Problem Set 0

All parts are due on September 8, 2019 at 6PM. Please write your solutions in the LaTeX and Python templates provided. Aim for concise solutions; convoluted and obtuse descriptions might receive low marks, even when they are correct. Solutions should be submitted on Gradescope, and any code should be submitted for automated checking on `alg.mit.edu`.

This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. You **may** ask for logistical help concerning LaTeX formatting and/or code submission.

Problem 0-1. Let $A = \{2^i \mid i \in \mathbb{N} \text{ and } 0 \leq i < 5\}$ and $B = \{2i - 1 \mid i \in \{0, 1, 2, 3\}\}$.

Evaluate: **(a)** $A \cap B$ **(b)** $|A \cup B|$ **(c)** $|A - B|$

Problem 0-2. Let X be the random variable representing the number of blue balls chosen, after choosing two balls uniformly at random without replacement from a bag containing exactly three blue balls and two red balls. Let Y be the random variable representing the number of heads seen after flipping a fair coin twice.

Evaluate: **(a)** $\mathrm{E}[X]$ **(b)** $\mathrm{E}[Y]$ **(c)** $\mathrm{E}[X + Y]$

Problem 0-3. Let $A = 600 + 6$ and $B = 60 \times (4 + 2)$. Are these statements True or False?

Evaluate: **(a)** $A \equiv B \bmod 2$ **(b)** $A \equiv B \bmod 3$ **(c)** $A \equiv B \bmod 4$

Problem 0-4. Prove **by induction** that $\sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a}$ when $a \neq 1$, for any integer $n \geq 0$.

Problem 0-5. Prove **by induction** that the vertices of a tree (a connected acyclic undirected graph) can each be colored either red or blue such that no edge connects two vertices of the same color. You may use the fact that every tree having more than one vertex contains a vertex with degree one.

Problem 0-6. Write a Python function `min_mod_tuple(A, k)` which accepts two arguments, Python List `A = [`$a_0, a_1, \ldots, a_{n-1}$`]` containing n positive integers and positive integer `k`, and returns a Python Tuple (i, j) of two array indices with $0 \leq i < j < n$ that minimizes $(a_i \times a_j) \bmod k$. You can download a code template containing some test cases from the website. Submit your code online at `alg.mit.edu`.