Assignment #1

Diego Oniarti

October 20, 2024

1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t"[t'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \quad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL\ t \Downarrow inL\ v} \quad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR\ t \Downarrow inR\ v} \quad \text{inRight}$$

$$\frac{t \Downarrow\ inL\ v' \quad t_1[v'/x_1] \Downarrow v}{\text{case } t \text{ of} \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching L}$$

$$\frac{t \Downarrow\ inR\ v' \quad t_2[v'/x_2] \Downarrow v}{\text{case } t \text{ of} \left| \begin{array}{l} inL\ x_1 \mapsto t_1 \\ inR\ x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \overset{def}{=} \lambda d : (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to \mathbb{N}$$
$$.d\ (\lambda a : \mathbb{N}.\lambda b : \mathbb{N}.b)\ (\lambda i : \mathbb{N}.\lambda j : \mathbb{N}.i)$$

Reduction 1

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 0\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 0\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 0\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 0\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.0)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 0 & \rightsquigarrow \\
0 &
\end{array}
$$

Reduction 2

$$
\begin{array}{ll}
(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 1\ 0)) & \rightsquigarrow \\
(\lambda x.\lambda y.x\ 0\ (y\ 1\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 1\ 0))\ (\lambda i.\lambda j.i) & \rightsquigarrow \\
(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 1\ 0) & \rightsquigarrow \\
(\lambda b.b)\ ((\lambda j.1)\ 0) & \rightsquigarrow \\
(\lambda b.b)\ 1 & \rightsquigarrow \\
1 &
\end{array}
$$

4 Safe untypable term

$$(\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1$$

typing derivation

$$
\dfrac{
\dfrac{
\dfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma}{x : \mathbb{N} \to \mathbb{N} \vdash x : \mathbb{N} \to \mathbb{N}}\,\text{var}
}{\emptyset \vdash \lambda x : \mathbb{N} \to \mathbb{N}.x : (\mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N})}\,\text{lam}
\qquad
\dfrac{}{\emptyset \vdash 1 : \mathbb{N}}\,\text{nat}
}{\emptyset \vdash (\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1 :}\,\cancel{app}
$$

COS-SM-CBV

$$(\lambda x : \mathbb{N} \to \mathbb{N}.x)\ 1 \rightsquigarrow 1$$

5 Typing derivation

$$
\dfrac{
\dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\,\text{var}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\,\text{var}
\qquad
\dfrac{}{\Gamma' \vdash 2 : \mathbb{N}}\,\text{nat}
}{\Gamma' \left\{ \begin{smallmatrix} f : \mathbb{N} \to \mathbb{N}, \\ x : \mathbb{N} \end{smallmatrix} \right. \vdash x + 2 : \mathbb{N}}\,\text{op}
}{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}\,\text{lam}
\qquad
\dfrac{}{\Gamma \vdash 4 : \mathbb{N}}\,\text{nat}
}{\Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}}\,\text{app}
}{\Gamma \left\{ f : \mathbb{N} \to \mathbb{N} \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N} \right.}\,\text{app}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N} \to \mathbb{N}}\text{var} \quad \dfrac{y : \mathbb{N} \in \Gamma'}{\Gamma' \vdash y : \mathbb{N}}\text{var}}{\Gamma' \left\{ \begin{smallmatrix} \Gamma, \\ x:\mathbb{N}\to\mathbb{N}, \\ y:\mathbb{N} \end{smallmatrix} \right. \vdash x\ y : \mathbb{N}}\text{app}}{\begin{smallmatrix} \Gamma, \\ x:\mathbb{N}\to\mathbb{N} \end{smallmatrix} \vdash \lambda y.x\ y : \mathbb{N} \to \mathbb{N}}\text{lam}}{\Gamma \vdash \lambda x.\lambda y.x\ y : (\mathbb{N} \to \mathbb{N}) \to \mathbb{N} \to \mathbb{N}}\text{lam} \quad \dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\text{var}}{\Gamma \vdash (\lambda x.\lambda y.x\ y)\ f : \mathbb{N} \to \mathbb{N}}\text{app} \quad \dfrac{}{\Gamma \vdash 3 : \mathbb{N}}\text{nat}}{\Gamma\,\{f : \mathbb{N} \to \mathbb{N} \vdash ((\lambda x.\lambda y.x\ y)\ f)\ 3 : \mathbb{N}}\text{app}
$$

6 Encoding

Sequencing

$$
\begin{gathered}
t ::= \cdots | t; t' \\
(\lambda x.t')\ t
\end{gathered}
$$

(Assuming x is a free variable not used in x)

Let-in

$$
\begin{gathered}
t ::= \cdots | let\ x = t\ in\ t' \\
(\lambda x.t')\ t
\end{gathered}
$$

Arrays of Length 4

$$
\begin{gathered}
t ::= \cdots | [t, t, t, t] \\
v ::= \cdots | [v, v, v, v] \\
[a, b, c, d] \equiv << a, b >, < c, d >>
\end{gathered}
$$

Array field access

$$
\begin{gathered}
t ::= \cdots | t.i\ (i \in 0..3) \\
t.0 \equiv t.0.0 \\
t.1 \equiv t.0.1 \\
t.2 \equiv t.1.0 \\
t.3 \equiv t.1.1
\end{gathered}
$$

Array update

$$
\begin{gathered}
t ::= \cdots | t.i = t\ (i \in 0..3) \\
t.0 = t' \equiv << t', t.0.1 >, < t.1.0, t.1.1 >>
\end{gathered}
$$