ECONOMICS 2

Tutorial 2

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https://personal.lse.ac.uk/BATTISTO/T2_slides.pdf

- Relation between Mg Product and Mg Cost

Q2. Suppose that labour is the only variable input to the production process. If the marginal cost of production is diminishing as more units of output are produced, what can you say about the marginal product of labour (the variable input)?

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Every extra unit is cheaper to produce than the previous one

=

Producing extra units requires less additional labour

=

Marginal product of labour must be increasing

- Relation between Total and Mg Cost

Q3. The marginal cost of production is given by $MC = 6Q + 1$. If the fixed cost of production is £10, find the total cost function and the variable cost function.

- Relation between Mg Cost and Avg Cost

Q4. Why does the short-run MC curve cut both the ATC and AVC curves at their minimum points?

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Very general idea:

- Suppose your average score is 80
- If in your next exam you get 95 , your avg score increases
- If in your next exam you get 60, your avg score decreases

- Relation between Mg Cost and Avg Cost

Q4. Why does the short-run MC curve cut both the ATC and AVC curves at their minimum points?

Translate the idea to costs:

- If next unit costs less than the average (i.e. MC<AC), then the average is decreasing
- If next unit costs more than the average (i.e. MC>AC), then the average is increasing
- Only way this can happen is if MC intersects AC at its minimum
- This is true for ATC and AVC

- Draw Isoquants based on firm's behaviour

Q6. Justin owns a lot of small urban parking lots and has a choice of either hiring an attendant or buying a parking machine for each lot. He finds that no matter how many cars use his parking lots or how the prices of machines or wages of attendants vary, he always minimizes his costs by buying only one or the other of the two. Draw Justin's isoquant map.

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Key info:

- No matter input prices (nor quantities), <u>fully specialize</u> in one input

- Draw Isoquants based on firm's behaviour

> **Q7.** A firm finds that no matter how much output it produces and no matter how input prices vary, it always minimizes its costs by buying half as many units of capital as of labour. Draw this firm's isoquant map.

Key info:

- No matter input prices (nor quantities), always combine inputs in <u>fixed proportion</u> $K = 0.5L$

- Optimal combination of inputs

Q8. Suppose that a firm has the following production function:

$$Q(K, L) = 2L\sqrt{K}.$$

a. If the price of labour is 2 and the price of capital is 4, what is the optimal ratio of capital to labour?

b. For an output level of $Q = 1000$, how much of each input will be used?

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This condition always gives us an OPTIMAL COMBINATION of L and K

L = 4K in this case

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b. For an output level of $Q = 1000$, how much of each input will be used?

b) Key: Substitute L = 4K in the production function

$$Q = 1000 = 2(4K)\sqrt{K}$$

$$\Rightarrow \boxed{K = 25}$$

- Optimal combination of inputs

Q10. A firm has a production function $Q = F(K, L)$ with constant returns to scale. Input prices are $r = 2$ and $w = 1$. The output-expansion path for this production function at these input prices is a straight line through the origin. When the firm produces 5 units of output, it uses 2 units of K and 3 units of L. How much K and L will it use when its long-run total cost is equal to 70?

Key info:

- Production function has CRS (does not matter for the exercise)
- r=2 and w=1
- Expansion path is a line
- For Q = 5 we have K = 2 and L=3

- Optimal combination of inputs

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i.e. to increase production, increase all inputs by same proportion

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i.e. to increase production, increase all inputs by same proportion

Suppose we multiply all inputs by 10. Then:

- Cost multiplies by 10: $2 \times 20 + 1 \times 30 = 70$

- Extra: Returns to Scale and Average Cost

What about CRS?

- K and L increase from 3 and 2 to 30 and 20
- CRS: production must increase from 5 to 50

… Total cost (long run) increases from 7 to 70

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- CRS: production must increase from 5 to 50

… Total cost (long run) increases from 7 to 70

$$AC = \frac{7}{5} = \frac{70}{50}$$

(constant AC)

CRS $\Rightarrow$ AC is constant

IRS $\Rightarrow$ AC is decreasing

DRS $\Rightarrow$ AC is increasing

- Short Run vs. Long Run costs

Q11. A firm employs a production function $Q = F(K, L)$ for which only two values of K are possible, K_1 and K_2. Its ATC curve when $K = K_1$ is given by $ATC_1 = Q^2 - 4Q + 6$. The corresponding curve for $K = K_2$ is $ATC_2 = Q^2 - 8Q + 18$. What is this firm's LAC curve?

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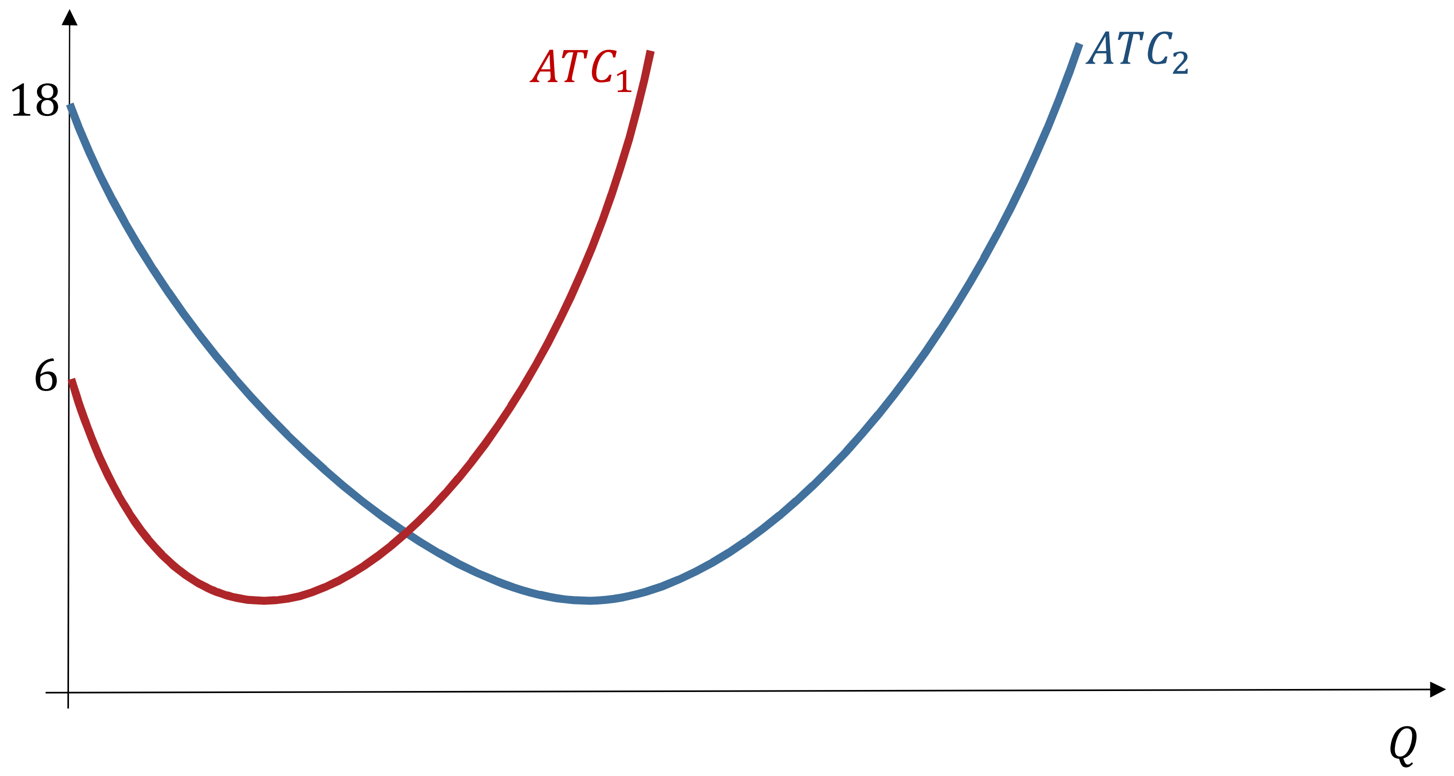

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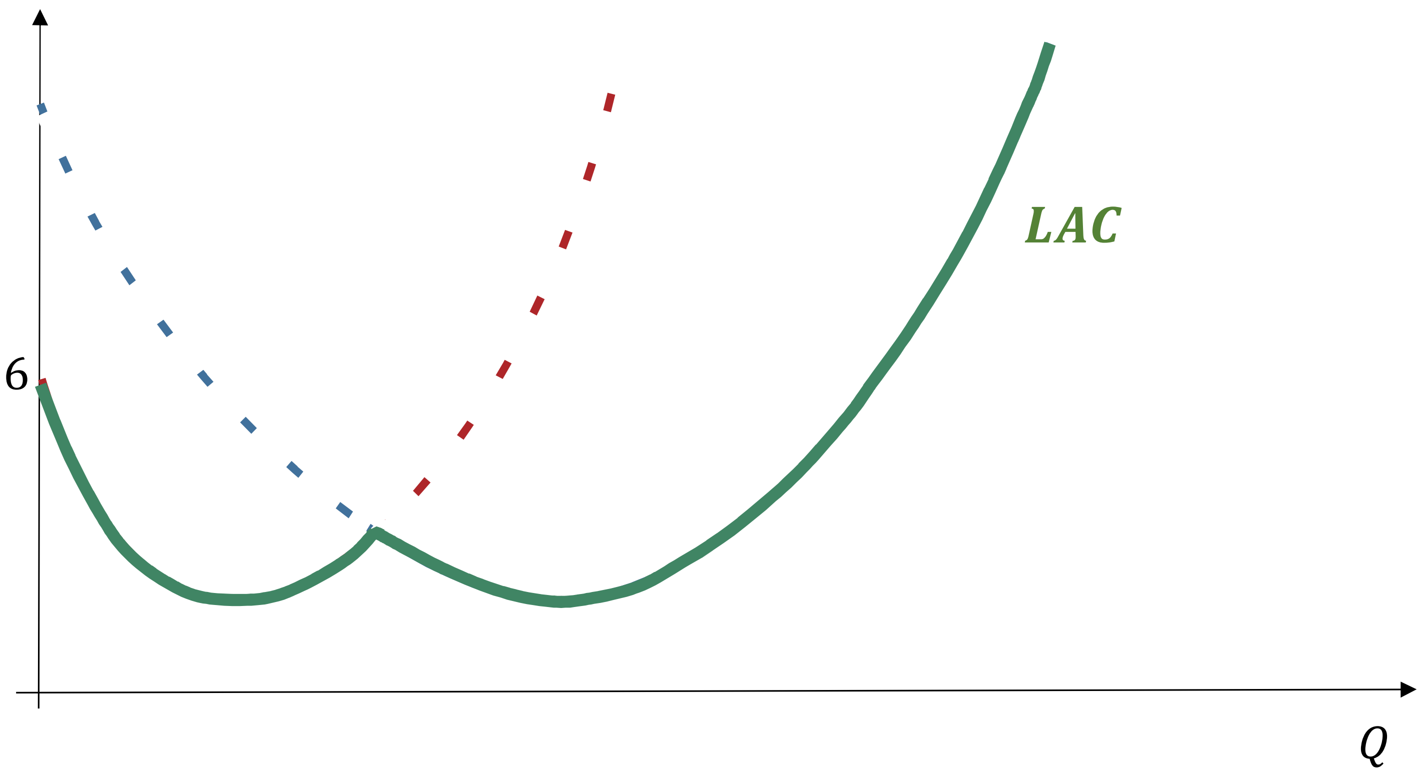

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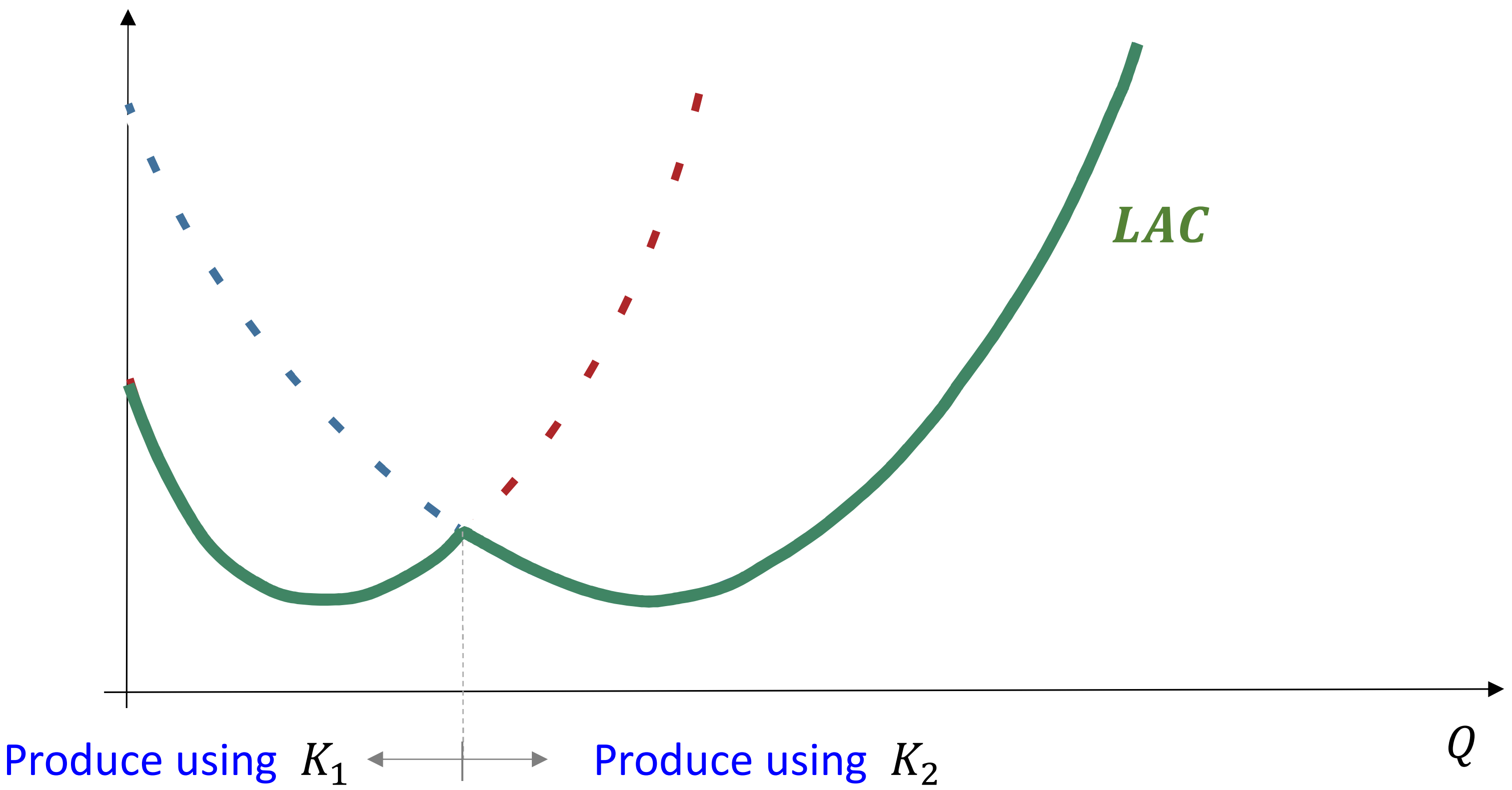

Math Questions

Q13. Find the rate of change of output with respect to time, if the production function is $Q = A(t)K^{\alpha}L^{\beta}$, where $A(t)$ is an increasing function of t, and $K = K_0 + \gamma t$, and $L = L_0 + bt$.

Output only depends on time indirectly, so we need to use chain rule when taking total derivative with respect to t

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial K}\frac{dK}{dt} + \frac{\partial Q}{\partial L}\frac{dL}{dt} + \frac{\partial Q}{\partial A}\frac{dA}{dt}$$

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Now just replace each element in the expression below

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We don't know $\frac{dA}{dt}$ *so leave it generically as* $A'(t)$

$$\frac{dQ}{dt} = \underbrace{\frac{\partial Q}{\partial K}}_{\alpha A K^{\alpha-1}L^{\beta}}\underbrace{\frac{dK}{dt}}_{\gamma} + \underbrace{\frac{\partial Q}{\partial L}}_{\beta A K^{\alpha}L^{\beta-1}}\underbrace{\frac{dL}{dt}}_{b} + \underbrace{\frac{\partial Q}{\partial A}}_{K^{\alpha}L^{\beta}}\underbrace{\frac{dA}{dt}}_{A'(t)}$$

Math Questions

Q15. Consider the following production function:

$$Q = 100KL - 5K^2 - 2L^2 + 3K + 5L$$

a. Use the differential of the production function to find the slope of any isoquant and show that this slope is given by the ratio of marginal products. Is the slope always negative?

b. Use the information from (a) to sketch some typical isoquants. Is this shape plausible?

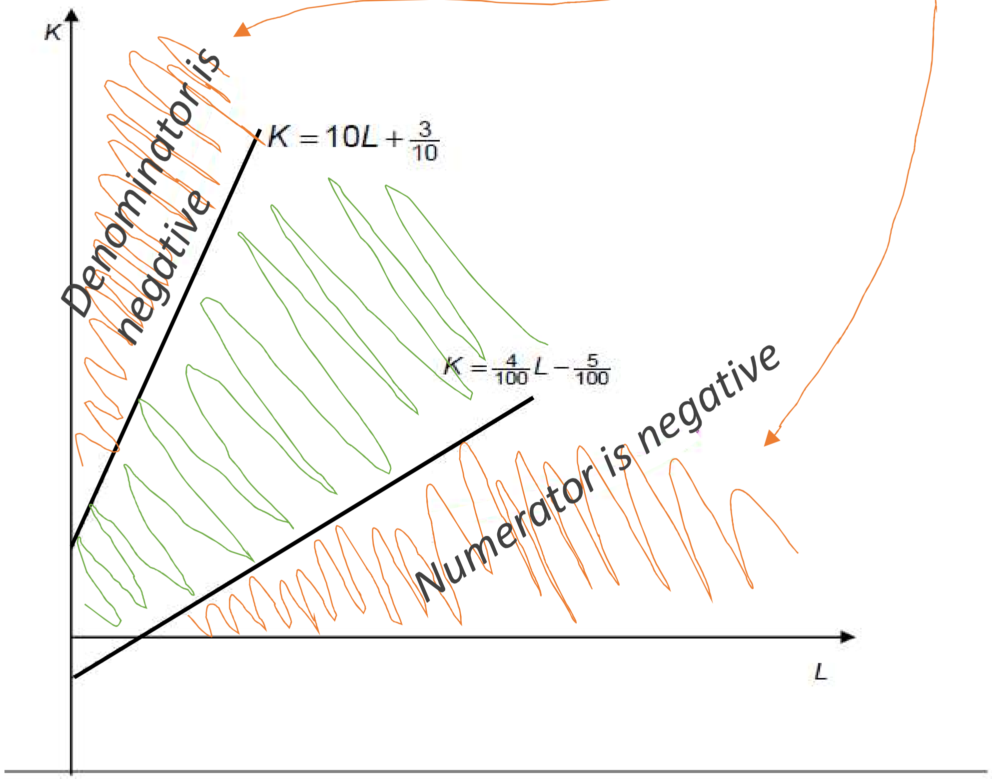
a) Slope of isoquant = –MRTS (we derived this last week)

$$Slope = -\frac{MP_L}{MP_K} = -\frac{100K - 4L + 5}{100L - 10K + 3}$$

Math Questions

Slope is Positive if e.g. numerator is negative and denominator positive, etc.

- *Numerator is negative if* $K < \frac{4}{100}L - \frac{5}{100}$
- *Denominator is negative if* $K > 10L + \frac{3}{10}$



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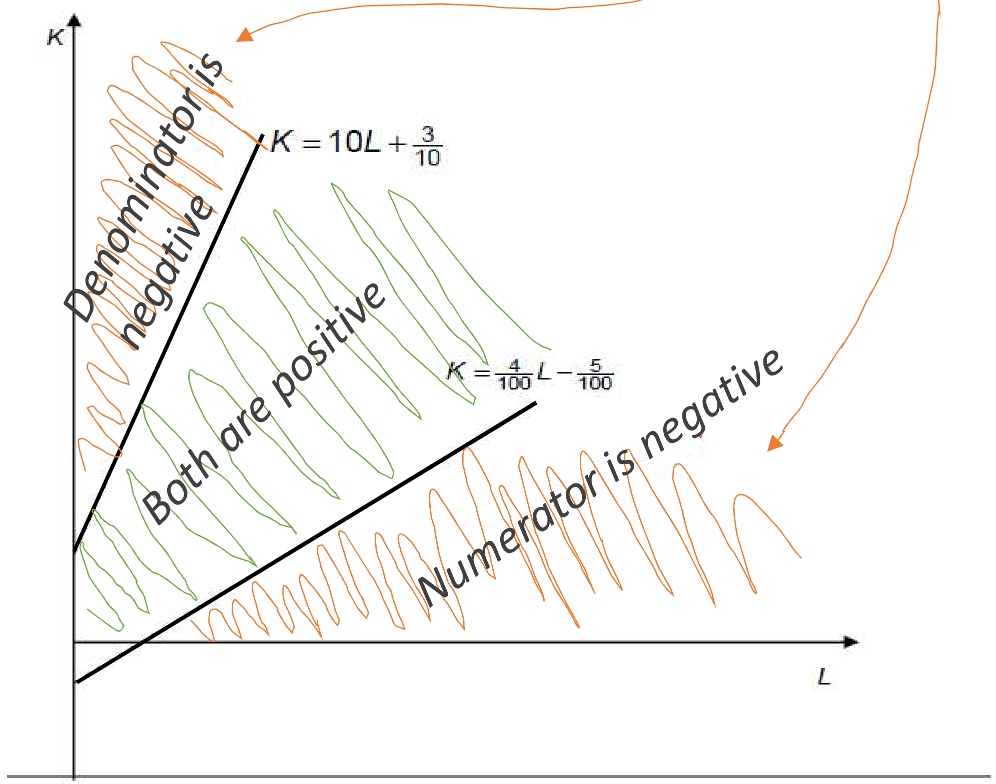

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