Assignment #1

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1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t"[t'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \quad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL\ t \Downarrow inL\ v} \quad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR\ t \Downarrow inR\ v} \quad \text{inRight}$$

$$\frac{t \Downarrow\ inL\ v' \quad t_1[v'/x_1] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching L}$$

$$\frac{t \Downarrow\ inR\ v' \quad t_2[v'/x_2] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to \mathbb{N} \\ .d\ (\lambda a : \mathbb{N}.\lambda b : \mathbb{N}.b)\ (\lambda i : \mathbb{N}.\lambda j : \mathbb{N}.i)$$

Reduction 1

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 0\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 0\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 0\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.0)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 0 & \rightsquigarrow \\
0 &
\end{array}
$$

Reduction 2

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 1\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 1\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 1\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.1)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 1 & \rightsquigarrow \\
1 &
\end{array}
$$

4 Safe untypable term

$$(\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1$$

typing derivation

$$
\dfrac{
\dfrac{
\dfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma}{x : \mathbb{N} \to \mathbb{N} \vdash x : \mathbb{N} \to \mathbb{N}}\,\text{var}
}{\emptyset \vdash \lambda x : \mathbb{N} \to \mathbb{N}.x : (\mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N})}\,\text{lam}
\qquad
\dfrac{}{\emptyset \vdash 1 : \mathbb{N}}\,\text{nat}
}{\emptyset \vdash (\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1 :}\,\cancel{app}
$$

COS-SM-CBV

$$(\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1 \rightsquigarrow 1$$

5 Typing derivation

$$
\dfrac{
\dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\,\text{var}
\quad
\dfrac{
\dfrac{
\dfrac{
\dfrac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\,\text{var}
\quad
\dfrac{}{\Gamma' \vdash 2 : \mathbb{N}}\,\text{nat}
}{\Gamma' \left\{ {f : \mathbb{N} \to \mathbb{N}, \atop x : \mathbb{N}} \right. \vdash x + 2 : \mathbb{N}}\,\text{op}
}{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}\,\text{lam}
\quad
\dfrac{}{\Gamma \vdash 4 : \mathbb{N}}\,\text{nat}
}{\Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}}\,\text{app}
}{\Gamma \left\{ f : \mathbb{N} \to \mathbb{N} \right. \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N}}\,\text{app}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x:\mathbb{N}\to\mathbb{N}\in\Gamma'}{\Gamma'\vdash x:\mathbb{N}\to\mathbb{N}}\text{var}\quad\dfrac{y:\mathbb{N}\in\Gamma'}{\Gamma'\vdash y:\mathbb{N}}\text{var}}{\Gamma'\begin{cases}\Gamma,\\ x{:}\mathbb{N}\to\mathbb{N},\\ y{:}\mathbb{N}\end{cases}\vdash x\ y:\mathbb{N}}\text{app}}{\begin{matrix}\Gamma,\\ x{:}\mathbb{N}\to\mathbb{N}\end{matrix}\vdash\lambda y.x\ y:\mathbb{N}\to\mathbb{N}}\text{lam}}{\Gamma\vdash\lambda x.\lambda y.x\ y:(\mathbb{N}\to\mathbb{N})\to\mathbb{N}\to\mathbb{N}}\text{lam}\quad\dfrac{f:\mathbb{N}\to\mathbb{N}\in\Gamma}{\Gamma\vdash f:\mathbb{N}\to\mathbb{N}}\text{var}}{\Gamma\vdash(\lambda x.\lambda y.x\ y)\ f:\mathbb{N}\to\mathbb{N}}\text{app}\quad\dfrac{}{\Gamma\vdash 3:\mathbb{N}}\text{nat}}{\Gamma\left\{f:\mathbb{N}\to\mathbb{N}\vdash((\lambda x.\lambda y.x\ y)\ f)\ 3:\mathbb{N}\right.}\text{app}
$$

6 Encoding

Sequencing

$$
\begin{gathered}
t ::= \cdots | t; t' \\
(\lambda x.t')\ t
\end{gathered}
$$

(Assuming x is a free variable not used in t')

Let-in

$$
\begin{gathered}
t ::= \cdots | let\ x = t\ in\ t' \\
(\lambda x.t')\ t
\end{gathered}
$$

Arrays of Length 4

$$
\begin{gathered}
t ::= \cdots | [t, t, t, t] \\
v ::= \cdots | [v, v, v, v] \\
[t_1, t_2, t_3, t_4] \equiv \lambda a.a\ t_1\ t_2\ t_3\ t_4
\end{gathered}
$$

Array field access

$$
\begin{gathered}
t ::= \cdots | t.i\ (i \in 0..3) \\
t.0 \equiv t\ (\lambda a.\lambda b.\lambda c.\lambda d.a) \\
t.1 \equiv t\ (\lambda a.\lambda b.\lambda c.\lambda d.b) \\
t.2 \equiv t\ (\lambda a.\lambda b.\lambda c.\lambda d.c) \\
t.3 \equiv t\ (\lambda a.\lambda b.\lambda c.\lambda d.d)
\end{gathered}
$$

Array update

$$t ::= \cdots | t.i = t \ (i \in 0..3)$$
$$t.0 = t' \equiv t \ (\lambda a.\lambda b.\lambda c.\lambda d.\lambda z.z \ t' \ b \ c \ d)$$
$$t.1 = t' \equiv t \ (\lambda a.\lambda b.\lambda c.\lambda d.\lambda z.z \ a \ t' \ c \ d)$$
$$t.2 = t' \equiv t \ (\lambda a.\lambda b.\lambda c.\lambda d.\lambda z.z \ a \ b \ t' \ d)$$
$$t.3 = t' \equiv t \ (\lambda a.\lambda b.\lambda c.\lambda d.\lambda z.z \ a \ b \ c \ t')$$