Assignment #2

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1 Missing Progress Cases

Write the proof for the progress theorem for the following cases

- $t \equiv \ inl\ t_1$
- $t \equiv \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right.$

Theorem:

$$\text{if } \emptyset \vdash t : \tau \text{ then either } \vdash t.VAL \text{ or } \exists t'.t \rightsquigarrow t'$$

Prof by induction on the typing derivation of t.
Base cases *t-var* and *t-nat* seen in class

1.1 $t \equiv \ inl\ t_1$

$$\text{if } \emptyset \vdash \ inl\ t : \tau_1 \uplus \tau_2 \text{ then either } \vdash \ inl\ t.\text{VAL or } \exists t'.\ inl\ t \rightsquigarrow t'$$

$$\begin{array}{rll} & \dfrac{\emptyset \vdash t : \tau_1}{\emptyset \vdash \ inl\ t : \tau_1 \uplus \tau_2}\text{T-inl} & \\ \text{t.s. either} & \vdash \ inl\ t.\text{VAL} & \\ \text{or} & \exists t'.\ inl\ t \rightsquigarrow t' & \\ \text{by I.H. either} & \vdash t.\text{VAL} & \text{I1} \\ \text{or} & \exists t".t \rightsquigarrow t" & \text{I2} \end{array}$$

Assuming I1 $\quad$ $inl\ t.$VAL by I1 and definition of inl . $\square$

Assuming I2

$$\begin{array}{rll} \text{by inversion on I2 w.h.} & t \equiv E[t_0] & \text{HE1} \\ & t" \equiv E[t_0"] & \text{HE1'} \\ & t_0 \rightsquigarrow^p t_0" & \text{HPR} \\ \text{by HE1, HE1' t.s.} & inl\ E[t_0] \rightsquigarrow \ inl\ E[t_0"] & \\ \text{by ctx with HPR and} & E' = \ inl\ E_\square & \end{array}$$

1.2 $t \equiv$ **case** t_0 **of** $\left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right.$

$$\text{if } \underbrace{\emptyset \vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. : \tau}_{\text{HY}} \text{ then either}$$

$$\cancel{\vdash \text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. .\text{VAL}}$$

$$\text{or } \exists t'.\text{case } t_0 \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow t'$$

$$\begin{array}{rll} by I.H.w.h. & \text{either} \vdash t_0.\text{VAL} & \text{I1} \\ & \text{or} \exists t_0'.t_0 \rightsquigarrow t_0' & \text{I2} \end{array}$$

Assuming I1

$$\text{by typing definition of case and HY } \begin{array}{rl} \text{either} & t_0 \equiv\ inl\ t_0^l \\ \text{or} & t_0 \equiv\ inr\ t_0^r \end{array}$$

- $t_0 \equiv\ inl\ t_0^l$

  by COS rule for case: case $inl\ t_0^l$ of $\left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow^p t_1[t_0^l/x_1]_\Box$

  (qui devo probabilmente inserire che $\mathbb{E}$=[] e tutta la tiritera)

- $t_0 \equiv\ inr\ t_0^r$

  by COS rule for case: case $inr\ t_0^r$ of $\left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow^p t_2[t_0^r/x_2]_\Box$

Assuming I2

$$\begin{array}{rll} \text{by inversion on I2 w.h.} & t_0 \equiv E[t_0^z] & \text{HE1} \\ & t_0' \equiv E[t_0'^z] & \text{HE1'} \\ & t_0^z \rightsquigarrow^p t_0'^z & \text{HPR} \\ \text{by HE1, HE1' t.s.} & \text{case } E[t_0^z] \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. \rightsquigarrow \text{case } E[t_0'^z] \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right. & \\ \text{by ctx with HPR and} & E' = \text{case } E \text{ of } \left| \begin{array}{l} inl\ x_1 \mapsto t_1 \\ inr\ x_2 \mapsto t_2 \end{array} \right._\Box & \end{array}$$

2 Missing Compatibility Lemmas

Write the proof for these compatibility lemmas:

- Pairs
  Assuming:
  - $\Gamma \vDash t_1 : \tau_1$
  - $\Gamma \vDash t_2 : \tau_2$

  Prove:
  - $\Gamma \langle t_1, t_2 \rangle : \tau_1 \times \tau_2$

3 Adding Cycles

Add `for` and `while` constructs to STLC. Add their syntax, their typing and their operational semantics in COS.

3.1 while

$$\begin{aligned} t &= \cdots |\text{while } t \text{ do } t \text{ end} \\ v &= \cdots |\Xi \\ \tau &= \cdots |\xi \\ \mathbb{E} &= \cdots |\text{while } \mathbb{E} \text{ do } t \text{ end} \end{aligned}$$

$$\frac{\Gamma \vdash t_0 : bool \quad \Gamma \vdash t_1 : \tau}{\Gamma \vdash \text{while } t_0 \text{ do } t_1 \text{ end} : \xi}\text{t-while}$$

$$\frac{}{\text{while } true \text{ do } t \text{ end} \leadsto^p t; \text{while } true \text{ do } t \text{ end}}\text{c-whileT}$$

$$\frac{}{\text{while } false \text{ do } t \text{ end} \leadsto^p \Xi}\text{c-whileF}$$

3.2 for

$$\begin{aligned} t &= \cdots |\text{for } i = t \text{ to } t \text{ do } t \text{ end} \\ v &= \cdots |\Xi \\ \tau &= \cdots |\xi \\ \mathbb{E} &= \cdots |\text{for } i = \mathbb{E} \text{ to } t \text{ do } t \text{ end} \\ & \quad\; |\text{for } i = n \text{ to } \mathbb{E} \text{ do } t \text{ end} \end{aligned}$$

$$\frac{\Gamma \vdash t_0 : \mathbb{N} \quad \Gamma \vdash t_1 : \mathbb{N} \quad \Gamma, i : \mathbb{N} \vdash t_2 : \tau}{\Gamma \vdash \text{for } i = t_0 \text{ to } t_1 \text{ do } t_2 \text{ end} : \xi}\text{t-for}$$

$$\frac{n_1 < n_2 \quad n_1' = n_1 + 1}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p t[n/i]; \text{for } i = n_1' \text{ to } n_2 \text{ do } t \text{ end}}\text{c-for1}$$

$$\frac{n_1 \geq n_2}{\text{for } i = n_1 \text{ to } n_2 \text{ do } t \text{ end} \rightsquigarrow^p \Xi}\text{c-for2}$$