ECONOMICS 1 (sem 2)
Tutorial 2

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https://diegobattiston.github.io/T2.pdf

Questions to cover today

- Q8
- Q9
- Q15
- Q16
- Q18
- Q19
- Q21

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Q8. Sketch the short-run TC, VC, FC, ATC, AVC, AFC, and MC curves for the production function

$$Q = 3KL$$

K is fixed at 2 units in the short run, with r = 3 and $w = 2$.

- Short Run: $Q = 6L \quad \Rightarrow \quad L = \frac{Q}{6}$
- $TC = rK + wL \quad \Rightarrow \quad TC = \underbrace{6}_{FC = 6} + \underbrace{2\frac{Q}{6}}_{VC = 2\frac{Q}{6}}$
- $ATC = \frac{TC}{Q} \quad \Rightarrow \quad ATC = \frac{6}{Q} + \frac{1}{3}$
- $MC = \frac{dTC}{dQ} \quad \Rightarrow \quad MC = \frac{1}{3}$
- etc.

Q9. When the average product of labour is the same as the marginal product of labour, how will marginal cost compare with average variable cost?

$$VC = wL$$

$$AVC = \frac{wL}{Q} = \frac{w}{AP}$$

If $MP = AP$,
then $MC = AVC$

$$MC = \frac{dVC}{dQ} = w\frac{dL}{dQ} = \frac{w}{MP}$$

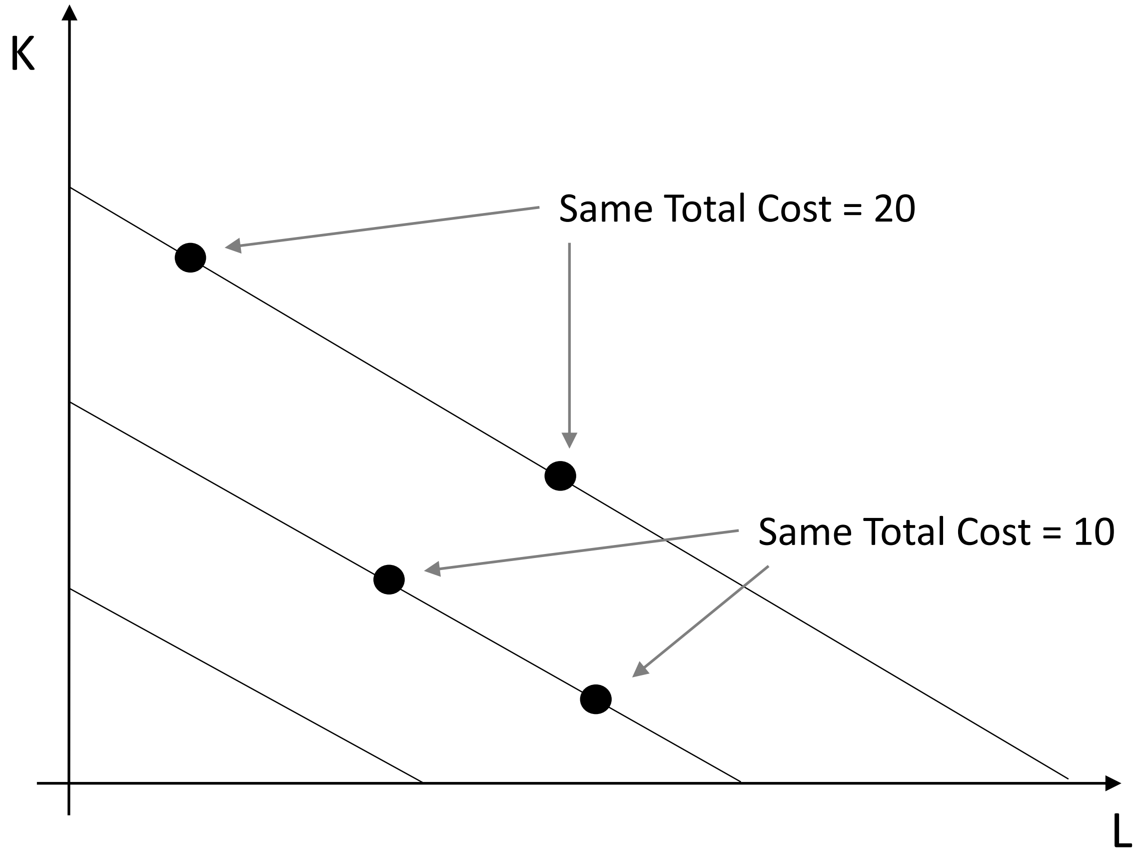
Q15. A firm purchases capital and labour in competitive markets at prices of r=6 and w=4, respectively. With the firm's current input mix, the marginal product of capital is 12 and the marginal product of labour is 18. Is this firm minimizing its costs? If so, explain how you know. If not, explain what the firm ought to do.

Key info:

- r=6 and w=4
- MPK=12 and MPL=18
- Is the firm minimizing costs?

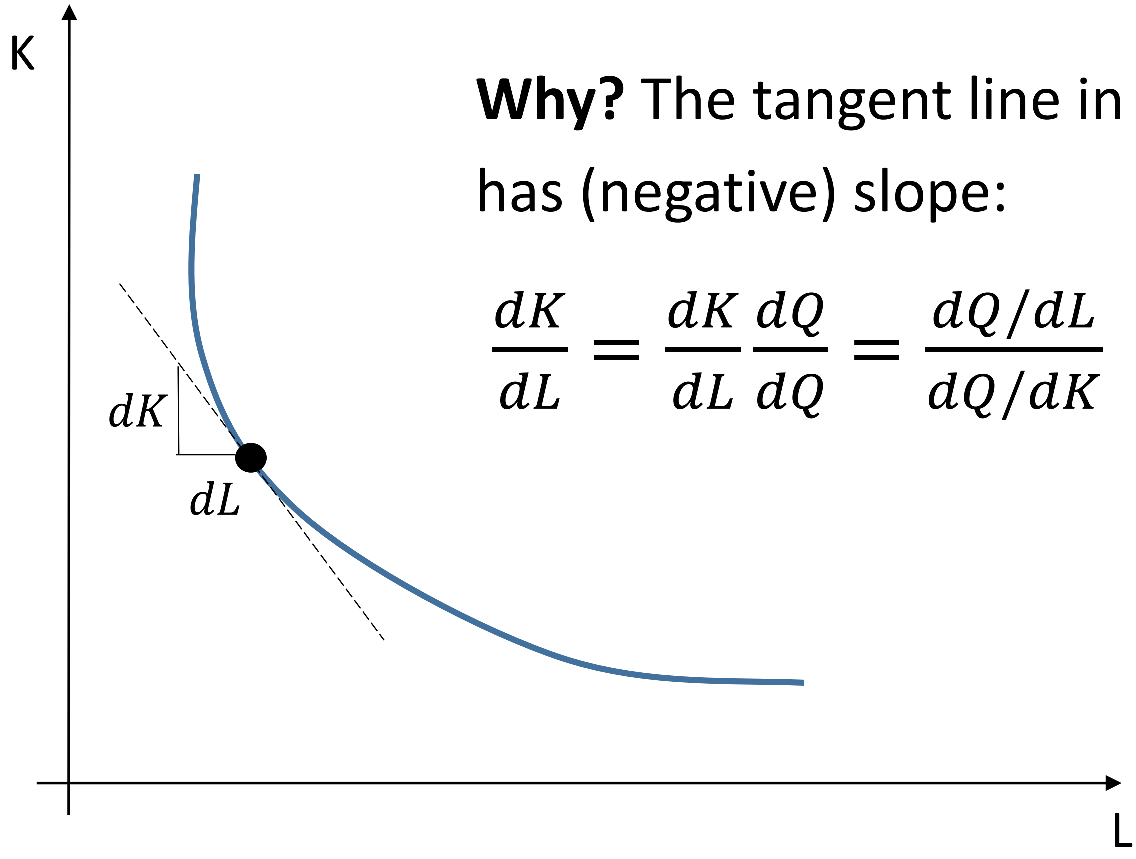
Isocosts

The slope of an Isocost line is $-\frac{w}{r}$



Isoquant

The slope of the isoquant in a point is $-\frac{MPL}{MPK}$

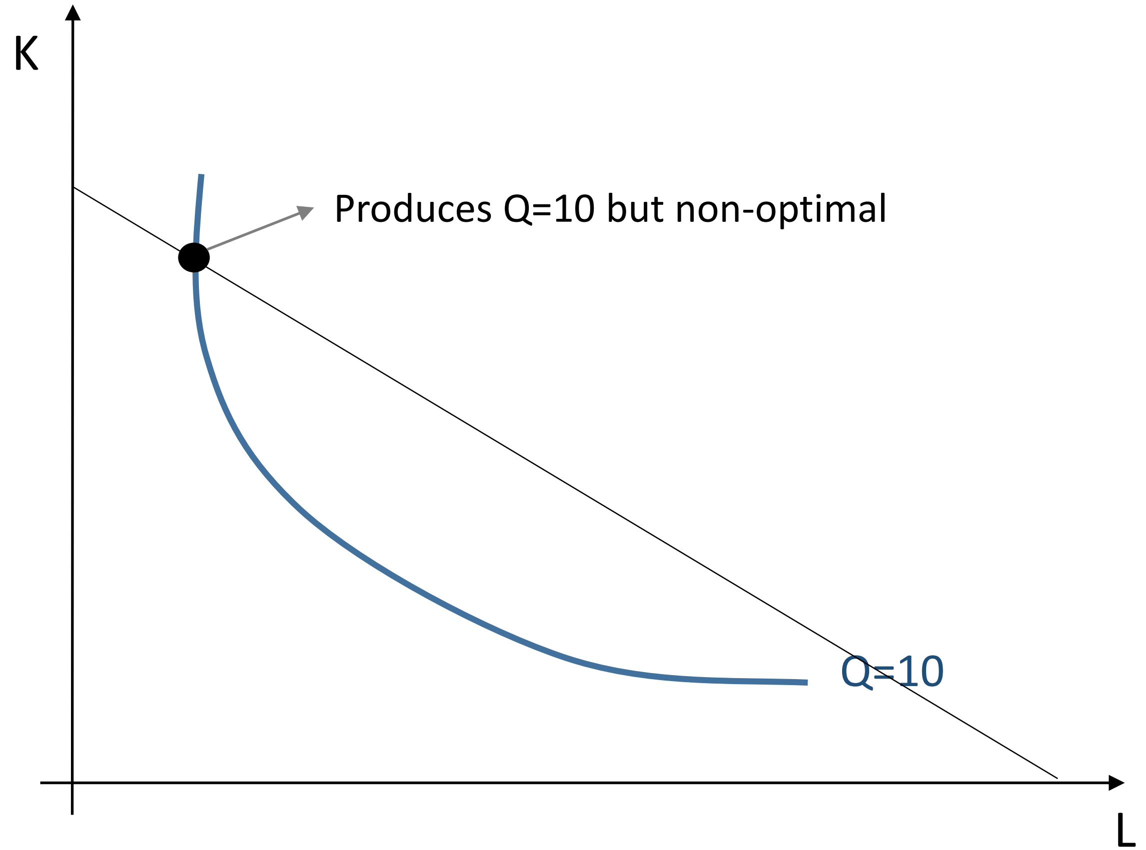


Why? The tangent line in a point has (negative) slope:

$$\frac{dK}{dL} = \frac{dK}{dL}\frac{dQ}{dQ} = \frac{dQ/dL}{dQ/dK}$$

Cost Minimization Problem

For given Q and input prices, find cheapest combination of K and L



Cost Minimization Problem

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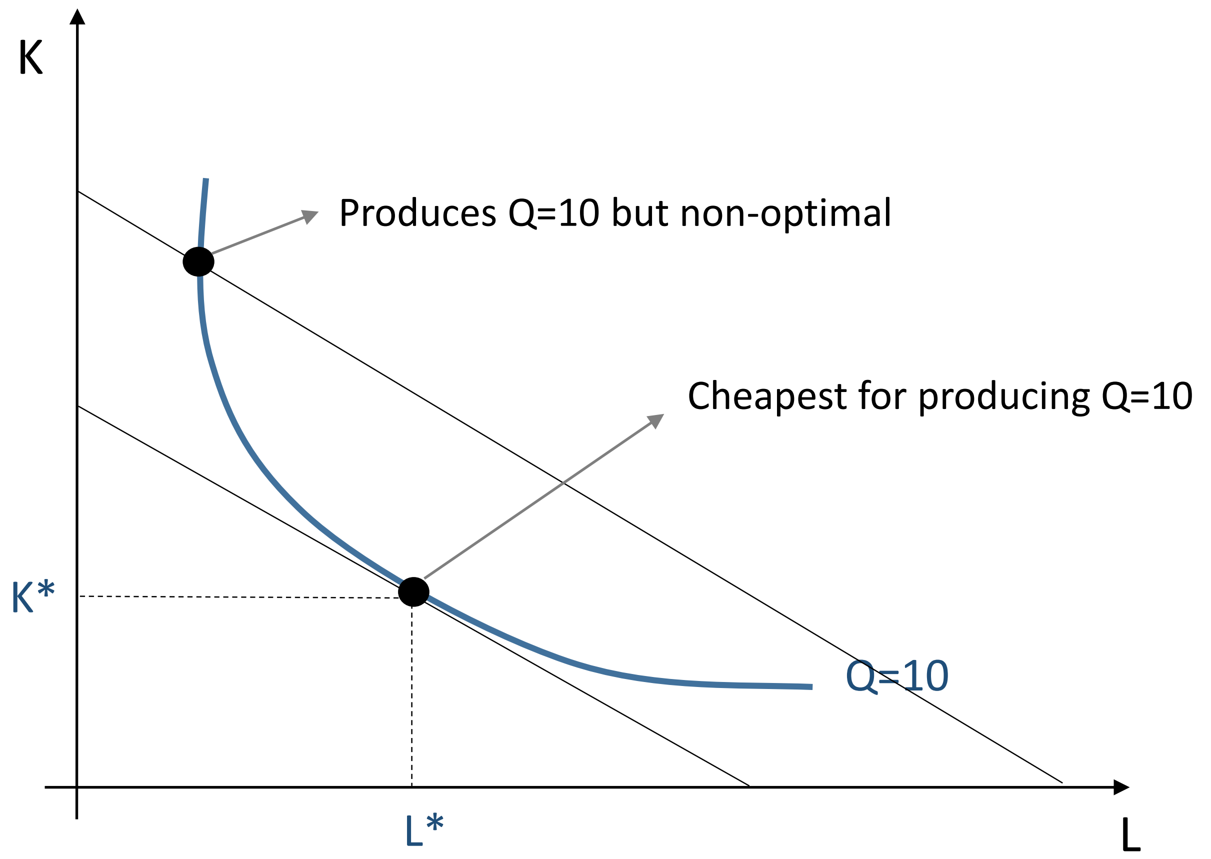
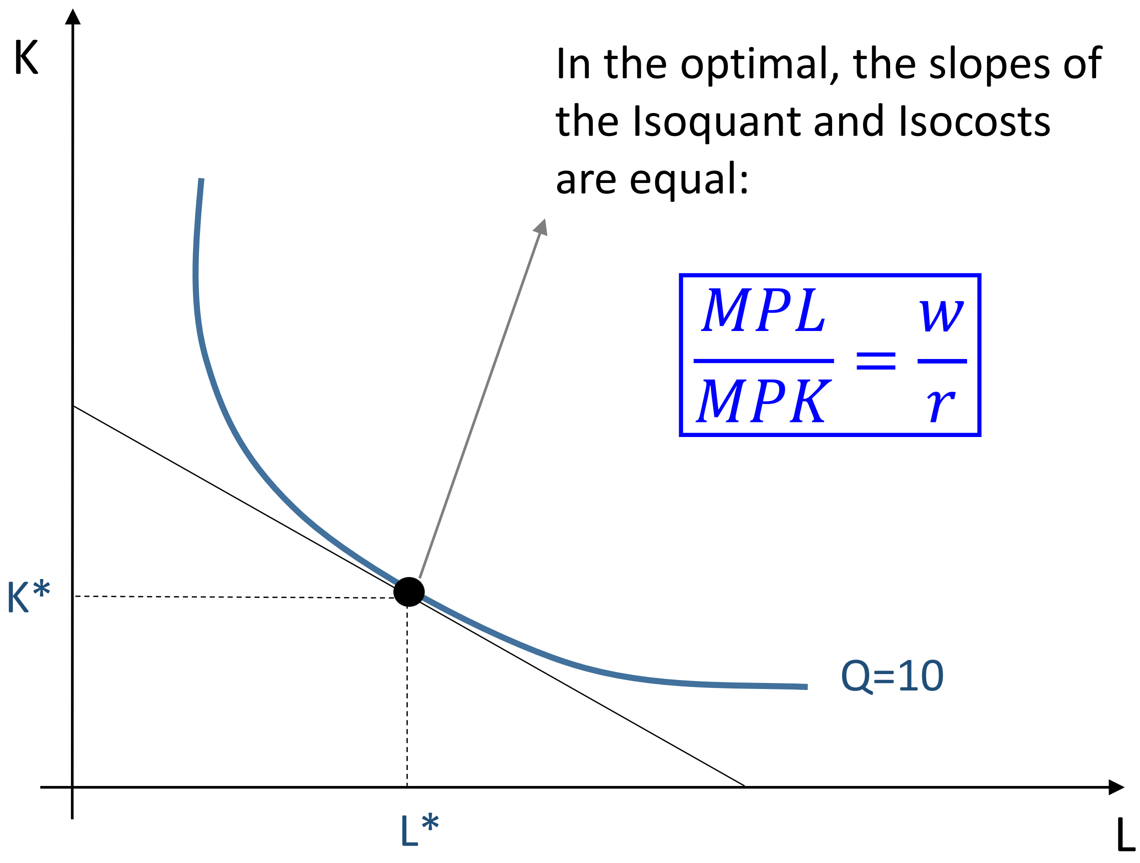


Cost Minimization Problem

For given Q and input prices, find cheapest combination of K and L

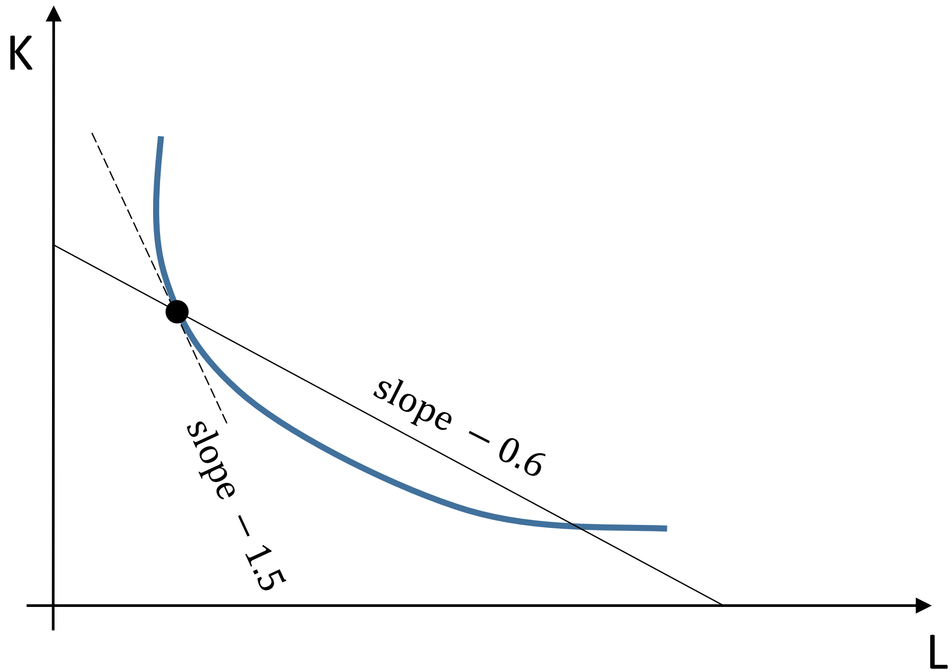


In the optimal, the slopes of the Isoquant and Isocosts are equal:

$$\boxed{\frac{MPL}{MPK} = \frac{w}{r}}$$

This Exercise

$$\frac{MPL}{MPK} = \frac{18}{12} = 1.5 \quad \text{and} \quad \frac{w}{r} = \frac{4}{6} = 0.6 \quad \Longrightarrow \quad \text{Non Optimal}$$



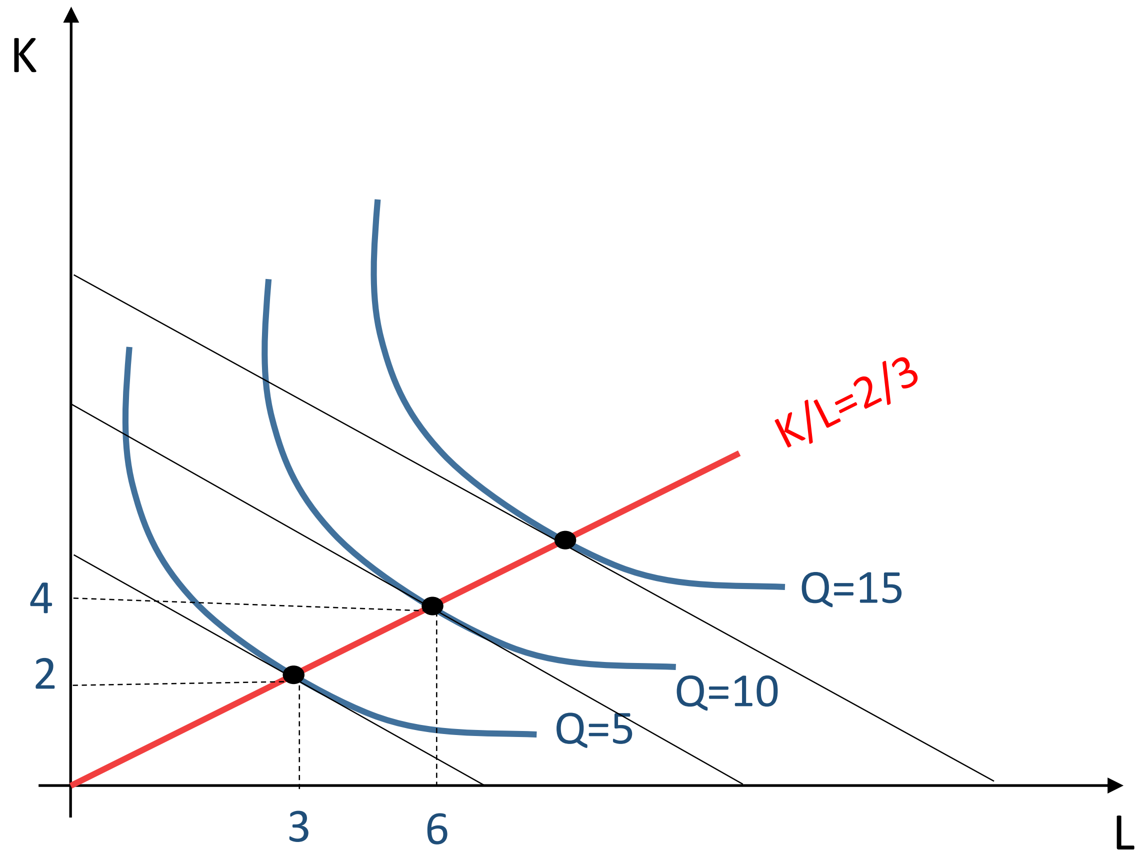
Q16. A firm has a production function $Q = F(K, L)$ with constant returns to scale. Input prices are $r = 2$ and $w = 1$. The output-expansion path for this production function at these input prices is a straight line through the origin. When it produces 5 units of output, it uses 2 units of K and 3 units of L. How much K and L will it use when its long-run total cost is equal to 70?

Key info:

- Production function has CRS
- r=2 and w=1
- Expansion path is a line trough the origin
- For Q = 5 we have K = 2 and L=3
- Question: Find K and L if TC = 70

Expansion Path

Optimal K/L as you increase production (at given prices)



Extra: How would this graph look with IRS or DRS?

Total Cost when Q = 5

$$rK + wL$$

$$2 \times 2 + 1 \times 3 = 7$$

CRS

Suppose we multiply <u>all</u> inputs by *g>1*. Then:

- Production increases by *g*
- Cost increases by *g:* $2 \times 2g + 1 \times 3g = 7g$
- K/L is still 2/3

New situation with TC = 70

Because the expansion path is linear <u>we know</u> that K and L must have increased by the same proportion g so K/L is remained constant

Then, the new cost 70 can be written as:

$$2 \times 2g + 1 \times 3g = 70$$

$$\Rightarrow g = 10$$

Which means that $K = 20$ and $L = 30$

Q18. Sketch LTC, LAC and LMC curves for the production function $Q=K+L$.
Does this production function have constant, increasing or decreasing returns to scale?

Key info:

- Perfect Substitutes (one-to-one substitution)
- Use only cheapest input

If $w < r \quad \Longrightarrow \quad Q = L \quad \Rightarrow \quad TC = wQ$

If $w > r \quad \Longrightarrow \quad Q = K \quad \Rightarrow \quad TC = rQ$

In either case, TC is linear, AC and MC are constant

The function has CRS:

$$Q(tK, tL) = tK + tL = t(K + L) = tQ(K, L)$$

Q19. A firm produces output with the production function

$$Q = \sqrt{KL}$$

If the price of labour is 1 and the price of capital is 4, what quantities of capital and labour should it employ if its goal is to produce 200 units of output?

Key info:

- w=1 and r=4
- Want to produce Q=200
- Question: Find L and K

Use optimal condition $\frac{MPL}{MPK} = \frac{w}{r}$

$$\frac{\sqrt{K}/2\sqrt{L}}{\sqrt{L}/2\sqrt{K}} = \frac{K}{L} = \frac{1}{4}$$

$$\boxed{L = 4K} \qquad (1)$$

Replace (1) into the production function

$$Q = \sqrt{K4K} = 2K$$

$\boxed{K = \frac{Q}{2} = 100}$ and from (1) $\boxed{L = 400}$

Extra: Find the Long Run TC function

Q21. Suppose that a firm has production function:

$$Q(K,L) = 2L^2 + K^2$$

a) If the price of labour is 6 and the price of capital is 5 what is the long-run average cost and marginal cost curve?

Key info:

- w=6 and r=5
- Find TC and MC long run

Key: Isoquants are concave!

$$Q = 2L^2 + K^2$$

Key: Isoquants are concave!

For $\frac{w}{r} = \frac{6}{5} \quad \Rightarrow \quad$ Use only L

$$Q = 2L^2 \quad \Rightarrow \quad \boxed{L = \left(\frac{Q}{2}\right)^{0.5}}$$

Replace into the definition of $TC = wL + rK = 6L$

$$\boxed{TC = 6\left(\frac{Q}{2}\right)^{0.5}}$$

b) Derive the short-run average total cost and marginal cost curve if K=4.

$$Q = 2L^2 + 16 \quad \Rightarrow \quad \boxed{L = \left(\frac{Q-16}{2}\right)^{0.5}}$$

Then, use definition of $TC = wL + rK = 6L + 20$

$$\boxed{TC^{SR} = 6\left(\frac{Q-16}{2}\right)^{0.5} + 20}$$