As $n \to \infty$, $\ln n \ll n^p \ll b^n \ll n! \ll n^n$, where $p > 0$ and $b > 1$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{conv} & p > 1 \\ \text{div} & p \leq 1 \end{cases}$$

(4p)

2. **Resolver:**

$$x^3y''' - 6x^2y'' + 10xy' + 28y = x^9 Ln\,(x^3) senh(Ln(x^3)) \cosh(Ln(x^3))$$

(4p)

Cauchy - Euler : $x^n y^{(n)} = D(D-1)\ldots(D-(n-1))\,y$

$D(D-1)(D-2)y - 6D(D-1)y + 10Dy + 28y = 0$

$(D^3 - 3D^2 + 2D - 6D^2 + 6D + 10D + 28)\,y = 0$

$(D^3 - 9D^2 + 18D + 28)\,y = 0$

$(D+1)(D^2 - 10D + 28)\,y = 0$

	1	-9	18	28
-1		-1	10	-28
	1	-10	28	

Ec. Carac. $(r+1)(r^2 - 10r + 28) = 0$

$r = -1$, $r = 5 + \sqrt{3}i$, $r = 5 - \sqrt{3}i$

$V = C_1 e^{-t} + e^{5t}\left(C_2 \cos(\sqrt{3}t) + C_3 \operatorname{sen}(\sqrt{3}t)\right)$

$V(x) = \frac{C_1}{x} + x^5\left(C_2 \cos(\sqrt{3}\ln x) + C_3 \operatorname{sen}(\sqrt{3}\ln x)\right)$

Sol. part.:

$f(x) = x^9 \ln(x^3) \operatorname{senh}(Ln(x^3)) \cosh(Ln(x^3))$, $x = e^t$

$f(e^t) = e^{9t}(3t) \operatorname{senh}(3t) \cosh(3t)$

$= \frac{3}{4}t\left(e^{15t} - e^{3t}\right)$

$Ly = (D+1)(D^2 - 10D + 28)\,y = \underbrace{\frac{3}{4}t e^{15t}}_{f_1(t)} - \underbrace{\frac{3}{4}t e^{3t}}_{f_2(t)}$

y_1 sol part de $Ly_1 = f_1(t) = \frac{3}{4}te^{15t}$

$y_1 = e^{15t}(a + bt)$

$(D+1)(D^2 - 10D + 28)\left[e^{15t}(a+bt)\right] = \frac{3}{4}te^{15t}$

$\cancel{e^{15t}}(D+16)(\underbrace{D^2 + 30D + 225 - 10D - 150 + 28}_{D^2 + 20D + 103})[a+bt] = \frac{3}{4}t\cancel{e^{15t}}$

$(\cdots 423D + 1648)(a+bt) = \frac{3}{4}t$

$\underbrace{423b + 1648a} + \underbrace{1648bt} = \underbrace{\frac{3}{4}t} \Rightarrow \begin{cases} 1648b = \frac{3}{4} \\ b = \frac{3}{6592} \end{cases}$

$423\left(\frac{3}{6592}\right) + 1648a = 0$

$a = \ldots$

$P(D)\left[e^{rt} q(t)\right] = e^{rt} P(D+r)\left[q(t)\right]$

$y_1 = a + \frac{3}{6\sqrt{92}}\,(t) \sim \ln x$ / $y_2 = \cdots$

$$\boxed{y = v + y_p}$$

Wronskiano

$$y'' + 2y' + 2y = e^{-x}\csc x$$

$$(D^2+2D+2)\,y = e^{-x}\csc x$$

e^{x}
sen x   cos x
p(x)

$\operatorname{senh} x \downarrow \frac{e^{x}-e^{-x}}{2}$ $\cosh x \downarrow \frac{e^{x}+e^{-x}}{2}$

Ec. carac. (homogénea)

$r^2 + 2r + 2 = 0 \qquad (r+1)^2 + 1$

$r_1 = -1 + i, \quad r_2 = -1 - i$

Sol. complementaria

$$\boxed{v = e^{-x}(c_1 \cos x + c_2 \operatorname{sen} x)}$$

$$\begin{cases} y_1 = e^{-x}\cos x \\ y_2 = e^{-x}\operatorname{sen} x \end{cases}$$

$$\begin{cases} u_1' h_1 + u_2' h_2 + \cdots + u_n' h_n = 0 \\ u_1' h_1' + u_2' h_2' + \cdots + u_n' h_n' = 0 \\ \vdots \\ u_n' h_1^{(n)} + u_2' h_2 + \cdots + u_n' h_n = R(x) \end{cases}$$

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = e^{-x}\csc x \end{cases}$$

$$W[y_1, y_2] = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x}\cos x & e^{-x}\operatorname{sen} x \\ e^{-x}(-\operatorname{sen} x - \cos x) & e^{-x}(\cos x - \operatorname{sen} x) \end{vmatrix} = e^{-2x} \neq 0$$

$$u_1' = \frac{\begin{vmatrix} 0 & e^{-x}\operatorname{sen} x \\ e^{-x}\csc x & \cdots \end{vmatrix}}{W} = -\frac{e^{-2x}}{e^{-2x}} = -1 \Rightarrow u_1 = x + \cancel{c_3}$$

$$u_2' = \frac{\begin{vmatrix} e^{-x}\cos x & 0 \\ \cdots & e^{-x}\csc x \end{vmatrix}}{W} = \frac{\cancel{e^{-2x}}\cot x}{\cancel{e^{-2x}}} = \cot x \Rightarrow u_2 = \ln(\operatorname{sen} x) + \cancel{c_4}$$

$$y_p = u_1 y_1 + u_2 y_2 = \boxed{(x + c_3)\,e^{-x}\cos x + (\ln(\operatorname{sen} x) + c_4)\,e^{-x}\operatorname{sen} x}$$

$$y = v + y_p = \boxed{c_1 e^{-x}\cos x + c_2 e^{-x}\operatorname{sen} x + x e^{-x}\cos x + \ln(\operatorname{sen} x)\, e^{-x}\operatorname{sen} x}$$

$$(D^2+D+1)^2 y = x^2 e^{2x}$$

Ec. carac.

$$(r^2+r+1)^2 = 0$$

$r_1 = -\frac{1}{2}+\frac{\sqrt{3}}{2}i \qquad r_2 = -\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (multiplicidad 2)

Sol. comp.

$$\boxed{U = e^{-\frac{x}{2}}\left[(c_1+c_2x)\cos\left(\tfrac{\sqrt{3}}{2}x\right)+(c_3+c_4x)\,\text{sen}\left(\tfrac{\sqrt{3}}{2}x\right)\right]}$$

$f(x) = x^2e^{2x} \Rightarrow y_p$ sol. part.

$$\boxed{y_p = e^{2x}(a+bx+cx^2)}$$

$$Ly_p = f(x)$$

$$(D^2+D+1)^2\left[e^{2x}(a+bx+cx^2)\right] = x^2e^{2x}$$

$$\cancel{e^{2x}}\left((D+2)^2+(D+2)+1\right)^2\left[a+bx+cx^2\right] = x^2\cancel{e^{2x}}$$

$$(D^2+5D+7)^2\left[a+bx+cx^2\right] = x^2$$

$$(D^4+10D^3+39D^2+70D+49)\left[a+bx+cx^2\right] = x^2$$

$$78c+70b+140cx+49a+49bx+49cx^2 = x^2$$

$$\underbrace{(78c+70b+49a)}_{0}+x\underbrace{(140c+49b)}_{0}+\underbrace{49c}_{1}x^2 = x^2$$

$$p(D)\left[e^{rx}q(x)\right] = e^{rx}p(D+r)\left[q(x)\right]$$

$$D(a+bx+cx^2) = b+2cx$$

$$D^2(a+bx+cx^2) = 2c$$

$49c = 1$, $c = \frac{1}{49}$ | $140\left(\frac{1}{49}\right)+49b = 0$, $b = -\frac{20}{343}$ | $78\left(\frac{1}{49}\right)+70\left(\frac{-20}{343}\right)+49a = 0$, $a = \frac{122}{2401}$

$$\boxed{y_p = \left(\frac{122}{2401}-\frac{20}{343}x+\frac{x^2}{49}\right)e^{2x}}$$

$$\therefore \boxed{y = U+y_p = e^{-\frac{x}{2}}\left[(c_1+c_2x)\cos\left(\tfrac{\sqrt{3}}{2}x\right)+(c_3+c_4x)\,\text{sen}\left(\tfrac{\sqrt{3}}{2}x\right)\right]+\left(\frac{122}{2401}-\frac{20}{343}x+\frac{x^2}{49}\right)e^{2x}}$$

$$(D+1)^2 y = (2x+1)e^{-x}$$

Sol. homog.

Ec. Caract.

$$(r+1)^2 = 0 \Rightarrow r = -1 \quad \text{(doble)}$$

$$\boxed{v = (c_1 + c_2 x)e^{-x}}$$

y_p sol part.

$$Ly_p = f(x) = (2x+1)e^{-x}$$

$$\boxed{y_p = x^2 e^{-x}(a+bx)} \quad ✓$$

$$(D+1)^2\left[x^2 e^{-x}(a+bx)\right] \stackrel{!}{=} (2x+1)e^{-x}$$

$$P(D)\left[e^{rx} q(x)\right] = e^{rx} P(D+r)\left[q(x)\right]$$

$$e^{-x}(D\cancel{-1}\cancel{+1})^2\left[(ax^2+bx^3)\right] = (2x+1)e^{-x}$$

$$\cancel{e^{-x}} D^2(ax^2+bx^3) = (2x+1)\cancel{e^{-x}}$$

$$2a + \underbrace{6b}x = \underbrace{2}x + 1$$

$6b = 2$, $b = \frac{1}{3}$ | $2a = 1$, $a = \frac{1}{2}$ $\Rightarrow y_p = x^2 e^{-x}\left(\frac{1}{2} + \frac{x}{3}\right)$

$$\boxed{y = v + y_p = e^{-x}(c_1 + c_2 x) + x^2 e^{-x}\left(\frac{1}{2} + \frac{1}{3}x\right)}$$

As $n \to \infty$, $\ln n \ll n^p \ll b^n \ll n! \ll n^n$, where $p > 0$ and $b > 1$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{conv} & p > 1 \\ \text{div} & p \leq 1 \end{cases} \quad \text{(Serie } p\text{)}$$

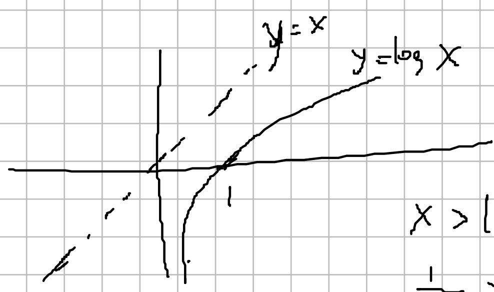
Crit. de comparación

$\sum a_n$ $\quad \sum b_n$

¿?

$$\begin{cases} a_n < b_n \wedge \sum b_n \text{ conv} \Rightarrow \sum a_n \text{ conv} \\ a_n > b_n \wedge \sum b_n \text{ div} \Rightarrow \sum a_n \text{ div} \end{cases}$$

$$S = \sum_{n=1}^{\infty} \underbrace{\frac{n^2+n+2}{\log(n+1)}}_{a_n} \quad ¿?$$



$x > \log x$, $\forall x > 1$

$$\frac{1}{\log x} > \frac{1}{x}$$

$$a_n = \frac{n^2+n+2}{\log(n+1)} > \frac{n^2+n+2}{n+1} > \frac{n^2+n}{n+1} = \frac{n\cancel{(n+1)}}{\cancel{n+1}} = n$$

$$a_n > n.$$

$$\sum_{n=1}^{\infty} n > \infty \quad \text{diverge}$$

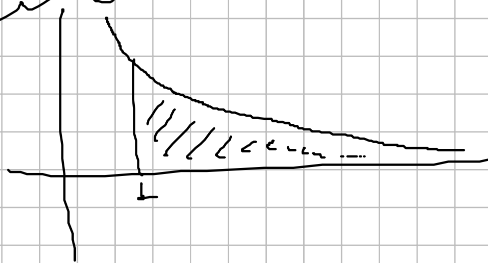
$$\Rightarrow \quad a_n > n \;\wedge\; \sum_{n=1}^{\infty} n > \infty \;\Rightarrow\; \boxed{\sum_{n=1}^{\infty} a_n > \infty}$$

$$\sum_{n=1}^{\infty} \frac{\log_{10} n}{n} \qquad \left(\underline{\text{Crit. de la Integral}}\right)$$

$f(x)$ decreciente $\quad f(x) < f(y)\,,\; x > y$

$$f(x) = \frac{\log_{10} x}{x}$$

$$\log_a b = \frac{\ln b}{\ln a}$$



$$\int_1^{\infty} \frac{\log_{10} x}{x}\, dx = \frac{1}{\ln 10}\int_1^{\infty} \frac{\ln x}{x}\, dx$$

$$= \frac{1}{\ln 10} \cdot \left[\frac{1}{2} \ln^2(x)\right] \Big|_1^{\infty}$$

$$\int_1^{\infty} \frac{\log_{10} x}{x}\, dx > \infty$$

$$\Rightarrow \quad \boxed{\sum_{n=1}^{\infty} \frac{\log_{10} x}{x} > \infty \qquad (\text{Diverge})}$$

Series que tienen términos negativos:

Teorema: $\sum^{\infty} |u_n| < \infty \Rightarrow \sum^{\infty} u_n < \infty$

Obs. La recíproca no siempre es verdad

$* \quad \sum \frac{(-1)^n}{n} < \infty$ pero $\sum \frac{1}{n} > \infty$

Si se da este caso $\Rightarrow$ CONVERGE CONDICIONALMENTE

Teorema: (Series alternadas)

$u_1, u_2, \ldots, u_n$ cumplen

1. Se alternan los signos $+, -$
2. $|u_{n+1}| \leq |u_n|$
3. $\lim_{n\to\infty} u_n = 0$

$\Rightarrow \sum^{\infty} u_n < \infty$

Ejemplo

$\sum_1^{\infty} \frac{(-1)^n}{n}$, términos alternados ① ✓

$|u_n| = \frac{1}{n}$ decreciente, $n < n+1 \Rightarrow \frac{1}{n+1} < \frac{1}{n} \Rightarrow |u_{n+1}| < |u_n|$ ② ✓

$\lim \frac{(-1)^n}{n} = 0$, pues $\forall \varepsilon > 0 \ \exists n_0 \in \mathbb{N}$, $n \geq n_0 \Rightarrow \left| \frac{(-1)^n}{n} - 0 \right| = \frac{1}{n} < \varepsilon \Rightarrow n > \frac{1}{\varepsilon} \Rightarrow n_0 = \left[\!\left[\frac{1}{\varepsilon} \right]\!\right] + 1$ ③ ✓

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{n} < \infty$, pero $\sum_{n=1}^{\infty} \frac{1}{n} > \infty$, entonces la serie CONVERGE CONDICIONALMENTE

Ej $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

- Términos alternados ① ✓
- $|u_{n+1}| \leq |u_n|$, pues $\sqrt{n+1} < \sqrt{n+2} \Rightarrow \frac{1}{\sqrt{n+2}} < \frac{1}{\sqrt{n+1}} \Rightarrow |u_{n+1}| < |u_n|$ ② ✓
- $\lim \frac{(-1)^n}{\sqrt{n+1}} = 0$, pues $\forall \varepsilon > 0 \ \exists n_0 \in \mathbb{N}$, $n \geq n_0 \Rightarrow \left| \frac{(-1)^n}{\sqrt{n+1}} - 0 \right| = \frac{1}{\sqrt{n+1}} < \varepsilon \Rightarrow n > \frac{1}{\varepsilon^2} - 1 \Rightarrow n_0 = \left[\!\left[\frac{1}{\varepsilon^2} \right]\!\right]$ ③ ✓

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}} < \infty$

Verificamos $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\sqrt{n+1}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$, $u_n = \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} = b_n$

$\sum \frac{1}{n^p} \begin{cases} p \leq 1 & \text{div} \\ p > 1 & \text{conv} \end{cases}$

$\Rightarrow \sum_1^{\infty} \frac{1}{\sqrt{n+1}} = \underbrace{\sum_1^{\infty} \frac{1}{\sqrt{n}}}_{\substack{> \infty \\ \text{serie } p}} - 1 > \infty$

$\therefore$ Conv. Cond.

Criterio de la razón

Sea $\sum_{n=1}^{\infty} u_n$ serie, $u_n \neq 0$

$$\lim \left|\frac{u_{n+1}}{u_n}\right| = p \quad \vee \quad \lim \left|\frac{u_{n+1}}{u_n}\right| > \infty$$

$$\begin{cases} p<1, & \text{CONVERGE ABSOLUTAMENTE} \checkmark \\ p>1 \text{ o } \left|\frac{u_{n+1}}{u_n}\right| \to \infty, & \text{DIVERGE} \checkmark \\ p=1, & \cancel{\text{fas cagao}}, \text{ otro } \cancel{\text{ciclo}} \text{ método} \Downarrow \end{cases}$$

Ej: $\sum_{1}^{\infty} \frac{2^n}{n!}$ $\qquad a^4,\ n!,\ (-1)^n$

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}\right| = \frac{2}{n+1}$$

$$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim \frac{2}{n+1} = \underbrace{0}_{p} < 1$$

$\therefore \sum_{1}^{\infty} \frac{2^n}{n!}$ CONV. ABSOLUTAMENTE $\checkmark$

Ej: $\sum_{1}^{\infty} \frac{(-1)^n n}{2^n}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{\cancel{(-1)^{n+1}}(n+1)}{\cancel{2^{n+1}}} \cdot \frac{\cancel{2^n}}{\cancel{(-1)^n}\, n}\right| = \frac{n+1}{2n}$$

$$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2} < 1$$

$\therefore \sum_{1}^{\infty} \frac{(-1)^n n}{2^n}$ CONV. ABSOLUTAMENTE $\checkmark$

Ej: $\sum_{n=1}^{\infty} \frac{(2n)!}{n^{100}}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{(2n+2)!}{(n+1)^{100}} \cdot \frac{n^{100}}{(2n)!} = (2n+2)(2n+1) \cdot \left(\frac{1}{1+\frac{1}{n}}\right)^{100}$$

$$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} (2n+1)(2n+2) \frac{1}{\left(1+\frac{1}{n}\right)^{100}} = \infty$$

$\therefore \sum_{n=1}^{\infty} \frac{(2n)!}{n^{100}}$ DIVERGE

Criterio de la Raíz

Si $\sum u_n$ cumple que $\lim_{n\to\infty} \sqrt[n]{|u_n|} = p$ $\begin{cases} p<1, & \text{CONV. ABSOLUTAMENTE} \\ p>1, & \text{DIVERGE} \end{cases}$

Ej: $\sum_{n=1}^{\infty} \frac{1}{n^n}$; $u_n = \frac{1}{n^n} \Rightarrow \sqrt[n]{|u_n|} = \frac{1}{n} \longrightarrow \underset{\substack{\Downarrow \\ p}}{0} < 1$, si $n \to +\infty$

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^n}$ CONV. ABSOLUTAMENTE ✓

1) $\sum \frac{n!}{10^n}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{(n+1)!}{10^{n+1}} \cdot \cancel{\frac{10^n}{n!}} = \frac{n+1}{10}$$

$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \frac{n+1}{10} = \infty$ $\therefore \sum \frac{n!}{10^n} > \infty$ DIVERGE ✓

2) $\sum \frac{(-1)^{n-1} n!}{10^n}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \frac{n+1}{10} \to +\infty \quad \text{si } n\to+\infty$$

$\therefore \sum \frac{(-1)^{n-1} n!}{10^n} > \infty$ DIVERGE ✓

3) $\sum n\left(\frac{3}{4}\right)^n$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{(n+1)\, 3^{\cancel{n+1}}}{\cancel{4}^{n+1}} \cdot \frac{4^n}{n\, \cancel{3^n}} = \frac{3}{4}\left(1+\frac{1}{n}\right)$$

$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim \frac{3}{4}\left(1+\frac{1}{n}\right) = \underset{\substack{\Downarrow \\ p}}{\frac{3}{4}} < 1$ $\therefore \sum n\left(\frac{3}{4}\right)^n$ CONV. ABSOLUTELY ✓

4) $\sum \frac{(-1)^n}{\sqrt{n}}$ términos alternantes ① ✓

· $|u_{n+1}| \leq |u_n|$, pues $\sqrt{n} < \sqrt{n+1} \Rightarrow \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} \Rightarrow |u_{n+1}| < |u_n|$ ② ✓

· $\lim \frac{(-1)^n}{\sqrt{n}} = 0$, pues $\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N},\ n \geq n_0 \Rightarrow \left|\frac{(-1)^n}{\sqrt{n}} - 0\right| = \frac{1}{\sqrt{n}} < \varepsilon \Rightarrow n > \frac{1}{\varepsilon^2} \Rightarrow n_0 = \left[\!\left[\frac{1}{\varepsilon^2}\right]\!\right]$ ③ ✓

$\therefore \sum \frac{(-1)^n}{\sqrt{n}} < \infty$, pero $\sum \frac{1}{\sqrt{n}} > \infty$ (Serie p) $\Rightarrow \sum \frac{(-1)^n}{\sqrt{n}}$ CONV. CONDITIONALLY

$p = \frac{1}{2} < 1$

⑤ $\sum \frac{(-1)^n}{n\sqrt{n}}$

Como $\sum \left| \frac{(-1)^n}{n\sqrt{n}} \right| = \sum \frac{1}{n^{3/2}} < \infty$ (Serie $p = \frac{3}{2} > 1$)

$\sum |u_n| < \infty \Rightarrow \sum u_n < \infty$

$\therefore \sum \frac{(-1)^n}{n\sqrt{n}}$ CONV. ABSOLUTELY

$\sum |a_n| < \infty$
$\Rightarrow \sum a_n < \infty$

⑥ $\sum \frac{(-1)^n n^2}{2^n}$

$\left| \frac{u_{n+1}}{u_n} \right| = \left| \frac{(n+1)^2}{\cancel{2^{n+1}}} \cdot \frac{\cancel{2^n}}{n^2} \right| = \frac{1}{2}\left(1+\frac{1}{n}\right)^2$

$\lim \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n\to\infty} \frac{1}{2}\left(1+\cancel{\frac{1}{n}}\right)^2 = \underbrace{\frac{1}{2}}_{p} < 1 \quad \therefore \sum \frac{(-1)^n n^2}{2^n}$ CONV. ABSOLUTELY ✓

⑦ $\sum \frac{(-1)^{n+1} \ln(n+1)}{n+1}$ términos alternados ① ✓

$\ln x < x \quad \forall x > 1$

- $|u_{n+1}| \leq |u_n|$ pues

- $\lim \frac{(-1)^{n+1} \ln(n+1)}{n+1} = 0$, pues $\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N},\ n \geq n_0 \Rightarrow \left| \frac{(-1)^{n+1} \ln(n+1)}{n+1} - 0 \right| = \frac{\ln(n+1)}{n+1}$

$\ldots\ 0 < \frac{\ln(n+1)}{n+1} = \frac{2 \ln\sqrt{n+1}}{n+1} < \frac{2\sqrt{n+1}}{n+1} = \frac{2}{\sqrt{n+1}} < \varepsilon \Rightarrow n_0 = \left[\!\left[\frac{4}{\varepsilon^2} \right]\!\right] - 1$ ③ ✓

$\sum \frac{(-1)^{n+1} \ln(n+1)}{n+1} < \infty$

Verificamos $\sum \left| \frac{(-1)^{n+1} \ln(n+1)}{n+1} \right| = \sum \frac{\ln(n+1)}{n+1} > \infty$ (Crit. de la integral)

pues: $f(x) = \frac{\ln(x+1)}{x+1}$; $\int_1^\infty \frac{\ln(x+1)}{x+1}\,dx = \frac{\ln^2(x+1)}{2} \Big|_1^\infty = \frac{\ln^2(\infty)}{2} - \frac{\ln^2(2)}{2} = \infty$

$\therefore \sum \frac{(-1)^{n+1} \ln(n+1)}{n+1} < \infty$, pero $\sum \frac{\ln(n+1)}{n+1} > \infty$, entonces CONV. CONDITIONALLY

⑧ $\sum \frac{2^n n!}{n^n}$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$

$\left| \frac{u_{n+1}}{u_n} \right| = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \frac{2 \cdot \cancel{(n+1)}}{\cancel{(n+1)}} \cdot \frac{n^n}{(n+1)^n} = 2\left(\frac{1}{1+\frac{1}{n}}\right)^n$

$\lim \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n\to\infty} 2\left(\frac{1}{1+\frac{1}{n}}\right)^n = \underbrace{\frac{2}{e}}_{p} < 1 \quad \therefore \sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ CONV. ABSOLUTELY

(9) $\sum \frac{3^n n!}{n^n}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{3^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{3^n n!} = \frac{3\cdot\cancel{(n+1)}}{\cancel{(n+1)}} \cdot \frac{n^n}{(n+1)^n} = 3\left(\frac{1}{1+\frac{1}{n}}\right)^n$$

$$\lim \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} 3\left(\frac{1}{1+\frac{1}{n}}\right)^n = \frac{3}{e} > 1 \quad \therefore \quad \sum \frac{3^n n!}{n^n} \text{ DIVERGE}$$

Serie de potencias

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \quad \text{o} \quad g(x) = \sum_{n=0}^{\infty} c_n x^n$$

Convergencia de SP

Usamos criterio de la razón

Ej: $f(x) = \sum \frac{1}{n} x^n$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{|x|^{n+1}}{n+1} \cdot \frac{n}{|x|^n} = |x| \cdot \frac{1}{1+\frac{1}{n}} \to |x| \text{ si } n \to +\infty$$

1) $|x| < 1 \Rightarrow -1 < x < 1$ CONV. ABSOLUTAMENTE

2) $|x| > 1$ DIVERGE

3) $|x| = 1$

$f(1) = \sum \frac{1}{n}$ DIVERGE

$f(-1) = \sum^{\infty} \frac{(-1)^n}{n}$ CONVERGE CONDITIONALLY

Intervalo de convergencia $x \in [-1, 1)$

Ej: $f(x) = \sum \frac{(-1)^n (x+1)^n}{2^n \cdot n^2}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{|x+1|^{n+1}}{2^{n+1}(n+1)^2} \cdot \frac{2^n \cdot n^2}{|x+1|^n} = \frac{|x+1|}{2}\left(\frac{1}{1+\frac{1}{n}}\right)^2 \Rightarrow \lim_{n\to\infty} \frac{|x+1|}{2}\left(\frac{1}{1+\frac{1}{n}}\right)^2 = \frac{|x+1|}{2}$$

1) $\frac{|x+1|}{2} < 1 \iff -2 < x+1 < 2 \iff -3 < x < 1$ CONV. ABSOLUTELY

2) $\frac{|x+1|}{2} > 1$ DIVERGE

3) $\frac{|x+1|}{2} = 1 \iff x = -3$ ó $x = 1$

* $\sum \frac{(-1)^n (-2)^n}{2^n \cdot n^2} = \sum \frac{1}{n^2} < \infty$ (Serie p)

* $\sum \frac{(-1)^n (2)^n}{2^n \cdot n^2} = \sum \frac{(-1)^n}{n^2} < \infty$ (Conv. absolutely)

$\therefore$ la serie converge en el intervalo $[-3, 1]$

Ej: $f(x) = \sum \frac{(-1)^n x^n}{n!}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(-1)^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n x^n}\right| = |x| \cdot \frac{1}{n+1} \Rightarrow \lim_{n\to\infty} |x| \cdot \frac{1}{n+1} = 0 < 1$$

$\therefore$ $f(x)$ converge absolutamente $\forall x \in \mathbb{R}$

Ej: $f(x) = \sum \frac{(-1)^n n! \, x^n}{10^n}$

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(-1)^{n+1}(n+1)! \, x^{n+1}}{10^{n+1}} \frac{10^n}{(-1)^n n! \, x^n}\right| = |x| \frac{n+1}{10} \Rightarrow \lim_{n\to\infty} \frac{|x|(n+1)}{10}$$

1) $x = 0 \Rightarrow \lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = 0 < 1$ CONV. ABSOLUTELY

2) $x \neq 0 \Rightarrow \lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \infty$ DIVERGE

$\therefore$ $f(x)$ converge para $x = 0$

Serie de Taylor

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Caso especial: Serie de Mc Laurin $(a = 0)$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Mc-Laurin

$f(x) = e^{-\frac{1}{x^2}} \rightarrow f(0) = 0$

$$f'(0) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{-\frac{1}{h^2}} - 0}{h} = 0 \quad , \quad f'(x) = e^{-\frac{1}{x^2}}\left(\frac{2}{x^3}\right)$$

$$f''(0) = \lim_{h\to 0} \frac{f'(h) - f'(0)}{h} = \lim_{h\to 0} \frac{e^{-\frac{1}{h^2}}\left(\frac{2}{h^3}\right) - 0}{h} = \lim_{h\to 0} \frac{2e^{-1/h^2}}{h^4} = 0 \quad , \quad f''(x) = e^{-\frac{1}{x^2}}\left(\frac{4}{x^6} - \frac{6}{x^4}\right)$$

$$f'''(0) = \lim_{h\to 0} \frac{f''(h) - f''(0)}{h} = \lim_{h\to 0}$$

$$f^{(n)}(x) = e^{-\frac{1}{x^2}} P_n\left(\frac{1}{x}\right), \quad P_n(x) = \sum_{i=0}^{n} a_i x^i$$

Hipótesis $\quad f^{(n)}(0) = 0$

$$\Rightarrow f^{(n+1)}(0) = \lim_{h\to 0} \frac{f^{(n)}(h) - \cancel{f^{(n)}(0)}}{h}$$

$$= \lim_{h\to 0} \frac{e^{-\frac{1}{x^2}} P_n\left(\frac{1}{x}\right)}{h}$$

$k = \frac{1}{h}$

$$\Rightarrow \lim_{|k|\to\infty} k e^{-k^2} P(k) = 0$$

$\Rightarrow$ Mc. Laurin

$$f(x) = 0 \neq \begin{cases} e^{-1/x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\Rightarrow f$ es C^∞ pero no es analítica.

Ejemplo 5.4.2 *Halle el intervalo de convergencia de la serie de potencias* $\sum_{n=0}^{+\infty} n^4(2x)^n$.

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{(n+1)^4\, 2^{n+1}\, |x|^{n+1}}{n^4\, 2^n\, |x|^n} = |x|\, 2\left(1+\frac{1}{n}\right)^4 \Rightarrow \lim 2|x|\left(1+\frac{1}{n}\right)^4 = 2|x|$$

Para que la serie sea convergente:

$$2|x| < 1 \Leftrightarrow -1 < 2x < 1 \Leftrightarrow -\frac{1}{2} < x < \frac{1}{2}$$

Si $2|x| = 1 \Rightarrow x = -\frac{1}{2}$ ó $x = \frac{1}{2}$

* $\sum n^4 (-1)^n > \infty$
* $\sum n^4 > \infty$

$\therefore$ Intervalo de convergencia: $\left(-\frac{1}{2}, \frac{1}{2}\right)$

1. $\sum \frac{1}{n\sqrt{n}} = \sum \frac{1}{n^{3/2}}$, $p = 3/2 > 1$

$\therefore \quad \sum \frac{1}{n\sqrt{n}} < \infty$

2. $\sum \frac{1}{(n+1)(n+2)}$

$a_n = \frac{1}{(n+1)(n+2)} < \frac{1}{n^2} = b_n \Rightarrow a_n < b_n$

Como $\sum \frac{1}{n^2} < \infty \Rightarrow \sum \frac{1}{(n+1)(n+2)} < \infty$

3. $\sum \frac{2n+3}{n^2+3n+2}$

Definimos $f(x) = \frac{2x+3}{x^2+3x+2}$

$\int_1^\infty \frac{2x+3}{x^2+3x+2}\,dx = \ln(x^2+3x+2)\Big|_1^\infty = \infty \Rightarrow \sum \frac{2n+3}{n^2+3n+2} > \infty$

4. $\sum \frac{n-1}{n^3}$

$a_n = \frac{n-1}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$

Como $\sum \frac{1}{n^2} < \infty \Rightarrow \sum \frac{n-1}{n^3} < \infty$

5. $\sum \frac{1}{\sqrt{n(n+1)}}$

$a_n = \frac{1}{\sqrt{n(n+1)}} > \frac{1}{\sqrt{(n+1)(n+1)}} = \frac{1}{n+1} = b_n \qquad a_n > b_n$

Como $\sum \frac{1}{n+1} > \infty$ (serie armónica) $\Rightarrow \sum \frac{1}{\sqrt{n(n+1)}} > \infty$

6. $\sum_{n=1}^{\infty} \frac{1}{n+100} = \underbrace{\left(\sum_{n=1}^{\infty} \frac{1}{n}\right)}_{> \infty} - \left(\sum_{n=1}^{100} \frac{1}{n}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n+100} > \infty$

3. Enuncie de forma precisa la ley de enfriamiento de Newton:

(a) Hallar el modelo que se ajuste a esta ley física (2p)

(b) Mi taza de café está a una tempereratura de 60°. Me traslado a una habitación que está a 20° y obsevo que después de 1 minuto la temperatura del café ha descendido a 50°. Estimar en cuanto tiempo se enfria hasta los 30°. (2p)

La velocidad de enfriamiento de un cuerpo en un medio es proporcional a la diferencia entre la temperatura T(t) del cuerpo y la temperatura del medio T A

a)
$$\frac{dT}{dt} = k(T - T_A)$$

$$\int \frac{dT}{T-T_A} = \int k\,dt \Rightarrow \ln|T-T_A| = kt + C_1$$

$$T - T_A = Ce^{kt}$$

$$T(t) = T_A + Ce^{kt}$$

$$t=0 : T(0) = T_0 = T_A + C \Rightarrow C = T_0 - T_A$$

$$\therefore T(t) = T_A + (T_0 - T_A)e^{kt}$$

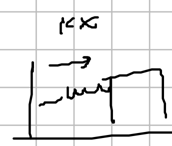
b) $T_0 = 60^\circ$, $T_A = 20^\circ$, $T(1) = 50^\circ$

$$T(1) = 50 = 20 + (40)e^{k}$$

$$\frac{3}{4} = e^{k} \Rightarrow k = \ln\left(\frac{3}{4}\right)$$

$$\Rightarrow T(t) = 20 + 40\left(\frac{3}{4}\right)^t = 30$$

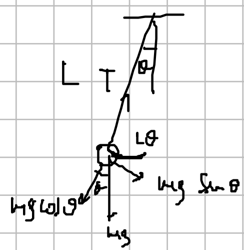
$$\left(\frac{3}{4}\right)^t = \frac{1}{4}$$



$F_R = ma$

$-kx = mx''$

$0 = mx'' + kx$



$\cancel{m}(L\theta)'' = -\cancel{m}g\sin\theta$

$L\theta'' + g\sin\theta = 0$

$\sin\theta \approx \theta$

$L\theta'' + g\theta = 0$

$\boxed{\theta'' + \frac{g}{L}\theta = 0}$

2. Si el wronskiano W de f y g es x^2e^x y $f(x) = x$. Hallar $g(x)$ (4p)

$$W[f,g] = x^2e^x$$

$$W[f,g] = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = \begin{vmatrix} x & g \\ 1 & g' \end{vmatrix} = x^2e^x$$

$$xg' - g = x^2e^x, \quad x \neq 0$$

$$g' - \frac{1}{x}g = xe^x$$

$$\mu(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$\frac{1}{x}g' - \frac{1}{x^2}g = e^x$$

$$\int d\left(\frac{1}{x}g\right) = \int e^x$$

$$\frac{g}{x} = e^x + c$$

$$g(x) = xe^x + cx$$

2. Aplique variación de parámetros para resolver

$$y'' - y = xe^x \qquad (4p)$$

$(D^2-1)y = xe^x$

Ec. caract.

$r^2 - 1 = 0$

$r = \pm 1$

$v = C_1e^x + C_2e^{-x}$

$y_1 = e^x \ , \ y_2 = e^{-x} \Rightarrow \quad y_p = u_1(x)\,y_1(x) + u_2(x)\,y_2(x)$

$$\begin{cases} u_1'e^x + u_2'e^{-x} = 0 \\ u_1'e^x - u_2'e^{-x} = xe^x \end{cases}$$

$$W[y_1, y_2] = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2 \neq 0$$

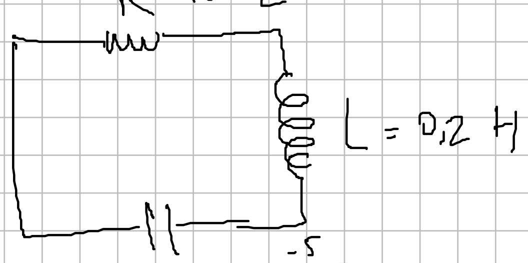
$$u_1' = \frac{\begin{vmatrix} 0 & e^{-x} \\ xe^x & -e^{-x} \end{vmatrix}}{-2} = \frac{x}{2} \Rightarrow u_1(x) = \frac{x^2}{4}$$

$$u_2' = \frac{\begin{vmatrix} e^x & 0 \\ e^x & xe^x \end{vmatrix}}{-2} = -\frac{xe^{2x}}{2} \Rightarrow u_2(x) = \frac{e^{2x}(1-2x)}{8}$$

$$\Rightarrow y_p = \frac{x^2}{4}e^x + \frac{e^x(1-2x)}{8} = e^x\left(\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\right)$$

$$\therefore \ y = v + y_p = C_1e^x + C_2e^{-x} + e^x\left(\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\right)$$

RLC

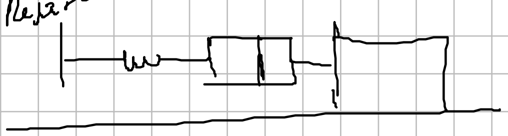


$$Lq'' + Rq' + \frac{q}{C} = 0$$

$$\frac{1}{5}q'' + 40q' + 100000\,q = 0$$

Ec ~ $\quad r = ??$

Resorte



$$mx'' + \beta x' + kx = 0$$