Assignment #3

Diego Oniarti - 257835

1 Polymorphic behaviour

Prove that for any closed term f of type $\forall\alpha.\forall\beta.\alpha \to (\alpha \uplus \beta)$ and for any closed types τ_1, τ_2 value $v : \tau_1$, we have $f\ \tau_1\ \tau_2\ v \rightsquigarrow^*\ inl\ v$

Assuming:

$$
\begin{array}{ll}
\Delta, \Gamma \vDash f : \forall\alpha.\forall\beta.\alpha \to (\alpha \uplus \beta) & HP1 \\
\Delta \vdash \tau_1 & HP2 \\
\Delta \vdash \tau_2 & HP3 \\
\vdash v : \tau_1 & HP4
\end{array}
$$

Prove:

$$
f\ \tau_1\ \tau_2\ v \rightsquigarrow^*\ inl\ v \qquad THS
$$

$$\text{by HP1 w.h. } \forall\delta \in D[\Delta], \forall\gamma \in G[\Gamma]^{\delta}, f\gamma\delta \in \mathcal{E}[\forall\alpha.\forall\beta.\alpha \rightarrow (\alpha \uplus \beta)]^{\delta}$$

$$\text{by def. of } \mathcal{E} \text{ w.h. } \exists v'.f\gamma\delta \rightsquigarrow^{*} v' \text{ and } v' \in \mathcal{V}[\forall\alpha.\forall\beta.\alpha \rightarrow (\alpha \uplus \beta)]^{\delta}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } v' \equiv \Lambda\alpha.t \text{ and } \forall\tau'.\forall S \in \text{semty}(\tau').t[\tau'/\alpha] \in \mathcal{E}[\forall\beta.\alpha \rightarrow (\alpha \uplus \beta)]^{\substack{\delta,\\ \alpha\mapsto\tau',S}}$$

$$\text{pick } \tau' = \tau_1, S = \{v\}$$

$$\text{by def. of } \mathcal{E} \text{ w.h. } \exists v''.t[\tau'/\alpha] \rightsquigarrow^{*} v'' \text{ and } v'' \in \mathcal{V}[\forall\beta.\alpha \rightarrow (\alpha \uplus \beta)]^{\substack{\delta,\\ \alpha\mapsto\tau_1,\{v\}}}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } v'' \equiv \Lambda\beta.t' \text{ and } \forall\tau''.\forall S' \in \text{semty}(\tau'').t'[\tau''/\beta] \in \mathcal{E}[\alpha \rightarrow (\alpha \uplus \beta)]^{\overbrace{\substack{\delta,\\ \alpha\mapsto\tau',S,\\ \beta\mapsto\tau'',S'}}^{\delta'}}$$

$$\text{pick } \beta = \tau_2, S' = \emptyset$$

$$\text{by def. of } \mathcal{E} \text{ w.h. } \exists v'''.t'[\tau''/\beta] \rightsquigarrow^{*} v''' \text{ and } v''' \in \mathcal{V}[\alpha \rightarrow (\alpha \uplus \beta)]^{\delta'}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } v''' \equiv \lambda x : \alpha.t'' \text{ and } \forall v'''' \in \mathcal{V}[\alpha]^{\delta'}.t''[v''''/x] \in \mathcal{E}[\alpha \uplus \beta]^{\delta'}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } v''' \equiv \lambda x : \alpha.t'' \text{ and } \forall v'''' \in \{v\}.t''[v''''/x] \in \mathcal{E}[\alpha \uplus \beta]^{\delta'}$$

$$\text{by } v'''' = v \text{ w.h. } v''' \equiv \lambda x : \alpha.t'' \text{ and } t''[v/x] \in \mathcal{E}[\alpha \uplus \beta]^{\delta'}$$

$$\text{by def. of } \mathcal{E} \text{ w.h. } \exists v'''''.t''[v/x] \rightsquigarrow^{*} v''''' \text{ and } v''''' \in \mathcal{V}[\alpha \uplus \beta]^{\delta'}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } \begin{array}{rl} \text{either} & v''''' \equiv \; \mathit{inl}\ v'''''' \text{ and } v'''''' \in \mathcal{V}[\alpha]^{\delta'} \\ \text{or} & v''''' \equiv \; \mathit{inr}\ v'''''' \text{ and } v'''''' \in \mathcal{V}[\beta]^{\delta'} \end{array}$$

$$\text{by def. of } \mathcal{V} \text{ w.h. } \begin{array}{rl} \text{either} & v''''' \equiv \; \mathit{inl}\ v'''''' \text{ and } v'''''' \in \{v\} \\ \text{or} & v''''' \equiv \; \mathit{inr}\ v'''''' \text{ and } v'''''' \in \emptyset \end{array}$$

$$\text{by no val in } \emptyset \text{ w.h. } v''''' \equiv \; \mathit{inl}\ v\square$$