# ECONOMICS 2

## Tutorial 6

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Questions 1,5,6,14,16

http://personal.lse.ac.uk/BATTISTO/T6_slides.pdf

**Question 1**

The Black Death – bubonic plague – wiped out between a third and a half of the population of medieval Western Europe. In England, the plague struck in 1348-1349, 1360-1361, 1369, and 1375. The average real wage in England rose by 25% in the second half of the fourteenth century compared to the first half. Explain why that happened. What do you think happened to real prices of capital and land?

**Optimal Hiring Rule:**

$$\boldsymbol{MRP_L} = \boldsymbol{MC_L}$$
$$\boldsymbol{MR} \times \boldsymbol{MP_L} = \boldsymbol{MC_L}$$

**In words:** *What you earn for one extra worker = what the extra worker costs*

- In competitive markets (of product) $\boldsymbol{MR}$ = p
- In competitive markets (of labour) $\boldsymbol{MC_L}$ = w

$$\boldsymbol{p.MP_L} = \boldsymbol{w}$$

**Question 1**

The Black Death – bubonic plague – wiped out between a third and a half of the population of medieval Western Europe. In England, the plague struck in 1348-1349, 1360-1361, 1369, and 1375. The average real wage in England rose by 25% in the second half of the fourteenth century compared to the first half. Explain why that happened. What do you think happened to real prices of capital and land?

$\frac{w}{p}$ **increased because ↑MPL** (see graph in whiteboard)

$\frac{r}{p}$ **decreased because ↓MPK** (see graph in whiteboard)

**Question 5**

A firm is Monopolist AND Monopsonist:

- Demand product $\mathrm{P} = 100 - \mathrm{Q}$,
- Production function $\mathrm{Q} = 4\mathrm{L}$
- Labour Supply $\mathrm{W} = 40 + 2\mathrm{L}$

Find L and W (you can also find Q and p)

**Optimal Hiring Rule:**

$$\boldsymbol{MR} \times \boldsymbol{MP_L} = \boldsymbol{MC_L}$$

- **Monopoly:** $\mathrm{MR} = 100 - 2\mathrm{Q}$
- **Monopsony:** Total cost of hiring is $\mathrm{w(L).L} = (40 + 2\mathrm{L})\mathrm{L}$

  Then, $MC_L = 40 + 4L$

Replace in the equation and solve for L

$$\underbrace{\boldsymbol{MR}}_{(100\ -\ 2\mathrm{Q})} \times \underbrace{\boldsymbol{MP_L}}_{4} = \underbrace{\boldsymbol{MC_L}}_{40+4L}$$

$$(100\ -8\mathrm{L})4 \quad = 40+4L$$

$$\boxed{\mathrm{L} = 10}$$

$$\boxed{\mathrm{W} = 60}$$

**Question 6**

a) Firm is Monopsonist in labour market BUT price taker in product market:

- $\mathrm{P} = 8$
- $\mathrm{MP_L} = 5$
- Labour Supply $\mathrm{W} = 10 + \mathrm{L}$

Find L, W, Q

Replace in the optimal condition equation and solve for L

$$\boldsymbol{MR \times MP_L = MC_L}$$

$$8 \times 5 = 10 + 2L$$

$$\boxed{\begin{array}{c} \mathrm{L} = 15 \\ \mathrm{W} = 25 \\ Q = 75 \end{array}}$$

**Question 6**

b) Firm is Monopsonist in labour market AND Monopolist in product market:

- $\mathrm{P} = 102 - 1.96\mathrm{Q}$
- $\mathrm{MP_L} = 5$
- Labour Supply $\mathrm{W} = 10 + \mathrm{L}$

Find L, W, Q, P

$$\boldsymbol{MR \times MP_L = MC_L}$$

$$(102 - 3.92\mathrm{Q}) \times 5 = 10 + 2L$$

$$(102 - 3.92(5\mathrm{L})) \times 5 = 10 + 2L$$

$$\boxed{\begin{gathered} \mathrm{L} = 5 \\ \mathrm{W} = 15 \\ Q = 25 \\ P = 53 \end{gathered}}$$

**Question 6**

c) Starting from L=5 and w=15: The firm can hire additional workers (at higher wage) but it does not have to pay more to already hired workers.

Will L increase?

**Extra worker:**

- Will cost W=17
- Will produce 5 units, but price will drop from 53 to $102 - 1.96 \times 30 = 43.2$
  - Raises revenues by $5 \times 43.2 = 216$
  - But decreases revenues of (previous units) by $25 \times (53 - 43.2) = 245$

Not convenient to hire an extra worker

Short answer: $\boldsymbol{MRP_L(L)} = \big(102 - 3.92(5\mathrm{L})\big) \times \ 5 \quad < 0$ if L=6

## Question 7

Competitive labour market:

$$\mathrm{L_S} = 50\mathrm{w} - 100$$
$$\mathrm{L_D} = 650 - 25\mathrm{w}$$

a) Equilibrium if no union

$$\boldsymbol{L_S = L_D}$$

$$650 - 25\mathrm{w} = 50\mathrm{w} - 100$$

$$w = 10$$

$$\mathrm{L} = 400$$

## Question 7

b) Union maximizes rents of workers

**This problem is similar to a Monopolist "selling labour" and facing whole demand for it**

- $\text{"MR"} = 26 - 0.08\text{L}$ (labour demand with x2 slope)
- $\text{"MC"} = 2 + 0.02\text{L}$ (labour supply, i.e. cost of providing one extra worker)

Equating both, we get $\mathbf{L = 240}$ and from the labour demand, $\mathbf{w = 16.4}$

**Question 7**

c) Union maximizes aggregated wages

**This problem is similar to a Monopolist maximizing revenues**

- $\text{Aggregated Wages} = \mathrm{w}^{\mathrm{d}}(\mathrm{L}).\,\mathrm{L} = (26 - 0.04\mathrm{L})\mathrm{L}$

- Derivative = 0 gives $\mathbf{L = 325}$ , then from the labour demand $\mathbf{w = 13}$

# Math Questions

**Question 14**

Given $\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & -3 & 1 \\ 1 & -1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & -5 & 3 \\ 1 & 4 & -1 \end{bmatrix}$, find $\mathbf{AB}$

## Matrix Multiplication

$$(2,3,5).\begin{pmatrix} 1 \\ 7 \\ 1 \end{pmatrix} = (2 \times 1) + (3 \times 7) + (5 \times 1) = 28$$

$$\begin{bmatrix} \boxed{2 \quad 3 \quad 5} \\ 1 \quad -3 \quad 1 \\ 1 \quad -1 \quad 2 \end{bmatrix} . \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 7 \\ 1 \end{matrix}} & \begin{matrix} 2 \\ -5 \\ 4 \end{matrix} & \begin{matrix} 3 \\ 3 \\ -1 \end{matrix} \end{bmatrix} = \begin{bmatrix} 28 & & \\ & & \\ & & \end{bmatrix}$$

**Question 14**

Given $\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & -3 & 1 \\ 1 & -1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & -5 & 3 \\ 1 & 4 & -1 \end{bmatrix}$, find $\mathbf{AB}$

**Matrix Multiplication**

$$(1, -3, 1) . \begin{pmatrix} 1 \\ 7 \\ 1 \end{pmatrix} = -19$$

$$\begin{bmatrix} 2 & 3 & 5 \\ \boxed{1 \quad -3 \quad 1} \\ 1 & -1 & 2 \end{bmatrix} . \begin{bmatrix} 1 & 2 & 3 \\ 7 & -5 & 3 \\ 1 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 28 & & \\ -19 & & \\ & & \end{bmatrix}$$

**Question 14**

Given $\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & -3 & 1 \\ 1 & -1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & -5 & 3 \\ 1 & 4 & -1 \end{bmatrix}$, find $\mathbf{AB}$

**Matrix Multiplication**

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & -3 & 1 \\ 1 & -1 & 2 \end{bmatrix} . \begin{bmatrix} 1 & 2 & 3 \\ 7 & -5 & 3 \\ 1 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 28 & 9 & 10 \\ -19 & 21 & -7 \\ -4 & 15 & -2 \end{bmatrix}$$

**Question 16**

Transition probability matrix is given by:

$$\mathbf{P} = \begin{bmatrix} Pr(E,E) & Pr(E,U) \\ Pr(U,E) & Pr(U,U) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}$$

Initial State $\mathbf{x}_0 = \begin{bmatrix} E \\ U \end{bmatrix} = \begin{bmatrix} 0 \\ 1200 \end{bmatrix}$

a) What will be the number of unemployed people after (i) 2 periods; (ii) 3 periods; (iii) 5 periods; (iv) 10 periods?

**First period:**

We start with 0 employed, so the only transitions are U→E (1200x0.7) = 840 and U→U (1200x0.3)=360

Note this is similar to doing:

$$\begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix} . \begin{bmatrix} 0 \\ 1200 \end{bmatrix} = \begin{bmatrix} 840 \\ 360 \end{bmatrix}$$

$$P . \begin{bmatrix} 0 \\ 1200 \end{bmatrix} = \begin{bmatrix} 840 \\ 360 \end{bmatrix}$$

**Second period:**

Now, all possible transitions:

E→E = 840x0.9, E→U = 840x0.1, U→E = 360x0.7, U→U = 360x0.3

$$P.\begin{bmatrix}840\\360\end{bmatrix} = \begin{bmatrix}1008\\192\end{bmatrix}$$

We can write it as

$$P.\left(P.\begin{bmatrix}0\\1200\end{bmatrix}\right) = \begin{bmatrix}1008\\192\end{bmatrix}$$

Or

$$P^2.\begin{bmatrix}0\\1200\end{bmatrix}$$

**After n periods :**

$$P^n . \begin{bmatrix} 0 \\ 1200 \end{bmatrix}$$

## Question 16

b) Steady State

**Option 1:** Keep iterating $P^n$ to see the convergency matrix

**Option 2:** Write the dynamic of unemployment

$$U_n = 0.1E_{n-1} + 0.3U_{n-1}$$

$$U_n = 0.1(1200 - U_{n-1}) + 0.3U_{n-1}$$

In steady State: $U_n = U_{n-1} = U$

$$U = 0.1(1200 - U) + 0.3U$$

$$U = 150$$