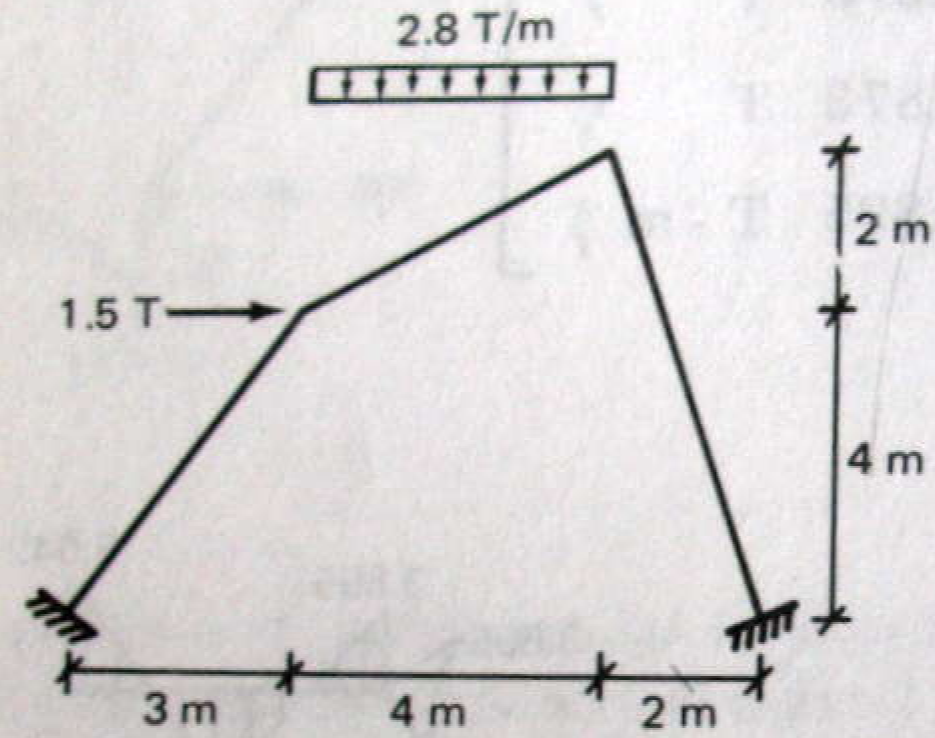
Ejemplo 11.23

Analice el pórtico de la figura:



Dimensiones ($b \times h$ *en cm*)

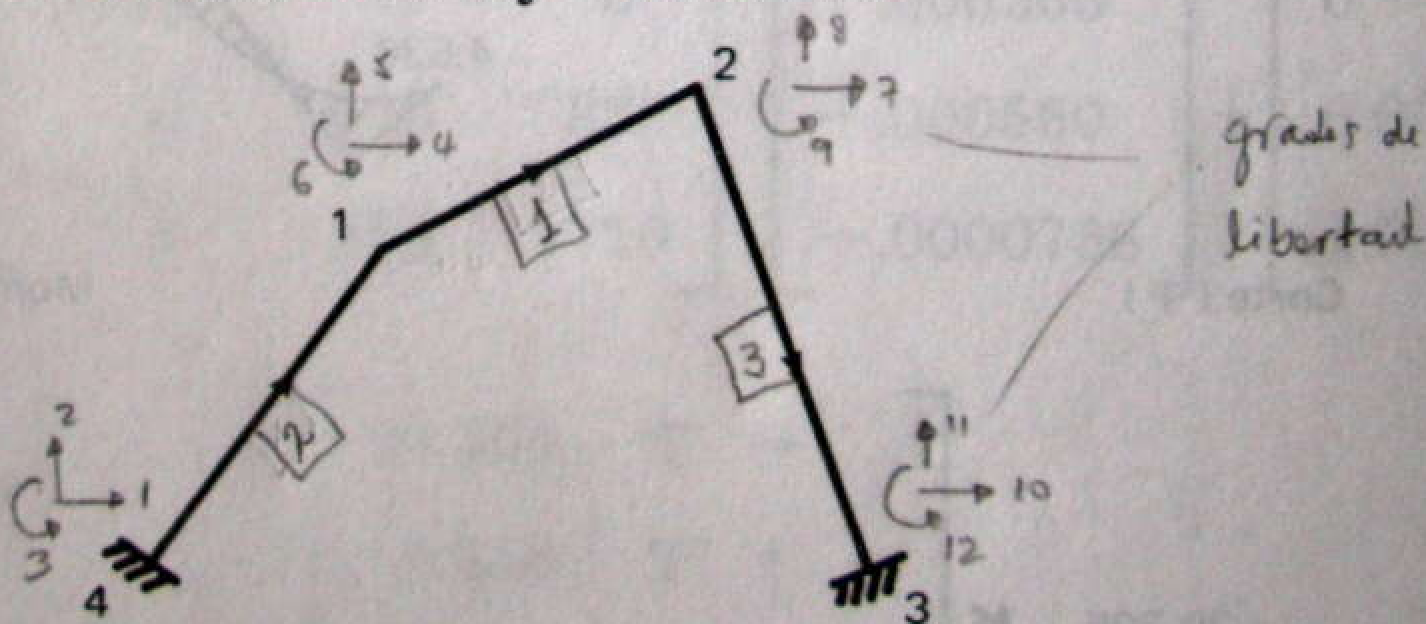
Viga: 30 × 35

Columnas: 30 × 30

$E = 190\ T/cm^2$

Solución

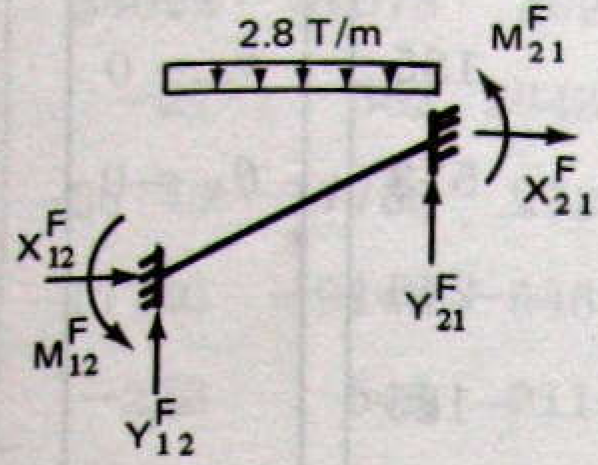
Se adopta la siguiente numeración y orientación:



Los miembros tienen las siguientes propiedades:

Miembro	$\theta°$	λ	μ	AE/L	EI	2 EI/L	4 EI/L	6 EI/L^2	12 EI/L^3
1 - 2	26.56	0.89443	0.44721	44610	2037	911	1822	611	273
4 - 1	53.13	0.60000	0.80000	34200	1282	513	1026	308	123
2 - 3	−71.56	0.31623	−0.94868	27037	1282	406	811	192	61

Las fuerzas de empotramiento de la viga cargada son:



$$X^F_{12} = X^F_{21} = 0$$

$$Y^F_{12} = Y^F_{21} = 2.8 \times 2 = 5.60 \quad \text{T}$$

$$M^F_{12} = -M^F_{21} = \frac{2.8 \times 16}{12} = 3.733 \text{ T - m}$$

Aplicando las ecuaciones (11.59) y (11.61) a cada miembro se obtiene:

barra 1

$$\begin{bmatrix} X_{12} \\ Y_{12} \\ M_{12} \\ X_{21} \\ Y_{21} \\ M_{21} \end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2 \end{array} \\ \begin{bmatrix} 35743 & 17735 & -273 & -35743 & -17735 & -273 \\ 17735 & 9141 & 546 & -17735 & -9141 & 546 \\ -273 & 546 & 1822 & 273 & -546 & 911 \\ -35743 & -17735 & 273 & 35743 & 17735 & 273 \\ -17735 & -9141 & -546 & 17735 & 9141 & -546 \\ -273 & 546 & 911 & 273 & -546 & 1822 \end{bmatrix} \end{array} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 5.60 \\ 3.733 \\ 0 \\ 5.60 \\ -3.733 \end{bmatrix} \quad \text{(a)}$$

$\rightarrow$ fe{2}

barra 2

$$\begin{bmatrix} X_{41} \\ Y_{41} \\ M_{41} \\ X_{14} \\ Y_{14} \\ M_{14} \end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} u_4 & v_4 & \theta_4 & u_1 & v_1 & \theta_1 \end{array} \\ \begin{bmatrix} | & | & | & -12391 & -16357 & -246 \\ | & | & | & -16357 & -21932 & 185 \\ | & | & | & 246 & -185 & 513 \\ | & | & | & 12391 & 16357 & 246 \\ | & | & | & 16357 & 21932 & -185 \\ | & | & | & 246 & -185 & 1026 \end{bmatrix} \end{array} \begin{bmatrix} u_4 = 0 \\ v_4 = 0 \\ \theta_4 = 0 \\ u_1 \\ v_1 \\ \theta_1 \end{bmatrix}$$

(b)

barra 3

$$\begin{bmatrix} X_{23} \\ Y_{23} \\ M_{23} \\ X_{32} \\ Y_{32} \\ M_{32} \end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} u_2 & v_2 & \theta_2 & u_3 & v_3 & \theta_3 \end{array} \\ \begin{bmatrix} 2758 & -8093 & 182 & | & | & | \\ -8093 & 24339 & 61 & | & | & | \\ 182 & 61 & 811 & | & | & | \\ -2758 & 8093 & -182 & | & | & | \\ 8093 & -24339 & -61 & | & | & | \\ 182 & 61 & 406 & | & | & | \end{bmatrix} \end{array} \begin{bmatrix} u_2 \\ v_2 \\ \theta_2 \\ u_3 = 0 \\ v_3 = 0 \\ \theta_3 = 0 \end{bmatrix}$$

(c)

Al ensamblar los términos correspondientes a los nudos libres se llega a:

$$\begin{bmatrix} X_1 & = & X_{12} + X_{14} & = & 1.5 \\ Y_1 & = & Y_{12} + Y_{14} & = & 0 \\ M_1 & = & M_{12} + M_{14} & = & 0 \\ X_2 & = & X_{21} + X_{23} & = & 0 \\ Y_2 & = & Y_{21} + Y_{23} & = & 0 \\ M_2 & = & M_{21} + M_{23} & = & 0 \end{bmatrix} =$$

$$
= \begin{bmatrix}
48134 & 34092 & -27 & -35743 & -17735 & -273 \\
34092 & 31073 & 362 & -17735 & -9141 & 546 \\
-27 & 362 & 2848 & 273 & -546 & 911 \\
-35743 & -17735 & 273 & 38502 & 9642 & 456 \\
-17735 & -9141 & -546 & 9642 & 33480 & -486 \\
-273 & 546 & 911 & 456 & -486 & 2633
\end{bmatrix}
\begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix}
+
\begin{bmatrix} 0 \\ 5.600 \\ 3.733 \\ 0 \\ 5.600 \\ -3.733 \end{bmatrix}
$$

en mi notación del programa utilizo el signo (−)

Y resolviendo el sistema resultante:

$$u_1 = 13.29 \times 10^{-3} \text{ m} \rightarrow$$

$$v_1 = -10.16 \times 10^{-3} \text{ m} \downarrow$$

$$\theta_1 = -1.655 \times 10^{-3} \text{ rad} \;)$$

$$u_2 = 7.09 \times 10^{-3} \text{ m} \rightarrow$$

$$v_2 = 2.10 \times 10^{-3} \text{ m} \uparrow$$

$$\theta_2 = 4.64 \times 10^{-3} \text{ rad} \;)$$

Reemplazando estos valores en las ecuaciones (a), (b) y (c) se obtienen las fuerzas internas, referidas a coordenadas generales:

barra 1

$X_{12} = 3.428$ T $\rightarrow$ $\qquad$ $X_{21} = -3.429$ T $\leftarrow$

$Y_{12} = 5.155$ T $\uparrow$ $\qquad$ $Y_{21} = 6.045$ T $\uparrow$

$M_{12} = -3.452$ T − m) $\qquad$ $M_{21} = -5.185$ T - m)

barra 2

$X_{41} = X_4 = 1.929$ T $\rightarrow$ $\qquad$ $X_{14} = -1.929$ T $\leftarrow$

$Y_{41} = Y_4 = 5.156$ T $\uparrow$ $\qquad$ $Y_{14} = -5.156$ T $\downarrow$

$M_{41} = M_4 = 4.301$ T -m) $\qquad$ $M_{14} = 3.452$ T - m)

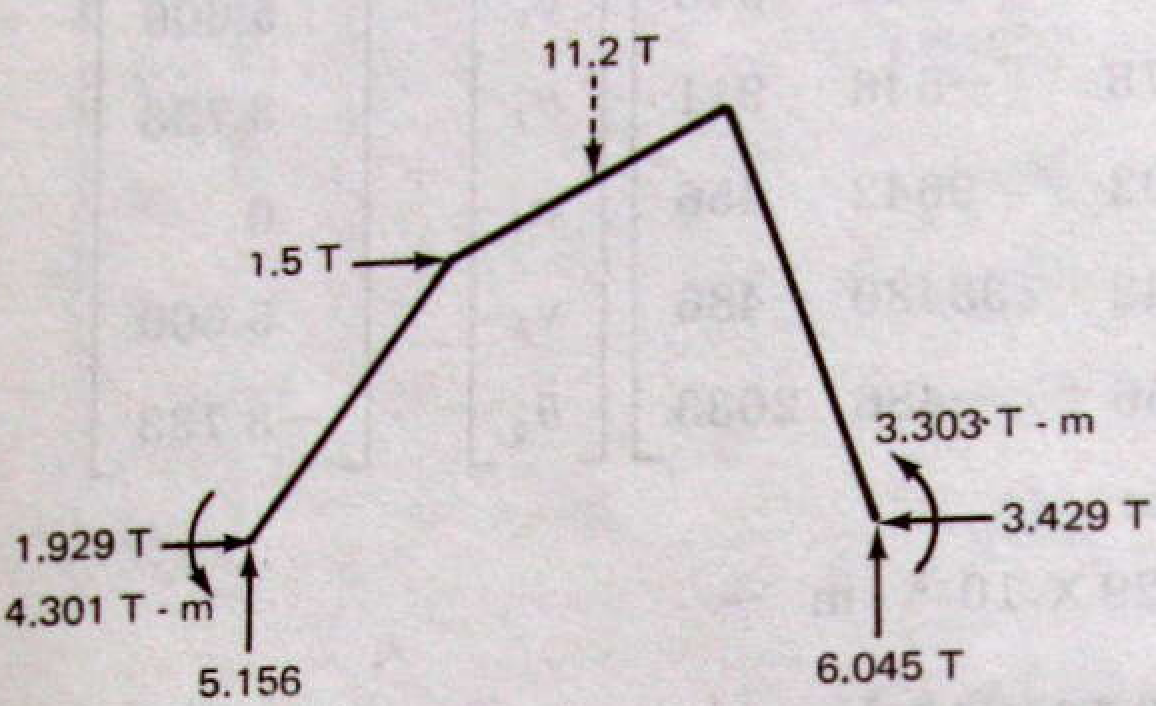
barra 3

$X_{23} = 3.429$ T $\rightarrow$ $\qquad$ $X_{32} = X_3 = -3.429$ T $\leftarrow$

$Y_{23} = -6.045$ T $\downarrow$ $\qquad$ $Y_{32} = Y_3 = 6.045$ T $\uparrow$

$M_{23} = 5.185$ T - m) $\qquad$ $M_{32} = M_3 = 3.303$ T - m)

Las fuerzas en los nudos 3 y 4 son las reacciones, con las cuales se puede verificar el equilibrio general:



$$\Sigma F_x = 0.000 \quad \text{Ton}$$

$$\Sigma F_y = 0.001 \quad \text{Ton}$$

$$\Sigma M_4 = 0.009 \quad \text{T - m}$$

Para hallar las fuerzas internas referidas a coordenadas locales se utilizan las matrices de transformación, $[\,\overline{F}\,] = [\,T\,]\ [\,F\,]$

$$\begin{bmatrix} \overline{X}_{12} \\ \overline{Y}_{12} \\ \overline{M}_{12} \\ \overline{X}_{21} \\ \overline{Y}_{21} \\ \overline{M}_{21} \end{bmatrix} = \begin{bmatrix} 0.89443 & 0.44721 & 0 & 0 & 0 & 0 \\ -0.44721 & 0.89443 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.89443 & 0.44721 & 0 \\ 0 & 0 & 0 & -0.44721 & 0.89443 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.428 \\ 5.155 \\ -3.452 \\ -3.429 \\ 6.045 \\ -5.185 \end{bmatrix}$$

$$= \begin{bmatrix} 5.372 & \text{T} \\ 3.078 & \text{T} \\ -3.452 & \text{T - m} \\ -0.364 & \text{T} \\ 6.940 & \text{T} \\ -5.185 & \text{T - m} \end{bmatrix}$$

barra I

$$\begin{bmatrix} \bar{X}_{41} \\ \bar{Y}_{41} \\ \bar{M}_{41} \\ \bar{X}_{14} \\ \bar{Y}_{14} \\ \bar{M}_{14} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ -0.8 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & -0.8 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.929 \\ 5.156 \\ -4.301 \\ -1.929 \\ -5.156 \\ 3.452 \end{bmatrix} =$$

$$= \begin{bmatrix} 5.282 & \mathrm{T} & \nearrow \\ 1.550 & \mathrm{T} & \nwarrow \\ 4.301 & \mathrm{T\text{-}m} & \\ -5.282 & \mathrm{T} & \swarrow \\ -1.550 & \mathrm{T} & \searrow \\ 3.452 & \mathrm{T\text{-}m} & \end{bmatrix}$$

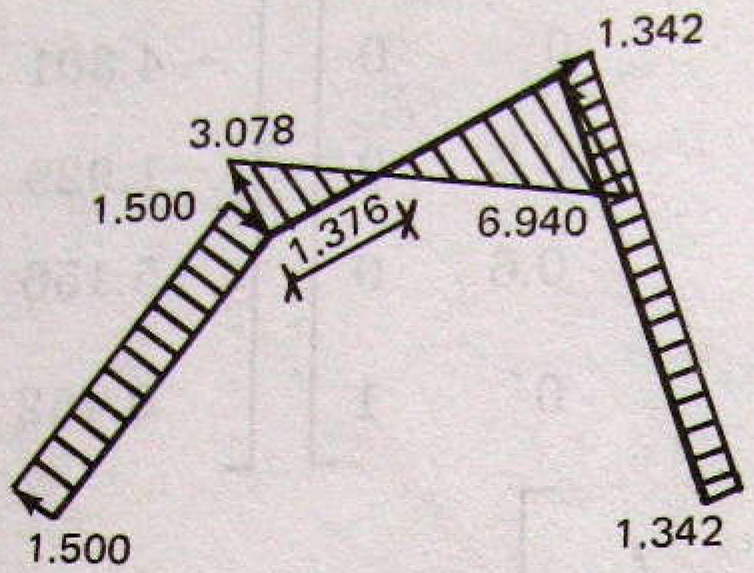
barra 2

$$\begin{bmatrix} \bar{X}_{23} \\ \bar{Y}_{23} \\ \bar{M}_{23} \\ \bar{X}_{32} \\ \bar{Y}_{32} \\ \bar{M}_{32} \end{bmatrix} = \begin{bmatrix} 0.31623 & -0.94868 & 0 & 0 & 0 & 0 \\ 0.94868 & 0.31623 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.31623 & -0.94868 & 0 \\ 0 & 0 & 0 & 0.94868 & 0.31623 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.429 \\ -6.045 \\ 5.185 \\ -3.429 \\ 6.045 \\ 3.303 \end{bmatrix} =$$

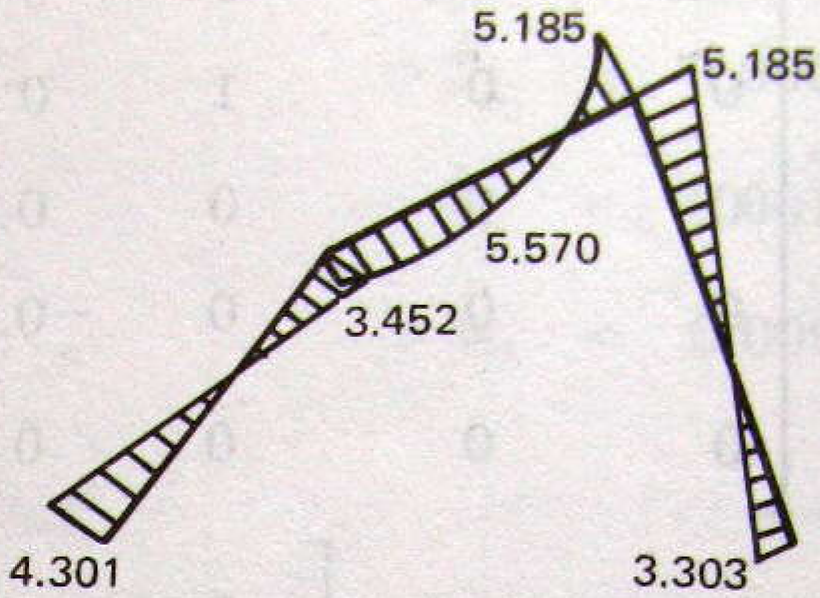
$$= \begin{bmatrix} 6.819 & \mathrm{T} & \searrow \\ 1.342 & \mathrm{T} & \nearrow \\ 5.185 & \mathrm{T\text{-}m} & \\ -6.819 & \mathrm{T} & \nwarrow \\ -1.342 & \mathrm{T} & \swarrow \\ 3.303 & \mathrm{T\text{-}m} & \end{bmatrix}$$

barra 3

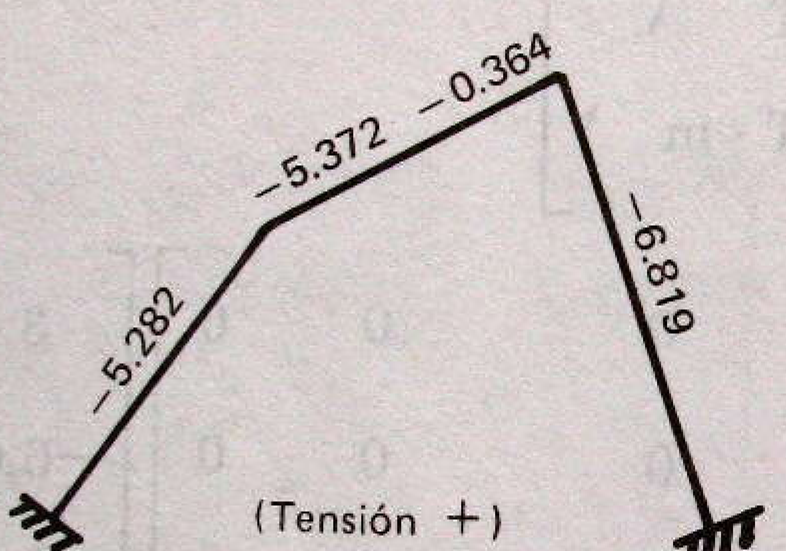
Finalmente se dibujan los diagra



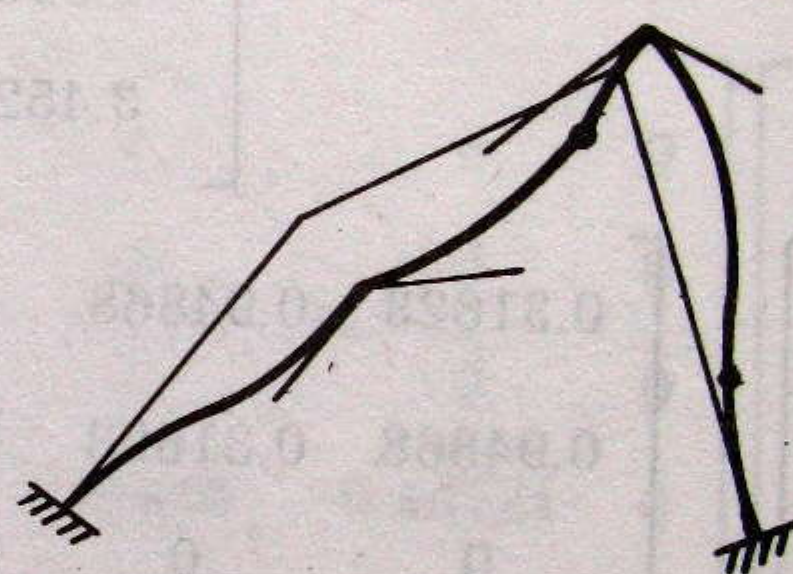
Corte (T)



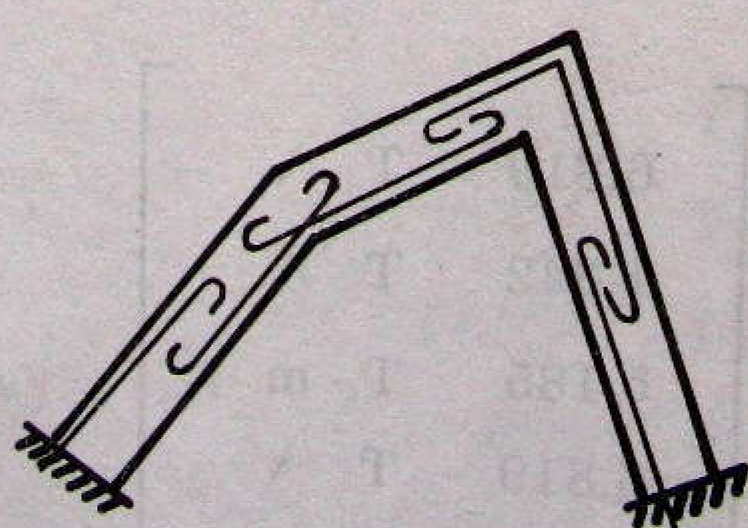
Momento (T - m)



Axial (T)



Elástica



Refuerzo Primario