinations which is listed in table **??**. The key concept is that the in-phase and quad-phase signal vector is always on the unit circle of complex

AGC MODE	AFC MODE	ARR
SFD-latch	Enable	93.3%
Continuous	Enable	95.5%
SFD-latch	Disable	94.5%
Continuous	Disable	95.1%

Table 2.1: Acknowledgment reception rate (ARR) for two constructively interfering transmitters with respect to different AGC modes and AFC mode. We transmitted 10,000 ACKs per transmitter per experiment. With AFC disable, carrier frequencies of TX and RX is off by 16.4KHz. In both case of AFC mode, continuous AGC works better. Enabling the AFC worsen the reception rate for SFD-Latch AGC simply because the baseband signal is attenuated by low-frequency envelope.

plane. Moreover, the signal vector is always rotating at angular velocity $\omega = \frac{\pi/2}{0.5\mu s}$. However, if the frequency of transmitter and receiver doesn't match, the angular velocities toward clockwise and counter-clockwise are different. Thus, by sampling the vector at constant rate and calculate angle difference toward two different direction, we are able to tell the frequency is leading or lagging. In usdr, we implemented a simple cordic core to measure the angle of input vector with sampling rate $= 250Ksps$. The delta angle should be $\pm\pi/4$ if frequencies are perfectly matched. By comparing the delta angle with $\pm\pi/4$, radio is able to self-align its carrier frequency to the incoming packet.

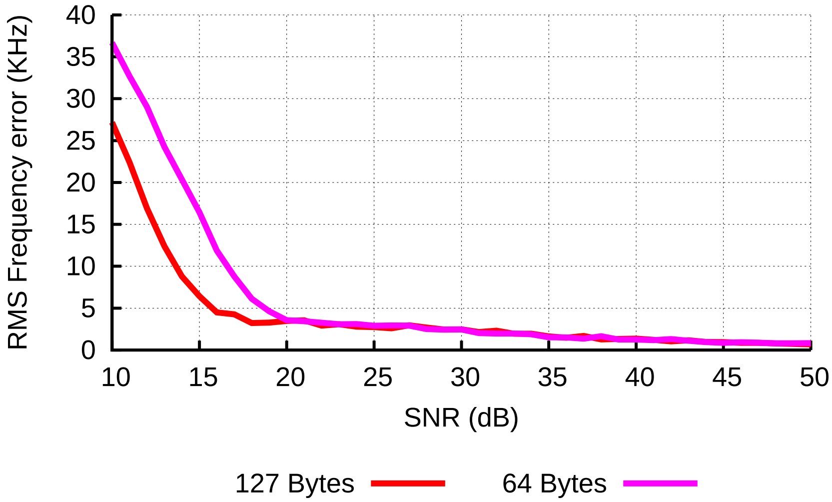


Figure 2.1

Figure 2.1 shows capability of AFC to correct the frequency mismatch. The original frequency mismatch is set to be 50 KHz. The figure shows how much frequency mismatch after automatic frequency compensation within a packet length. In these simulations, the packet lengths are set to be 127 and 64 bytes. The frequency step size in both simulation is 300 Hz. Thus, if the RMS frequency error < 300 Hz, the AFC successfully compensated all frequency mismatch. We can clearly see that for larger SNR, AFC works better. In addition, more opportunities to compensate the frequency in longer packet. Hence, frequency error is smaller in 127 bytes packet.

2.1.2 Demodulation with frequency mismatch

Recall the demodulation equation $d(t) = S_Q(t) \times S_I(t-\tau) - S_Q(t-\tau) \times S_I(t)$. Under the frequency mismatch scenario, the recieved $r_{IB}(t)$ and $r_{QB}(t)$ can be substituded by $S_I(t)$ and $S_Q(t)$. $d_f(t)$ can be written in following:

$$\begin{aligned} d_f(t) = & \left[-\tfrac{1}{2}S_I(t)sin(\Delta\omega t) + \tfrac{1}{2}S_Q(t)cos(\Delta\omega t)\right] \times \left[\tfrac{1}{2}S_I(t-\tau)cos(\Delta\omega(t-\tau)) + \tfrac{1}{2}S_Q(t-\tau)sin(\Delta\omega(t-\tau))\right] \\ & - \left[-\tfrac{1}{2}S_I(t-\tau)sin(\Delta\omega(t-\tau)) + \tfrac{1}{2}S_Q(t-\tau)cos(\Delta\omega(t-\tau))\right] \times \left[\tfrac{1}{2}S_I(t)cos(\Delta\omega t) + \tfrac{1}{2}S_Q(t)sin(\Delta\omega t)\right] \end{aligned} \tag{2.7}$$

Assuming that the $\Delta\omega \times \tau$ is small enough.

$$cos(\Delta\omega(t-\tau)) \approx cos(\Delta\omega t) \tag{2.8}$$

$$sin(\Delta\omega(t-\tau)) \approx sin(\Delta\omega t) \tag{2.9}$$

The equation can be further simplified.

$$
\begin{aligned}
d_f(t) \approx & \left[-\tfrac{1}{2}S_I(t)sin(\Delta\omega t) + \tfrac{1}{2}S_Q(t)cos(\Delta\omega t)\right] \times \left[\tfrac{1}{2}S_I(t-\tau)cos(\Delta\omega t) + \tfrac{1}{2}S_Q(t-\tau)sin(\Delta\omega t)\right] \\
& - \left[-\tfrac{1}{2}S_I(t-\tau)sin(\Delta\omega t) + \tfrac{1}{2}S_Q(t-\tau)cos(\Delta\omega t)\right] \times \left[\tfrac{1}{2}S_I(t)cos(\Delta\omega t) + \tfrac{1}{2}S_Q(t)sin(\Delta\omega t)\right] && (2.10) \\
\approx & \left[-\tfrac{1}{4}S_I(t)S_Q(t-\tau)sin^2(\Delta\omega t) + \tfrac{1}{4}S_I(t-\tau)S_Q(t)cos^2(\Delta\omega t)\right] \\
& - \left[\tfrac{1}{4}S_I(t)S_Q(t-\tau)cos^2(\Delta\omega t) - \tfrac{1}{4}S_I(t-\tau)S_Q(t)sin^2(\Delta\omega t)\right] && (2.11) \\
\approx & \tfrac{1}{2}\left[S_Q(t)S_I(t-\tau) - S_Q(t-\tau)S_I(t)\right] && (2.12)
\end{aligned}
$$

The decoded equation $d_f(t)$ is linear scaling of $d(t)$. The sign of $d_f(t)$ and $d(t)$ are identical. Therefore, the decoder scheme still function if there is a mismatch between transmitter and receiver.

2.2 Carrier frequency mismatch in double TX, single RX

In this scenario, two TX node transmit the identical baseband signal simultaneously. Assuming the TX nodes have different carrier frequency f_{c1} and f_{c2}, and receiver's frequency is f_{cr}. We simply assume that $R(t) = S(t)$.

$$R(t) = S(t) = S_I(t)\left[cos(\omega_{c1}t) + cos(\omega_{c2}t)\right] + S_Q(t)\left[sin(\omega_{c1}t) + sin(\omega_{c2}t)\right] \tag{2.13}$$

Simplifying the equation by sum-to-product identities.

$$\begin{aligned} R(t) &= S_I(t)\left[2cos(\tfrac{\omega_{c1}+\omega_{c2}}{2}t)cos(\tfrac{\omega_{c1}-\omega_{c2}}{2}t)\right] + S_Q(t)\left[2sin(\tfrac{\omega_{c1}+\omega_{c2}}{2}t)cos(\tfrac{\omega_{c1}-\omega_{c2}}{2}t)\right] && (2.14)\\ &= cos(\tfrac{\omega_{c1}-\omega_{c2}}{2}t)\left[2S_I(t)cos(\tfrac{\omega_{c1}+\omega_{c2}}{2}t) + 2S_Q(t)sin(\tfrac{\omega_{c1}+\omega_{c2}}{2}t)\right] && (2.15) \end{aligned}$$

The received In-phase signal $r_I(t)$ can be expressed as following:

$$r_I(t) = R(t) \times cos(\omega_{cr}t) \tag{2.16}$$

$$\begin{aligned} r_I(t) = cos(\tfrac{\omega_{c1}-\omega_{c2}}{2}t)\{S_I(t)&\left[cos(\tfrac{\omega_{c1}+\omega_{c2}-2\omega_{cr}}{2}t) + cos(\tfrac{\omega_{c1}+\omega_{c2}+2\omega_{cr}}{2}t)\right] + \\ S_Q(t)&\left[sin(\tfrac{\omega_{c1}+\omega_{c2}-2\omega_{cr}}{2}t) + sin(\tfrac{\omega_{c1}+\omega_{c2}+2\omega_{cr}}{2}t)\right]\} && (2.17) \end{aligned}$$

Let $\Delta\omega_1 = (\omega_{c1} - \omega_{cr})$ and $\Delta\omega_2 = (\omega_{c2} - \omega_{cr})$. By removing high frequency tems, $r_{IB}(t)$ can be simplified.

$$r_{IB}(t) = cos(\tfrac{\Delta\omega_1-\Delta\omega_2}{2}t)\left[S_I(t)cos(\tfrac{\Delta\omega_1+\Delta\omega_2}{2}t) + S_Q(t)sin(\tfrac{\Delta\omega_1+\Delta\omega_2}{2}t)\right] \tag{2.18}$$

Similarily, the received Quad-phase baseband signal $r_{QB}(t)$ can be derived from $R(t) \times sin(\omega_{cr}t)$.

$$r_{QB}(t) = cos(\tfrac{\Delta\omega_1-\Delta\omega_2}{2}t)\left[-S_I(t)sin(\tfrac{\Delta\omega_1+\Delta\omega_2}{2}t) + S_Q(t)cos(\tfrac{\Delta\omega_1+\Delta\omega_2}{2}t)\right] \tag{2.19}$$

From section 2.1.2, we can conclude that if $(\Delta\omega_1 - \Delta\omega_2)t$ is small enough, the decoding equation just simply scaling by $cos^2((\Delta\omega_1 - \Delta\omega_2)t)$, which is always greater than zero. The sign of decoding equation remains the same as no frequency mismatch.

2.3 Envelop Modeling

2.3.1 Theoretical Method

Considering a simplified model, there are two transmitters and one receiver with their carrier frequency f_{c1}, f_{c2} and f_{cr} respectively. Assuming that $f_{cr} = f_{c1}$ and $S_Q(t) = 0$, the received in-phase baseband signal can be expressed as:

$$r_I(t) = S_I(t) \times cos^2((\Delta\omega_2/2) \times t) \tag{2.20}$$

Thus, from equation 2.20, the low-frequency envelop can be observed intuitively. The period T of the envelope is equal to $\frac{1}{\Delta f_2/2} \times \frac{1}{2} = \frac{1}{\Delta f_2}$. Where $\Delta f_2 = \frac{\Delta\omega_2}{2\pi}$. We can define a threshold α which the receiver makes false decision once the received signal's amplitude below α. The time (t) which the signal's amplitude is queal to threshold (α) can be expressed as:

$$\begin{aligned} cos^2(2\pi\tfrac{\Delta\omega_2}{2} \times t) &= \alpha && (2.21)\\ \tfrac{\Delta\omega_2 t}{2} &= cos^{-1}(\sqrt{\alpha}) && (2.22)\\ t &= \tfrac{cos^{-1}(\sqrt{\alpha})}{\Delta\omega_2/2} && (2.23)\\ t &= \tfrac{cos^{-1}(\sqrt{\alpha})}{\pi\Delta f_2} && (2.24)\\ & && (2.25) \end{aligned}$$

Thus, we defined γ, which is the proportion of signal below threshold (α) to the entire period (T) can be calculated as:

$$\begin{aligned} \gamma &= \tfrac{T/2-t}{T/2} && (2.26)\\ &= \frac{\frac{1}{2\Delta f_2} - \frac{cos^{-1}(\sqrt{\alpha})}{\pi\Delta f_2}}{\frac{1}{2\Delta f_2}} && (2.27)\\ &= 1 - \tfrac{2}{\pi}cos^{-1}(\sqrt{\alpha}) && (2.28) \end{aligned}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0		14	15	14	16	13	15	13	31	17	16	17	15	18	16	18
1	14		13	16	14	15	13	15	17	31	18	15	17	16	18	16
2	15	13		13	15	14	16	14	16	18	31	18	16	17	15	17
3	14	16	13		14	15	13	15	17	15	18	31	17	16	18	16
4	16	14	15	14		13	15	13	15	17	16	17	31	18	16	18
5	13	15	14	15	13		14	16	18	16	17	16	18	31	17	15
6	15	13	16	13	15	14		14	16	18	15	18	16	17	31	17
7	13	15	14	15	13	16	14		18	16	17	16	18	15	17	31
8	31	17	16	17	15	18	16	18		14	15	14	16	13	15	13
9	17	31	18	15	17	16	18	16	14		13	16	14	15	13	15
10	16	18	31	18	16	17	15	17	15	13		13	15	14	16	14
11	17	15	18	31	17	16	18	16	14	16	13		14	15	13	15
12	15	17	16	17	31	18	16	18	16	14	15	14		13	15	13
13	18	16	17	16	18	31	17	15	13	15	14	15	13		14	16
14	16	18	15	18	16	17	31	17	15	13	16	13	15	14		14
15	18	16	17	16	18	15	17	31	13	15	14	15	13	16	14	

Table 2.2: Chip distance table. This table shows the distance between any two chip sequence in codebook. The minimum distance two sequences is 13 chips

From equation 2.28, we find that the proportion γ is a function of α only. In other words, the proportion remains the same no matter what Δf_2 is. Thus, in this simplified model, assuming that all the false decoding comes from the amplitude attenuation by $cos^2(\Delta\omega_2/2)$. The time duration D while signal below threshold (α) can be expressed as:

$$
\begin{aligned}
D &= \gamma \times T && (2.29)\\
&= \left[1 - \tfrac{2}{\pi} cos^{-1}(\sqrt{\alpha})\right] \times \tfrac{1}{\Delta f_2}(s) && (2.30)
\end{aligned}
$$

In 802.15.4, the period of each chip is $0.5\mu s$ and each byte occupies $32\mu s$ in time. Therefore, the relationship between number of chips n in each duration D and total number of envelopes m in a given lengh packet L bytes can be written as:

$$
\begin{aligned}
n &= D/0.5\mu s && (2.31)\\
m &= L \times 32\mu s/T && (2.32)
\end{aligned}
$$

Given $\alpha, \Delta f_2$ and L, we can calculate n, m. For example, $\alpha = 0.05$, $\Delta f_2 = 15$ KHz and $L = 127$ bytes. $n = 4.24, m = 60.96$. Decoding a 802.15.4 symbol requires a distance calculation between a received sequence and 16 possible sequence in codebook. From table 2.2, any chip sequence can be correctly decoded if the number of bit errors is less than 13. Furthermore, we define a probability p_{single} which is the probability that a single bit error while the amplitude is below threshold α. In the duration D, the number of bit errors should less than 13 for symbol to be able to decode correctly. We derive a probability $p_{correct}$, which is the probability that the symbol is able to be decoded correctly.

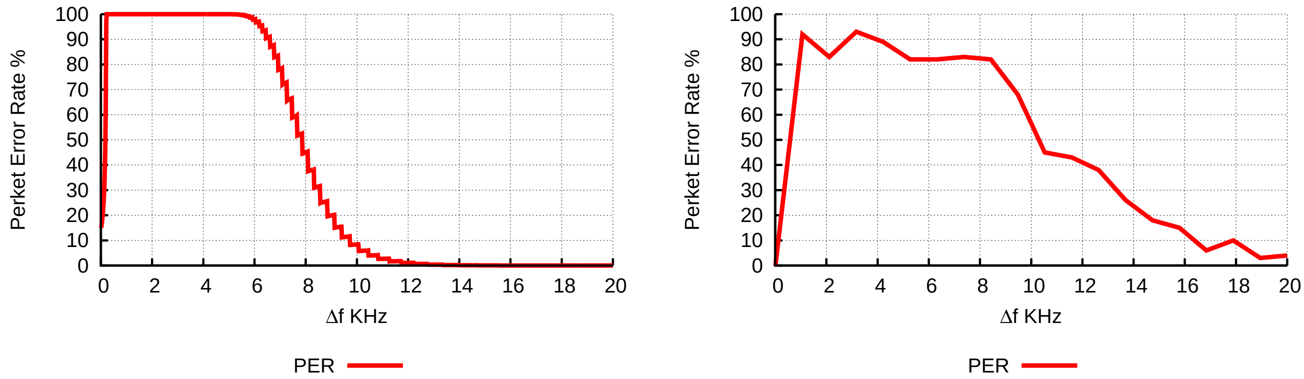
$$
p_{correct} = \sum_{i=0}^{12} \binom{n}{i} \times (p_{single})^i \times (1 - p_{single})^{n-i} \tag{2.33}
$$

Thus, the packet error rate (PER) and be can be expressed by $p_{correct}$ and m.

$$
PER = 1 - (p_{correct})^m \tag{2.34}
$$

Figure 2.2(a) shows the simulation result of packet error rate (PER) versus Δf_2. Parameters of this simulation are: $L = 127$ bytes, $\alpha = 0.05$, $p_{single} = 0.2$ and Δf_2 from 100 to 20 KHz. Since number of chips n, which below threshold α drops while increasing the frequency. Even number of evnelops m increases but the correct probability $p_{correct}$ is close or equal to 1 (if $n <$ chip distance, 13). In contrast, low frequency separation dramatically deteriorates the $p_{correct}$. The PER is much worse even if the number of envelop m is small. However, the equations we derived above don't hold under a special case, which is the period of envelop (T) is somehow greater than packet length. Imaging when $m \to \infty$, the PER should be 0 instead of 1. Thus, if the condition $(1-\gamma)T + (6.5\mu s(chipdistance)/p_{single}) > L \times 32\mu s$ meets, the PER can be written as:

$$
\begin{aligned}
PER &= 1 - p_{correct} && (2.35)\\
&= 1 - \tfrac{(1-\gamma)T - L \times 32\mu s + 6.5\mu s/p_{single}}{T - L \times 32\mu s} && (2.36)
\end{aligned}
$$



(a) Theoretical Modeling (b) Simulated Modeling

Figure 2.2: Figure 2.2(a) simulates the PER using equation 2.34. Whereas the figure 2.2(b) uses 802.15.4 transceiver architectural model. The parameters for both plots are: $L = 127$ Bytes, $\alpha = 0.05$, $\Delta f_1 = 0, \Delta f_2$ from 10 to 20 KHz. we can find the trend that the PER drops while Δf_2 increases in both plots. Since the number of consecutive contaminated bits get shorter in larger frequency separation, the possibility for receiver to false decode a packet become smaller.

Where the $6.5\mu s/p_{single}$ is the expection value of numbers of error chip, and $(1-\gamma)T - L \times 32\mu s + 6.5\mu s/p_{single}$ is the length for correct decoding. Namely, if the packet starts within the envelop any moment from $0 \sim$ numerator, the total error chips is less than 13 (chip distance). The denominator is the total possible starting point.

2.3.2 Architectural Method

From section 2.2, we derived the in-phase and quad-phase signal if the frequency present in two transmitters and one receiver. We build a complete models of uSDR's transceiver. Figure 2.3 shows the received in-phase and quad-phase signal with $\Delta f_2 = 15$ KHz. Thus, the period T of envelops is $\frac{1}{\Delta f_2} = 66.67\mu$s.

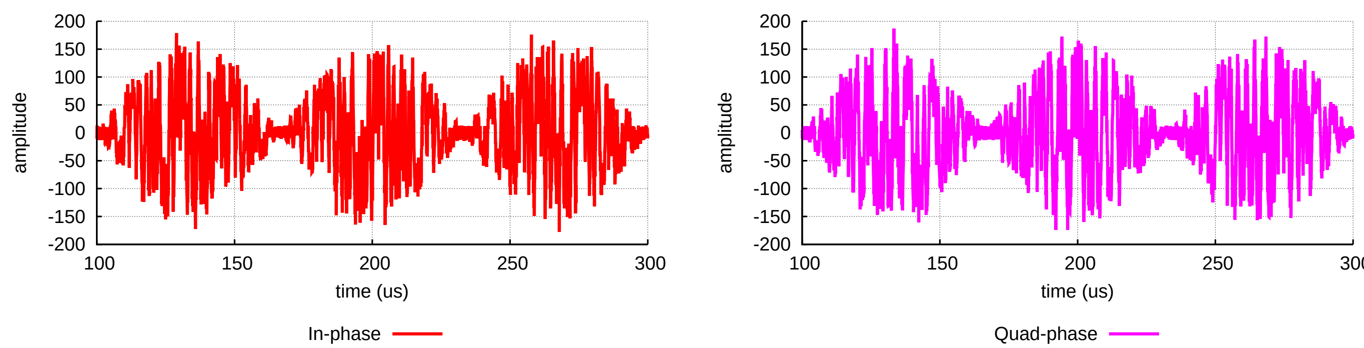


Figure 2.3: This figure shows the received in-phase and quad-phase baseband signals in simulation. The parameters of this simulation are: $\Delta f_1 = 0, \Delta f_2 = 15$ KHz. We can see that the period of envelop $T = \frac{200\mu s}{3} = 66.67\mu s = \frac{1}{15KHz} = \frac{1}{\Delta f_2}$

In the architectural based simulation, we add Gaussian noise to the signal and setting the signal to noise ration (SNR) to 15dB. In addition to that, we set a threshold, which similar to α. If the amplitude less than α, instead of original signal, we randomly generate the noise bounded by α. From figure 2.2(b), we can see trend of PER from architectural simulation is similar to the theoretical method. However, in architectural simulation, the PER never goes to 0 even the Δf is large enough. The resaon is the AWGN present and the random noise we added. The random noise wee add not only exist on low-frequency envelop but the entire signal. Namely, every chip in the sequence is affected by the noise. Thus, the architectural method has less "effective" chip distance.