ECONOMICS 2

Tutorial 5

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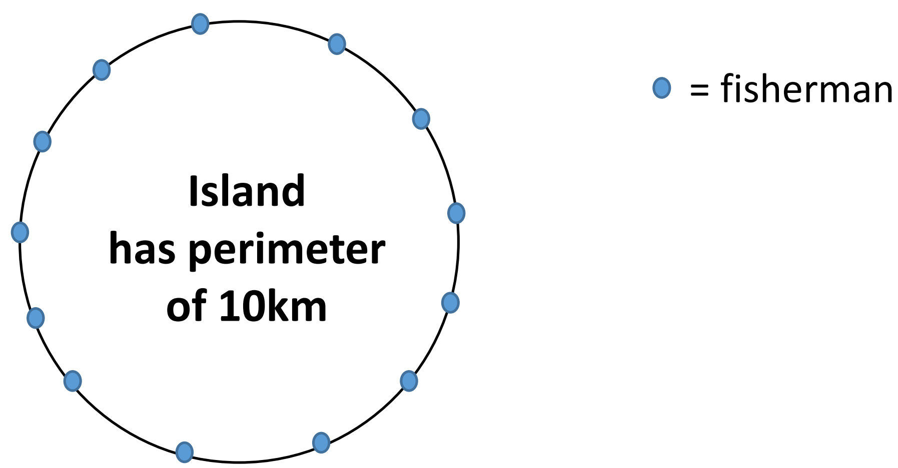
Questions: 4,5,8,10,13,14

http://personal.lse.ac.uk/BATTISTO/T5_slides.pdf

Question 4

The 1,000 residents of Great Donut Island are all fishermen. Every morning they go to the nearest port to launch their fishing boats and then return in the evening with their catch. The residents are evenly distributed along the 10-kilometer perimeter of the island. Each port has a fixed cost of €1,000/day. If the optimal number of ports is 2, what must be the per kilometre travel cost?



We need to find t = cost per kilometre knowing that optimal N = 2

Key:

- The total cost for the island is:

TRAVEL COSTS + PORT'S COST

(A) + (B)

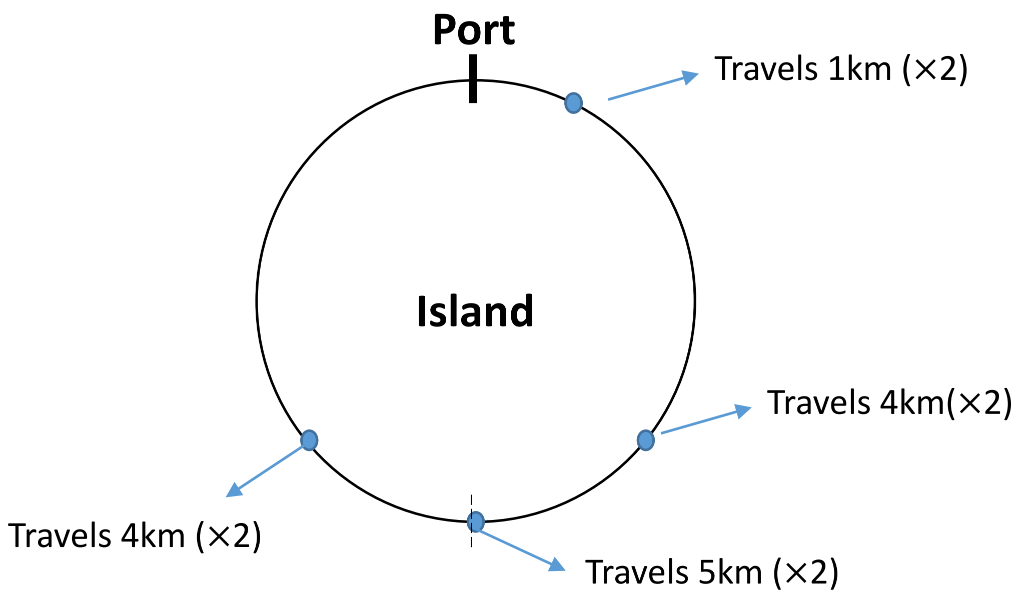
(A) is reduced if we build more ports

(B) is increased if we build more ports

Then, there should be an optimal N of ports

Travel Costs:

With one port

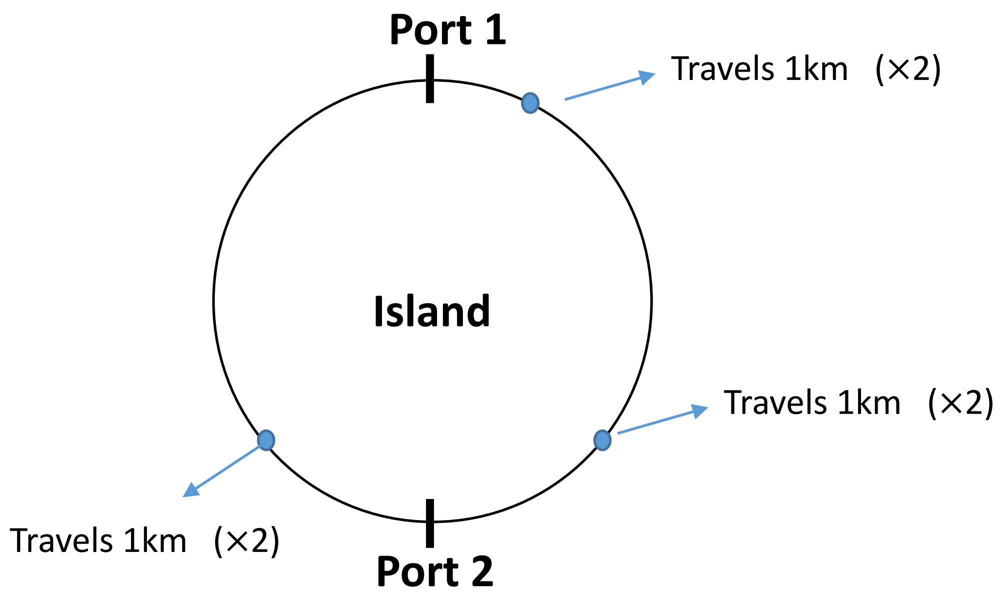


The average trip is 5km

The total cost (all fishermen) is 5km x 1000 x t

Travel Costs:

With two ports

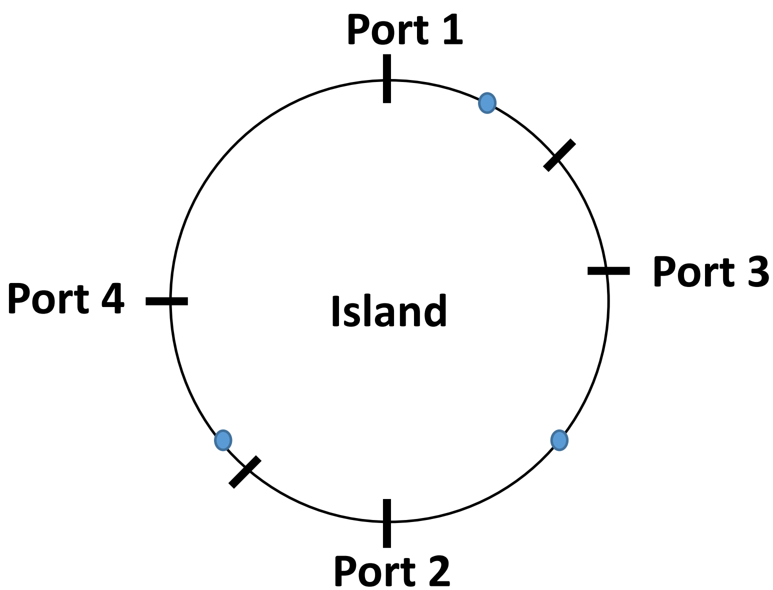


The average trip is 2.5km

The total cost (all fishermen) is 2.5km x 1000 x t

Travel Costs:

With N ports



The average trip is $\frac{10}{2N}$ km

The total cost (all fishermen) is $\frac{10}{2N}$ x 1000 x t

Total Costs:

TC = TRAVEL COSTS + PORT'S COST

$$= \frac{10}{2N}\, 1000t \quad + \quad 1000N$$

Optimal N of ports $\frac{\partial TC}{\partial N} = 0$

$$\frac{-5000t}{N^2} + 1000 = 0 \qquad \Longrightarrow \qquad t = \frac{N^2}{5} = \frac{4}{5}$$

Question 5
What is the fundamental difference among the Cournot, Bertrand, and Stackelberg models of oligopoly? Consider the competition between Uber and Lyft, which model of oligopoly best summarises the way that the firms interact? (Read this short article: Link)

- **Cournot:** Compete in quantities. Simultaneous
- **Bertrand:** Compete in prices. Simultaneous
- **Stackelberg:** Compete in quantities. Dynamic

Extra question: What happen if we do Bertrand dynamic?

Shared monopoly

- Both firms behave as a single Monopoly
- MC = MR rule
- Then, just split quantities (and profits)

Cournot

- Each firm max profits taking “as given” what the other produce
- That results in a Best Response function for each firm
  - Q1* as a function of Q2
  - Q2* as a function of Q1
- **Nash Equilibrium:** Q1* is a BR to Q2* which is a BR to Q1*, …

Bertrand

- Firms choose prices
- Subtle but important difference r/Cournot: The lowest price firm gets all the consumers
- Don’t need any calculation: A price that give profits > 0 can’t be a equilibrium, a firm is strictly better if reduces a bit the price.
  - Only possible equilibrium is each firm sets P = MC

Stackelberg

- Is Cournot but one firm plays first
- Player 2 makes optimal decision after observing Player 1 move
- We solve by **backward induction**: Start from last player and find her optimal decision. First player will "anticipate" that behaviour
  - $Q2^*(Q1)$ is similar to Cournot
  - But first player "controls" what the second plays, so her maximization is different from Cournot's

Model	Q_1	Q_2	$Q_1 + Q_2$	P	π_1	π_2	$\pi_1 + \pi_2$
Shared monopoly	3	3	6	9	18	18	36
Cournot	4	4	8	7	16	16	32
Bertrand	6	6	12	3	0	0	0
Stackelberg	6	3	9	6	18	9	27

Question 10

Cournot Model with:

- Market demand $Q = 53 - P$
- MC = 5
- n firms

Calculate quantities, profits, etc. depending on n

Check what happens when n is very large

- Profits for firm 1:

$$\pi_1 = PQ_1 - 5Q_1 = [53 - (Q_1 + Q_2 + \cdots + Q_n)]Q_1 - 5Q_1$$

- Get reaction function

- Firms are identical, then:

$$Q_1 = Q_2 = \cdots = Q_n$$

Question 13

Alfred and Barry each own a restaurant on a busy high street. At the moment they each earn £100 of profit. They each have the option of running an advertising campaign. If Alfred advertises and Barry does not then Alfred's profits would be £110 and Barry's £105. If Barry advertises and Alfred does not then Alfred's profits would be 115 and Barry's 101. If both advertise their profits would be 104. If both have to decide simultaneously whether to advertise, what are the Nash equilibrium outcomes?

- When strategies are discrete options => Write payoff matrix
- Then check Best Responses

		Barry	
		A	**NA**
Alfred	**A**	104, 104	110, 105
	NA	115, 101	100, 100

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		Barry	
		A	**NA**
Alfred	**A**	104, 104	110, 105
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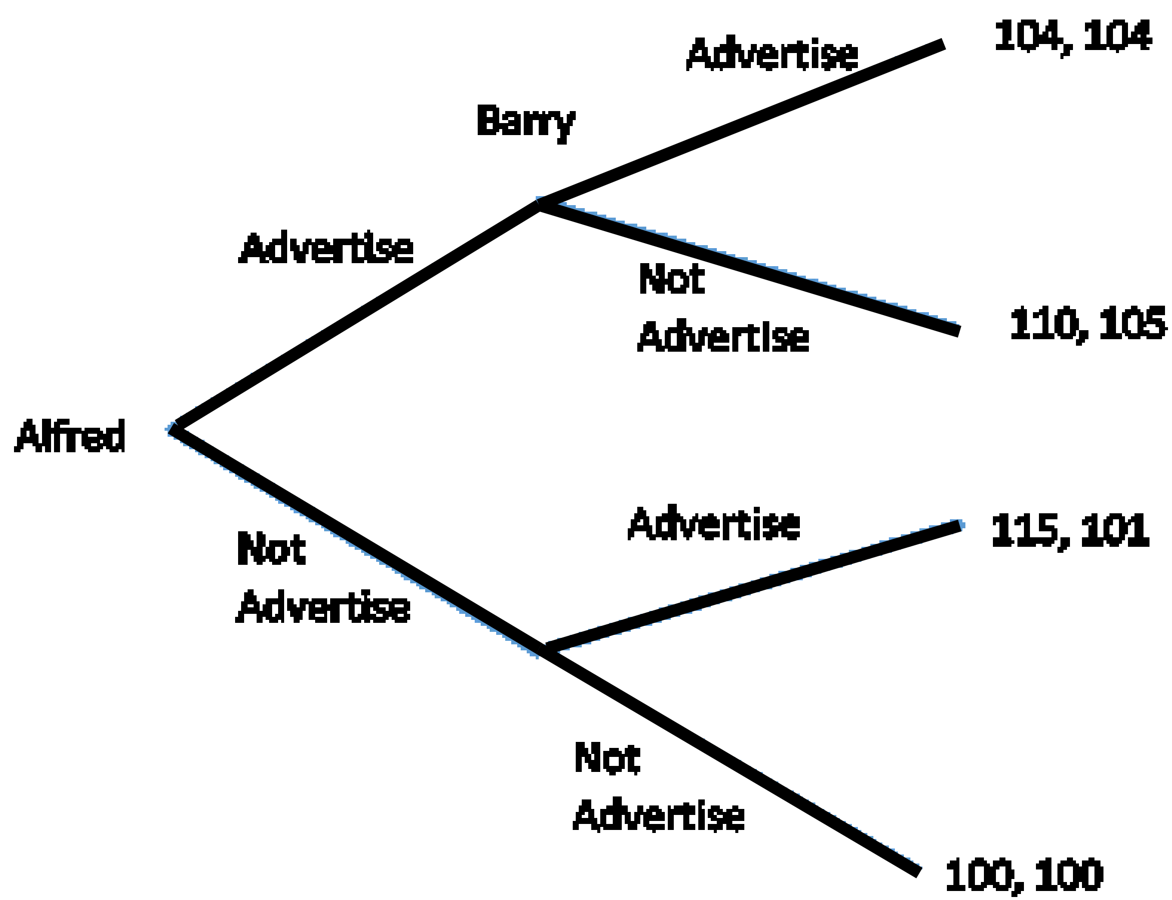
NE when both playing Best Response to each other

Question 14

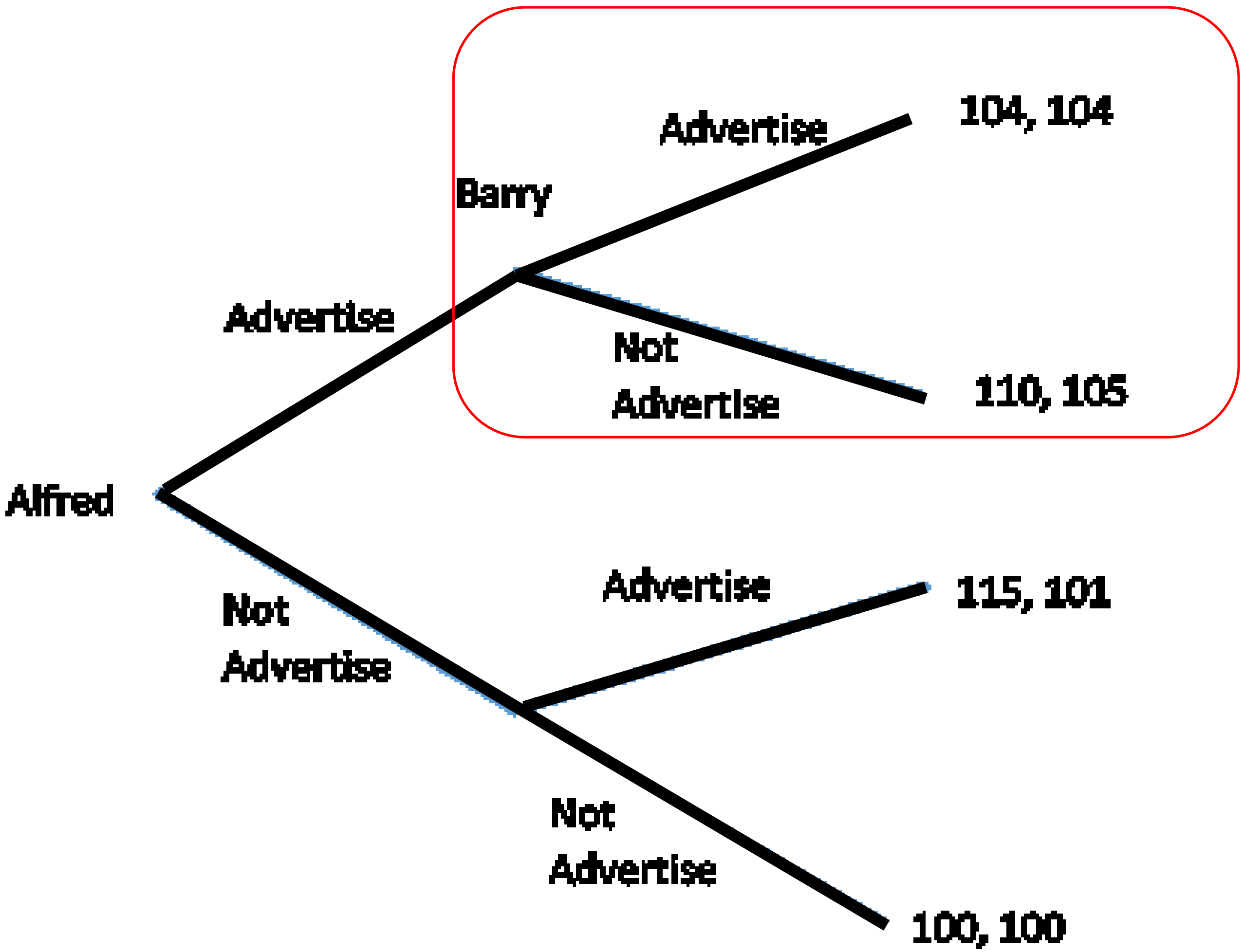
Same game than Q13 but dynamic (Alfred play first)

Find SPE and NE

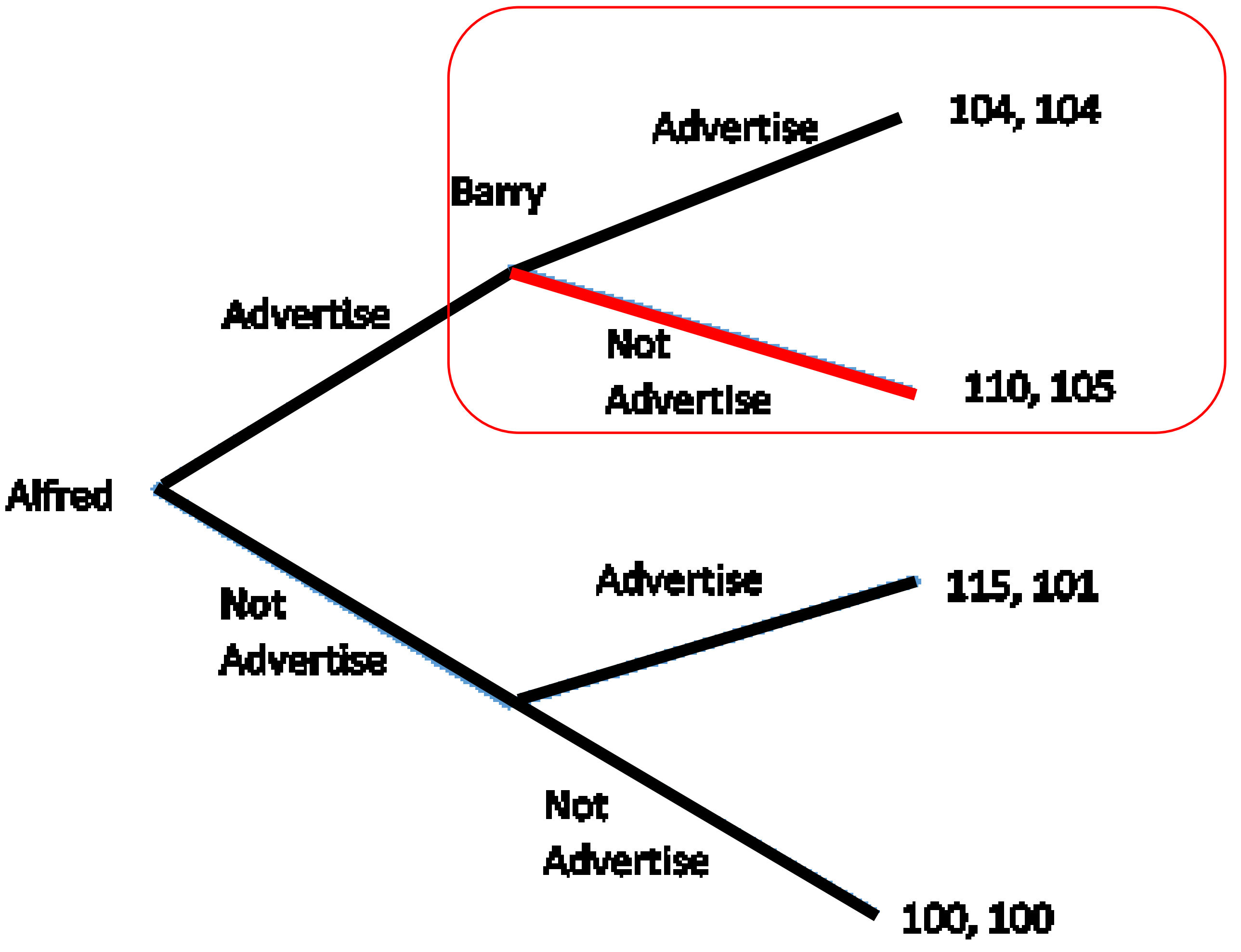
- **Subgame Perfect Equilibrium**: Is a Nash Equilibrium with the additional property of being a NE in every sub-game
- In other words:
  - Rational moves if we look at any part of the game
  - No one plays a “non-credible threat”
- We **always** find the SPE using **backward induction**


Alfred
Advertise
Not Advertise
Barry
Advertise
Not Advertise
104, 104
110, 105
Advertise
Not Advertise
115, 101
100, 100

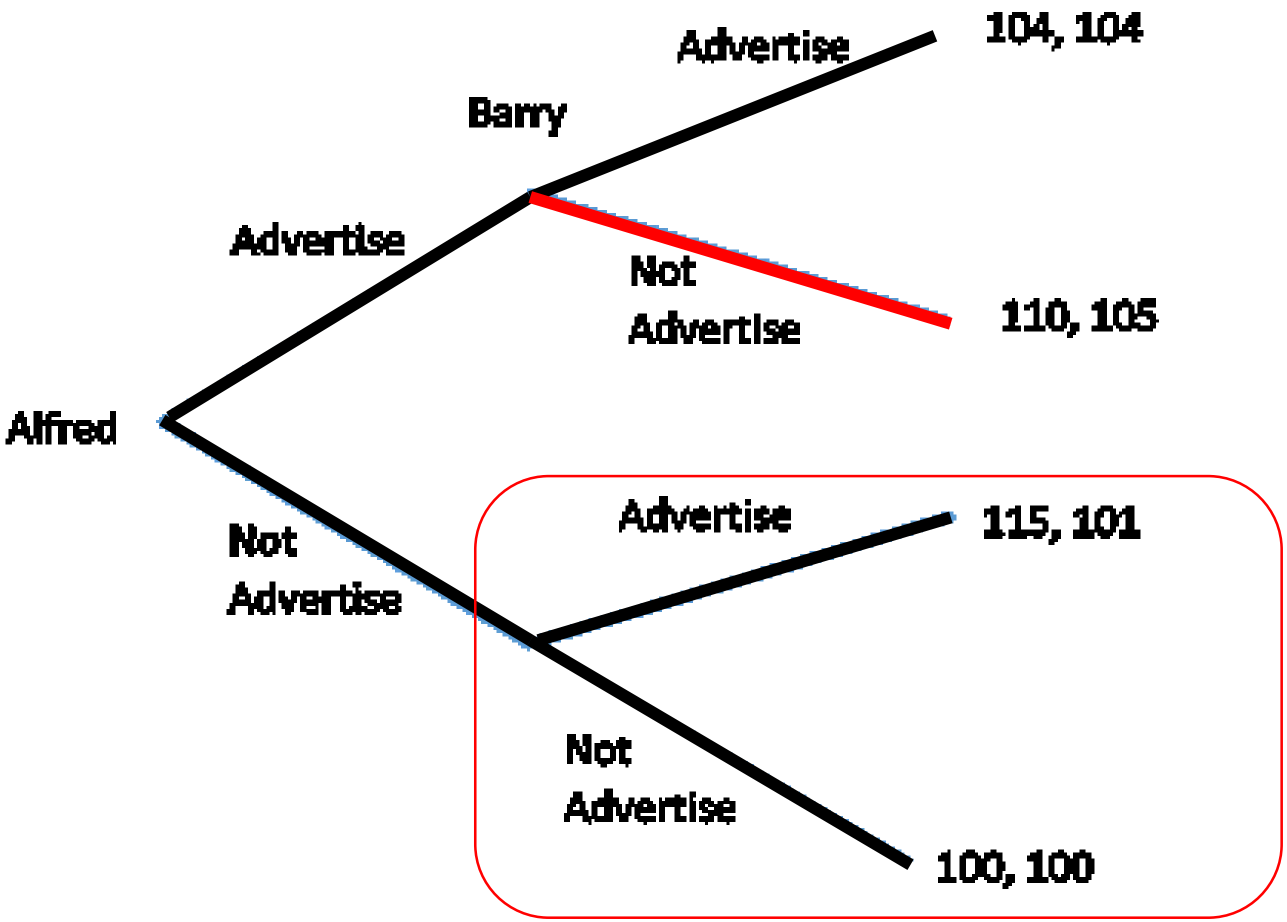
Start with last (or one of the last) subgames



Barry prefers 105 > 104 in such a subgame



In the other final subgame, Barry prefers 101>100



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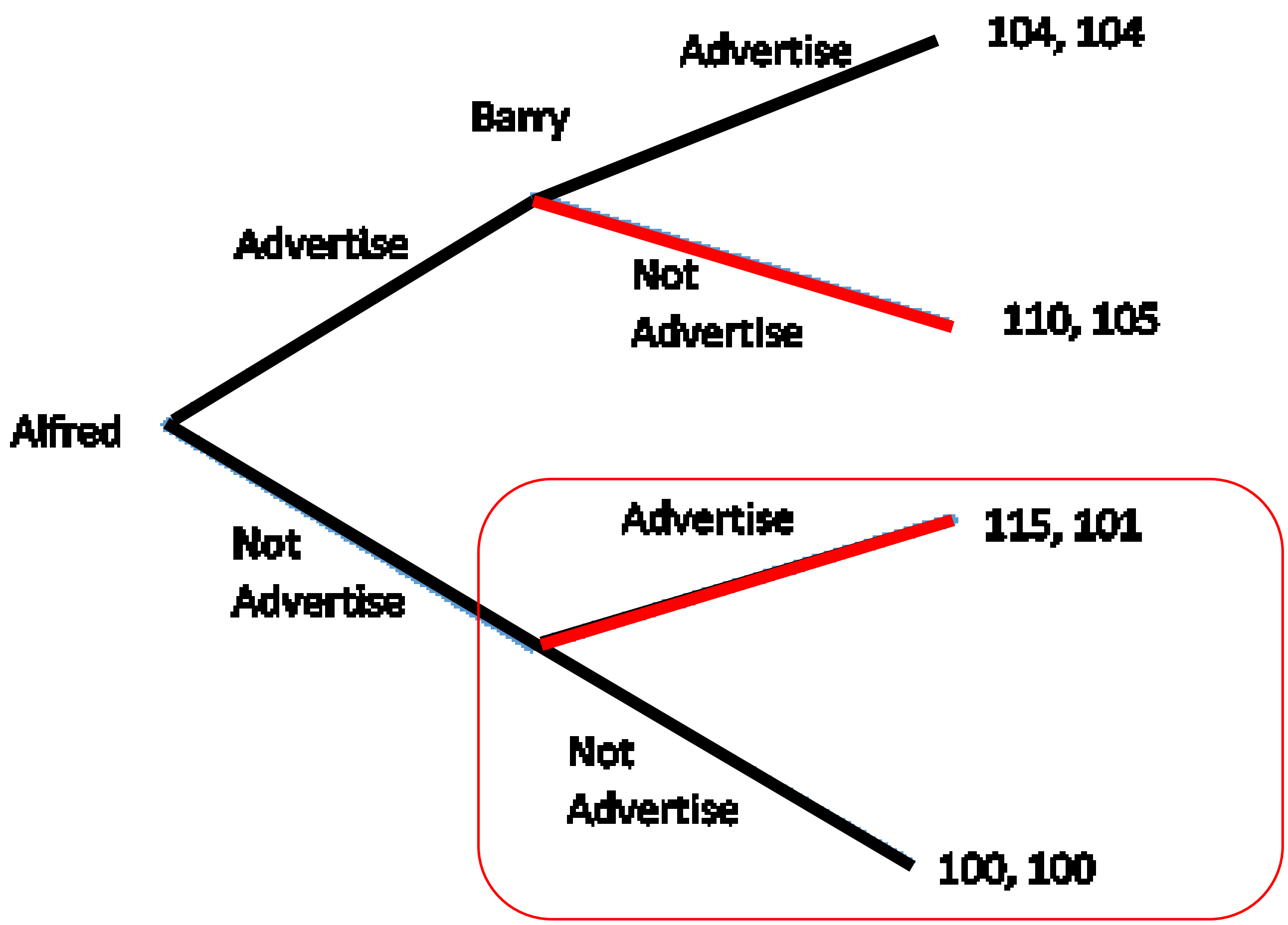
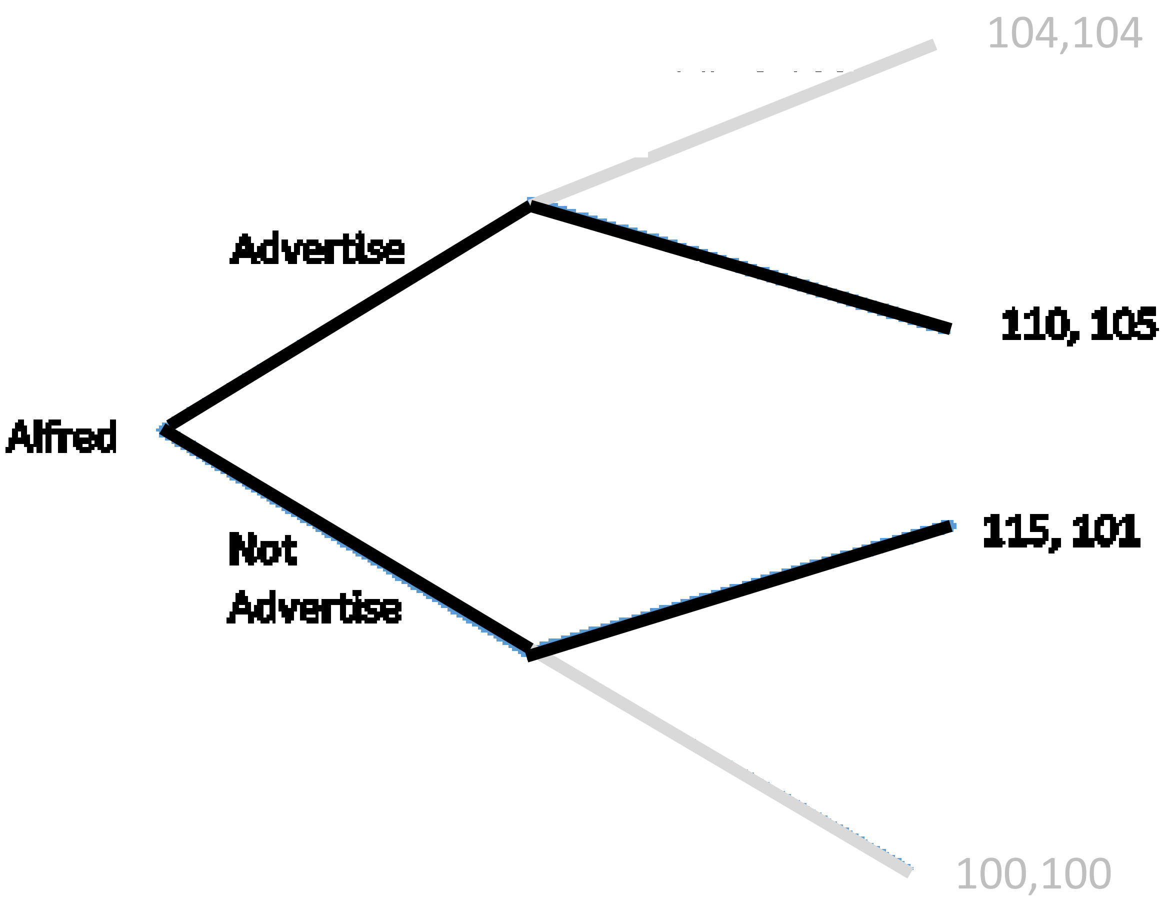
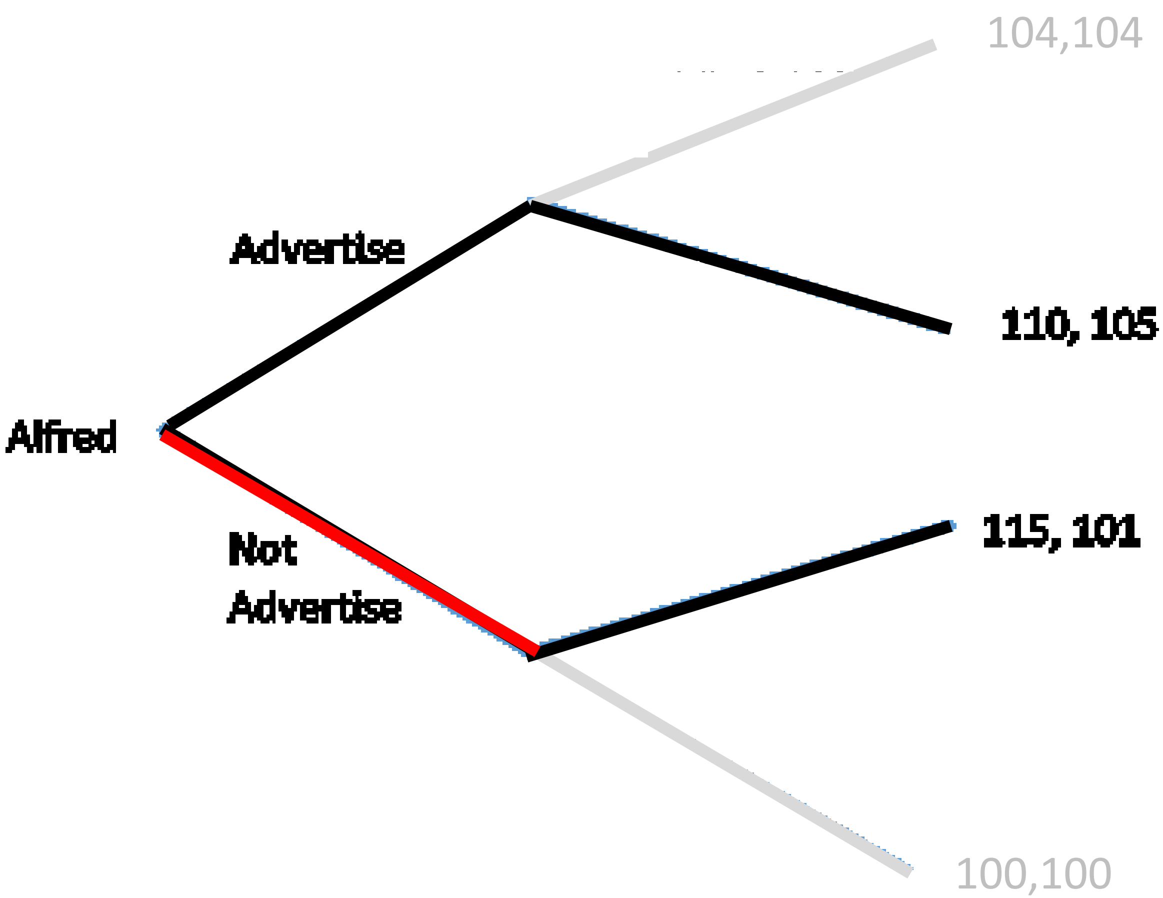


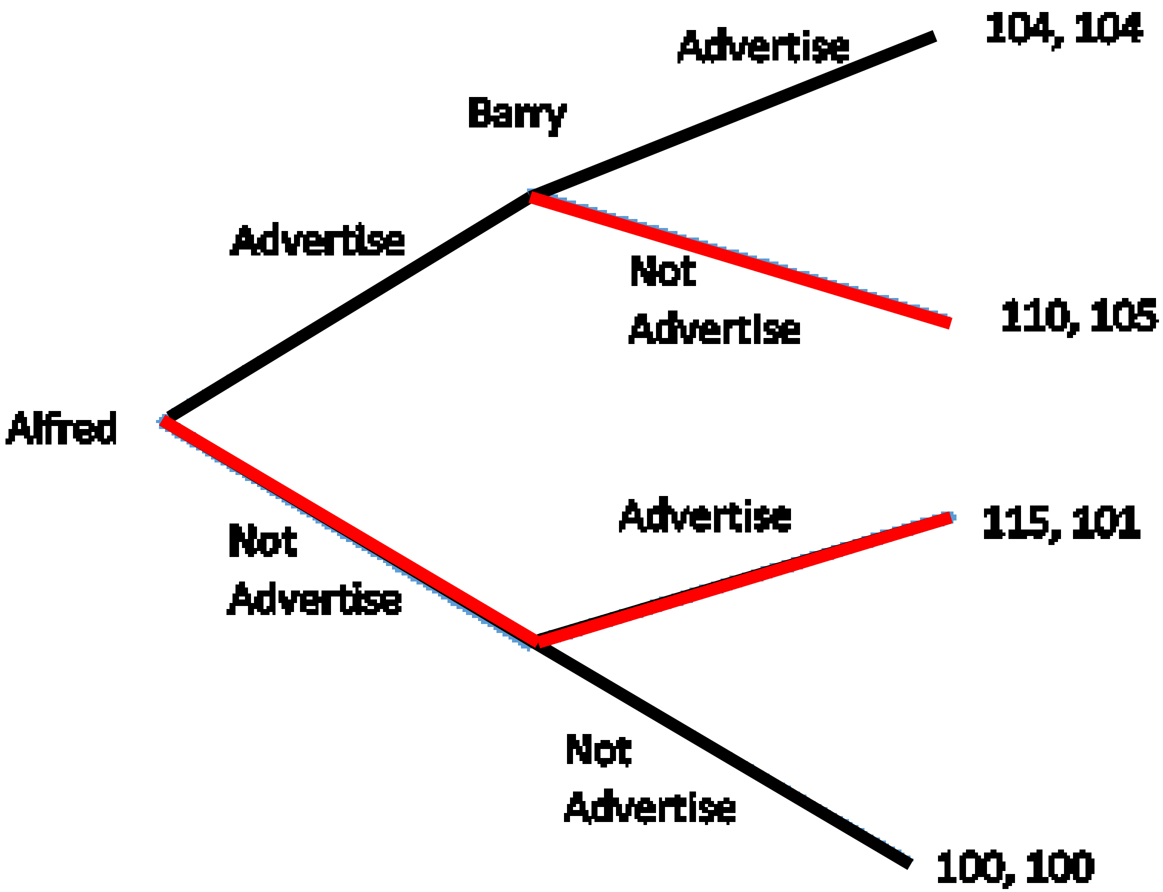
Now Alfred “anticipates” Barry’s moves in each node



And prefers 115 > 110



Put all the moves together to see the SPE



SPE: Alfred plays NA. Barry plays A in response to NA and NA in response to A

Important Notes:

- Barry’s strategy indicates what he plays IN ANY situation, even those that never occur (e.g. Alfred playing A)
- The fact that Barry plays NA in the upper node is a “credible threat”
- There are some NE that are not SPE

- Write the payoff matrix
- Barry’s strategy has two elements (what to do in the upper node and what to do in the lower node)

		Barry			
		A-A	**A-NA**	**NA-A**	**NA-NA**
Alfred	**A**	104, 104	104, 104	110, 105	110, 105
	NA	115, 101	100, 100	115, 101	100, 100

- Write the payoff matrix
- Barry's strategy has two elements (what to do in the upper node and what to do in the lower node)
- Find Best Responses

		Barry			
		A-A	**A-NA**	**NA-A**	**NA-NA**
Alfred	**A**	104, 104	104, 104	110, 105	110, 105
	NA	115, 101	100, 100	115, 101	100, 100

NE but not SPE

SPE

Additional Exercises (only if time allows)

Question 8

- Two firms compete for a franchise
- Benefit if win = R
- Probability of win depends on lobbying expenditures (I_1 and I_2)

  Probs are $\frac{I_1}{I_1+I_2}$ and $\frac{I_2}{I_1+I_2}$ for each firm

Find equilibrium level of lobbying (assume firms are risk neutral)

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Find equilibrium level of lobbying (assume firms are risk neutral)

- Conceptually similar to a Cournot Game
- Write expected profits for firm 1 (firm 2 is symmetric):

$$\Pr(\text{win})R - I_1$$

$$= \frac{I_1}{I_1+I_2}R - I_1$$

- Then find NE by solving reaction functions or using $I_1 = I_2$

Question 13

Present a table which shows the probability distribution and the cumulative probability distribution of the number of heads obtained when three fair coins are tossed independently.

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- Call X to the number of heads

$$P(X=0)=P(TTT)=0.5^3=0.125$$
$$P(X=1)=P(HTT)+P(THT)+P(TTH)=3\times 0.5^3=0.375$$
$$P(X=2)=P(HHT)+P(HTH)+P(THH)=3\times 0.5^3=0.375$$
$$P(X=3)=P(HHH)=0.5^3=0.125$$

x : n of heads	$P(x)$	$F(x)$
0	0.125	0.125
1	0.375	0.5
2	0.375	0.875
3	0.125	1

Question 15

Suppose X and Y are two random variables with $E(X) = 1.50, E(Y) = 0.55$, $E(XY) = 0.80, Var(X) = 0.25$, and $Var(Y) = 0.2475$.

a) What is the value of $Cov(X, Y)$?

b) What is the value of $Corr(X, Y)$?

c) Would you say there is a strong or a weak relationship between X and Y?

d) What is the value of $E(2X + 3Y)$?

a) $Cov(X,Y) = E[XY] - \mu_X\mu_Y \qquad = 0.80 - (1.5)(0.55) = -0.025$

b) $Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} \qquad = \frac{-0.025}{\sqrt{0.25}\sqrt{0.2475}} = -0.1005$

c) Linear association is low (but depends on the context)

d) Expectation is a linear operation, then:

$$\mathrm{E}(2\mathrm{X} + 3\mathrm{Y}) = 2E(X) + 3E(Y) = 3 + 1.65 = 4.65$$