Algoritmo de Deutsch-JOZSA

Luis Daniel Benavides Navarro, 27-01-2020

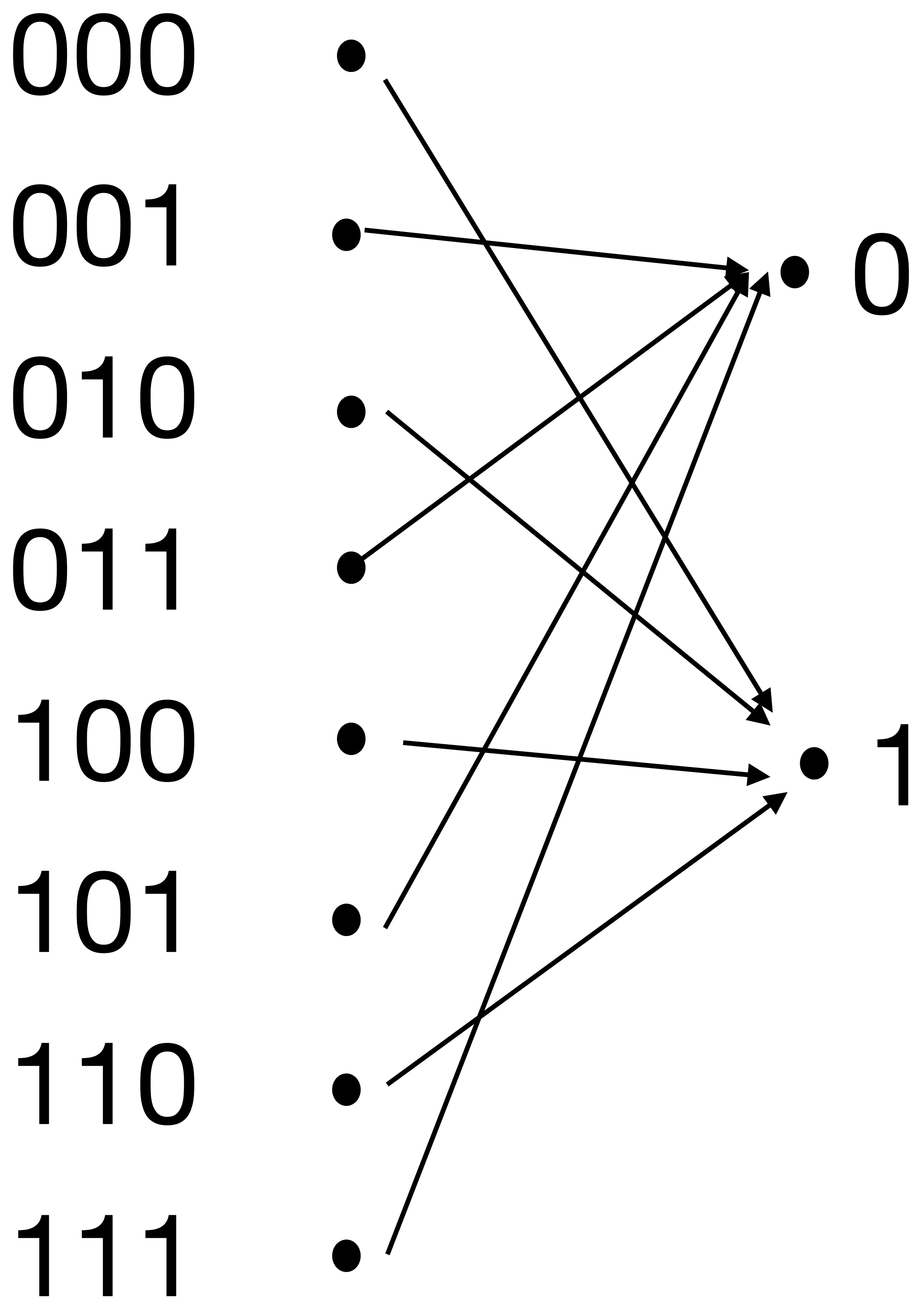
Estrategia General de los Algoritmos cuánticos

- Iniciar los qubits en un estado clásico.
- Poner los qubits en superposición
- Realizar las operaciones en los qubits.
- Realizar una observación

Problema que deseamos resolver

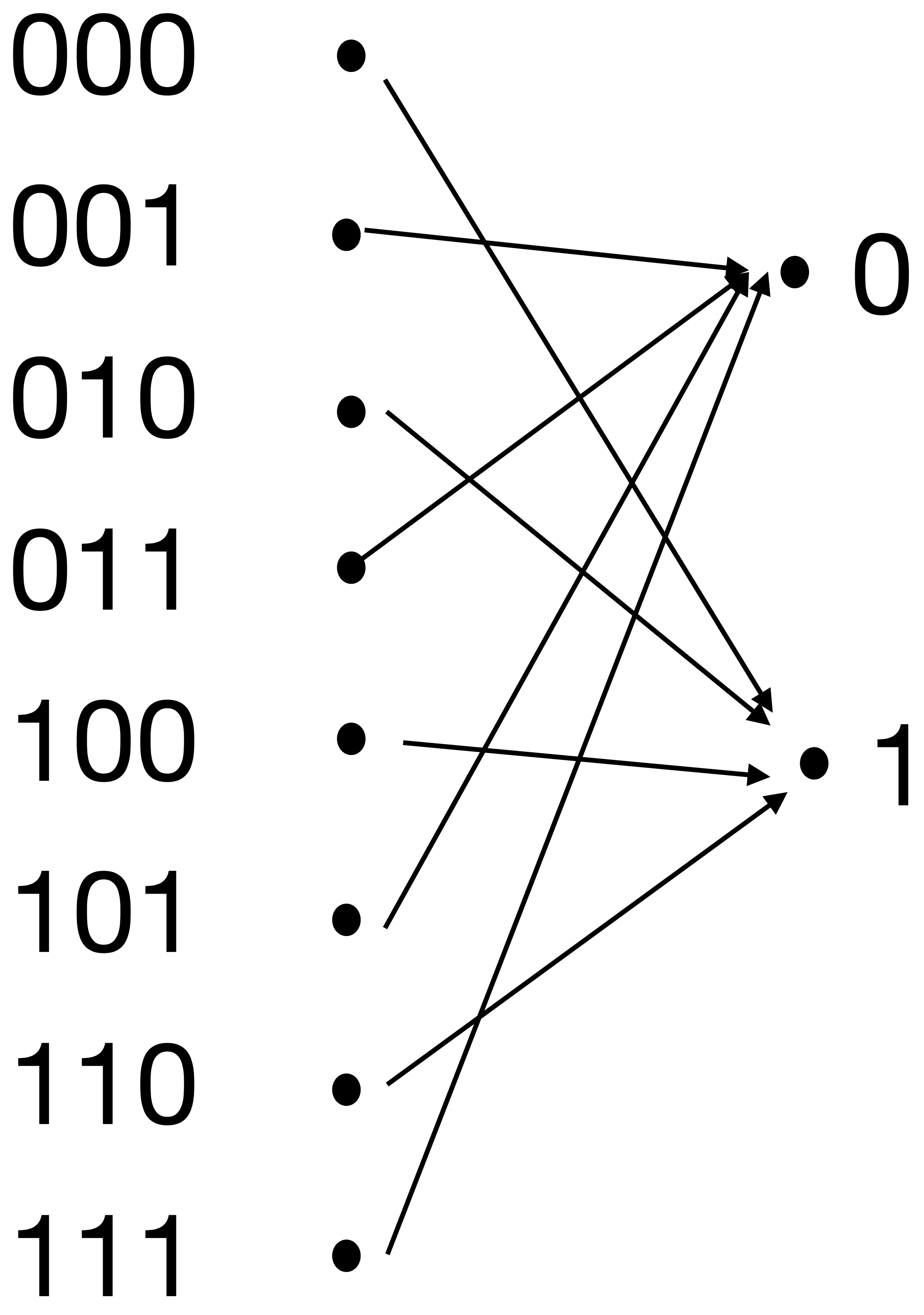
- Imagine que le dan una caja negra que implementa una función de $\{0,1\}^n$ hacia {0,1}.
- Es decir, $f : \{0,1\}^n \to \{0,1\}$
- Ahora el problema es determinar si la función es balanceada o constante si:
  - Le aseguran que siempre le dan o una función balanceada o una constante, nunca le dan otra de otro tipo.
  - Balanceada si exactamente la mitad de las entradas van a 0 y la otra mitad a 1
  - Constante si todas las entradas van a 0 o todas van a 1

¿Cuántas funciones hay?



Ejercicio ¿Cuántas funciones hay de $\{0,1\}^n$ hacia {0,1}?, ¿cuántas balanceadas?, ¿cuantas constantes?.

¿Cuántas funciones hay?

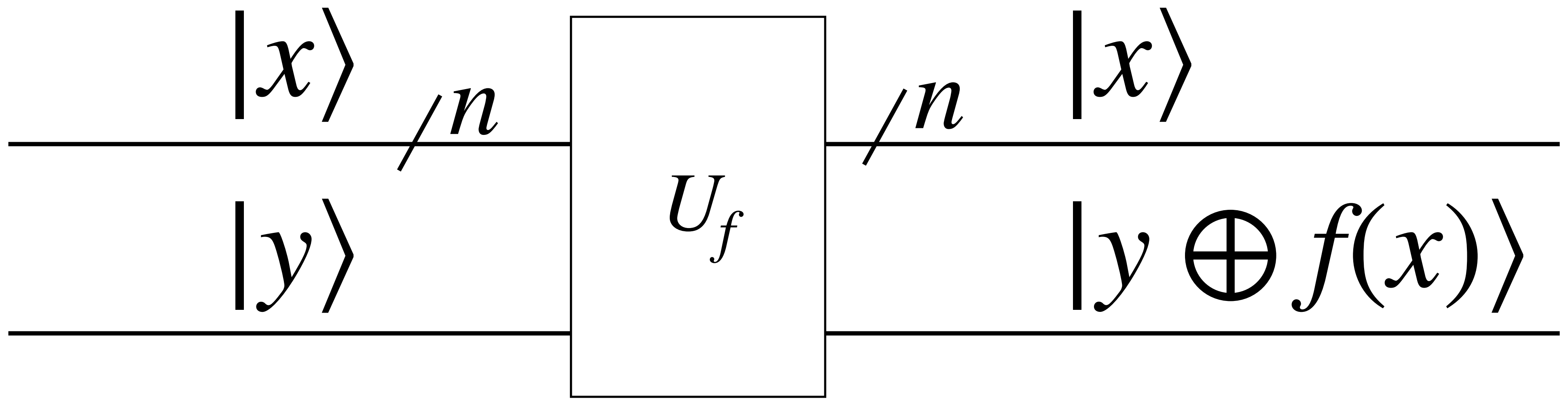


Ejercicio ¿Cuántas funciones hay de $\{0,1\}^n$ hacia {0,1}?, ¿cuántas balanceadas?, ¿cuantas constantes?.

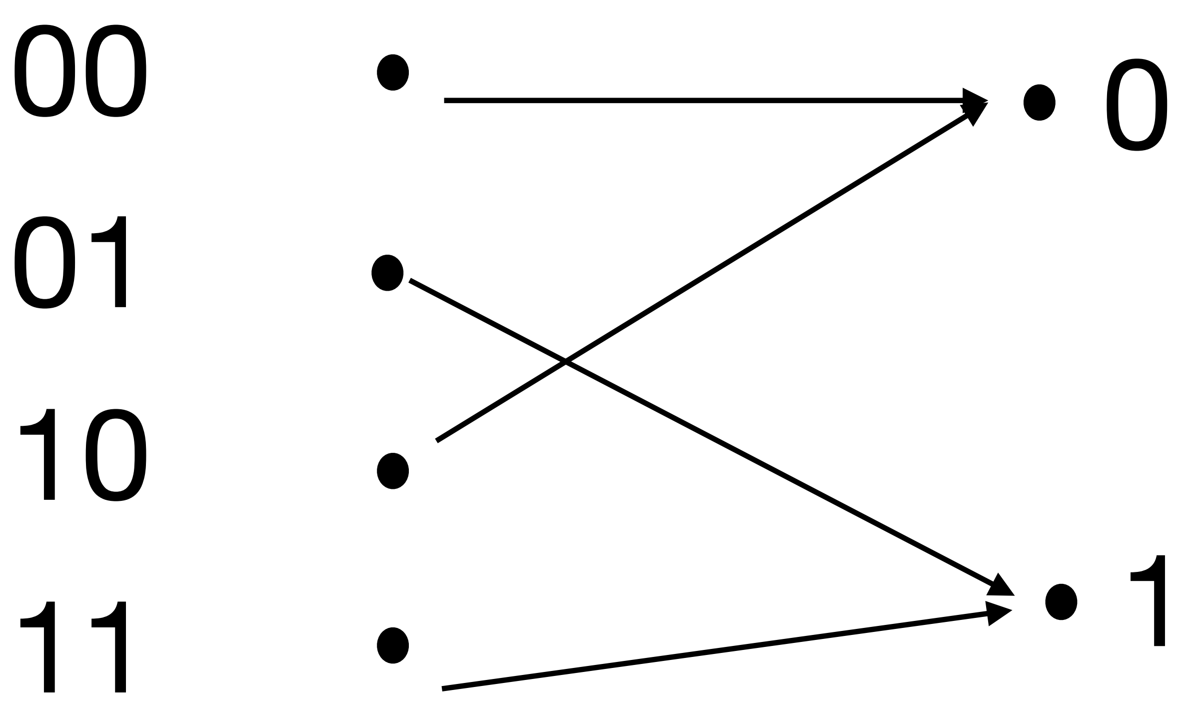
Número de funciones: 2^{2^n}

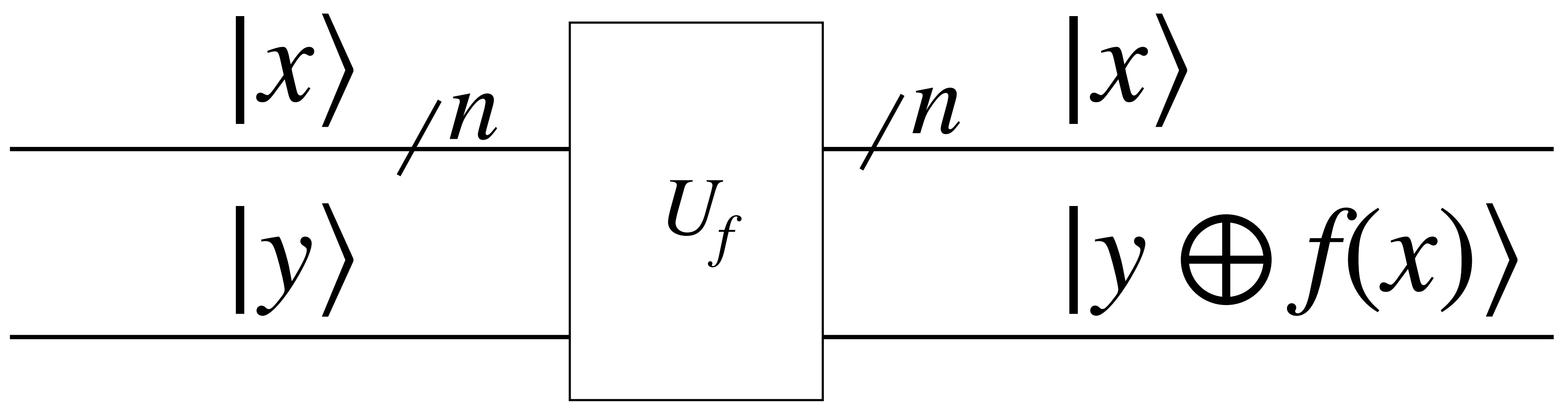
Número de funciones balanceadas: $\binom{2^n}{2^{(n-1)}}$

¿Podemos representar el problema con compuertas y matrices?



¿Podemos representar el problema con compuertas y matrices?

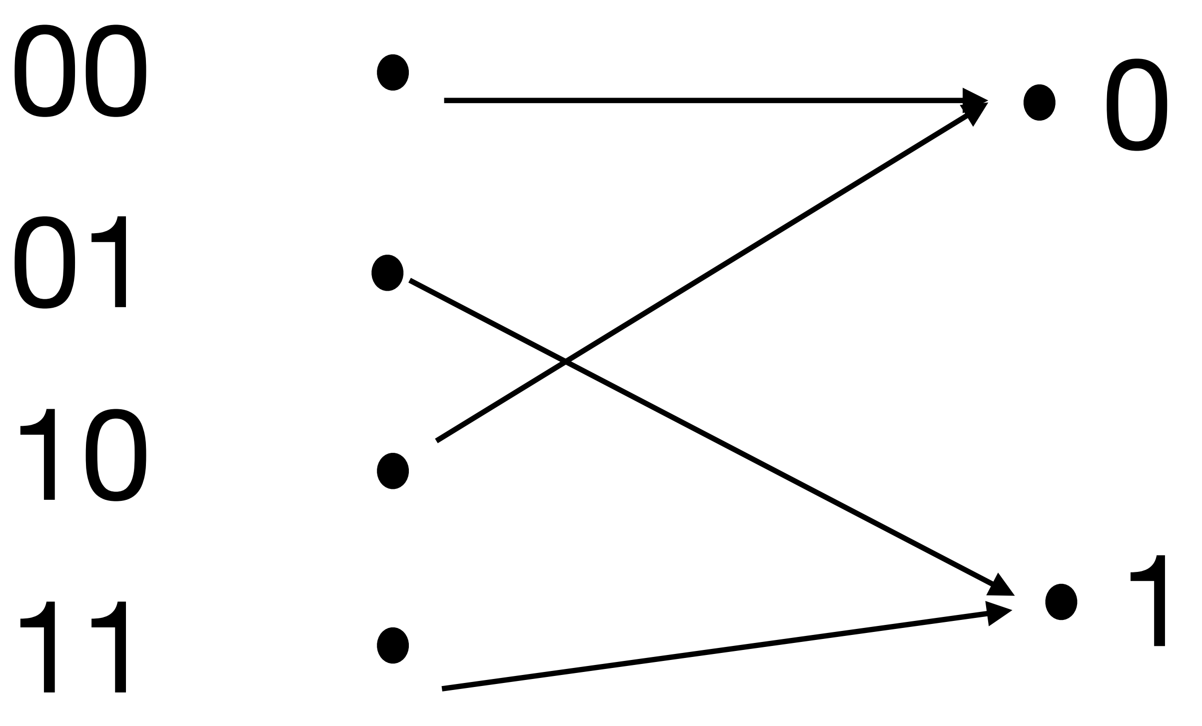


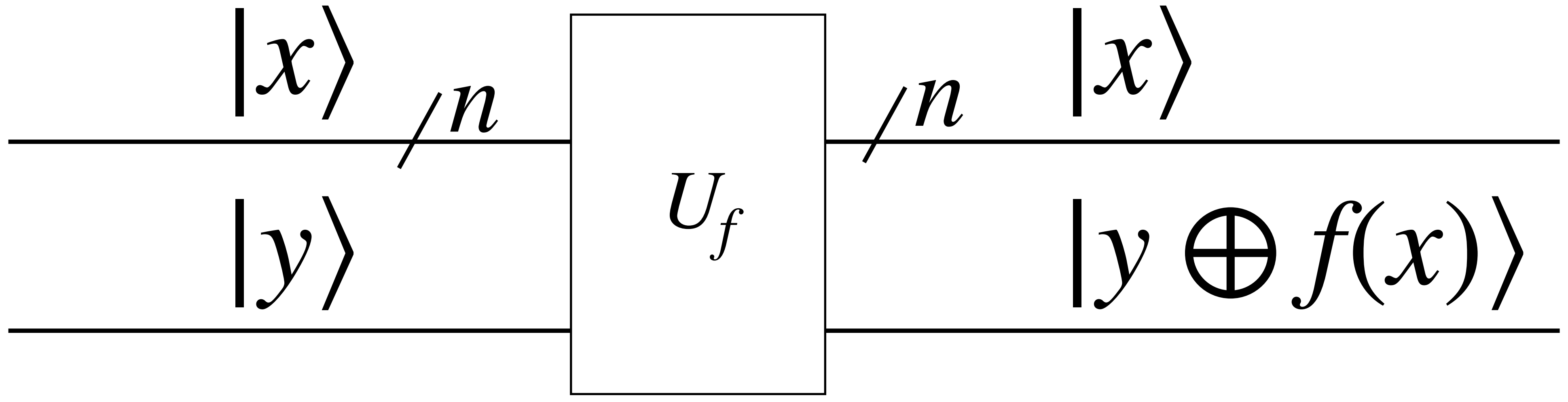


	000	001	010	011	100	101	110	111
000								
001								
010								
011								
100								
101								
110								
111								

Ejercicio.
Representar
otras funciones.

¿Podemos representar el problema con compuertas y matrices?





	000	**001**	**010**	**011**	**100**	**101**	**110**	**111**
000	1							
001		1						
010				1				
011			1					
100					1			
101						1		
110								1
111							1	

Ejercicio. Representar algunas otras funciones.

Vamos a resolverlo en computador clásico

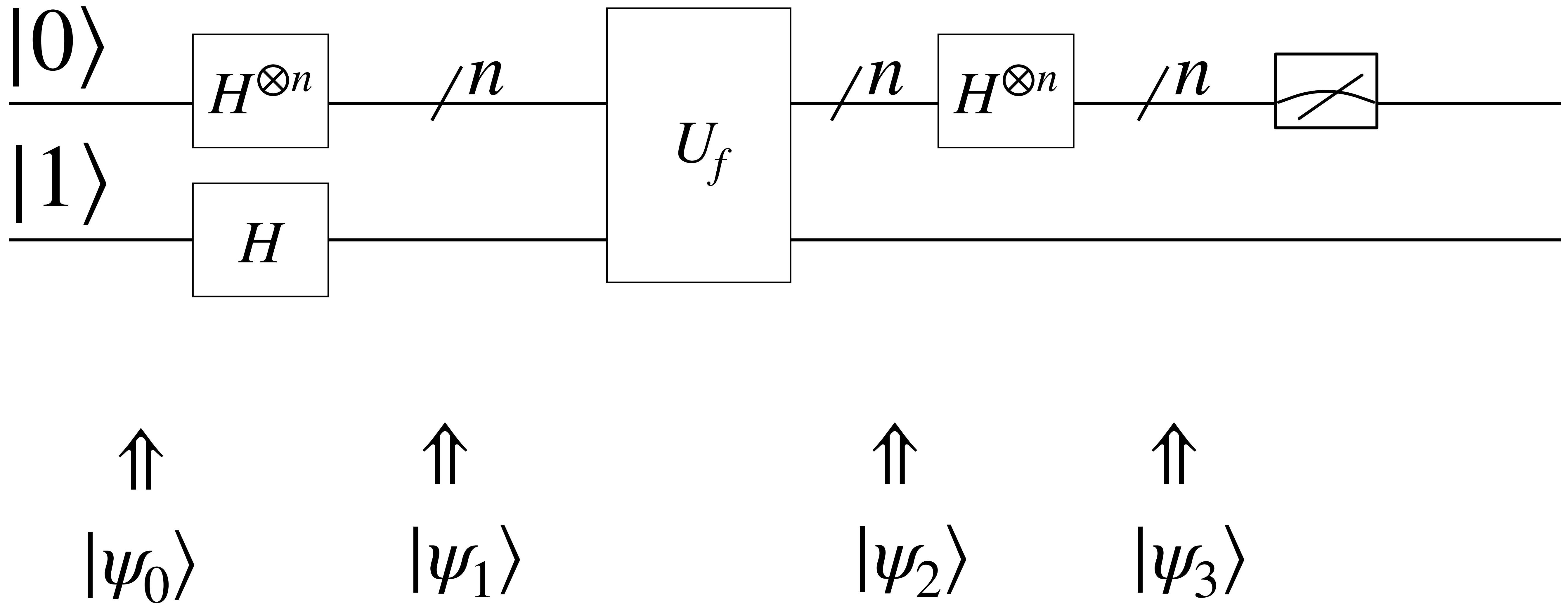
- Si sabemos que la función es balanceada o constante
- Debemos evaluar la función indiferentes puntos para determinar que función es
  - Mejor caso: los dos primeros casos son diferentes ya se que es balanceada
  - Peor caso: Para saber que es constante debe evaluarla en la mitad más uno de los casos.

Algoritmo en computador clásico

```
//Precondición: la función f es constante o balanceada, no puede ser de otro tipo, sino el algoritmo
falla.

//El dominio de la función es {0,1}^n, y n es el segundo parámetro y n >0.

//Poscondición: El algoritmo retorna verdadero si es balanceada y falso si es constante

boolean esBalanceada(function f, n){

   Integer numMáximoIteraciones = 2^(n-1) + 1; //La mitad de elementos más uno

   Arreglo<bit[n]> dominio = new ArregloOrdenadoConElementosBinariosDeltamañoDelParámetro<>(n);

   Int val inicial= f(dominio[0]);

   for(int i = 1; i <= numMáximoIteraciones; i++){

      If (inicial != f(dominio[i])){

         return true; // sale del ciclo ya sabe que es balanceada

      }

   }

   return false; // es función constante

}
```

¿Podemos hacerlo mejor con un sistema cuántico?

El algoritmo



$$|\psi_3\rangle = \begin{cases} |0\rangle \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] & \text{si } f(x) \text{ es constante} \\ |\phi\rangle \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] & \text{, con } \phi \neq |0\rangle \text{ y si } f(x) \text{ es balanceada} \end{cases}$$

Poner en superposición n qubits

Un formula para $H^{\otimes 2}$ basada en la paridad

$$H = 1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Un formula para $H^{\otimes 2}$ basada en la paridad

$H = 1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$H[i,j] = 1/\sqrt{2}(-1)^{i \wedge j}$

Un formula para $H^{\otimes 2}$ basada en la paridad

$$H = 1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H[i,j] = 1/\sqrt{2}(-1)^{i \wedge j}$$

$$H = 1/\sqrt{2}\begin{bmatrix} (-1)^{0\wedge 0} & (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1} \end{bmatrix}$$

Un formula para $H^{\otimes 2}$ basada en la paridad

$$H = 1/\sqrt{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad H[i,j] = 1/\sqrt{2}(-1)^{i\wedge j} \qquad H = 1/\sqrt{2}\begin{bmatrix}(-1)^{0\wedge 0} & (-1)^{0\wedge 1}\\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1}\end{bmatrix}$$

$$H^{\otimes 2} = (H \otimes H) = 1/2\begin{bmatrix}(-1)^{0\wedge 0} * (-1)^{0\wedge 0} & (-1)^{0\wedge 0} * (-1)^{0\wedge 1} & (-1)^{0\wedge 1} * (-1)^{0\wedge 0} & (-1)^{0\wedge 1} * (-1)^{0\wedge 1}\\ (-1)^{0\wedge 0} * (-1)^{1\wedge 0} & (-1)^{0\wedge 0} * (-1)^{1\wedge 1} & (-1)^{0\wedge 1} * (-1)^{1\wedge 0} & (-1)^{0\wedge 1} * (-1)^{1\wedge 1}\\ (-1)^{1\wedge 0} * (-1)^{0\wedge 0} & (-1)^{1\wedge 0} * (-1)^{0\wedge 1} & (-1)^{1\wedge 1} * (-1)^{0\wedge 0} & (-1)^{1\wedge 1} * (-1)^{0\wedge 1}\\ (-1)^{1\wedge 0} * (-1)^{1\wedge 0} & (-1)^{1\wedge 0} * (-1)^{1\wedge 1} & (-1)^{1\wedge 1} * (-1)^{1\wedge 0} & (-1)^{1\wedge 1} * (-1)^{1\wedge 1}\end{bmatrix}$$

Un formula para $H^{\otimes 2}$ basada en la paridad

$$H = 1/\sqrt{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad H[i,j] = 1/\sqrt{2}(-1)^{i\wedge j} \qquad H = 1/\sqrt{2}\begin{bmatrix}(-1)^{0\wedge 0} & (-1)^{0\wedge 1}\\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1}\end{bmatrix}$$

$$H^{\otimes 2} = (H \otimes H) = 1/2\begin{bmatrix}(-1)^{0\wedge 0} * (-1)^{0\wedge 0} & (-1)^{0\wedge 0} * (-1)^{0\wedge 1} & (-1)^{0\wedge 1} * (-1)^{0\wedge 0} & (-1)^{0\wedge 1} * (-1)^{0\wedge 1}\\ (-1)^{0\wedge 0} * (-1)^{1\wedge 0} & (-1)^{0\wedge 0} * (-1)^{1\wedge 1} & (-1)^{0\wedge 1} * (-1)^{1\wedge 0} & (-1)^{0\wedge 1} * (-1)^{1\wedge 1}\\ (-1)^{1\wedge 0} * (-1)^{0\wedge 0} & (-1)^{1\wedge 0} * (-1)^{0\wedge 1} & (-1)^{1\wedge 1} * (-1)^{0\wedge 0} & (-1)^{1\wedge 1} * (-1)^{0\wedge 1}\\ (-1)^{1\wedge 0} * (-1)^{1\wedge 0} & (-1)^{1\wedge 0} * (-1)^{1\wedge 1} & (-1)^{1\wedge 1} * (-1)^{1\wedge 0} & (-1)^{1\wedge 1} * (-1)^{1\wedge 1}\end{bmatrix}$$

Al multiplicar $(-1)^x$ por $(-1)^y$ estamos interesados en la paridad y no en $(-1)^{x+y}$.
Es decir si son iguales o no. Si son iguales son siempre 1 y si son diferentes son -1.
Entonces,

Un formula para $H^{\otimes 2}$ basada en la paridad

$$H = 1/\sqrt{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad H[i,j] = 1/\sqrt{2}(-1)^{i\wedge j} \qquad H = 1/\sqrt{2}\begin{bmatrix}(-1)^{0\wedge 0} & (-1)^{0\wedge 1}\\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1}\end{bmatrix}$$

$$H^{\otimes 2} = (H \otimes X) = 1/2\begin{bmatrix}(-1)^{0\wedge 0}*(-1)^{0\wedge 0} & (-1)^{0\wedge 0}*(-1)^{0\wedge 1} & (-1)^{0\wedge 1}*(-1)^{0\wedge 0} & (-1)^{0\wedge 1}*(-1)^{0\wedge 1}\\ (-1)^{0\wedge 0}*(-1)^{1\wedge 0} & (-1)^{0\wedge 0}*(-1)^{1\wedge 1} & (-1)^{0\wedge 1}*(-1)^{1\wedge 0} & (-1)^{0\wedge 1}*(-1)^{1\wedge 1}\\ (-1)^{1\wedge 0}*(-1)^{0\wedge 0} & (-1)^{1\wedge 0}*(-1)^{0\wedge 1} & (-1)^{1\wedge 1}*(-1)^{0\wedge 0} & (-1)^{1\wedge 1}*(-1)^{0\wedge 1}\\ (-1)^{1\wedge 0}*(-1)^{1\wedge 0} & (-1)^{1\wedge 0}*(-1)^{1\wedge 1} & (-1)^{1\wedge 1}*(-1)^{1\wedge 0} & (-1)^{1\wedge 1}*(-1)^{1\wedge 1}\end{bmatrix}$$

Al multiplicar $(-1)^x$ por $(-1)^y$ estamos interesados en la paridad y no en $(-1)^{x+y}$. Es decir si son iguales o no. Si son iguales son siempre 1 y si son diferentes son -1. Entonces,

$$(-1)^x * (-1)^y = (-1)^{x\oplus y}$$

Un formula para $H^{\otimes 2}$ basada en la paridad II

$(-1)^x * (-1)^y = (-1)^{x \oplus y}$

Un formula para $H^{\otimes 2}$ basada en la paridad II

$$(-1)^x * (-1)^y = (-1)^{x \oplus y}$$

$$H^{\otimes 2} = (H \otimes H) = 1/2 \begin{bmatrix} (-1)^{0\wedge 0} * (-1)^{0\wedge 0} & (-1)^{0\wedge 0} * (-1)^{0\wedge 1} & (-1)^{0\wedge 1} * (-1)^{0\wedge 0} & (-1)^{0\wedge 1} * (-1)^{0\wedge 1} \\ (-1)^{0\wedge 0} * (-1)^{1\wedge 0} & (-1)^{0\wedge 0} * (-1)^{1\wedge 1} & (-1)^{0\wedge 1} * (-1)^{1\wedge 0} & (-1)^{0\wedge 1} * (-1)^{1\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{0\wedge 0} & (-1)^{1\wedge 0} * (-1)^{0\wedge 1} & (-1)^{1\wedge 1} * (-1)^{0\wedge 0} & (-1)^{1\wedge 1} * (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{1\wedge 0} & (-1)^{1\wedge 0} * (-1)^{1\wedge 1} & (-1)^{1\wedge 1} * (-1)^{1\wedge 0} & (-1)^{1\wedge 1} * (-1)^{1\wedge 1} \end{bmatrix}$$

Un formula para $H^{\otimes 2}$ basada en la paridad II

$$(-1)^x * (-1)^y = (-1)^{x \oplus y}$$

$$H^{\otimes 2} = (H \otimes H) = 1/2 \begin{bmatrix} (-1)^{0\wedge 0} * (-1)^{0\wedge 0} & (-1)^{0\wedge 0} * (-1)^{0\wedge 1} & (-1)^{0\wedge 1} * (-1)^{0\wedge 0} & (-1)^{0\wedge 1} * (-1)^{0\wedge 1} \\ (-1)^{0\wedge 0} * (-1)^{1\wedge 0} & (-1)^{0\wedge 0} * (-1)^{1\wedge 1} & (-1)^{0\wedge 1} * (-1)^{1\wedge 0} & (-1)^{0\wedge 1} * (-1)^{1\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{0\wedge 0} & (-1)^{1\wedge 0} * (-1)^{0\wedge 1} & (-1)^{1\wedge 1} * (-1)^{0\wedge 0} & (-1)^{1\wedge 1} * (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{1\wedge 0} & (-1)^{1\wedge 0} * (-1)^{1\wedge 1} & (-1)^{1\wedge 1} * (-1)^{1\wedge 0} & (-1)^{1\wedge 1} * (-1)^{1\wedge 1} \end{bmatrix}$$

$$H^{\otimes 2} = (H \otimes H) = 1/2 \begin{bmatrix} (-1)^{0\wedge 0 \oplus 0\wedge 0} & (-1)^{0\wedge 0 \oplus 0\wedge 1} & (-1)^{0\wedge 1 \oplus 0\wedge 0} & (-1)^{0\wedge 1 \oplus 0\wedge 1} \\ (-1)^{0\wedge 0 \oplus 1\wedge 0} & (-1)^{0\wedge 0 \oplus 1\wedge 1} & (-1)^{0\wedge 1 \oplus 1\wedge 0} & (-1)^{0\wedge 1 \oplus 1\wedge 1} \\ (-1)^{1\wedge 0 \oplus 0\wedge 0} & (-1)^{1\wedge 0 \oplus 0\wedge 1} & (-1)^{1\wedge 1 \oplus 0\wedge 0} & (-1)^{1\wedge 1 \oplus 0\wedge 1} \\ (-1)^{1\wedge 0 \oplus 1\wedge 0} & (-1)^{1\wedge 0 \oplus 1\wedge 1} & (-1)^{1\wedge 1 \oplus 1\wedge 0} & (-1)^{1\wedge 1 \oplus 1\wedge 1} \end{bmatrix} = 1/2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Un formula para $H^{\otimes 3}$ basada en la paridad

$$H^{\otimes 2} = 1/2 \begin{bmatrix} (-1)^{0\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 1} \\ (-1)^{0\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 1} \\ (-1)^{1\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 1} \\ (-1)^{1\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 1} \end{bmatrix}$$

$$H = 1/\sqrt{2} \begin{bmatrix} (-1)^{0\wedge 0} & (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1} \end{bmatrix}$$

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}} \begin{bmatrix} (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 0\wedge 1} \\ (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 1\wedge 1} \\ (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 0\wedge 1} \\ (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 1\wedge 1} \\ (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 0\wedge 1} \\ (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 1\wedge 1} \\ (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 0\wedge 1} \\ (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 1\wedge 1} \end{bmatrix}$$

Un formula para $H^{\otimes 3}$ basada en la paridad II

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}} \begin{bmatrix}
(-1)^{0\wedge 0\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 0\wedge 1} \\
(-1)^{0\wedge 0\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 0\wedge 1\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 0\wedge 1\oplus 1\wedge 1} \\
(-1)^{0\wedge 0\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 0\wedge 1} \\
(-1)^{0\wedge 0\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 0\oplus 1\wedge 1\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{0\wedge 1\oplus 1\wedge 1\oplus 1\wedge 1} \\
(-1)^{1\wedge 0\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 0\wedge 1} \\
(-1)^{1\wedge 0\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 0\wedge 1\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 0\wedge 1\oplus 1\wedge 1} \\
(-1)^{1\wedge 0\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 0\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 0\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 0\wedge 1} \\
(-1)^{1\wedge 0\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 0\oplus 1\wedge 1\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 0\oplus 1\wedge 1} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 1\wedge 0} & (-1)^{1\wedge 1\oplus 1\wedge 1\oplus 1\wedge 1}
\end{bmatrix}$$

Sea $x = x_0x_1x_2x_3\ldots x_{n-1}$, $y = y_0y_1y_2y_3\ldots y_{n-1}$ dos cadenas de la misma longitud y

$$\langle , \rangle : \{0,1\}^n \times \{0,1\}^n \to \{0,1\} : \langle x, y\rangle = x_0 \wedge y_0 \oplus x_1 \wedge y_1 \oplus \ldots \oplus x_{n-1} \wedge y_{n-1}$$

Esta función determina la paridad de los 1 que coinciden, es decir si coinciden un número par de veces o impar de veces. Cuando el número de coincidencias es par, el exponente es 0, y cuando eñ número de coincidencias es impar, el exponente es 1.

Un formula para $H^{\otimes 3}$ basada en la paridad III

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}} \begin{bmatrix} (-1)^{\langle 000,000\rangle} & (-1)^{\langle 000,001\rangle} & (-1)^{\langle 000,010\rangle} & (-1)^{\langle 000,011\rangle} & (-1)^{\langle 000,100\rangle} & (-1)^{\langle 000,101\rangle} & (-1)^{\langle 000,110\rangle} & (-1)^{\langle 000,111\rangle} \\ (-1)^{\langle 001,000\rangle} & (-1)^{\langle 001,001\rangle} & (-1)^{\langle 001,010\rangle} & (-1)^{\langle 001,011\rangle} & (-1)^{\langle 001,100\rangle} & (-1)^{\langle 001,101\rangle} & (-1)^{\langle 001,110\rangle} & (-1)^{\langle 001,111\rangle} \\ (-1)^{\langle 010,000\rangle} & (-1)^{\langle 010,001\rangle} & (-1)^{\langle 010,010\rangle} & (-1)^{\langle 010,011\rangle} & (-1)^{\langle 010,100\rangle} & (-1)^{\langle 010,101\rangle} & (-1)^{\langle 010,110\rangle} & (-1)^{\langle 010,111\rangle} \\ (-1)^{\langle 011,000\rangle} & (-1)^{\langle 011,001\rangle} & (-1)^{\langle 011,010\rangle} & (-1)^{\langle 011,011\rangle} & (-1)^{\langle 011,100\rangle} & (-1)^{\langle 011,101\rangle} & (-1)^{\langle 011,110\rangle} & (-1)^{\langle 011,111\rangle} \\ (-1)^{\langle 100,000\rangle} & (-1)^{\langle 100,001\rangle} & (-1)^{\langle 100,010\rangle} & (-1)^{\langle 100,011\rangle} & (-1)^{\langle 100,100\rangle} & (-1)^{\langle 100,101\rangle} & (-1)^{\langle 100,110\rangle} & (-1)^{\langle 100,111\rangle} \\ (-1)^{\langle 101,000\rangle} & (-1)^{\langle 101,001\rangle} & (-1)^{\langle 101,010\rangle} & (-1)^{\langle 101,011\rangle} & (-1)^{\langle 101,100\rangle} & (-1)^{\langle 101,101\rangle} & (-1)^{\langle 101,110\rangle} & (-1)^{\langle 101,111\rangle} \\ (-1)^{\langle 110,000\rangle} & (-1)^{\langle 110,001\rangle} & (-1)^{\langle 110,010\rangle} & (-1)^{\langle 110,011\rangle} & (-1)^{\langle 110,100\rangle} & (-1)^{\langle 110,101\rangle} & (-1)^{\langle 110,110\rangle} & (-1)^{\langle 110,111\rangle} \\ (-1)^{\langle 111,000\rangle} & (-1)^{\langle 111,001\rangle} & (-1)^{\langle 111,010\rangle} & (-1)^{\langle 111,011\rangle} & (-1)^{\langle 111,100\rangle} & (-1)^{\langle 111,101\rangle} & (-1)^{\langle 111,110\rangle} & (-1)^{\langle 111,111\rangle} \end{bmatrix}$$

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Un formula para $H^{\otimes n}$ basada en la paridad

$$H^{\otimes n}[i,j] = \frac{1}{\sqrt{2}^n}(-1)^{\langle i,j \rangle}$$

¿Qué pasa cuando multiplico por un vector?

$$H^{\otimes n}[i,j] = \frac{1}{\sqrt{2}^n}(-1)^{\langle i,j \rangle}$$

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$H^{\otimes n}|\mathbf{0}\rangle = H^{\otimes n} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = H^{\otimes n}[-,0] = \frac{1}{\sqrt{2}^n} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}^n}(|0000...00\rangle + |0000...01\rangle + |0000...10\rangle + |0000...11\rangle + \ldots |1111...11\rangle)$$

¿Qué pasa cuando multiplico por un vector?II

$$H^{\otimes n}[i,j] = \frac{1}{\sqrt{2}^n}(-1)^{\langle i,j \rangle}$$

$$H^{\otimes 3} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Si $y \in \{0,1\}^n$ $|y\rangle$ es un vector columna con un solo 1 en alguna posición. Al multiplicar lo que hago es extraer la columna y de la matriz.

$$H^{\otimes n}|y\rangle = H^{\otimes n}[-,y] = \frac{1}{\sqrt{2}^n}\sum_{x \in \{0,1\}^n}(-1)^{\langle x,y \rangle}|x\rangle$$

Tarea

- El algoritmo completo
- Ejercicios

Recordatorio de propiedades de $\otimes$

$$(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$$

$$(A \star A') \otimes (B \star B') = (A \otimes B) \star (A' \otimes B')$$

$$(A \otimes B) \star (V \otimes V') = (A \star V) \otimes (B \star V')$$

Producto tensor respeta adición en $\mathbb{V}$ y en $\mathbb{V}'$

$$(V_i + V_j) \otimes V'_k = V_i \otimes V'_k +'' V_j \otimes V'_k$$

$$V_i \otimes (V'_j +' V'_k) = V_i \otimes V'_j +'' V_i \otimes V'_k$$

Producto tensor respeta la multiplicación escalar en $\mathbb{V}$ y en $\mathbb{V}'$

$$c \cdot'' (V_j \otimes V'_k) = (c \cdot V_j) \otimes V'_k = V_j \otimes (c \cdot' V'_k)$$

Recordatorio de producto tensor de H

$$H^{\otimes n}[i,j] = \frac{1}{\sqrt{2}^n}(-1)^{\langle i,j \rangle}$$

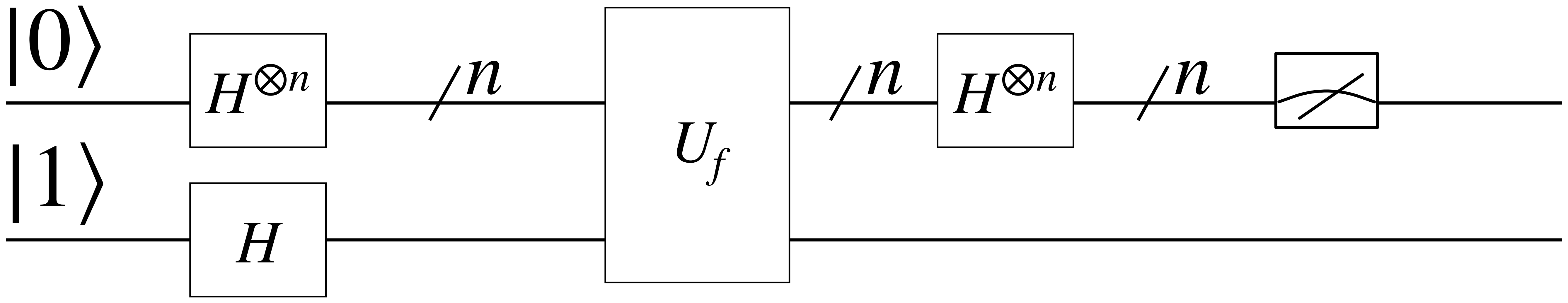
$$H^{\otimes 3} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Si $y \in \{0,1\}^n$ $|y\rangle$ es un vector columna con un solo 1 en alguna posición. Al multiplicar lo que hago es extraer la columna y de la matriz.

$$H^{\otimes n}|y\rangle = H^{\otimes n}[\,-\,, y] = \frac{1}{\sqrt{2}^n}\sum_{x \in \{0,1\}^n}(-1)^{\langle x,y \rangle}|x\rangle$$

El algoritmo de Deutsch-JOZSA

Paso 1: $|\psi_0\rangle$

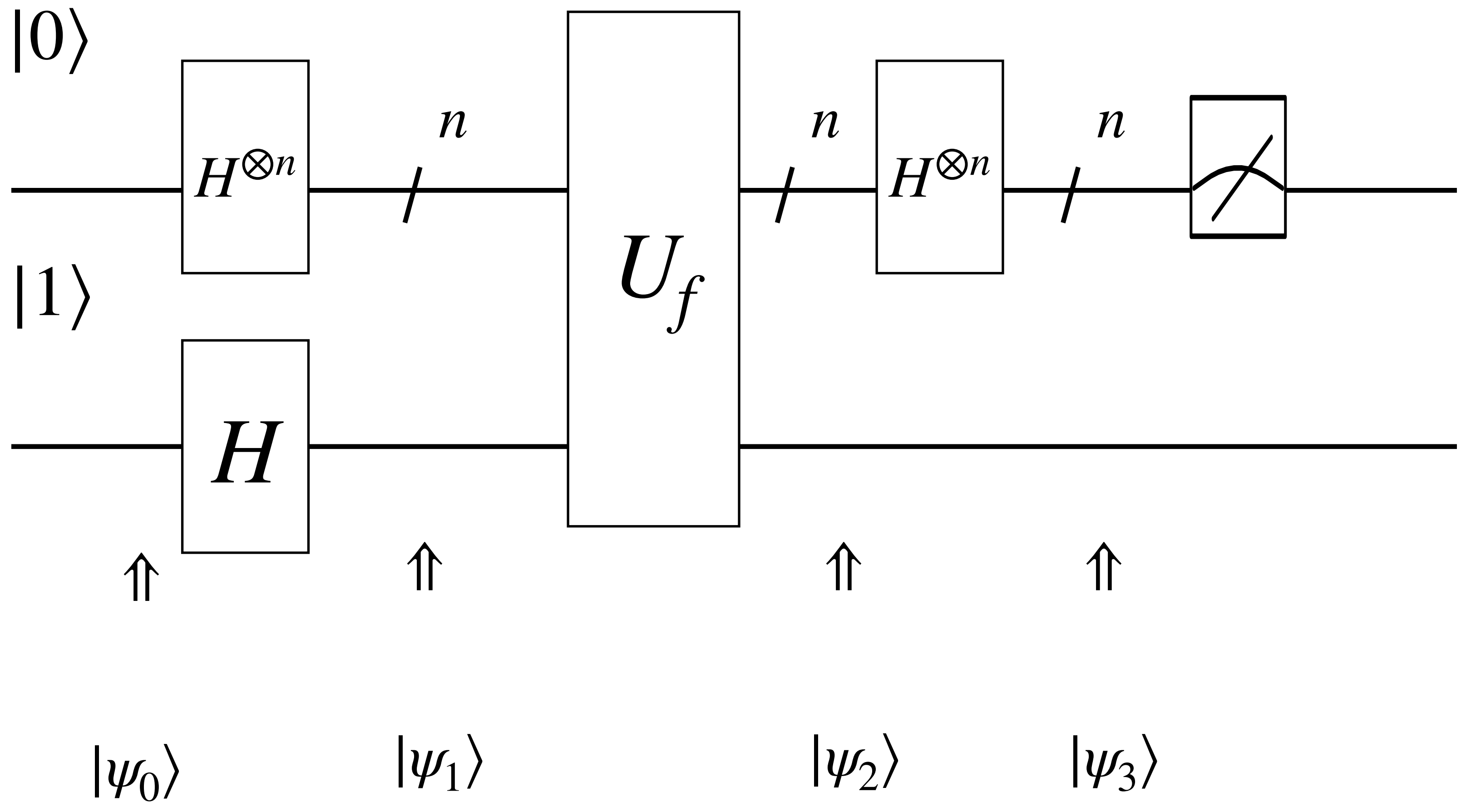


$\Uparrow$ $|\psi_0\rangle$ $\Uparrow$ $|\psi_1\rangle$ $\Uparrow$ $|\psi_2\rangle$ $\Uparrow$ $|\psi_3\rangle$

$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

El algoritmo de Deutsch-JOZSA

Paso 2: $|\psi_1\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

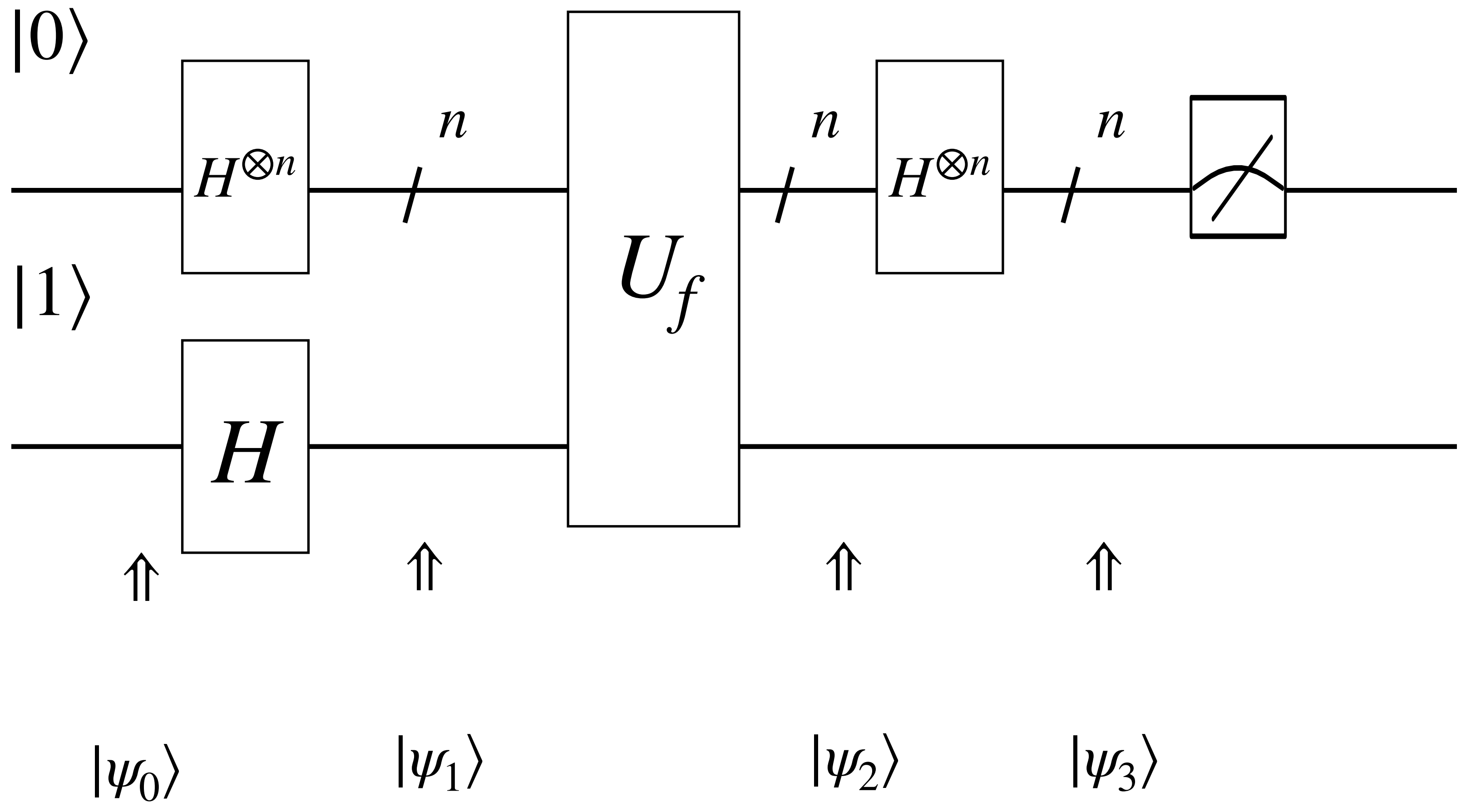
$$\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle = \frac{1}{\sqrt{2}^n}(|0000...000\rangle + |0000...001\rangle + |0000...010\rangle + |0000...011\rangle + |0000...100\rangle + ... + |1111...110\rangle + |1111...111\rangle)$$

El algoritmo de Deutsch-JOZSA

Paso 3: $|\psi_2\rangle$

$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

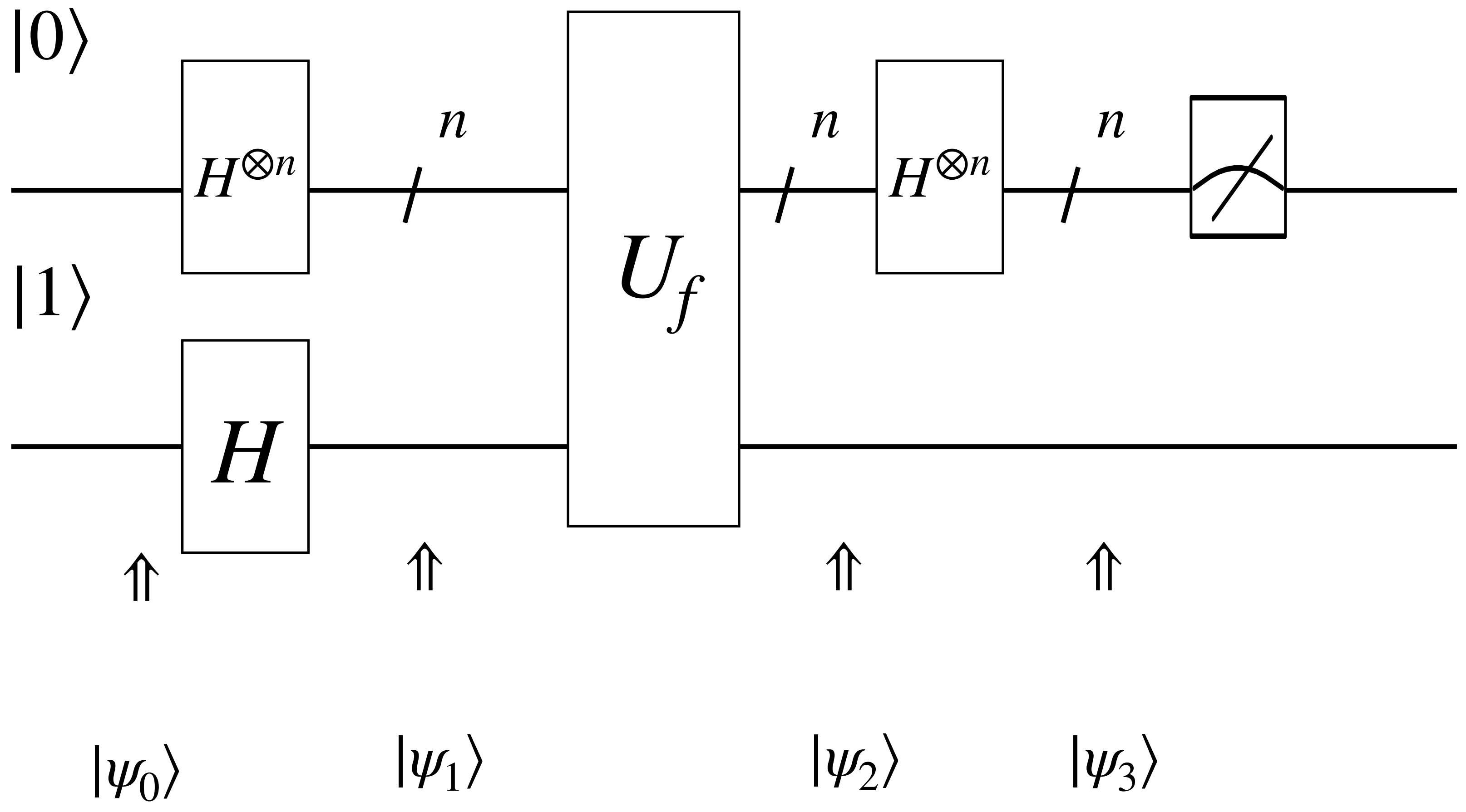


$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|f(\boldsymbol{x}) \oplus 0\rangle - |f(\boldsymbol{x}) \oplus 1\rangle}{\sqrt{2}}]$$

$$= [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|f(\boldsymbol{x})\rangle - |\neg f(\boldsymbol{x})\rangle}{\sqrt{2}}]$$

El algoritmo de Deutsch-JOZSA

Paso 3: $|\psi_2\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

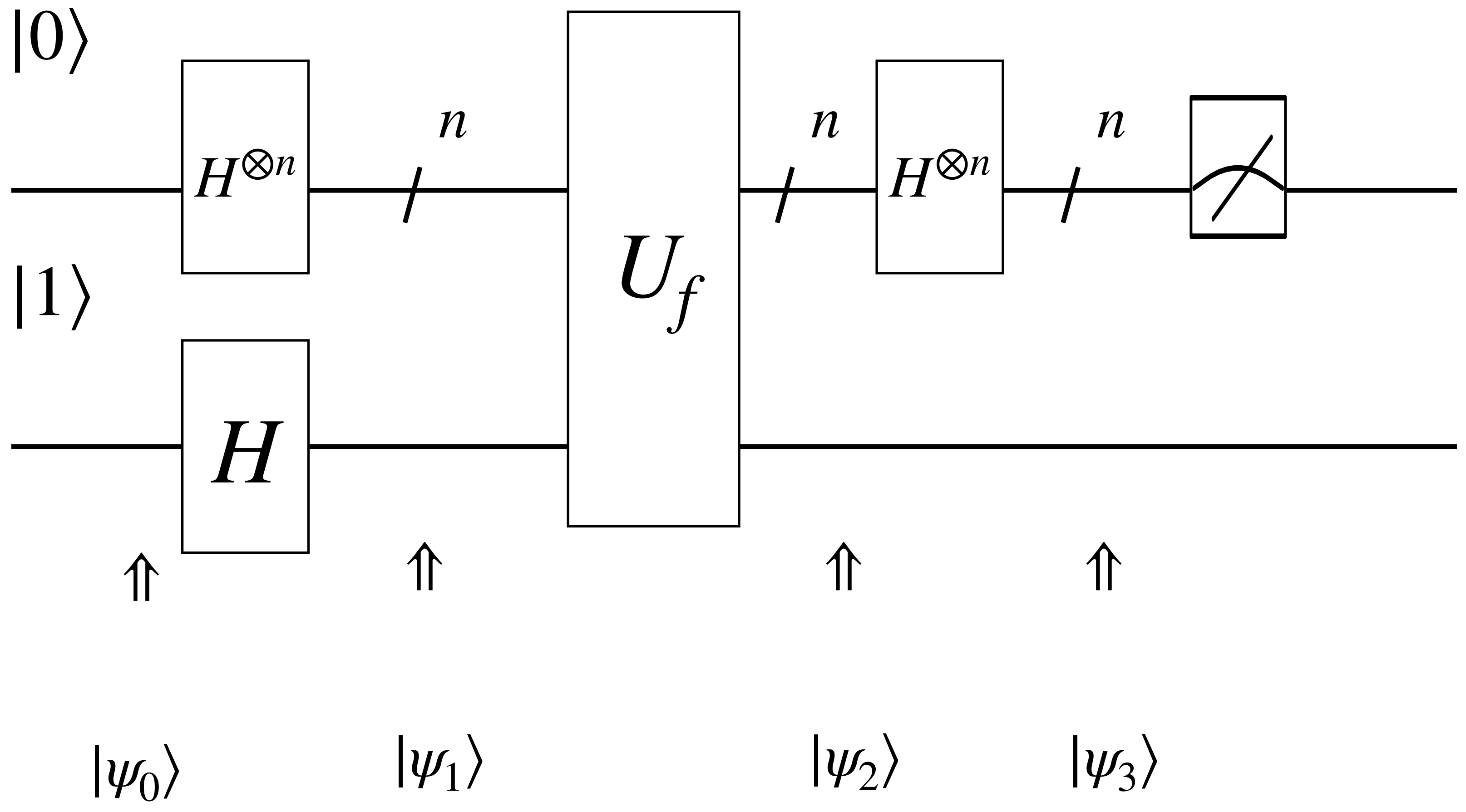
$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|f(\boldsymbol{x})\rangle - |\neg f(\boldsymbol{x})\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = \begin{cases} [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] & \text{if } f(x) = 0 \\ [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|1\rangle - |0\rangle}{\sqrt{2}}] & \text{if } f(x) = 1 \end{cases}$$

$$= (-1)^{f(x)} [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

El algoritmo de Deutsch-JOZSA

Paso 3: $|\psi_2\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

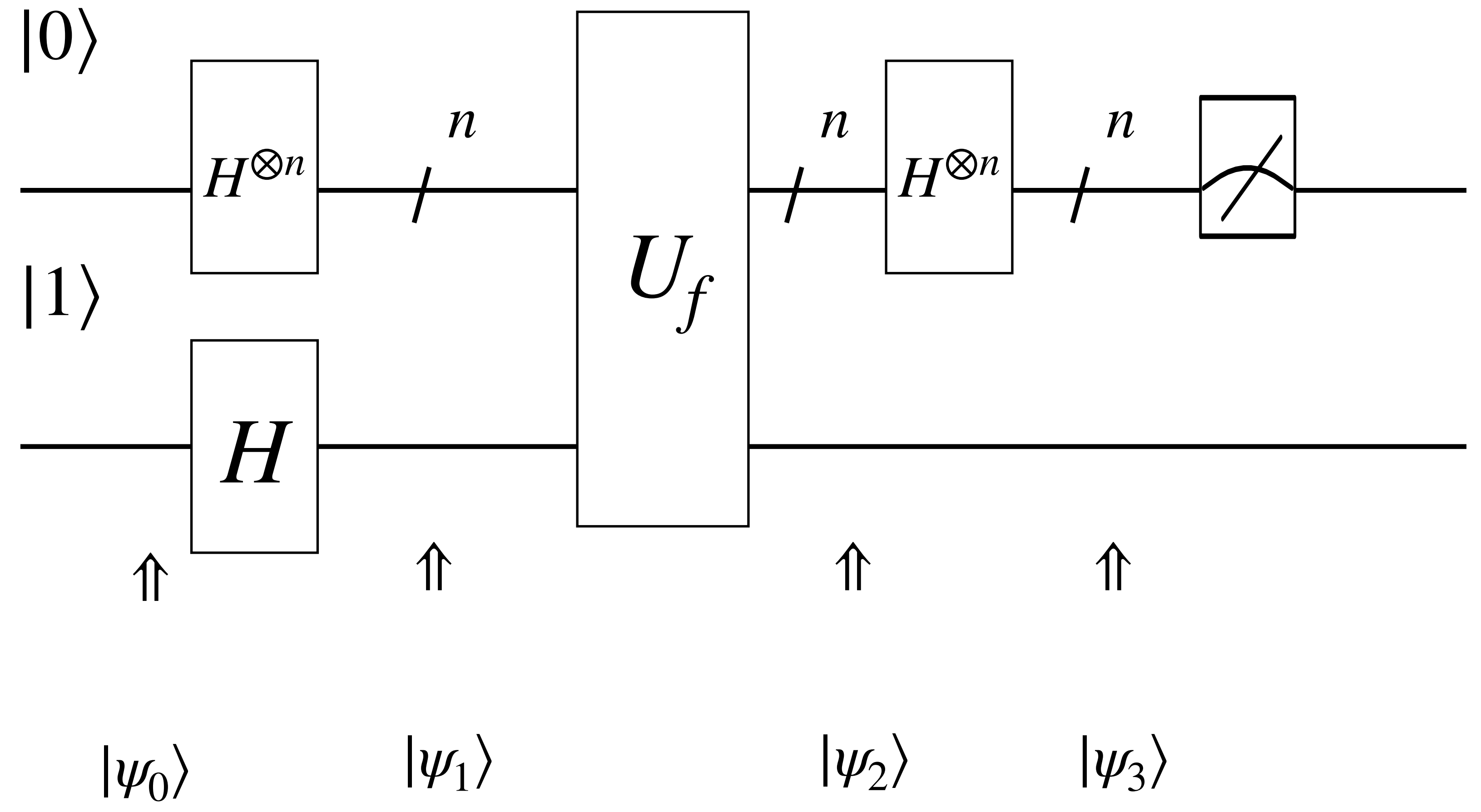
$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = (-1)^{f(x)}[\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$= [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)}|x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

El algoritmo de Deutsch-JOZSA

Paso 4: $|\psi_3\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$\begin{aligned}
|\psi_3\rangle = (H^{\otimes n} \otimes I) * U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) &= [H^{\otimes n} * [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle]] \otimes [I * [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]] \\
&= [H^{\otimes n} * [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle]] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] \\
&= [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} [H^{\otimes n} * |x\rangle]] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] \\
&= [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} [\frac{1}{\sqrt{2}^n} \sum_{z \in \{0,1\}^n} (-1)^{\langle z,x \rangle} |z\rangle]] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]
\end{aligned}$$

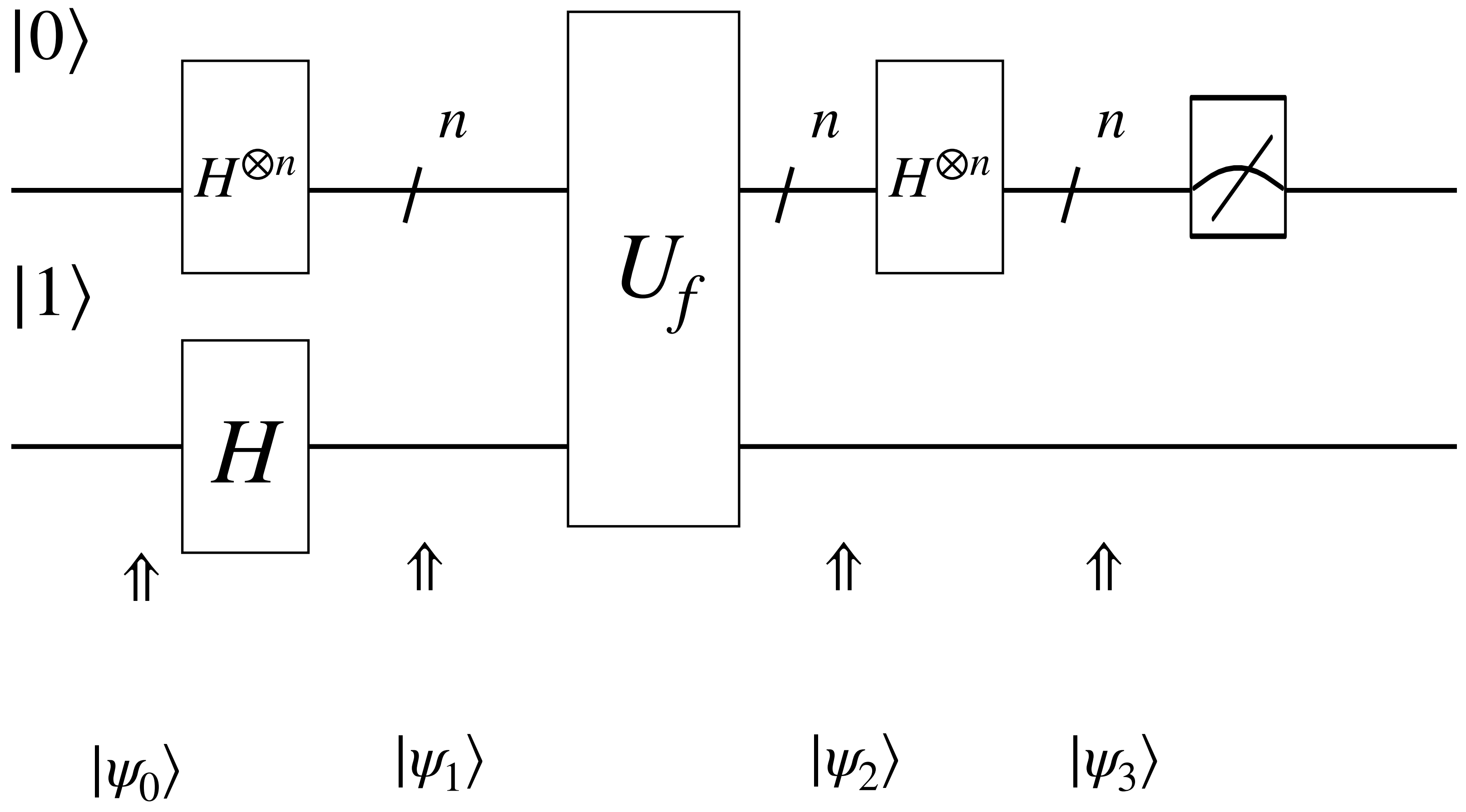
El algoritmo de Deutsch-JOZSA

Paso 4: $|\psi_3\rangle$

$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$



$$|\psi_3\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} [\frac{1}{\sqrt{2}^n} \sum_{z \in \{0,1\}^n} (-1)^{\langle z,x \rangle} |z\rangle]] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_3\rangle = [\frac{1}{2^n} \sum_{x \in \{0,1\}^n} \sum_{z \in \{0,1\}^n} (-1)^{f(x)} (-1)^{\langle z,x \rangle} |z\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_3\rangle = [\frac{1}{2^n} \sum_{x \in \{0,1\}^n} \sum_{z \in \{0,1\}^n} (-1)^{f(x) \oplus \langle z,x \rangle} |z\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

El algoritmo de Deutsch-JOZSA

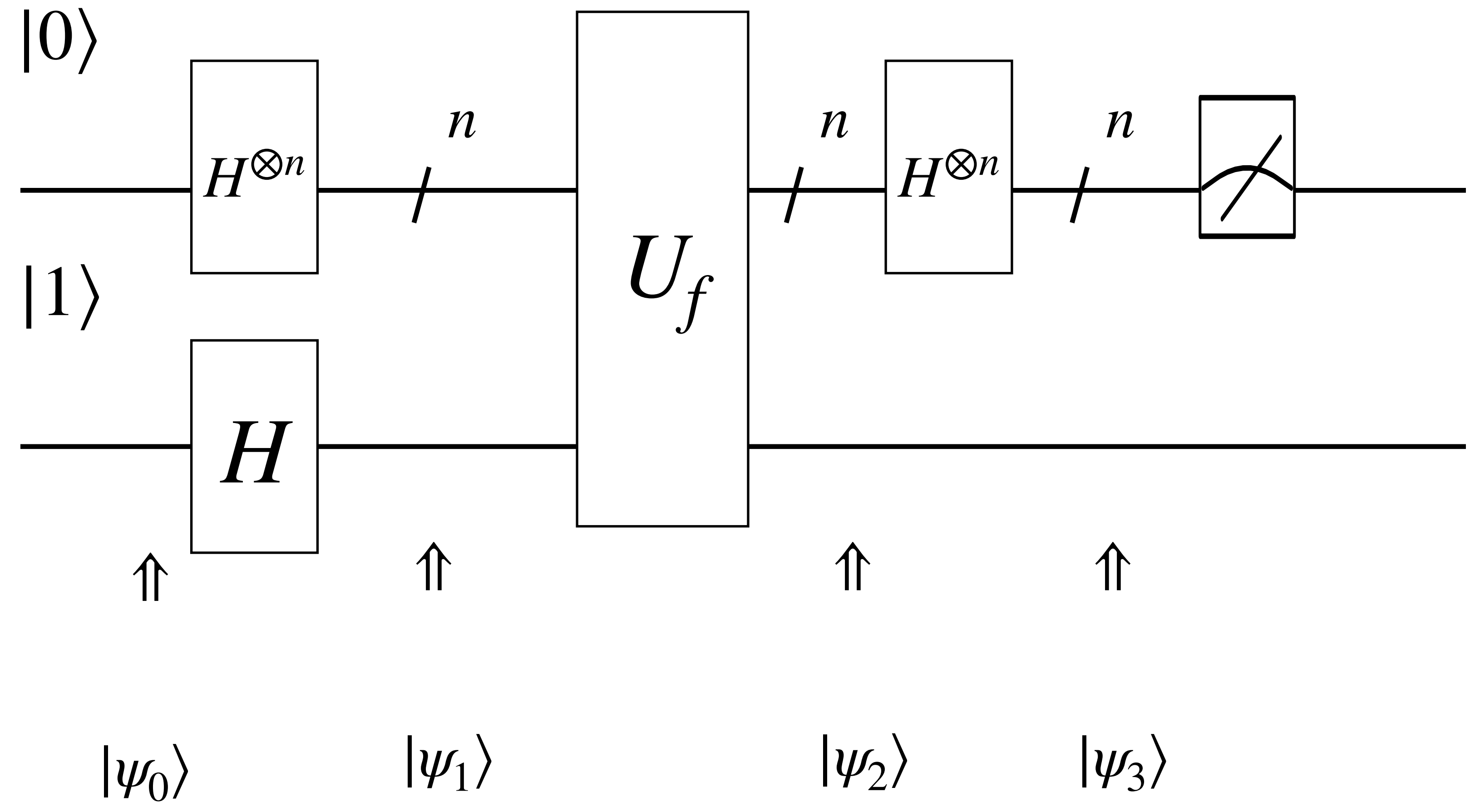
Paso 5: Cuál es la probabilidad de $|\psi_3\rangle$ colapse a $|0\rangle$

$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_3\rangle = [\frac{1}{2^n} \sum_{x \in \{0,1\}^n} \sum_{z \in \{0,1\}^n} (-1)^{f(x) \oplus \langle z,x\rangle} |z\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$



Cuál es la probabilidad de $|\psi_3\rangle$ colapse a $|0\rangle$? Es decir $z = |0\rangle$, entonces $\langle z, x\rangle = \langle 0,x\rangle = 0$

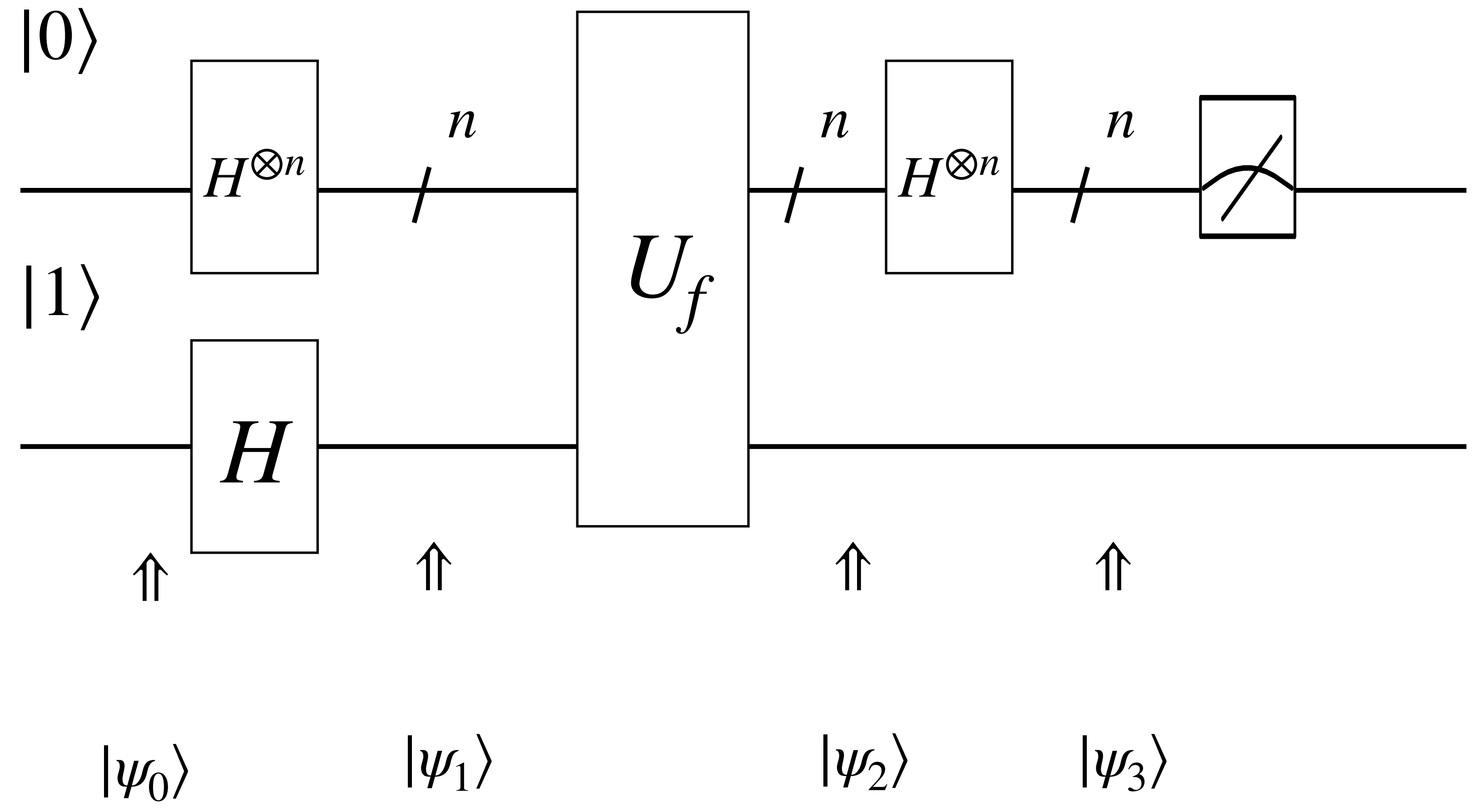
La probabilidad estaría determinada por este término : $[\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |0\rangle]$

Si $f(x) = 1$ **(constante en 1), entonces** $[\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)|0\rangle] = \frac{-(2^n)|0\rangle}{2^n} = -1|0\rangle$

Si $f(x) = 0$ **(constante en 0), entonces** $[\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (1)|0\rangle] = \frac{(2^n)|0\rangle}{2^n} = 1|0\rangle$

El algoritmo de Deutsch-JOZSA

Paso 5: Cuál es la probabilidad de $|\psi_3\rangle$ colapse a $|0\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_3\rangle = [\frac{1}{2^n} \sum_{x \in \{0,1\}^n} \sum_{z \in \{0,1\}^n} (-1)^{f(x) \oplus \langle z,x\rangle} |z\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

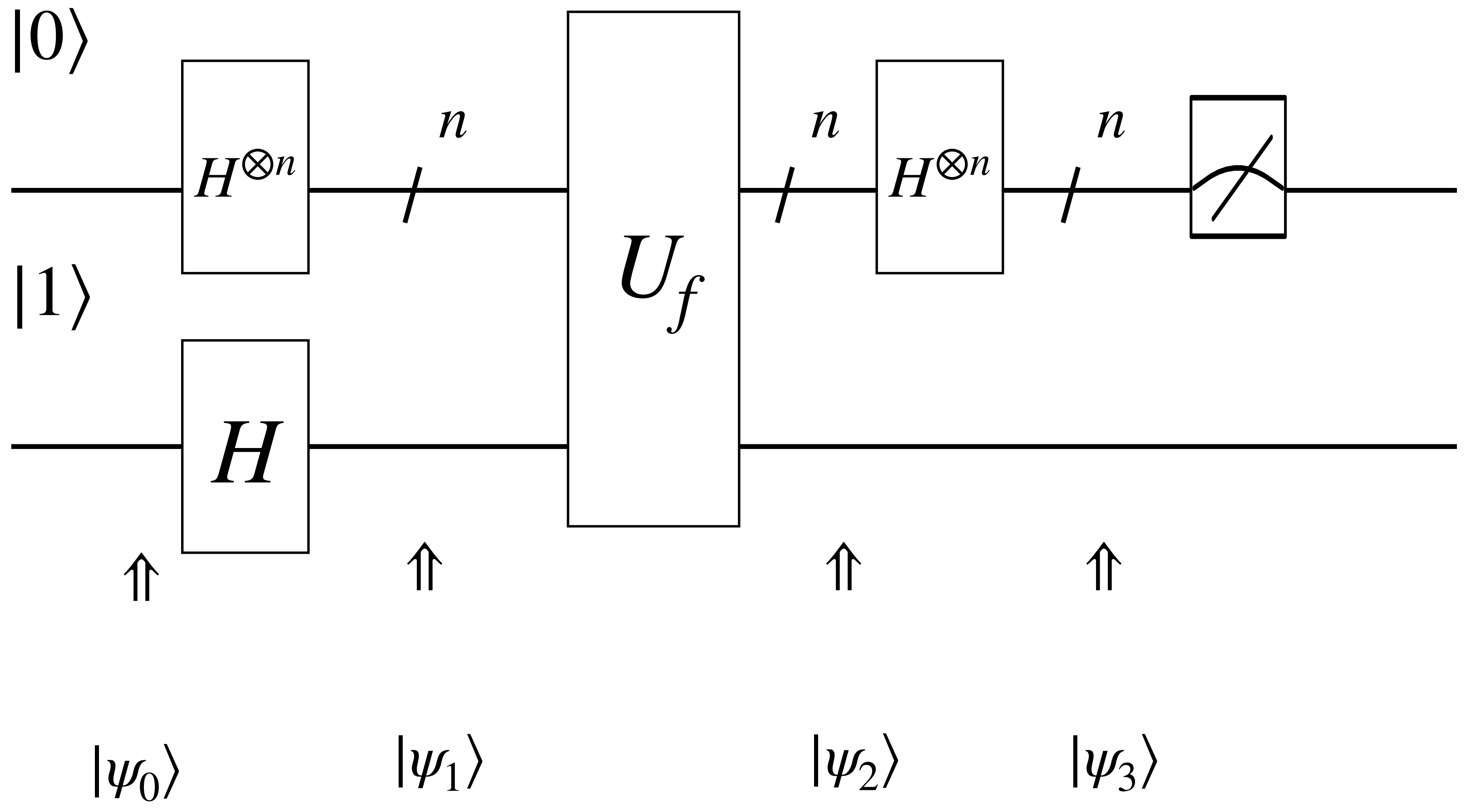
Cuál es la probabilidad de $|\psi_3\rangle$ colapse a $|0\rangle$? Es decir $z = |0\rangle$, entonces $\langle z, x\rangle = \langle 0,x\rangle = 0$

La probabilidad estaría determinada por este término : $[\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |0\rangle]$

Si $f(x)$ es balanceada, la sumatoria se cancela entonces $[\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |0\rangle] = 0|0\rangle$

El algoritmo de Deutsch-JOZSA

Paso 5: Cuál es la probabilidad de $|\psi_3\rangle$ colapse a $|0\rangle$



$$|\psi_0\rangle = |\mathbf{0}\rangle \otimes |1\rangle = |\mathbf{0},1\rangle = |00000...000,1\rangle$$

$$|\psi_1\rangle = H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_2\rangle = U_f * (H^{\otimes n} * |\mathbf{0}\rangle \otimes H * |1\rangle) = [\frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle] \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}]$$

$$|\psi_3\rangle = \begin{cases} |0\rangle \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] & \text{si } f(x) \text{ es constante} \\ |\phi\rangle \otimes [\frac{|0\rangle - |1\rangle}{\sqrt{2}}] & \text{, con } \phi \neq |0\rangle \text{ y si } f(x) \text{ es balanceada} \end{cases}$$

¿Preguntas?