Robust Adaptive Model Predictive Control for Forklift Operations with uncertain load dynamics

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1. Introduction

In modern warehouses, forklifts play a critical role in transporting and stacking goods. However, their performance and safety can be significantly affected by uncertain load dynamics, such as: Varying Load Mass: Different weights of items lead to changes in the forklift's inertia. Shifting Center of Gravity: Uneven or poorly secured loads may cause instability. Dynamic Friction: Changes in load weight affect tire-floor interactions, especially on uneven or slippery surfaces.

These uncertainties make it difficult for traditional controllers (e.g., PID or non-adaptive MPC) to achieve optimal and safe performance, especially when precision is required in tight warehouse environments.

While AMPC can adapt to uncertainties, it assumes reasonably accurate parameter estimates during operation.

A robust component ensures safety and stability even if parameter estimation is delayed or imperfect.

Load mass and center of gravity are often unknown and change as the forklift lifts, moves, or stacks items.

The forklift must remain stable and maintain control performance under worst-case scenarios (e.g., sudden load shifts, high disturbances). The controller must continuously adapt to updated parameter estimates in real-time. Forklifts operate in environments with dynamic obstacles (e.g., workers, other forklifts), requiring precise path tracking and collision avoidance. Actuator saturation (e.g., limited steering angles, acceleration) and environmental factors (e.g., slippery floors) impose constraints on control inputs. This report is limited to addressing the issue of variable load mass in the larger forklift warehouse operation problem.

2. Approach

2.1. Dynamics

This report the forklift motion and load dynamics as a discrete time linear system:

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + w_k \quad (1)$$

Where x_k is the system state, u_k is the control state, w_k is the external disturbance and k is the discrete time index.

$$x_{k+1} = A\big(\hat{\theta} + \Delta\theta\big)x_k + B\big(\hat{\theta} + \Delta\theta\big)u_k + w_k \quad (1)$$

Where $\hat{\theta}$ is the estimated parameter and $\Delta\theta$ is the bounded uncertainty. The robust AMPC problem is to minimize the cost function:

$$J = \sum_{k=0}^{N} \big(\big(x_k - x_{ref,k}\big)^T Q\big(x_k - x_{ref,k}\big) + u_k^T R u_k\big) \quad (3)$$

To estimate the uncertain parameters θ, defined as the variable total mass of the vehicle and load mass, a Recursive Least Squares approach is proposed.

For the MPC problem, the kinematics of the forklift are simplified to:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \times \cos(\varphi) \\ v \times \cos(\varphi) \\ v \times \tan\left(\frac{\delta}{Ld}\right) \\ \frac{f}{m} \end{bmatrix} \quad (3)$$

Where the state vector $X = [x, y, \varphi, v]^T$ represents the vehicle's coordinate x and y position, the vehicles heading angle, and the vehicles velocity respectively. Ld represents the rear wheel to front wheel distance and m represents the total variable mass of the vehicle and any load it carries. The control vector $u = [f \quad \delta]^T$ is made up of f representing the forward longitudinal force applied to the vehicle and δ representing the steering angle.

Discretizing the model, $\tilde{A}$ and $\tilde{B}$ matrices are defined to represent the forklift state and control matrices:

$$\tilde{A} = \begin{bmatrix} 1 & 0 & -Ts \times v \times \sin(\varphi) & Ts \times \cos(\varphi) \\ 0 & 1 & Ts \times v \times \cos(\varphi) & Ts \times \sin(\varphi) \\ 0 & 0 & 1 & Ts \times \frac{\tan(\delta)}{Ld} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\tilde{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & Ts \times \dfrac{v}{Ld \times cos^2(\delta)} \\ \dfrac{Ts}{m} & 0 \end{bmatrix} \quad (4)$$

Where Ts is the sampling time.

2.2. Constraints

The following constraints are introduced to the MPC problem, based on the forklift system dynamics:

$$\begin{aligned} x_{min} &= [-10, -10, -\pi, 0]^T \\ x_{max} &= [10, 10, \pi, 5]^T \\ u_{min} &= [-1000, -0.5]^T \\ u_{max} &= [1000, 0.5]^T \end{aligned} \quad (4)$$

$$\begin{aligned} &x(k+1) = f(X(k), U(k)) \\ &x_{min} \le x_k \le x_{max} \qquad , k = 0,1, \dots, N \\ &u_{min} \le u_k \le u_{max} \end{aligned} \quad (5)$$

Where N is the prediction horizon and k is the current time step.

2.3. Reference signals

To establish the adaptive control properties of the proposed solution, several reference signals are generated. These are x, y coordinate paths representing different paths for the forklift to traverse in a warehouse environment. The paths are predetermined and would not be computed online in this implementation. Three basic paths are proposed to evaluate the tracking properties of the MPC solution:

1. Straight line with constant velocity
2. S curve with constant velocity
3. Straight line with weight change (offloading)

2.4. Recursive least squares mass estimation

Variable load mass will affect the forklift system dynamics by changing the acceleration output given a forward longitudinal force on the vehicle. The simplified dynamics proposed have mass only affect the acceleration as a function of force and mass. While the mass of the vehicle is known, estimation is still necessary to determine the total mass including any load the forklift is carrying. To perform this estimation the following recursive least squares approach is proposed at every MPC time step:

$$\begin{aligned} K &= \frac{P \times a_m}{\lambda + {a_m}^T \times P \times a_m} \\ \theta_{est} &= \theta_{est} + K \times f - {a_m}^T \times \theta_{est} \\ P &= \frac{P - K \times {a_m}^T \times P}{\lambda} \end{aligned} \quad (7)$$

Where K is the adaptive gain, P is the covariance matrix, θ_{est} is the estimated parameter ($\frac{1}{m}$ in this MPC problem), f is the current applied force and λ is the forgetting factor determining the weight of past data to the current estimate.

3. Theoretical Results

Under the bounded uncertainties $\Delta\theta$, Lyapunov theory can be used to prove the stability of the closed loop system. By defining bounds for $\Delta\theta$ and disturbances w_k, robustness of the system can be established under worst case scenarios. These operating bounds should then be respected in the real-world implementation of an automated forklift as stability will be assured within these bounds.

4. Implementation

Implementing this MPC solution on a Forklift would require outfitting a standard 4-wheel forklift with the following sensors:

1. GPS for position tracking
2. Tachometer
3. Accelerometer
4. Steering potentiometer

For computing an Nvidia Jetson GPU and 4GB of RAM would benefit the system with the possibility of performing online optimization for time sensitive path planning and tracking.

An electric power steering system and electric motor drives for acceleration/braking are needed.

In this report the dynamic constraints are estimated, in a real-life application these would depend on the actuators used and the mechanical limitations of the real forklift.

5. Preliminary results

The following simulation results show preliminary positional tracking, states and control inputs for the three reference signals mentioned in 2.3.

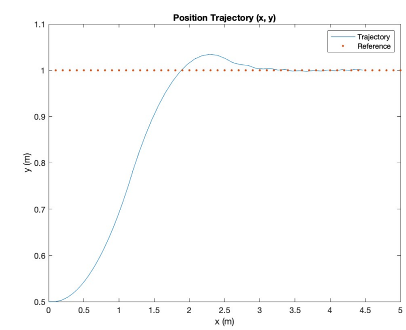

Figure 1: Position tracking for straight path reference trajectory

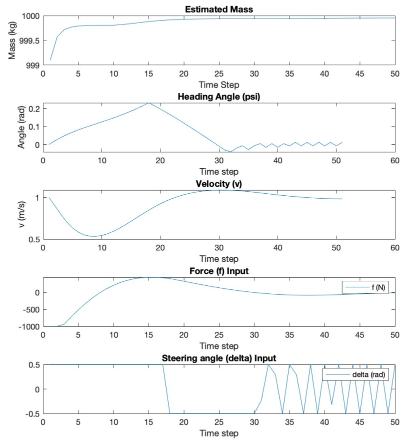

Figure 2: Estimated Mass, heading and velocity states, control inputs for straight path reference trajectory

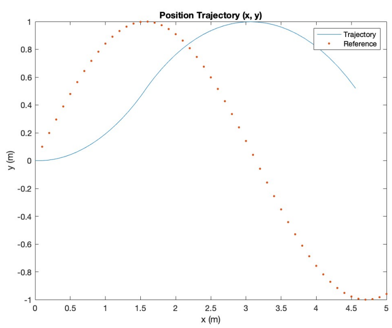

Figure 3: Position tracking for s curve reference trajectory

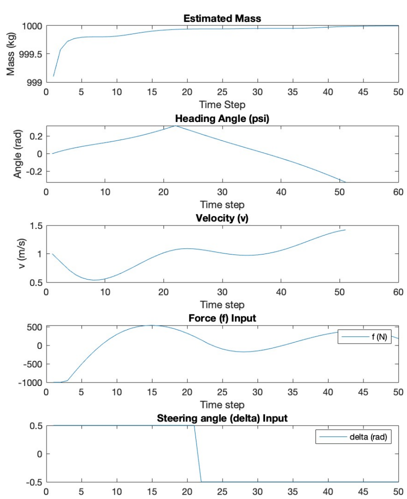

Figure 4: Estimated Mass, heading and velocity states, control inputs for L turn and stop reference trajectory

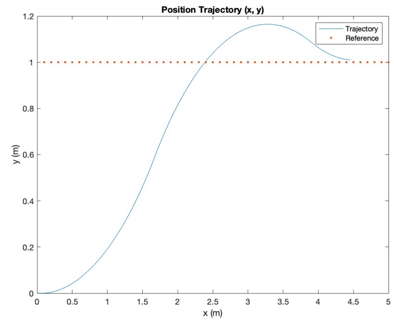

Figure 5: Position tracking for straight path reference trajectory with 200 kg offloading.

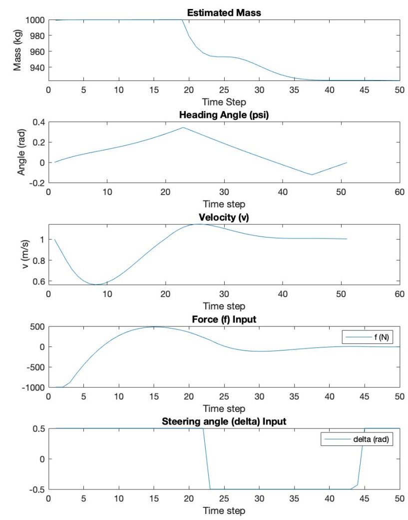

Figure 6: Estimated Mass, heading and velocity states, control inputs for straight path reference trajectory with 200 kg offloading

6. Conclusions

This project implemented an adaptive MPC framework to a forklift with variable load mass system. The objective was to create a robust solution to a high impact problem in modern warehouses. The preliminary results from this work demonstrate the need for more work on this problem. While tracking results were acceptable as initial results to the problem, greater accuracy and faster time responses are desired. Expanding the dynamic model to include center of mass considerations would also further the scope of the problem to be robust enough for additional edge case scenarios. Further theoretical work is also required in the current implementation to ensure robustness of the controller in worst case scenario operation.

References

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[2] B. Zhu and X. Xia. "Adaptive Model Predictive Control for Unconstrained Discrete-Time Linear Systems With Parametric Uncertainties." *IEEE Transactions on Automatic Control,* vol. 61, no. 10, pp. 3171-3176.

[3] Wang, Y., Sun, K., Zhang, W., & Jin, X. (2024). A Velocity-Adaptive MPC-Based Path Tracking Method for Heavy-Duty Forklift AGVs. *Machines*, *12*(8), 558.