Assignment #1

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1   Big step - call by name

Write the operational semantics rules for a big-step, call-by-name reduction for
ULC. Write the semantically correct ones only, but write them all

$$\frac{}{v \Downarrow v} \qquad \text{val}$$

$$\frac{t \Downarrow n' \quad t' \Downarrow n'' \quad n' \oplus n'' = n}{t \oplus t' \Downarrow n} \qquad \text{bs-bop}$$

$$\frac{t \Downarrow \lambda x.t'' \quad t''[t'/x] \Downarrow v}{t \; t' \Downarrow v} \qquad \text{bs-app}$$

$$\frac{t_1 \Downarrow v_1 \quad t_2 \Downarrow v_2}{\langle t_1, t_2 \rangle \Downarrow \langle v_1, v_2 \rangle} \qquad \text{pair}$$

$$\frac{t \Downarrow \langle v, v' \rangle}{t.1 \Downarrow v} \qquad \text{first-projection}$$

$$\frac{t \Downarrow \langle v', v \rangle}{t.2 \Downarrow v} \qquad \text{first-projection}$$

$$\frac{t \Downarrow v}{inL \; t \Downarrow inL \; v} \qquad \text{inLeft}$$

$$\frac{t \Downarrow v}{inR \; t \Downarrow inR \; v} \qquad \text{inRight}$$

$$\frac{t \Downarrow \; inL \; v' \quad t_1[v'/x_1] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL \; x_1 \mapsto t_1 \\ inR \; x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \qquad \text{pattern matching L}$$

$$\frac{t \Downarrow \; inR \; v' \quad t_2[v'/x_2] \Downarrow v}{\text{case } t \text{ of } \left| \begin{array}{l} inL \; x_1 \mapsto t_1 \\ inR \; x_2 \mapsto t_2 \end{array} \right. \Downarrow v} \qquad \text{pattern matching R}$$

2 Equivalence of SOS and COS

3 Distinguish terms

$$t \stackrel{def}{=} \lambda d : (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N} \to \mathbb{N}) \to \mathbb{N}$$
$$.d \; (\lambda a : \mathbb{N}.\lambda b : \mathbb{N}.b) \; (\lambda i : \mathbb{N}.\lambda j : \mathbb{N}.i)$$

Reduction 1

$(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 0\ 0)) \to\ \beta \qquad E = []$

$(\lambda x.\lambda y.x\ 0\ (y\ 0\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) \qquad \to \frac{\beta}{app1} \quad E = [](\lambda i.\lambda j.i)$

$(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 0\ 0))\ (\lambda i.\lambda j.i) \qquad \to\ \beta \qquad E = []$

$(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 0\ 0) \qquad \to \frac{\beta}{app1} \quad E = []((\lambda i.\lambda j.i)\ 0\ 0)$

$(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 0\ 0) \qquad \to \frac{\overline{\frac{\beta}{app1}}}{app2} \quad E = (\lambda b.b)\ ([]\ 0)$

$(\lambda b.b)\ ((\lambda j.0)\ 0) \qquad \to \frac{\beta}{app2} \quad E = (\lambda b.b)\ []$

$(\lambda b.b)\ 0 \qquad \to\ \beta \qquad E = []$

$\quad 0$

Reduction 2

$(\lambda d.d\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i))\ (\lambda x.\lambda y.x\ 0\ (y\ 1\ 0)) \to\ \beta \qquad E = []$

$(\lambda x.\lambda y.x\ 0\ (y\ 1\ 0))\ (\lambda a.\lambda b.b)\ (\lambda i.\lambda j.i) \qquad \to \frac{\beta}{app1} \quad E = [](\lambda i.\lambda j.i)$

$(\lambda y.(\lambda a.\lambda b.b)\ 0\ (y\ 1\ 0))\ (\lambda i.\lambda j.i) \qquad \to\ \beta \qquad E = []$

$(\lambda a.\lambda b.b)\ 0\ ((\lambda i.\lambda j.i)\ 1\ 0) \qquad \to \frac{\beta}{app1} \quad E = []((\lambda i.\lambda j.i)\ 0\ 0)$

$(\lambda b.b)\ ((\lambda i.\lambda j.i)\ 1\ 0) \qquad \to \frac{\overline{\frac{\beta}{app1}}}{app2} \quad E = (\lambda b.b)\ ([]\ 0)$

$(\lambda b.b)\ ((\lambda j.1)\ 0) \qquad \to \frac{\beta}{app2} \quad E = (\lambda b.b)\ []$

$(\lambda b.b)\ 1 \qquad \to\ \beta \qquad E = []$

$\quad 1$

4 Safe untypable term

typing derivation

$$\cfrac{\cfrac{\cfrac{???}{x:\tau \vdash x\ x:\tau'}\text{Err}}{\emptyset \vdash \lambda x.x\ x:\tau \to \tau'}\text{lam} \quad \emptyset \vdash \lambda y.y\ y:\tau}{\emptyset \vdash (\lambda x.x\ x)\ (\lambda y.y\ y):\tau'}\text{app}$$

COS-SM-CBV

$(\lambda x.x\ x)\ (\lambda y.y\ y) \to \beta \quad E = []$

$(\lambda y.y\ y)\ (\lambda y.y\ y) \to \beta \quad E = []$

$(\lambda y.y\ y)\ (\lambda y.y\ y) \to \beta \quad E = []$

$... \qquad\qquad\qquad \to$

5 Typing derivation

$$
\cfrac{
  f : \mathbb{N} \to \mathbb{N} \in \Gamma
  \quad
  \cfrac{
    \cfrac{
      \cfrac{x : \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N}}\ \text{var}
      \quad
      \cfrac{}{\Gamma' \vdash 2 : \mathbb{N}}\ \text{nat}
    }{
      \cfrac{\Gamma' \left\{ \begin{smallmatrix} f:\mathbb{N}\to\mathbb{N}, \\ x:\mathbb{N} \end{smallmatrix} \right. \vdash x + 2 : \mathbb{N}}{\Gamma \vdash \lambda x.x + 2 : \mathbb{N} \to \mathbb{N}}\ \text{lam}
    }\ \text{op}
    \quad
    \cfrac{}{\Gamma \vdash 4 : \mathbb{N}}\ \text{nat}
  }{
    \Gamma \vdash (\lambda x.x + 2)\ 4 : \mathbb{N}
  }\ \text{app}
}{
  \cfrac{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}{\Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash f\ ((\lambda x.x + 2)\ 4) : \mathbb{N}}\ \text{var}
}\ \text{app}
$$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{x : \mathbb{N} \to \mathbb{N} \in \Gamma'}{\Gamma' \vdash x : \mathbb{N} \to \mathbb{N}}\ \text{var}
        \quad
        \cfrac{y : \mathbb{N} \in \Gamma'}{\Gamma' \vdash y : \mathbb{N}}\ \text{var}
      }{
        \Gamma' \left\{ \begin{smallmatrix} \Gamma, \\ x:\mathbb{N}\to\mathbb{N}, \\ y:\mathbb{N} \end{smallmatrix} \right. \vdash x\ y : \mathbb{N}
      }\ \text{app}
    }{
      \cfrac{\Gamma,_{x:\mathbb{N}\to\mathbb{N}} \vdash \lambda y.x\ y : \mathbb{N} \to \mathbb{N}}{\Gamma \vdash \lambda x.\lambda y.x\ y : (\mathbb{N} \to \mathbb{N}) \to \mathbb{N} \to \mathbb{N}}\ \text{lam}
    }\ \text{lam}
    \quad
    \cfrac{f : \mathbb{N} \to \mathbb{N} \in \Gamma}{\Gamma \vdash f : \mathbb{N} \to \mathbb{N}}\ \text{var}
  }{
    \Gamma \vdash (\lambda x.\lambda y.x\ y)\ f : \mathbb{N} \to \mathbb{N}
  }\ \text{app}
  \quad
  \cfrac{}{\Gamma \vdash 3 : \mathbb{N}}\ \text{nat}
}{
  \Gamma \{ f : \mathbb{N} \to \mathbb{N} \vdash ((\lambda x.\lambda y.x\ y)\ f)\ 3 : \mathbb{N}
}\ \text{app}
$$