CSE306 - Computer Graphics

# free-surface 2D fluid solver

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# Contents

# 1 Introduction

Fluid simulation refers to the sub-field of Computer Graphics which is interested in simulating liquids and gases. In the following report an approach we will showcased on how to simulate free-surface 2D fluids. This method relies on the incompressible Euler's equations, which read

$$\frac{\partial u}{\partial t} + u.\Delta u + \frac{1}{\rho}\Delta p = g + \nu \Delta u$$

$$divu = 0$$

First of all, we will start by showcasing the representation we choose for our fluid. Then we will explain the method that was used to simulate the liquid. In particular the goal of this project was to be able to create a 2D-fluid animation. All the code, pictures, and animations are available on the git repository associated to this report.

# 2 Diagrams

In the following we will showcase two different methods that will allow us to represent cells in a plane.

## 2.1 Voronoï Diagrams

A first approach is to use Voronoï cells to represent air and liquid cells in the animation we seek to construct. To compute these cells we use the Sutherland-Hodgman (with complexity $O(n^2)$). We can see an example in Figure 1.

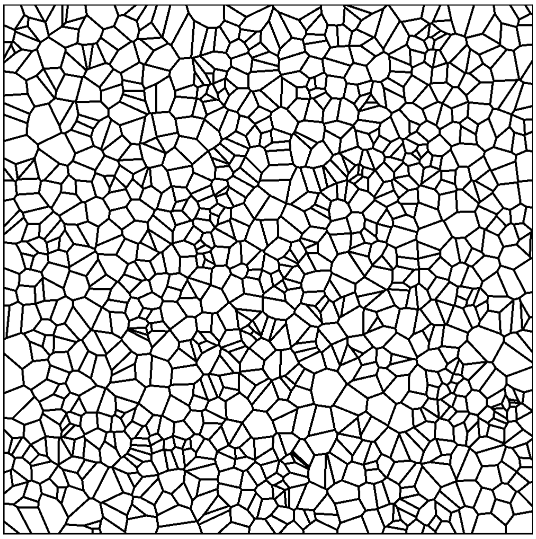

Figure 1: Here we have a Voronoï Diagram composed of 1k points. It was generated in around 2s.

A Centroidal Voronoï Tessellation is a Voronoï diagram in which sites coincide with cell barycenters. This Diagram is useful to create well-spaced air cells that we will need in the future. A strategy to obtain this diagram is to start with random points $x_i$'s, compute the Voronoï Diagram, and move the $x_i$'s to the center of their respective cell. This process is also referred to as a Lloyd iteration. We can see an example in Figure 2.

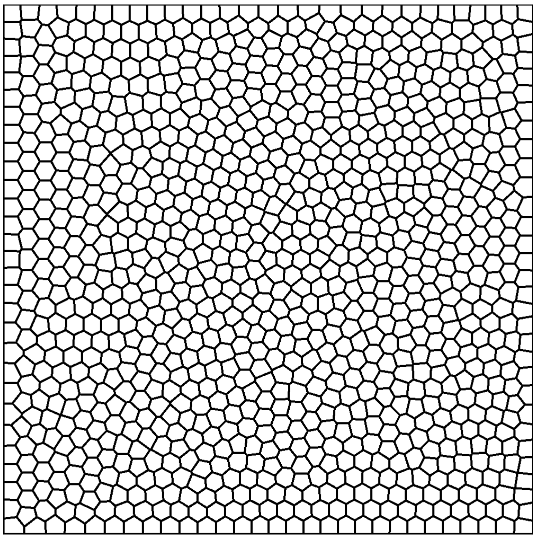

Figure 2: Here we have a Voronoï Diagram composed of 1k points after 100 Lloyd Iterations. Each Lloyd iteration takes about 140ms.

### 2.2 Power Diagrams and Optimal Transport

Power Diagrams are an extension of Voronoï Diagrams that add a parameter weights that help us influence the size of the of each point. Nevertheless, these weights do not directly correspond to the area of the cell. This now becomes an Optimal Transport Problem. In order to solve this, it was given to us a function to maximize, which we do thanks to the quasi_Newton solver (L-BFGS) in our repository. We can see the example in Figure 3.

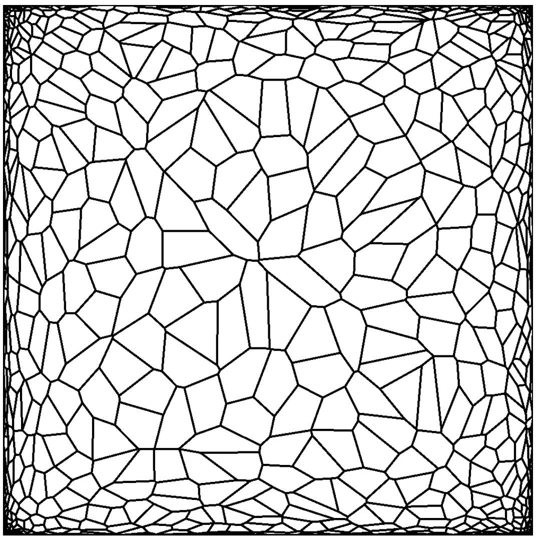

Figure 3: Here we have a Voronoï Diagram composed of 2k points. The area of each cell is proportional to $\exp \frac{||y_i - C||^2}{0.02}$ for each point $y_i$

## 3 Fluid Simulation

Now that we have all of these tools, it is simple to perform our fluid simulation. We use the Gallouët Mérigot scheme. The main idea is that, we can now implement some forces to move our points every given time step. Then, thanks to our method to compute Power Diagrams we can set an constant area for the liquid in the scene, and maintain it for all water cells throughout our animation. This results then in the simulation of in-compressible fluids in 2D. Results can be seen in the `Animations` folder in our repository. One can find 3 main animations. First, `test1.mp4` is the most detailed one with 700 fluid particles, 2500 Air particles, nevertheless with our current Naïve algorithms it took about one night to generate 5s of video. The `space` animation is composed of 250 particles and 70 fluid particles. It took 10 min to generate 17s of video.

(We used $m_i = 200, = 0.04, dt = 0.02$ for more information about these variables and detailed definitions to the methods and mathematical objects used refer to the Textbook for the CSE306 course.)