ECONOMICS 1 (sem 2)
Tutorial 1

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You can download these slides from

https://diegobattiston.github.io/T1.pdf

Plan:

- Discuss the exercises highlighting the key concepts
- We can’t solve all the exercises in 1 hour
- You’ll get solutions, so I will try to address things not explained there
  - E.g. I won’t spend much time doing algebra

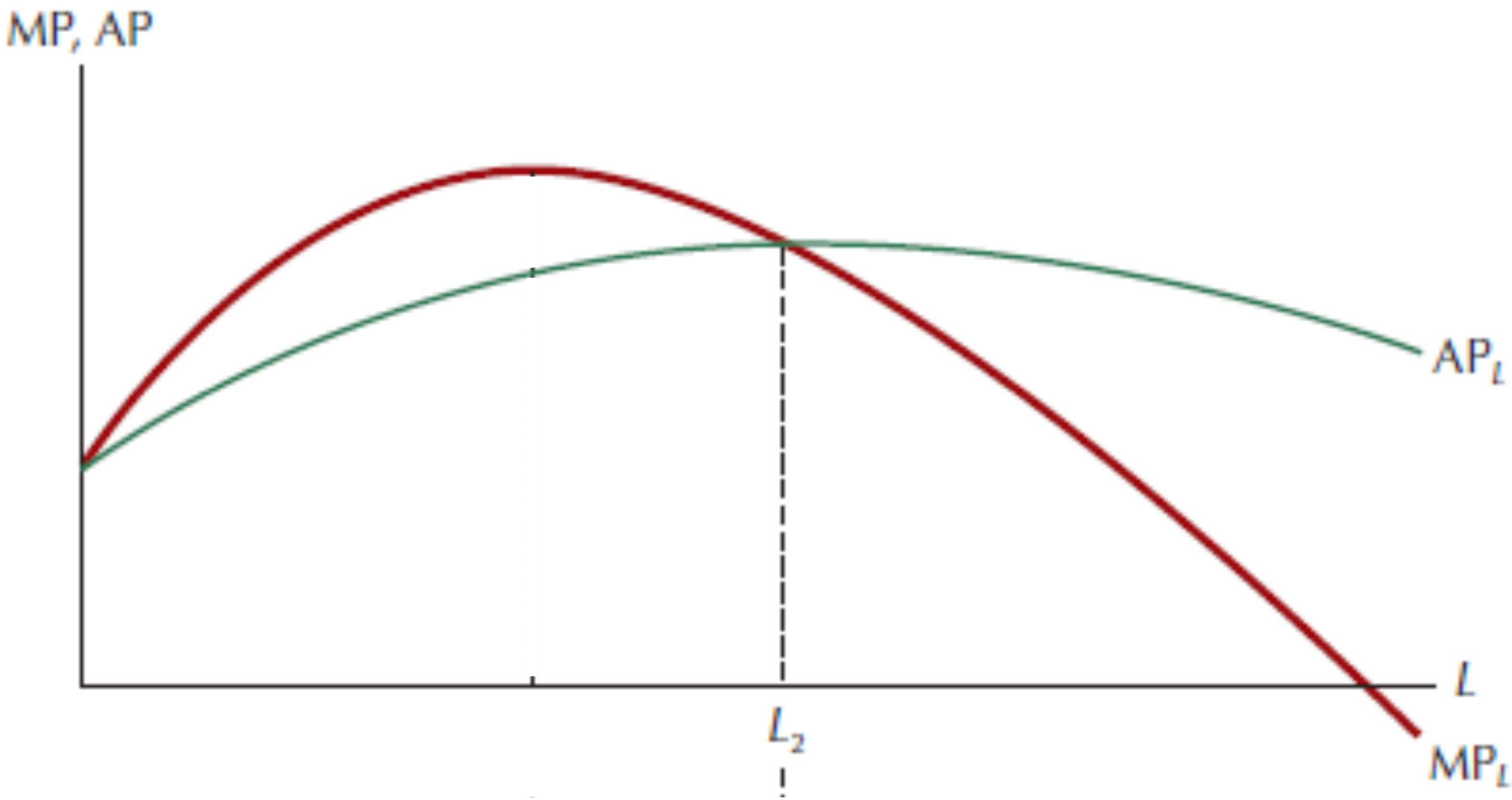
Q3. Should a person in charge of hiring productive inputs care more about marginal products or about average products?

- Marginal vs. Average Product

Q6. True or false: If the marginal product is decreasing, then the average product must also be decreasing.

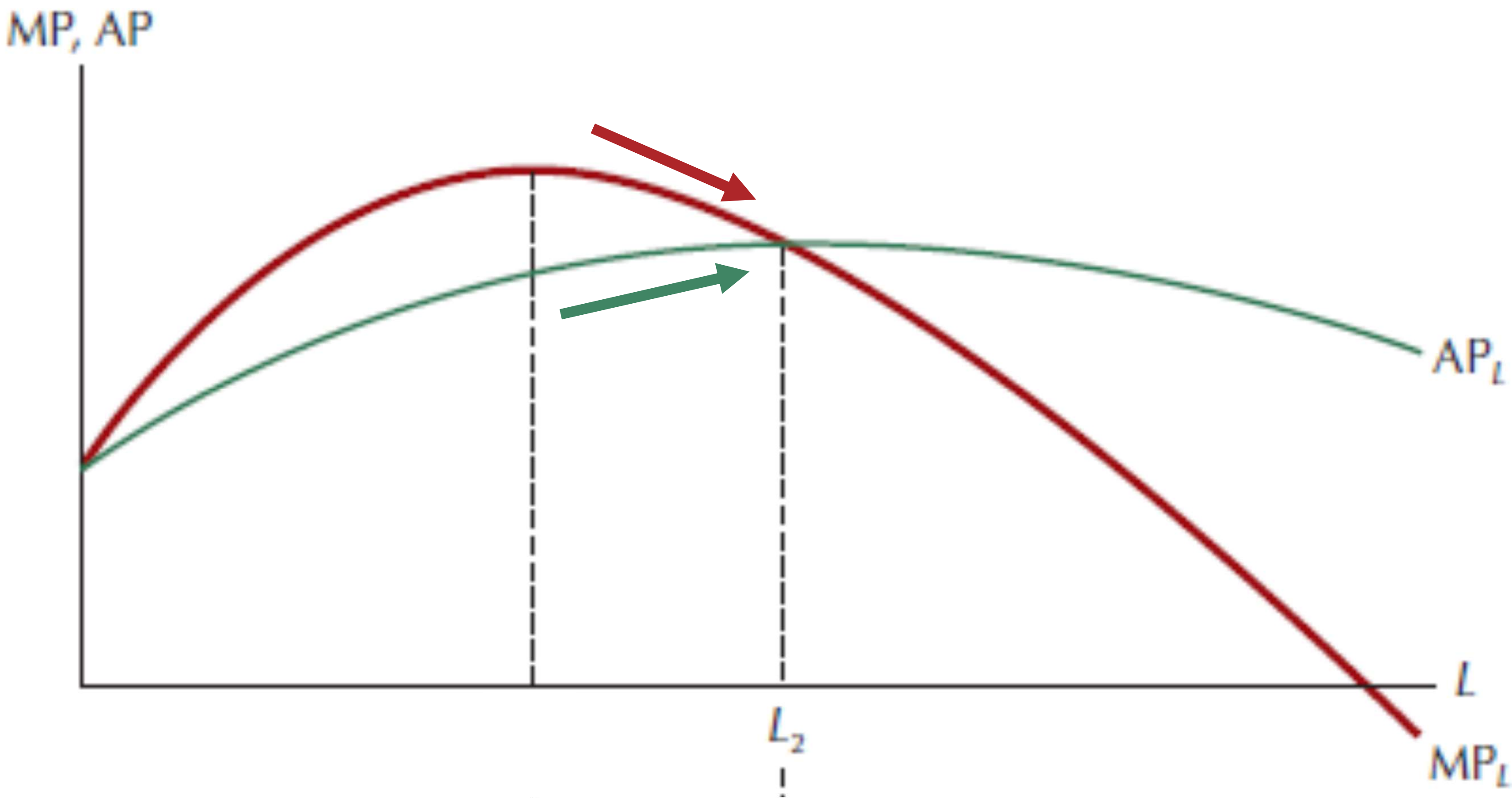
- Marginal vs. Average Product
- Solve in two ways: Graphically and Analytically

Remember graph from the book



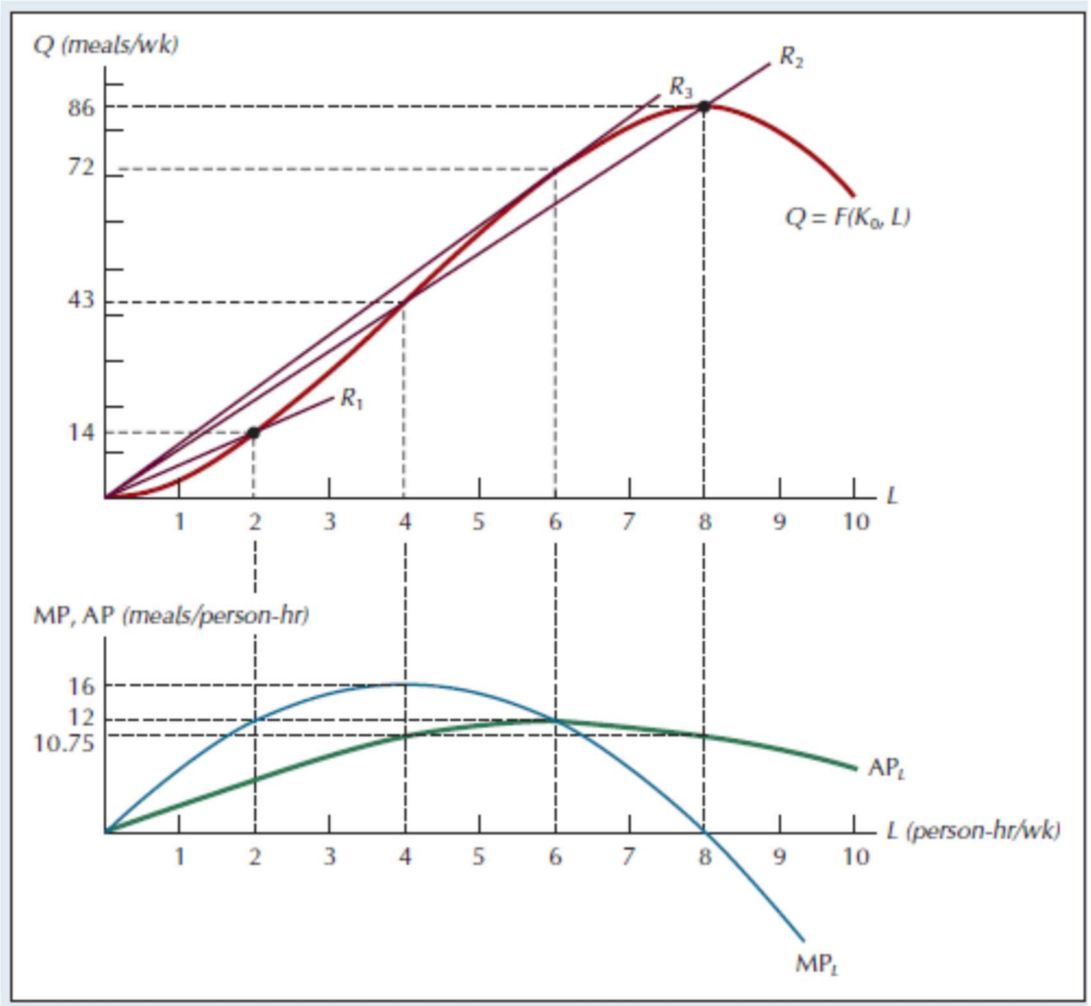
False:

There is a range where MP is DECREASING but AVP is INCREASING



If you don't remember how to derive the previous graph, it is explained in the book

(Figure 10.6 or 9.8 depending on the edition)



With Math (not in the solutions):

First, find the maximum of $AVP = \frac{Y(L)}{L}$

First Order Condition: $$\frac{\partial \frac{Y(L)}{L}}{\partial L} = 0$$

Use quotient rule: $$\frac{Y'(L)L - Y(L)}{L^2} = 0$$

$$\Rightarrow \quad \underbrace{Y'(L)}_{MP} = \underbrace{Y(L)/L}_{AVP}$$

Thus, the Max AVP occurs when MP = AVP

- The AVP can’t decrease if the MP is not decreasing.
- Then, if you move to the right of the max AVP (where AVP is decreasing), MP must be decreasing.
- Which also implies that MP is cuts AVP from above

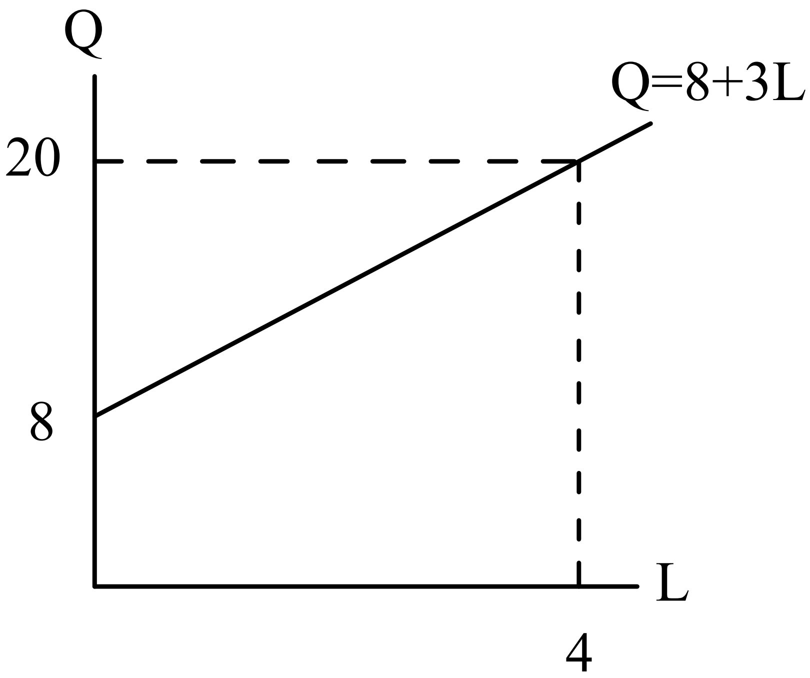
Q8. Graph the short-run total product curves for each of the following production functions if K is fixed at K_0=4. Are there diminishing returns to labour?

- Decreasing Returns to an Input

Intuition: Adding extra units of it increases total output but the increase is each time lower

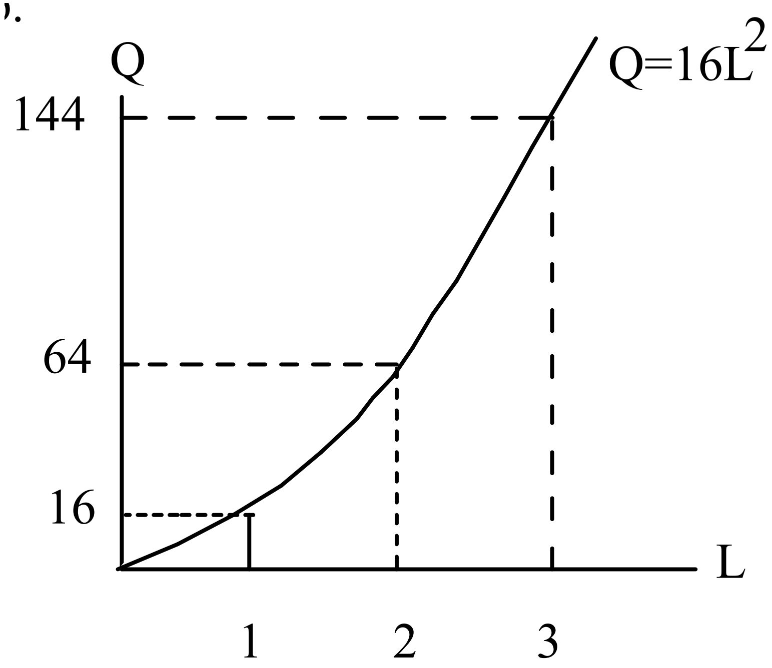
Math: Partial derivative of the production function is decreasing (second derivative negative)

a) $Q = F(K, L) = 2K + 3L$



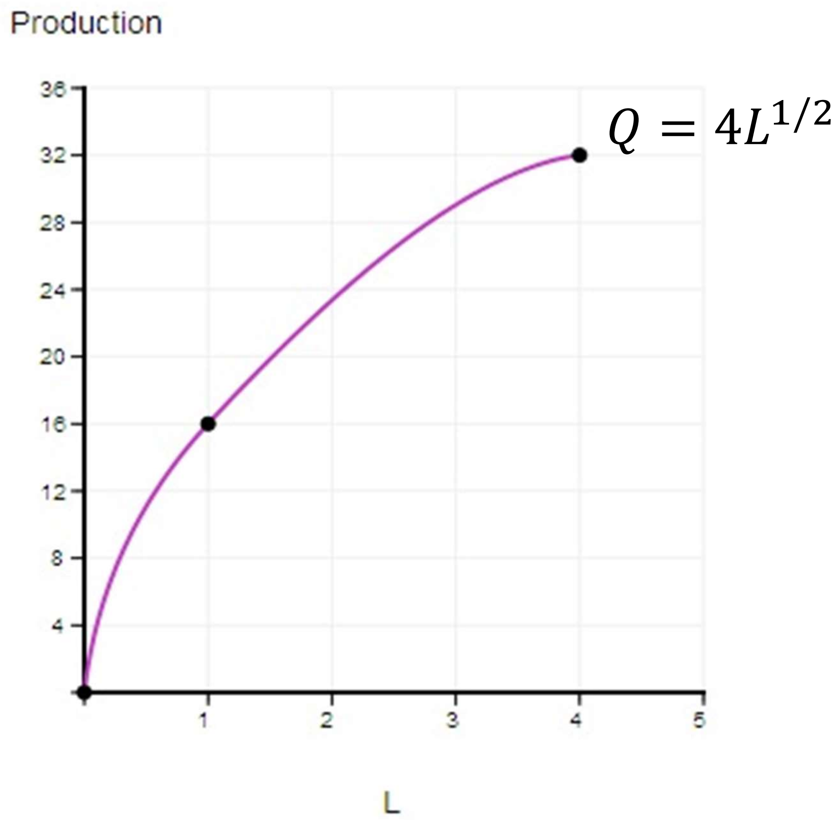
Constant returns to Labour

b) $Q = F(K, L) = K^2 L^2$



Increasing returns to Labour

c) $Q = F(K, L) = K^2 L^{1/2}$



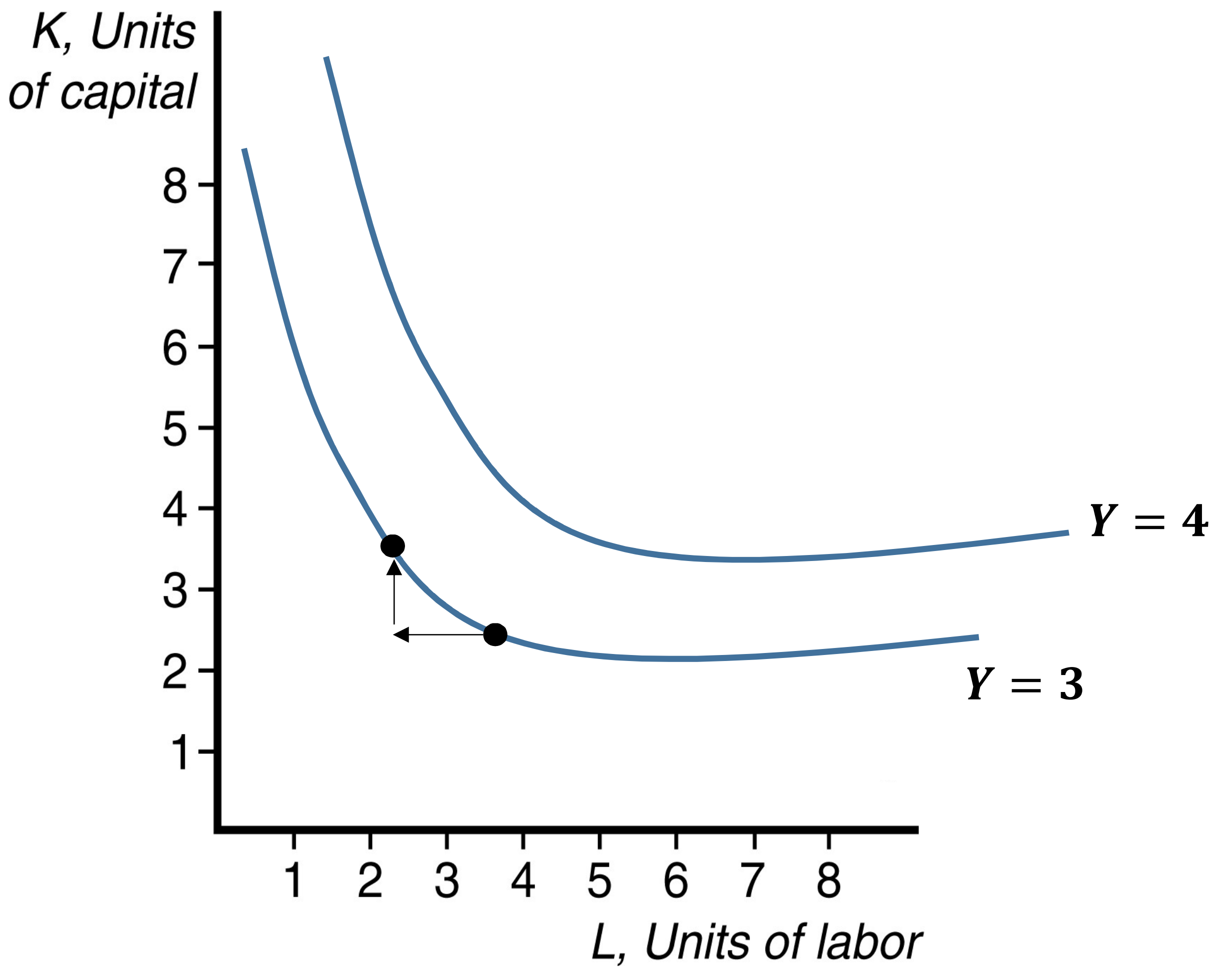
Decreasing returns to Labour

Q13. Suppose that as long as neither input exceeds four times the other, capital and labour are perfect substitutes at a one-to-one ratio. However, once the input ratio reaches four to one in favour of either input, no further substitution is possible. Draw the isoquants.

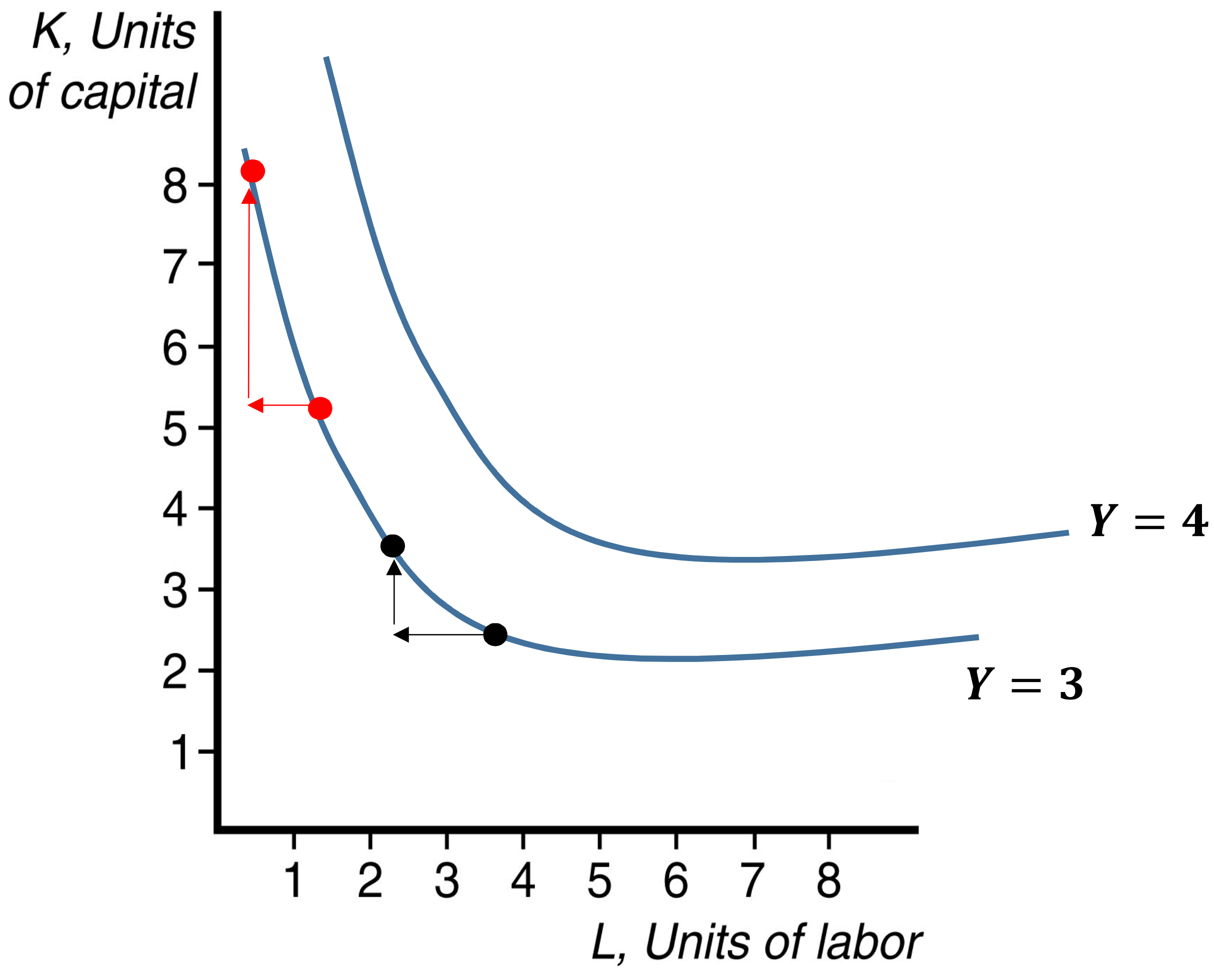
- Isoquants

  Idea: Combination of inputs that produce the same output

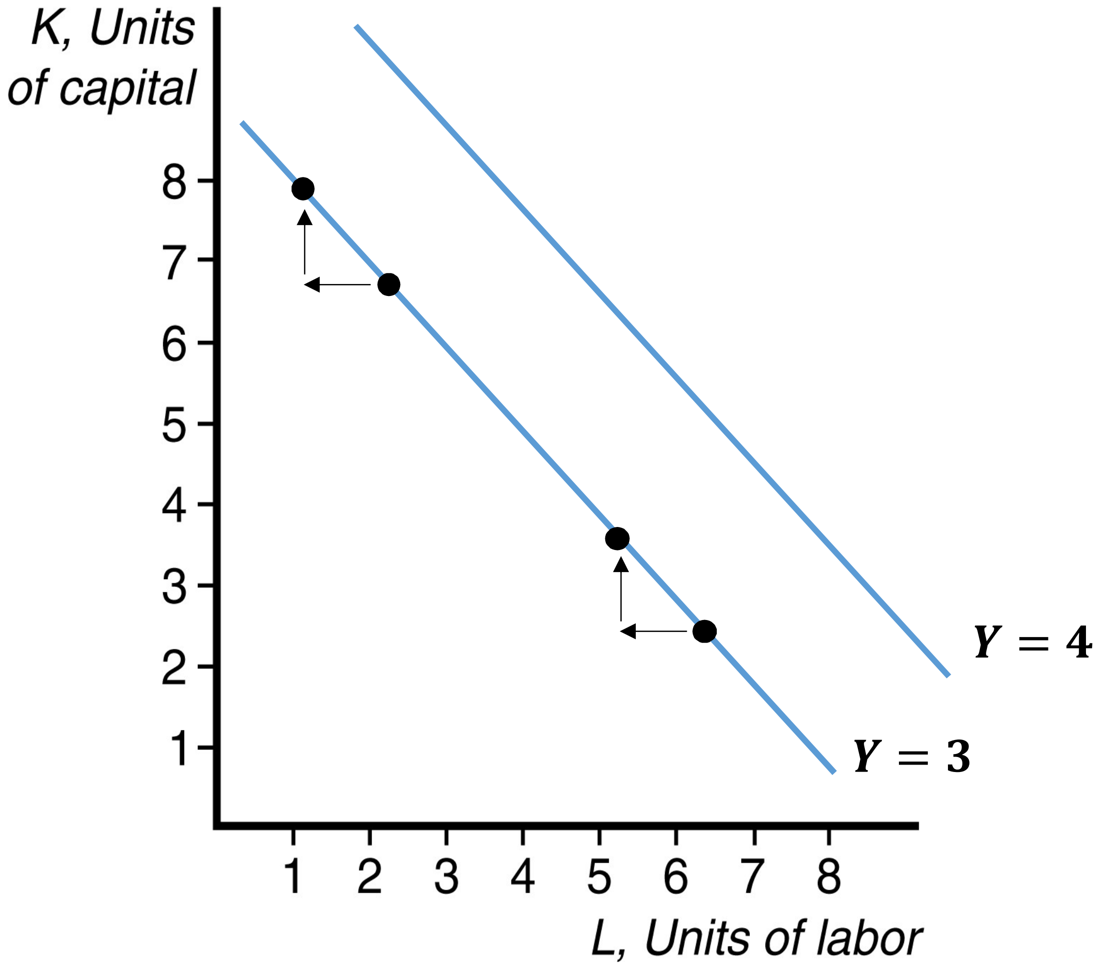
Isoquants (generic case)



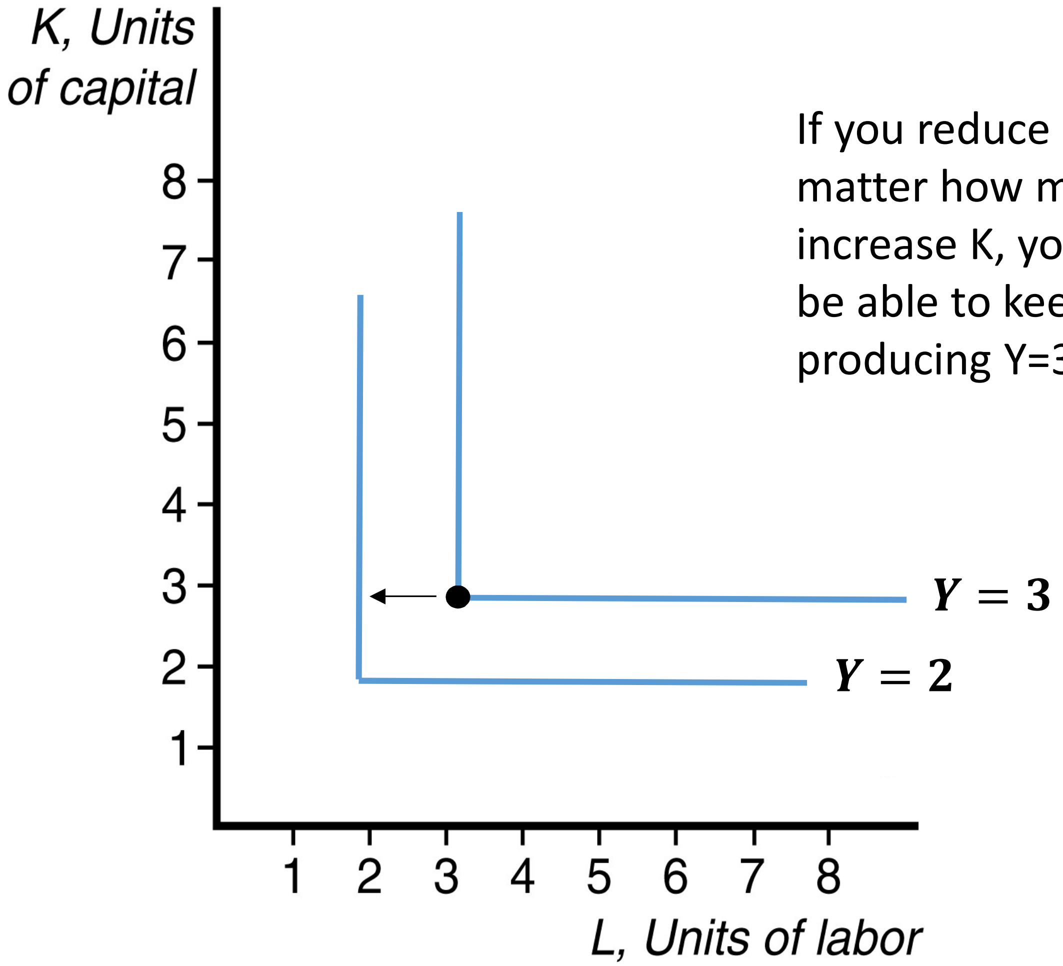
In the example below, it is more difficult to substitute L with K when L is relatively low



Perfect Substitutes: You can substitute K for L at the same rate, independently on how much K or L you are using

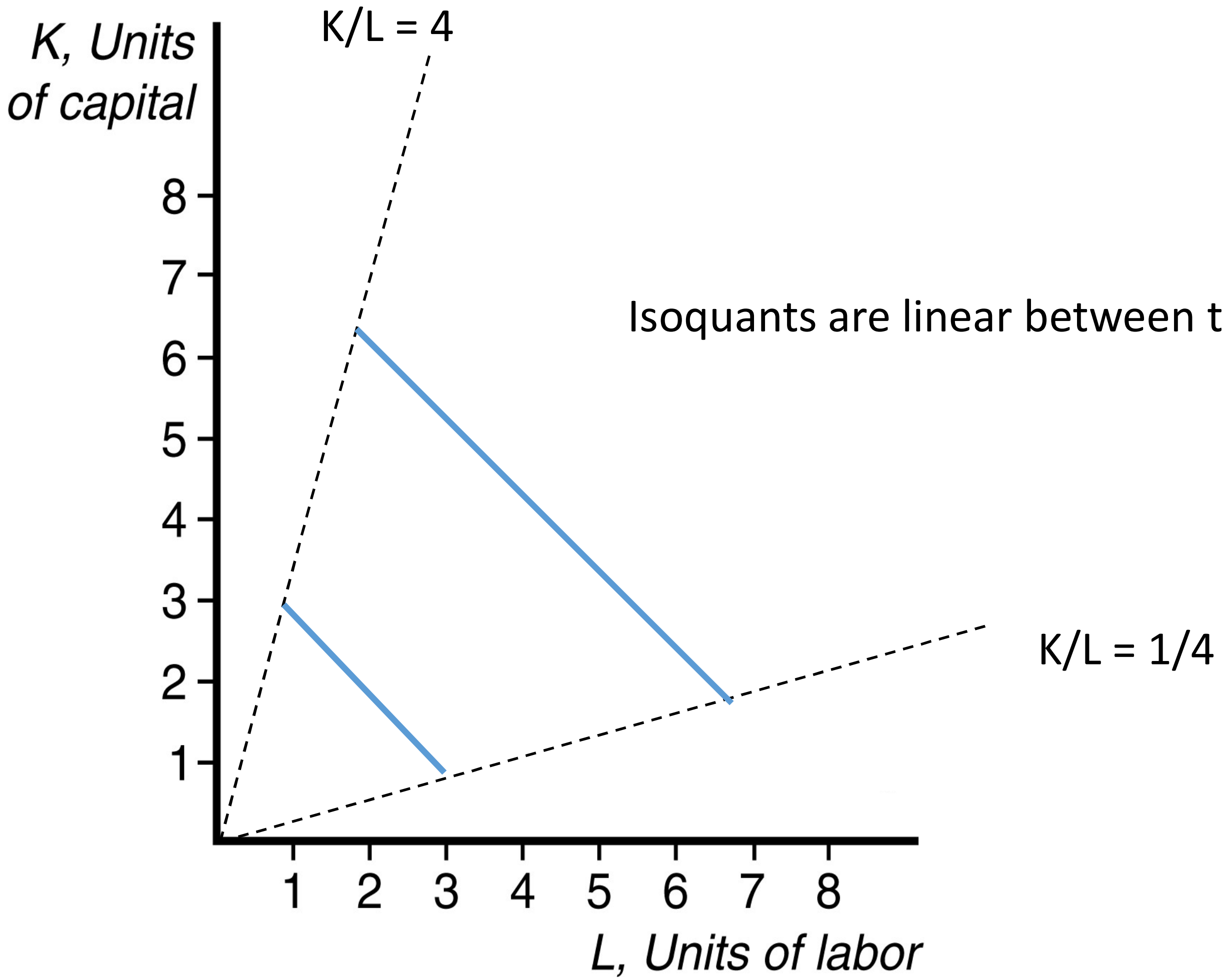


Perfect Complements: You <u>can't</u> substitute K for L

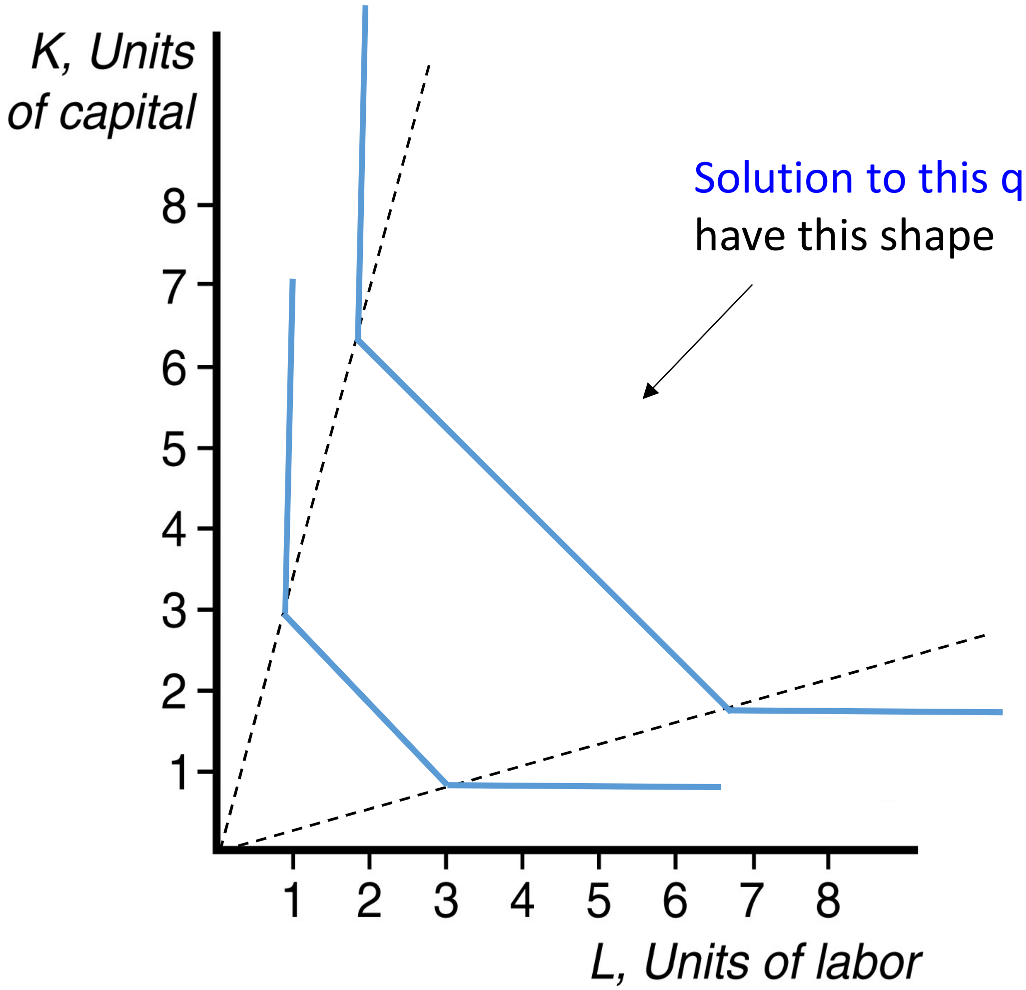


If you reduce L, no matter how much you increase K, you will not be able to keep producing Y=3

This question: Perfect substitutes (one-to-one) if the ratios K/L or L/K are less than 4



This question: But no substitution otherwise



Solution to this question: Isoquants have this shape

Q14. The Cobb-Douglas production $f(x_1, x_2) = Ax_1^a x_2^b$. It turns out that the type of returns to scale of this function will depend on the magnitude of a+b. Which values of a+b will be associated with the different kinds of returns to scale?

- Idea of Returns to Scale

Increase **ALL** inputs by *g*%. How much output does increase?

More than *g%* → IRS

Exactly *g%* → CRS

Less than *g%* → DRS

Start with a production of $\boldsymbol{f}(\boldsymbol{x_1}, \boldsymbol{x_2})$ and duplicate the inputs:

$$
\begin{aligned}
f(2x_1, 2x_2) &= A(2x_1)^a(2x_2)^b \\
&= 2^{a+b} A x_1^a x_2^b \\
&= 2^{a+b} \underbrace{f(x_1, x_2)}_{\text{initial production}}
\end{aligned}
$$

new production

*Then, new production will be **more/less/equal** than double of initial production depending on $\boldsymbol{a} + \boldsymbol{b}$ being **more/less/equal** than 1*

Q15. Is it possible to have decreasing marginal product in an input and yet increasing returns to scale?

- Yes, they are different concepts

Example of such a function:

$$f(x_1, x_2) = x_1^{0.6}\, x_2^{0.6}$$

Q18. Josip is leading a guerrilla unit of 1000 troops. Josip can send his soldiers to collect berries, hunt rabbits or to collect birds' eggs.

Suppose that each activity yields the following levels of output (measured in kilocalories):

- Egg collection yields $Q_e = 8000L_e - 10L_e^2$
- Berry collection yields $Q_b = 4000L_b - 5L_b^2$
- Rabbit hunting yields $Q_r = 2000L_r$

a) If Josip's objective is to maximise the total number of calories, how should he distribute them among these tasks?

- Allocate troops to equalize MP
- Otherwise, you can do better by moving troops from the lower MP task to the high MP task

$$Q'_e = 8000 - 20L_e \qquad (1)$$
$$Q'_b = 4000 - 10L_b \qquad (2)$$
$$Q'_r = 2000 \qquad (3)$$

Equate (1) and (2) to 2000 to get $L_e = 300,\ L_b = 200.$
The remaining 500 work as L_r

b) What if Josip's army actually had 400 more or fewer troops in it?

- Just assign them to/from L_r (they still have MP = 2000 and all the MP remain equalized)

c) What if Josip's army actually had 650 less troops in it?

- Now, Q'_e and Q'_b is larger than 2000 even if all the troops work in either task
- So, nobody should do rabbit hunt
- Equate (1) and (2) and use condition $L_e + L_b = 350$

$$8000 - 20L_e = 4000 - 10L_b$$
$$L_e + L_b = 350$$

System has solution $L_e = 250$ and $L_b = 100$

d) What if Josip's army actually consisted entirely of 1000 vegetarians?

- Similar to part c), ignore rabbit hunting and solve using the same steps