Assignment #2

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1 Missing Progress Cases

Write the proof for the progress theorem for the following cases

- $t \equiv$ *inl* t_1
- $t \equiv \text{case } t_0 \text{ of } \left| \begin{array}{l} \textit{inl } x_1 \mapsto t_1 \\ \textit{inr } x_2 \mapsto t_2 \end{array} \right.$

Theorem:

$$\text{if } \emptyset \vdash t : \tau \text{ then either } \vdash t.VAL \text{ or } \exists t'.t \rightsquigarrow t'$$

Prof by induction on the typing derivation of t.
Base cases *t-var* and *t-nat* seen in class

1.1 $t \equiv$ *inl* t_1

$$\text{if } \emptyset \vdash \textit{ inl } t : \tau_1 \uplus \tau_2 \text{ then either } \vdash \textit{ inl } t.\text{VAL or } \exists t'. \textit{ inl } t \rightsquigarrow t'$$

$$\frac{\emptyset \vdash t : \tau_1}{\emptyset \vdash \textit{ inl } t : \tau_1 \uplus \tau_2}\text{T-inl}$$

$$\begin{array}{rll}
\text{t.s. either} & \vdash \textit{ inl } t.\text{VAL} & \\
\text{or} & \exists t'. \textit{ inl } t \rightsquigarrow t' & \\
\text{by I.H. either} & \vdash t.\text{VAL} & \text{I1} \\
\text{or} & \exists t".t \rightsquigarrow t" & \text{I2}
\end{array}$$

Assuming I1 *inl* t.VAL by I1 and definition of *inl* . $\square$

Assuming I2

$$\begin{array}{rll}
\text{by inversion on I2 w.h.} & t \equiv E[t_0] & \text{HE1} \\
 & t" \equiv E[t_0"] & \text{HE1'} \\
 & t_0 \rightsquigarrow^p t_0" & \text{HPR} \\
\text{by HE1, HE1' t.s.} & \textit{inl } E[t_0] \rightsquigarrow \textit{ inl } E[t_0"] & \\
\text{by ctx with HPR and} & E' = \textit{ inl } E_\square &
\end{array}$$

1.2 $t \equiv$ **case** t_0 **of** $\left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right.$

$$\text{if } \emptyset \vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. : \tau \text{ then either}$$

$$\cancel{\vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. .\text{VAL}}$$

$$\text{or } \exists t'.\text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow t'$$

2 Missing Compatibility Lemmas

Write the proof for these compatibility lemmas:

3 Adding Cycles

Add `for` and `while` constructs to STLC. Add their syntax, their typing and their operational semantics in COS.

3.1 while

$$\begin{aligned} t &= \cdots | \text{while } t \text{ do } t \text{ end} \\ v &= \cdots | \Xi \\ \tau &= \cdots | \xi \\ \mathbb{E} &= \cdots | \text{while } \mathbb{E} \text{ do } t \text{ end} \end{aligned}$$

$$\frac{\Gamma \vdash t_0 : bool \quad \Gamma \vdash t_1 : \tau}{\Gamma \vdash \text{while } t_0 \text{ do } t_1 \text{ end} : \xi}\text{t-while}$$

$$\frac{}{\text{while } true \text{ do } t \text{ end} \rightsquigarrow^p t; \text{while } true \text{ do } t \text{ end}}\text{c-whileT}$$

$$\frac{}{\text{while } false \text{ do } t \text{ end} \rightsquigarrow^p \Xi}\text{c-whileF}$$

3.2 for

$$
\begin{aligned}
t &= \cdots | \text{for } i = t \text{ to } t \text{ do } t \text{ end} \\
v &= \cdots | \Xi \\
\tau &= \cdots | \xi \\
\mathbb{E} &= \cdots | \text{for } i = \mathbb{E} \text{ to } t \text{ do } t \text{ end} \\
&\quad\;\, | \text{for } i = n \text{ to } \mathbb{E} \text{ do } t \text{ end}
\end{aligned}
$$

$$
\frac{\Gamma \vdash t_0 : \mathbb{N} \quad \Gamma \vdash t_1 : \mathbb{N} \quad \Gamma, i : \mathbb{N} \vdash t_2 : \tau}{\Gamma \vdash \text{for } i = t_0 \text{ to } t_1 \text{ do } t_2 \text{ end} : \xi}\text{t-for}
$$

$$
\frac{n_1 < n_2 \quad n_1' = n_1 + 1}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p t[n/i]; \text{for } i = n_1' \text{ to } n_2 \text{ do } t \text{ end}}\text{c-for1}
$$

$$
\frac{n_1 \geq n_2}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p \Xi}\text{c-for2}
$$