BY
VFI

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From the previous documents, I know that:

$$V_t = \left[(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}$$

Given the homogeneity of preferences and technology, optimal V_t and C_t take the form:

$$V_t = \phi(S_t)W_t \equiv \phi_t W_t$$
$$C_t = b(S_t)W_t \equiv b_t W_t$$

Therefore:

$$V_t = \frac{\phi_t}{b_t}C_t$$

If I define $\nu_t = \frac{\phi_t}{b_t}$:

$$\Leftrightarrow \nu_t C_t = \left[(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t\left[(\nu_{t+1}C_{t+1})^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}$$

$$\Leftrightarrow (\nu_t C_t)^{\frac{1-\gamma}{\theta}} = \left[(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t\left[(\nu_{t+1}C_{t+1})^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta\left(\mathbb{E}_t\left[\left(\nu_{t+1}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta\left(\mathbb{E}_t\left[\left(\nu_{t+1}e^{\ln(\frac{C_{t+1}}{C_t})}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta\left(\mathbb{E}_t\left[\left(\nu_{t+1}e^{\Delta c_{t+1}}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$

Because $\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}$:

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta\left(\mathbb{E}_t\left[\left(\nu_{t+1}e^{(\mu+x_t+\sigma_t\eta_{t+1})}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta\left(\mathbb{E}_t\left[\left(\nu_{t+1}^{1-\gamma}e^{(1-\gamma)(\mu+x_t+\sigma_t\eta_{t+1})}\right)\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta e^{(\frac{1-\gamma}{\theta})(\mu+x_t)}\left(\mathbb{E}_t\left[\left(\nu_{t+1}^{1-\gamma}e^{(1-\gamma)(\sigma_t\eta_{t+1})}\right)\right]\right)^{\frac{1}{\theta}}\right]$$

Since $\eta_{t+1} \sim N(0,1)$ and is uncorrelated with other shocks (in particular ϵ_{t+1}):

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta e^{(\frac{1-\gamma}{\theta})(\mu+x_t)}\left(\mathbb{E}_t\left[\nu_{t+1}^{1-\gamma}\right]\mathbb{E}_t\left[e^{(1-\gamma)(\sigma_t\eta_{t+1})}\right]\right)^{\frac{1}{\theta}}\right]$$

$$\Leftrightarrow \nu_t^{\frac{1-\gamma}{\theta}} = \left[(1-\beta) + \beta e^{(\frac{1-\gamma}{\theta})(\mu+x_t)+\frac{1}{2\theta}(1-\gamma)^2\sigma_t^2}\left(\mathbb{E}_t\left[\nu_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]$$