ECONOMICS 2
Tutorial 1

Diego Battiston

Plan:

- Discuss the exercises highlighting the key concepts
- You'll get solutions, so I will try to address things not explained there
    - E.g. I won't spend much time doing algebra
    - **Today:** 1,3,5,7,8,10,11,12

- Short Run vs. Long Run in production

Q1. Give an example of production in which the short run lasts at least one year.

- Short Run vs. Long Run in production

Q1. Give an example of production in which the short run lasts at least one year.

Q3. Graph the short-run total product curves for each of the following production functions if K is fixed at $K_0 = 4$.

a. $Q = F(K,L) = K^{1/2}L^{1/2}$

b. $Q = F(K,L) = 2K + 0.5L$

- Marginal Returns vs. Returns to Scale
  - Given a function F(K,L): How do we find about?

Q3. Graph the short-run total product curves for each of the following production functions if K is fixed at $K_0 = 4$. Are there diminishing returns to labour?

a. $Q = F(K, L) = K^{1/2}L^{1/2}$

b. $Q = F(K, L) = 2K + 0.5L$

- Marginal Returns vs. Returns to Scale
  - Given a function F(K,L): How do we find about?

Q7. In a production process is it possible to have decreasing marginal product in an input and yet increasing returns to scale?

- Marginal Returns vs. Returns to Scale
  - Given a function F(K,L): How do we find about?

Q7. In a production process is it possible to have decreasing marginal product in an input and yet increasing returns to scale?

Q10. Determine if the following production functions have increasing, decreasing, or constant returns to scale.

a. $Q = 4K^{1/2}L$

b. $Q = 4K + 2L$

- Marginal Rate of Technical Substitution

Q7. In a production process is it possible to have decreasing marginal product in an input and yet increasing returns to scale?

Q10. Determine if the following production functions have increasing, decreasing, or constant returns to scale. Also calculate the marginal rate of technical substitution for each production function.

a. $Q = 4K^{1/2}L$

b. $Q = 4K + 2L$

- Marginal Rate of Technical Substitution

a)

$$\frac{MP_L}{MP_K} = \frac{4K^{1/2}}{2K^{-1/2}L} = \frac{2K}{L}$$

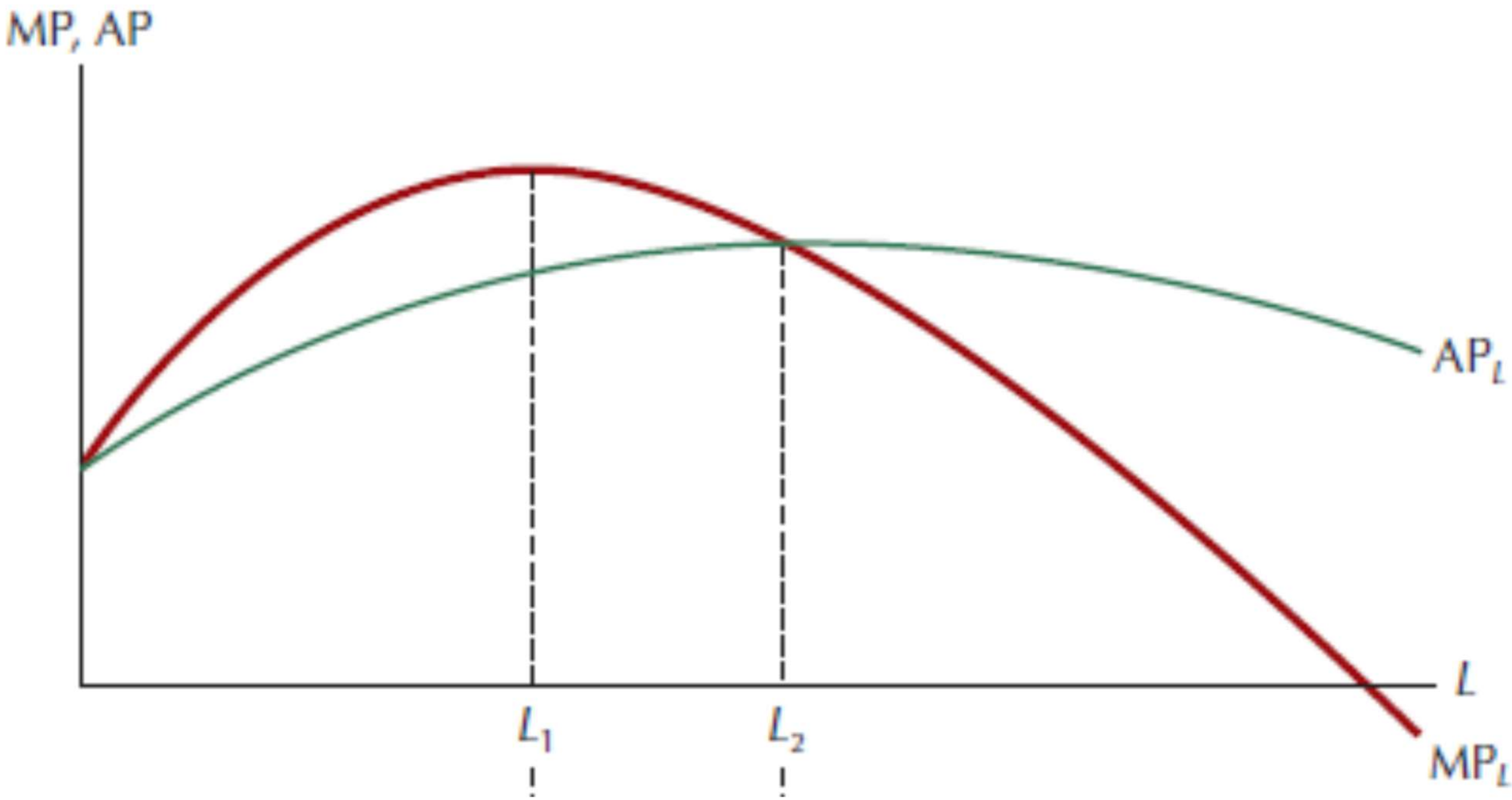
b)

$$\frac{MP_L}{MP_K} = \frac{1}{2}$$

- Relation between Mg Product and Avg Product

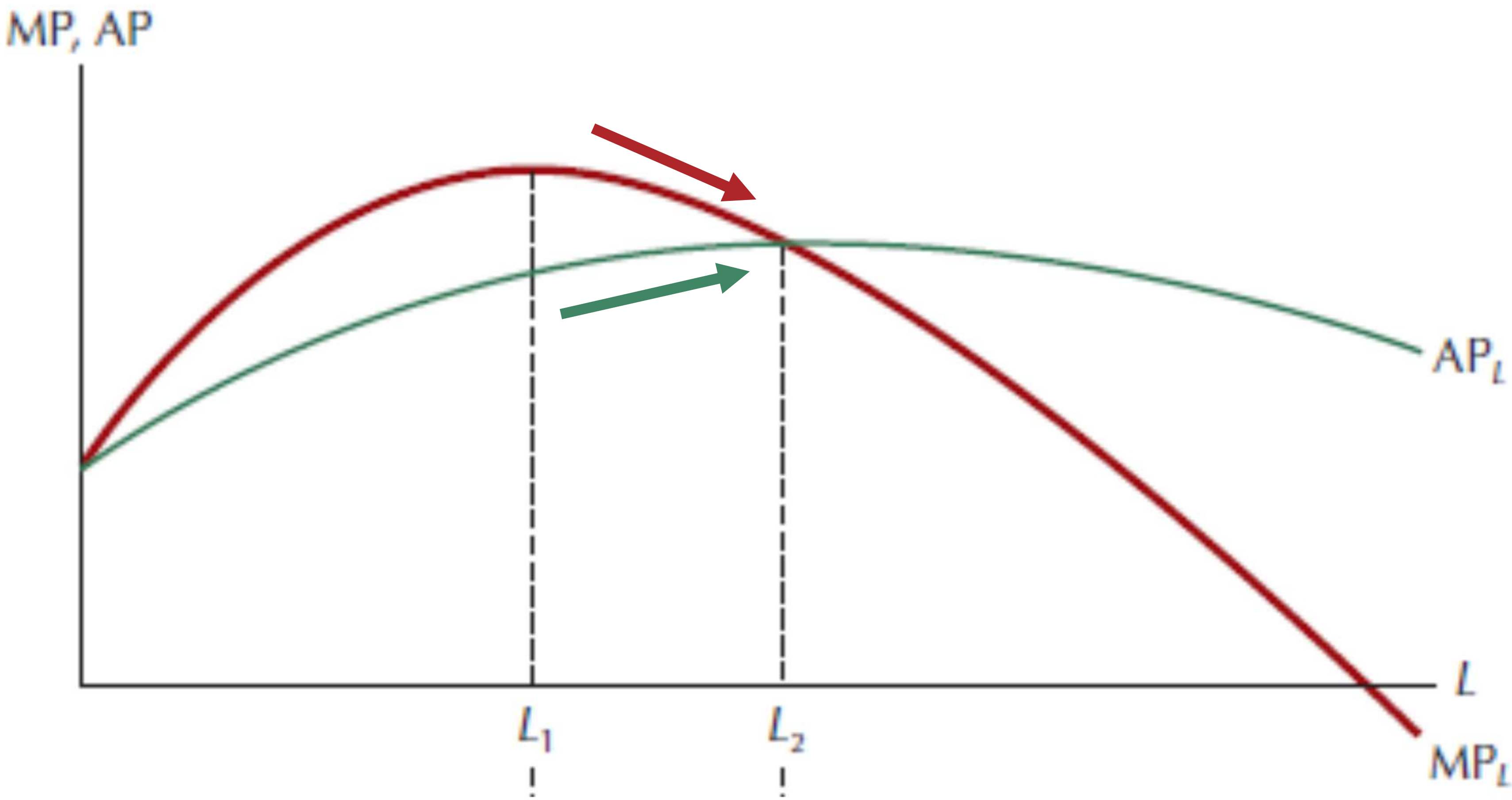
Q5. True or False? If the marginal product is decreasing, then the average product must also be decreasing. Explain.

Remember the following graph (taken from the book)



If you don’t remember how to derive this graph, it is explained in the book (Figure 10.6 or 9.8 depending on the edition)

There is a range where MP is DECREASING but AP is INCREASING



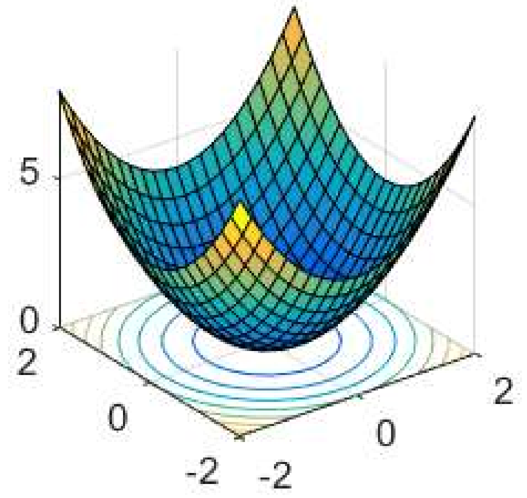
- Isoquant: Concept and graph

Q8. What would an upward sloping isoquant imply?

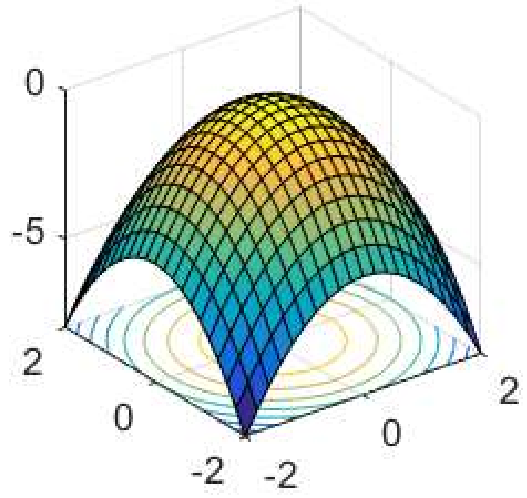
- Optimize a function of two independent variables
    1. Get critical points
    2. Decide if point is min, max, or saddle point

Q11. Show that $z = 2x^3 + y^3 - 18x - 12y + 50$ has a minimum at $x = 1.732,\ y = 2$, a maximum at $x = -1.732,\ y = -2$, a saddle point at $x = 1.732,\ y = -2$ and another saddle point at $x = -1.732,\ y = 2$.

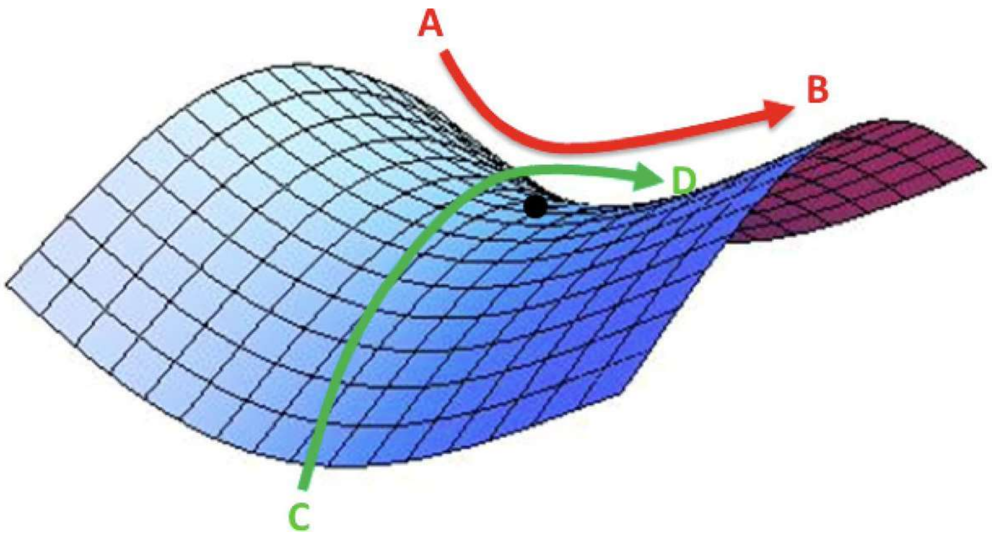
local min



local max



saddle point



- Optimize a function of two independent variables
  1. Get critical points
  2. Decide if point is min, max, or saddle point

Q11. Show that $z = 2x^3 + y^3 - 18x - 12y + 50$ has a minimum at $x = 1.732,\ y = 2$, a maximum at $x = -1.732,\ y = -2$, a saddle point at $x = 1.732,\ y = -2$ and another saddle point at $x = -1.732,\ y = 2$.

This table is in the solutions:

x	y	z_{xx}	z_{yy}	$z_{xx}z_{xx} - z_{xy}z_{yx}$	*Conclusion*
+1.732	*+2*	*>0*	*>0*	*>0*	*min*
-1.732	*-2*	*<0*	*<0*	*>0*	*max*
+1.732	*-2*	*>0*	*<0*	*<0*	*saddle pt.*
-1.732	*+2*	*<0*	*>0*	*<0*	*saddle pt.*

- Calculate total differential for a function
  - Procedure
  - Error we make if we use total differential instead of actual change

Q12. Find the total differential (dz) of the following functions:

a. $z = 3x^2y^3 + \frac{x+y}{3y^2}$

b. $z = (x^2 + 5y^3)^{0.5}$

c. $z = e^{2x-3y}$

Use a simpler example: $z = x^2y^3 + x$

- Calculate total differential for a function
  - Procedure
  - Error we make if we use total differential instead of actual change

If for instance we move from point $(0,0)$ to $(1,2)$, compare:

$$dz = f_x(0,0)(1-0) + f_y(0,0)(2-0)$$

vs.

Real change in z = $f(1,2) - f(0,0)$