Assignment #1

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1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t' \Downarrow v' \quad t"[v'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \tau \to \tau') \to (\mathbb{N} \to \mathbb{N} \to \tau) \to \tau'$$
$$.d\ (\lambda m : \mathbb{N}.\lambda b : \tau.b)\ (\lambda i : \mathbb{N}.\lambda n : \mathbb{N}.i)$$

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \tau \to \tau') \to (\mathbb{N} \to \mathbb{N} \to \tau)$$
$$\to \tau'.d\ (\lambda m : \mathbb{N}.\lambda b : \tau.b)\ (\lambda i : \mathbb{N}.\lambda n : \mathbb{N}.i * n_1 + (1 - i) * n_2)$$

4 Typing derivation

$$\frac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\text{var} \quad \frac{\frac{\frac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\text{var} \quad \frac{}{\Gamma' \vdash 2 : \mathbb{N}}\text{nat}}{\Gamma'\ \{^{f:\mathbb{N}\to\mathbb{N},}_{x:\mathbb{N}}\ \vdash x + 2 : \mathbb{N}}\text{op}}{\frac{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}{\Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}}\text{lam} \quad \frac{}{\Gamma \vdash 4 : \mathbb{N}}\text{nat}}\text{app}}{\Gamma\ \{f : \mathbb{N} \to \mathbb{N} \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N}}\text{app}$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{x:\mathbb{N}\to\mathbb{N}\in\Gamma'}{\Gamma'\vdash x:\mathbb{N}\to\mathbb{N}}\text{var} \quad \cfrac{y:\mathbb{N}\in\Gamma'}{\Gamma'\vdash y:\mathbb{N}}\text{var}}{\Gamma'\left\{{}^{\Gamma,}_{{}^{x:\mathbb{N}\to\mathbb{N},}_{y:\mathbb{N}}}\vdash x\ y:\mathbb{N}\right.}\text{app}}{{}^{\Gamma,}_{x:\mathbb{N}\to\mathbb{N}}\vdash\lambda y.x\ y:\mathbb{N}\to\mathbb{N}}\text{lam}}{\Gamma\vdash\lambda x.\lambda y.x\ y:(\mathbb{N}\to\mathbb{N})\to\mathbb{N}\to\mathbb{N}}\text{lam} \quad \cfrac{f:\mathbb{N}\to\mathbb{N}\in\Gamma}{\Gamma\vdash f:\mathbb{N}\to\mathbb{N}}\text{var}}{\Gamma\vdash(\lambda x.\lambda y.x\ y)\ f:\mathbb{N}\to\mathbb{N}}\text{app} \quad \cfrac{}{\Gamma\vdash 3:\mathbb{N}}\text{nat}}{\Gamma\left\{f:\mathbb{N}\to\mathbb{N}\vdash((\lambda x.\lambda y.x\ y)\ f)\ 3:\mathbb{N}\right.}\text{app}$$