Assignment #1

Diego Oniarti

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1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t"[t'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \quad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL\ t \Downarrow inL\ v} \quad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR\ t \Downarrow inR\ v} \quad \text{inRight}$$

$$\frac{t \Downarrow\ inL\ v' \quad t_1[v'/x_1] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching L}$$

$$\frac{t \Downarrow\ inR\ v' \quad t_2[v'/x_2] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to \mathbb{N}$$
$$.d\ (\lambda a : \mathbb{N}.\lambda b : \mathbb{N}.b)\ (\lambda i : \mathbb{N}.\lambda j : \mathbb{N}.i)$$

Reduction 1

$$\begin{array}{lll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 0\ 0)) & \rightarrow \beta & E = [] \\
(\lambda x.\lambda y.x\ 0\ (y\ 0\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightarrow \frac{\beta}{app1} & E = [](\lambda i.\lambda j.i) \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 0\ 0))\ (\lambda i.\lambda j.i) & \rightarrow \beta & E = [] \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightarrow \frac{\beta}{app1} & E = []((\lambda i.\lambda j.i)\ 0\ 0) \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightarrow \frac{\frac{\beta}{app1}}{app2} & E = (\lambda b.b)\ ([]\ 0) \\
(\lambda b.b)\ ((\lambda j.0)\ 0) & \rightarrow \frac{\beta}{app2} & E = (\lambda b.b)\ [] \\
(\lambda b.b)\ 0 & \rightarrow \beta & E = []
\end{array}$$

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Reduction 2

$$\begin{array}{lll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 1\ 0)) & \rightarrow \beta & E = [] \\
(\lambda x.\lambda y.x\ 0\ (y\ 1\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightarrow \frac{\beta}{app1} & E = [](\lambda i.\lambda j.i) \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 1\ 0))\ (\lambda i.\lambda j.i) & \rightarrow \beta & E = [] \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightarrow \frac{\beta}{app1} & E = []((\lambda i.\lambda j.i)\ 0\ 0) \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightarrow \frac{\frac{\beta}{app1}}{app2} & E = (\lambda b.b)\ ([]\ 0) \\
(\lambda b.b)\ ((\lambda j.1)\ 0) & \rightarrow \frac{\beta}{app2} & E = (\lambda b.b)\ [] \\
(\lambda b.b)\ 1 & \rightarrow \beta & E = []
\end{array}$$

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4 Safe untypable term

typing derivation

$$\dfrac{\dfrac{\dfrac{???}{x : \tau \vdash x\ x : \tau'}\text{Err}}{\emptyset \vdash \lambda x.x\ x : \tau \rightarrow \tau'}\text{lam} \quad \emptyset \vdash \lambda y.y\ y : \tau}{\emptyset \vdash (\lambda x.x\ x)\ (\lambda y.y\ y) : \tau'}\text{app}$$

COS-SM-CBV

$$\begin{array}{lll}
(\lambda x.x\ x)\ (\lambda y.y\ y) & \rightarrow \beta & E = [] \\
(\lambda y.y\ y)\ (\lambda y.y\ y) & \rightarrow \beta & E = [] \\
(\lambda y.y\ y)\ (\lambda y.y\ y) & \rightarrow \beta & E = [] \\
... & \rightarrow &
\end{array}$$

5 Typing derivation

$$\dfrac{\dfrac{f:\mathbb{N}\to\mathbb{N}\in\Gamma}{\Gamma\vdash f:\mathbb{N}\to\mathbb{N}}\text{var} \quad \dfrac{\dfrac{\dfrac{\dfrac{x:\mathbb{N}\in\Gamma'}{\Gamma'\vdash x:\mathbb{N}}\text{var}\quad \dfrac{}{\Gamma'\vdash 2:\mathbb{N}}\text{nat}}{\Gamma' \left\{ \begin{smallmatrix} f:\mathbb{N}\to\mathbb{N}, \\ x:\mathbb{N}\end{smallmatrix}\right. \vdash x+2:\mathbb{N}}\text{op}}{\Gamma\vdash \lambda x.x+2:\mathbb{N}\to\mathbb{N}}\text{lam}\quad \dfrac{}{\Gamma\vdash 4:\mathbb{N}}\text{nat}}{\Gamma\vdash(\lambda x.x+2)\ 4:\mathbb{N}}\text{app}}{\Gamma\left\{f:\mathbb{N}\to\mathbb{N}\right. \vdash f\ ((\lambda x.x+2)\ 4):\mathbb{N}}\text{app}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x:\mathbb{N}\to\mathbb{N}\in\Gamma'}{\Gamma'\vdash x:\mathbb{N}\to\mathbb{N}}\text{var}\quad\dfrac{y:\mathbb{N}\in\Gamma'}{\Gamma'\vdash y:\mathbb{N}}\text{var}}{\Gamma'\left\{\begin{smallmatrix}\Gamma,\\ x:\mathbb{N}\to\mathbb{N},\\ y:\mathbb{N}\end{smallmatrix}\right.\vdash x\ y:\mathbb{N}}\text{app}}{\begin{smallmatrix}\Gamma,\\ x:\mathbb{N}\to\mathbb{N}\end{smallmatrix}\vdash\lambda y.x\ y:\mathbb{N}\to\mathbb{N}}\text{lam}}{\Gamma\vdash\lambda x.\lambda y.x\ y:(\mathbb{N}\to\mathbb{N})\to\mathbb{N}\to\mathbb{N}}\text{lam}\quad\dfrac{f:\mathbb{N}\to\mathbb{N}\in\Gamma}{\Gamma\vdash f:\mathbb{N}\to\mathbb{N}}\text{var}}{\Gamma\vdash(\lambda x.\lambda y.x\ y)\ f:\mathbb{N}\to\mathbb{N}}\text{app}\quad\dfrac{}{\Gamma\vdash 3:\mathbb{N}}\text{nat}}{\Gamma\left\{f:\mathbb{N}\to\mathbb{N}\right.\vdash((\lambda x.\lambda y.x\ y)\ f)\ 3:\mathbb{N}}\text{app}$$

6 Encoding

Sequencing

$$t ::= \cdots | t;t'$$
$$t;t' \leadsto^p (\lambda x.t)t'$$

Let-in

$$t ::= \cdots | let\ x = t\ in\ t'$$
$$\dfrac{}{let\ x = t\ in\ t' \leadsto}\text{let-in}$$