

# Assignment #2

Diego Oniarti - 257835

## 1 Missing Progress Cases

Write the proof for the progress theorem for the following cases

- $t \equiv \text{inl } t_1$
- $t \equiv \text{case } t_0 \text{ of } \left\{ \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right.$

Theorem:

if  $\emptyset \vdash t : \tau$  then either  $\vdash t.\text{VAL}$  or  $\exists t'. t \rightsquigarrow t'$

Prof by induction on the typing derivation of  $t$ .  
Base cases  $t\text{-var}$  and  $t\text{-nat}$  seen in class

### 1.1 $t \equiv \text{inl } t_1$

if  $\emptyset \vdash \text{inl } t : \tau_1 \uplus \tau_2$  then either  $\vdash \text{inl } t.\text{VAL}$  or  $\exists t'. \text{inl } t \rightsquigarrow t'$

	$\frac{\emptyset \vdash t : \tau_1}{\emptyset \vdash \text{inl } t : \tau_1 \uplus \tau_2}$	T-inl
t.s. either	$\vdash \text{inl } t.\text{VAL}$	
	or $\exists t'. \text{inl } t \rightsquigarrow t'$	
by I.H. either	$\vdash t.\text{VAL}$	I1
	or $\exists t''. t \rightsquigarrow t''$	I2

**Assuming I1**  $\text{inl } t.\text{VAL}$  by I1 and definition of  $\text{inl}$ .  $\square$

**Assuming I2**

by inversion on I2 w.h.	$t \equiv E[t_0]$	HE1
	$t'' \equiv E[t_0'']$	HE1'
	$t_0 \rightsquigarrow^p t_0''$	HPR
by HE1, HE1' t.s.	$\text{inl } E[t_0] \rightsquigarrow \text{inl } E[t_0'']$	
by ctx with HPR and	$E' = \text{inl } E_\square$	

$$1.2 \quad t \equiv \text{case } t_0 \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right|$$

$$\underbrace{\text{if } \emptyset \vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| : \tau \text{ then either}}_{\text{HY}}$$

$$\vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| . \text{VAL}$$

$$\text{or } \exists t'. \text{case } t_0 \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| \rightsquigarrow t'$$

$$\text{by } I.H.w.h. \text{ either } \vdash t_0.\text{VAL} \quad \text{I1}$$

$$\text{or } \exists t'_0. t_0 \rightsquigarrow t'_0 \quad \text{I2}$$

**Assuming I1**

$$\text{by typing definition of case and HY} \quad \begin{array}{l} \text{either } t_0 \equiv \text{inl } t_0^l \\ \text{or } t_0 \equiv \text{inr } t_0^r \end{array}$$

$$\bullet \quad t_0 \equiv \text{inl } t_0^l$$

$$\text{by COS rule for case: case } \text{inl } t_0^l \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| \rightsquigarrow^p t_1[t_0^l/x_1] \square$$

(qui devo probabilmente inserire che  $\mathbb{E} = \square$  e tutta la tiritera)

$$\bullet \quad t_0 \equiv \text{inr } t_0^r$$

$$\text{by COS rule for case: case } \text{inr } t_0^r \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| \rightsquigarrow^p t_2[t_0^r/x_2] \square$$

**Assuming I2**

$$\begin{array}{lll} \text{by inversion on I2 w.h.} & t_0 \equiv E[t_0^z] & \text{HE1} \\ & t'_0 \equiv E[t_0'^z] & \text{HE1'} \\ & t_0^z \rightsquigarrow^p t_0'^z & \text{HPR} \end{array}$$

$$\text{by HE1, HE1' t.s. case } E[t_0^z] \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| \rightsquigarrow \text{case } E[t_0'^z] \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right|$$

$$\text{by ctx with HPR and } E' = \text{case } E \text{ of } \left| \begin{array}{l} \text{inl } x_1 \mapsto t_1 \\ \text{inr } x_2 \mapsto t_2 \end{array} \right| \square$$

## 2 Missing Compatibility Lemmas

Write the proof for these compatibility lemmas:

- Pairs  
Assuming:

- $\Gamma \vdash t_1 : \tau_1$
- $\Gamma \vdash t_2 : \tau_2$

Prove:

- $\Gamma \langle t_1, t_2 \rangle : \tau_1 \times \tau_2$

## 3 Adding Cycles

Add **for** and **while** constructs to STLC. Add their syntax, their typing and their operational semantics in COS.

### 3.1 while

$t = \dots \mid \text{while } t \text{ do } t \text{ end}$   
 $v = \dots \mid \Xi$   
 $\tau = \dots \mid \xi$   
 $\mathbb{E} = \dots \mid \text{while } \mathbb{E} \text{ do } t \text{ end}$

$$\frac{\Gamma \vdash t_0 : \text{bool} \quad \Gamma \vdash t_1 : \tau}{\Gamma \vdash \text{while } t_0 \text{ do } t_1 \text{ end} : \xi} \text{t-while}$$

$$\frac{}{\text{while } \text{true} \text{ do } t \text{ end} \rightsquigarrow^p t; \text{while } \text{true} \text{ do } t \text{ end}} \text{c-whileT}$$

$$\frac{}{\text{while } \text{false} \text{ do } t \text{ end} \rightsquigarrow^p \Xi} \text{c-whileF}$$

### 3.2 for

$t = \dots \mid \text{for } i = t \text{ to } t \text{ do } t \text{ end}$   
 $v = \dots \mid \Xi$   
 $\tau = \dots \mid \xi$   
 $\mathbb{E} = \dots \mid \text{for } i = \mathbb{E} \text{ to } t \text{ do } t \text{ end}$   
 $\quad \mid \text{for } i = n \text{ to } \mathbb{E} \text{ do } t \text{ end}$

$$\frac{\Gamma \vdash t_0 : \mathbb{N} \quad \Gamma \vdash t_1 : \mathbb{N} \quad \Gamma, i : \mathbb{N} \vdash t_2 : \tau}{\Gamma \vdash \text{for } i = t_0 \text{ to } t_1 \text{ do } t_2 \text{ end} : \xi} \text{t-for}$$

$$\frac{n_1 < n_2 \quad n'_1 = n_1 + 1}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p t[n/i]; \text{for } i = n'_1 \text{ to } n_2 \text{ do } t \text{ end}} \text{c-for1}$$

$$\frac{n_1 \geq n_2}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p \Xi} \text{c-for2}$$