Assignment #1

Diego Oniarti

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1 Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \quad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n" \quad n' \oplus n" = n}{t \oplus t' \Downarrow n} \quad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t" \quad t' \Downarrow v' \quad t"[v'/x] \Downarrow v}{t\ t' \Downarrow v} \quad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \quad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \quad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL\ t \Downarrow inL\ v} \quad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR\ t \Downarrow inR\ v} \quad \text{inRight}$$

$$\frac{t \Downarrow \ inL\ v' \quad t_1[v'/v_1] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ v_1 \mapsto t_1 \\ inR\ v_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching L}$$

$$\frac{t \Downarrow \ inR\ v' \quad t_2[v'/v_2] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL\ v_1 \mapsto t_1 \\ inR\ v_2 \mapsto t_2 \end{array} \right. \Downarrow v} \quad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \tau \to \tau') \to (\mathbb{N} \to \mathbb{N} \to \tau) \to \tau' \\ .d\ (\lambda m : \mathbb{N}.\lambda b : \tau.b)\ (\lambda i : \mathbb{N}.\lambda n : \mathbb{N}.i)$$

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \tau \to \tau') \to (\mathbb{N} \to \mathbb{N} \to \tau) \\ \to \tau'.d\ (\lambda m : \mathbb{N}.\lambda b : \tau.b)\ (\lambda i : \mathbb{N}.\lambda n : \mathbb{N}.i * n_1 + (1 - i) * n_2)$$

4 Typing derivation

$$\dfrac{\dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\text{var} \quad \dfrac{\dfrac{\dfrac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\text{var} \quad \dfrac{}{\Gamma' \vdash 2 : \mathbb{N}}\text{nat}}{\Gamma' \left\{ \begin{smallmatrix} f:\mathbb{N}\to\mathbb{N}, \\ x:\mathbb{N} \end{smallmatrix} \right. \vdash x + 2 : \mathbb{N}}\text{op}}{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}\text{lam} \quad \dfrac{}{\Gamma \vdash 4 : \mathbb{N}}\text{nat}}{\Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}}\text{app}}{\Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N}}\text{app}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N} \to \mathbb{N}}\text{var} \quad \dfrac{y : \mathbb{N} \in \Gamma'}{\Gamma' \vdash y : \mathbb{N}}\text{var}}{\Gamma' \left\{ \begin{smallmatrix} \Gamma, \\ x:\mathbb{N}\to\mathbb{N}, \\ y:\mathbb{N} \end{smallmatrix} \right. \vdash x\ y : \mathbb{N}}\text{app}}{\Gamma, \atop x:\mathbb{N}\to\mathbb{N} \vdash \lambda y.x\ y : \mathbb{N} \to \mathbb{N}}\text{lam}}{\Gamma \vdash \lambda x.\lambda y.x\ y : (\mathbb{N} \to \mathbb{N}) \to \mathbb{N} \to \mathbb{N}}\text{lam} \quad \dfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\text{var}}{\Gamma \vdash (\lambda x.\lambda y.x\ y)\ f : \mathbb{N} \to \mathbb{N}}\text{app} \quad \dfrac{}{\Gamma \vdash 3 : \mathbb{N}}\text{nat}}{\Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash ((\lambda x.\lambda y.x\ y)\ f)\ 3 : \mathbb{N}}\text{app}$$