Assignment #2

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1 Missing Progress Cases

Write the proof for the progress theorem for the following cases

- $t \equiv inl\ t_1$

- $t \equiv \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right.$

Theorem:

$$\text{if } \emptyset \vdash t : \tau \text{ then either } \vdash t.VAL \text{ or } \exists t'.t \rightsquigarrow t'$$

Prof by induction on the typing derivation of t.
Base cases *t-var* and *t-nat* seen in class

1.1 $t \equiv inl\ t_1$

if $\emptyset \vdash\ inl\ t : \tau_1 \uplus \tau_2$ then either $\vdash\ inl\ t.\text{VAL}$ or $\exists t'.\ inl\ t \rightsquigarrow t'$

$$\cfrac{\emptyset \vdash t : \tau_1}{\emptyset \vdash\ inl\ t : \tau_1 \uplus \tau_2}\text{T-inl}$$

$$\begin{array}{rll} \text{t.s. either} & \vdash\ inl\ t.\text{VAL} \\ \text{or} & \exists t'.\ inl\ t \rightsquigarrow t' \\ \text{by I.H. either} & \vdash t.\text{VAL} & \text{I1} \\ \text{or} & \exists t".t \rightsquigarrow t" & \text{I2} \end{array}$$

Assuming I1 $inl\ t.\text{VAL}$ by I1 and definition of inl . $\square$

Assuming I2

$$\begin{array}{rll} \text{by inversion on I2 w.h.} & t \equiv E[t_0] & \text{HE1} \\ & t" \equiv E[t_0"] & \text{HE1'} \\ & t_0 \rightsquigarrow^p t_0" & \text{HPR} \\ \text{by HE1, HE1' t.s.} & inl\ E[t_0] \rightsquigarrow\ inl\ E[t_0"] \\ \text{by ctx with HPR and} & E' =\ inl\ E_\square \end{array}$$

1.2 $t \equiv \textbf{case } t_0 \textbf{ of } \left| \begin{array}{l} inl \ x_1 \mapsto t_1 \\ inr \ x_2 \mapsto t_2 \end{array} \right.$

if $\emptyset \vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} inl \ x_1 \mapsto t_1 \\ inr \ x_2 \mapsto t_2 \end{array} \right. : \tau$ then either

$\vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} \cancel{inl \ x_1 \mapsto t_1} \\ inr \ x_2 \mapsto t_2 \end{array} \right. .\text{VAL}$

or $\exists t'.\text{case } t_0 \text{ of } \left| \begin{array}{l} inl \ x_1 \mapsto t_1 \\ inr \ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow t'$

2 Missing Compatibility Lemmas

Write the proof for these compatibility lemmas:

3 Adding Cycles

Add `for` and `while` constructs to STLC. Add their syntax, their typing and their operational semantics in COS.

3.1 while

$$t = \cdots | \text{while } t \text{ do } t \text{ end}$$
$$\tau = \cdots | \xi$$
$$\mathbb{E} = \cdots | \text{while } \mathbb{E} \text{ do } t \text{ end}$$
$$| \text{while } true \text{ do } \mathbb{E} \text{ end}$$

$$\frac{\Gamma \vdash t_0 : bool \quad \Gamma \vdash t_1 : \tau}{\Gamma \vdash \text{while } t_0 \text{ do } t_1 \text{ end} : \xi}\text{t-while}$$

$$\frac{}{\text{while } \text{ do } \text{ end}}\text{c-while}$$