Assignment #1

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1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t"[t'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \quad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL\ t \Downarrow inL\ v} \quad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR\ t \Downarrow inR\ v} \quad \text{inRight}$$

$$\frac{t \Downarrow\ inL\ v' \quad t_1[v'/x_1] \Downarrow v}{\text{case } t \text{ of} \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching L}$$

$$\frac{t \Downarrow\ inR\ v' \quad t_2[v'/x_2] \Downarrow v}{\text{case } t \text{ of} \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \overset{def}{=} \lambda d : (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to \mathbb{N}$$
$$.d\ (\lambda a : \mathbb{N}.\lambda b : \mathbb{N}.b)\ (\lambda i : \mathbb{N}.\lambda j : \mathbb{N}.i)$$

Reduction 1

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 0\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 0\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 0\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.0)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 0 & \rightsquigarrow
\end{array}
$$

$$0$$

Reduction 2

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 1\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 1\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 1\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.1)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 1 & \rightsquigarrow
\end{array}
$$

$$1$$

4 Safe untypable term

$$(\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1$$

typing derivation

$$
\dfrac{\dfrac{}{\emptyset \vdash \lambda x : \mathbb{N} \to \mathbb{N}.x : (\mathbb{N} \to \mathbb{N}) \to \tau} \qquad \dfrac{}{\emptyset \vdash 1 : \mathbb{N}}\,\text{nat}}{\emptyset \vdash (\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1 :}\,\text{app}
$$

COS-SM-CBV

$$(\lambda x \mathbb{N} \to \mathbb{N}.x)\ 1 \rightsquigarrow 1$$

5 Typing derivation

$$
\dfrac{\dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\,\text{var} \qquad \dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\,\text{var} \qquad \dfrac{}{\Gamma' \vdash 2 : \mathbb{N}}\,\text{nat}}{\Gamma' \left\{ \begin{smallmatrix} f:\mathbb{N}\to\mathbb{N}, \\ x:\mathbb{N} \end{smallmatrix} \right. \vdash x + 2 : \mathbb{N}}\,\text{op}}{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}\,\text{lam} \qquad \dfrac{}{\Gamma \vdash 4 : \mathbb{N}}\,\text{nat}}{\Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}}\,\text{app}}{\Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N}}\,\text{app}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N} \to \mathbb{N}}\text{var} \quad \dfrac{y : \mathbb{N} \in \Gamma'}{\Gamma' \vdash y : \mathbb{N}}\text{var}}{\Gamma' \begin{cases} \Gamma, \\ x{:}\mathbb{N}\to\mathbb{N}, \\ y{:}\mathbb{N} \end{cases} \vdash x\ y : \mathbb{N}}\text{app}}{\substack{\Gamma, \\ x:\mathbb{N}\to\mathbb{N}} \vdash \lambda y.x\ y : \mathbb{N} \to \mathbb{N}}\text{lam}}{\Gamma \vdash \lambda x.\lambda y.x\ y : (\mathbb{N} \to \mathbb{N}) \to \mathbb{N} \to \mathbb{N}}\text{lam} \quad \dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\text{var}}{\Gamma \vdash (\lambda x.\lambda y.x\ y)\ f : \mathbb{N} \to \mathbb{N}}\text{app} \quad \dfrac{}{\Gamma \vdash 3 : \mathbb{N}}\text{nat}}{\Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash ((\lambda x.\lambda y.x\ y)\ f)\ 3 : \mathbb{N}}\text{app}
$$

6 Encoding

Sequencing

$$
\begin{aligned}
t &::= \cdots | t; t' \\
t; t' &\rightsquigarrow^{p} (\lambda x.t)t'
\end{aligned}
$$

Let-in

$$
t ::= \cdots | let\ x = t\ in\ t'
$$

$$
\dfrac{}{let\ x = t\ in\ t' \rightsquigarrow}\text{let-in}
$$