Laboratory of Computational Physics 2022-2023
Astrophysics Project

The Formation of Binary Black Holes

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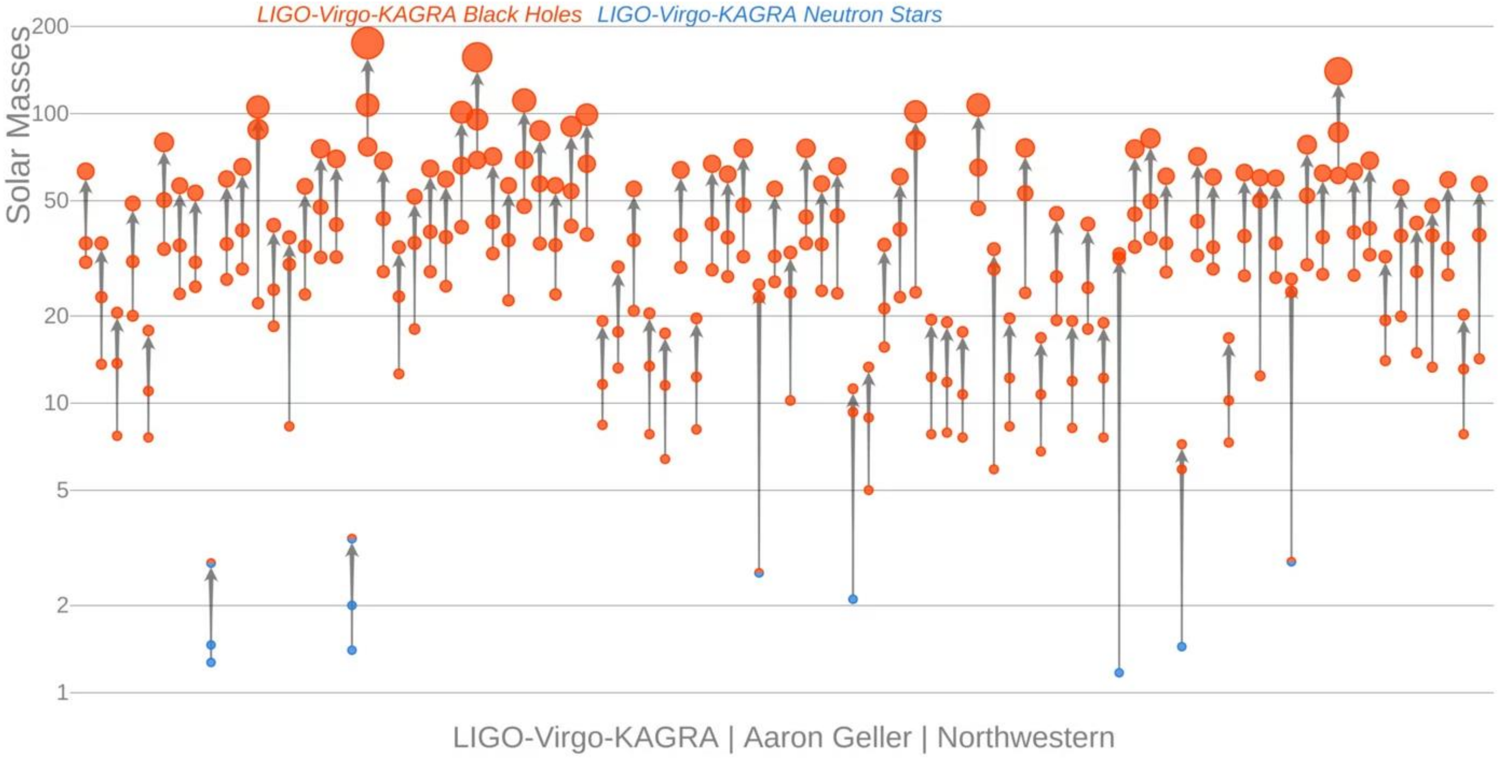
Binary black holes and gravitational waves

90 events of **gravitational waves**

80 mergers of **binary black holes**



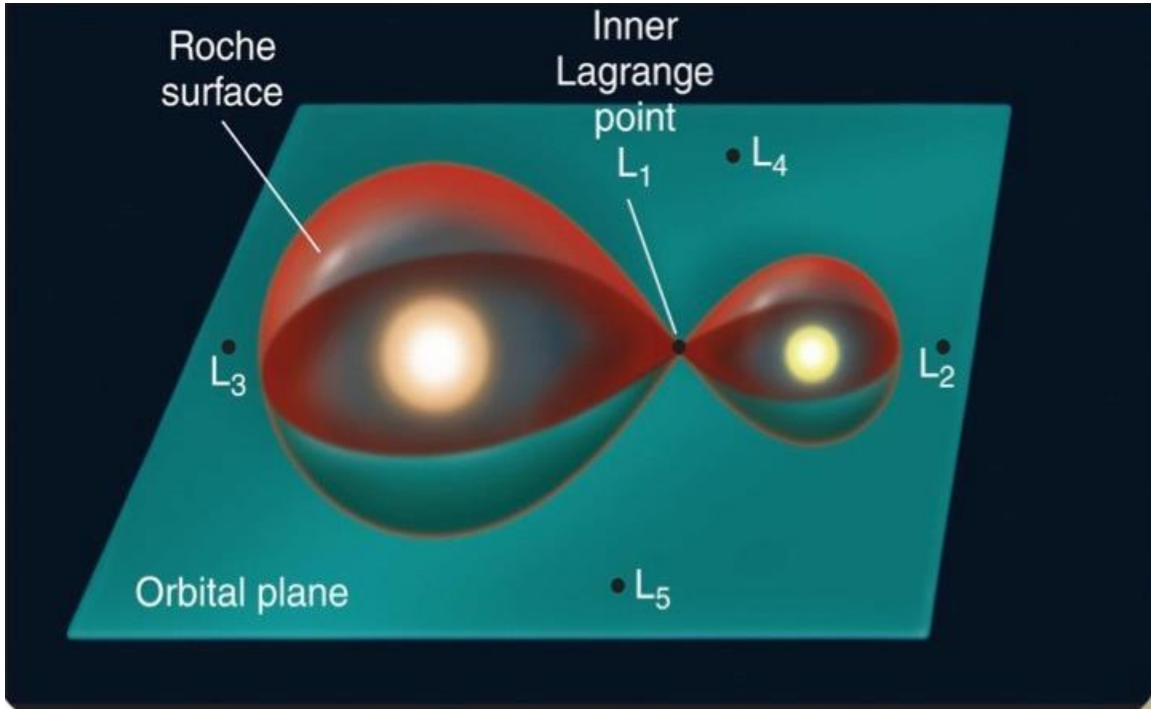
Masses in the stellar graveyard

Goal of the project

Goal of the project:
finding **what features** have the highest impact on the evolution of a binary system into a Binary Black Hole

Credits: ESO/M. Kornmesser/S.E. de Mink

Roche Lobe: region where orbiting material is gravitationally bound



Credits: Cengage Learning (2016)

Mass transfer via Roche Lobe Overflow

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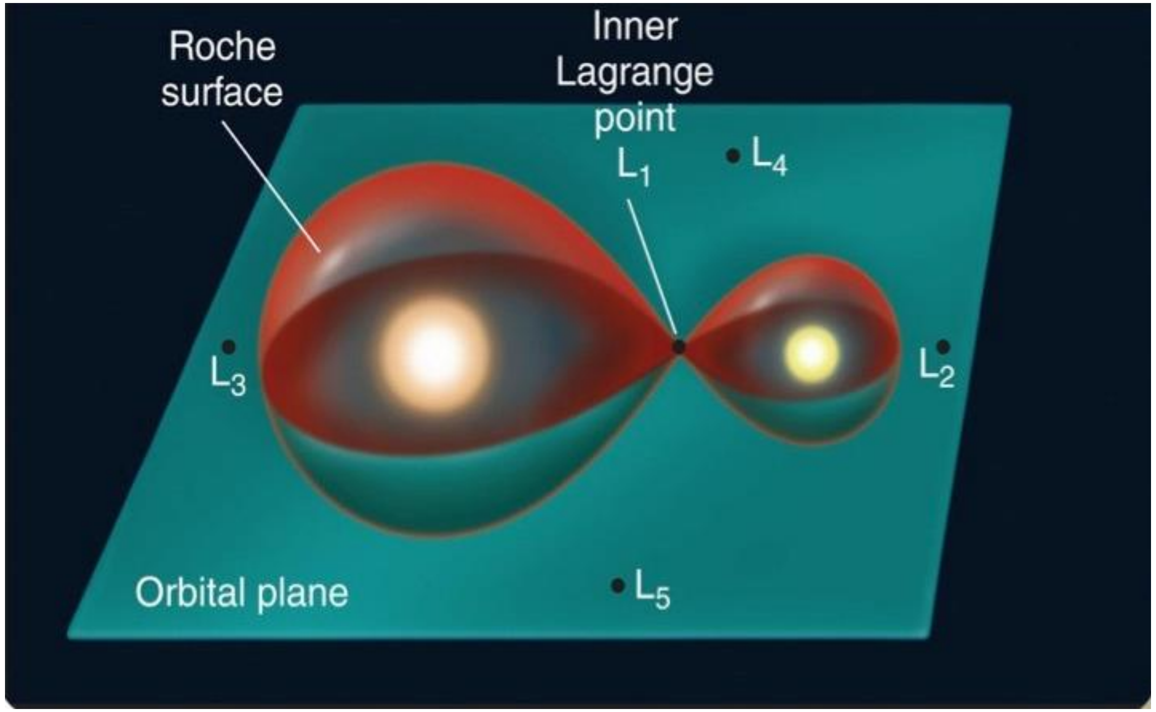

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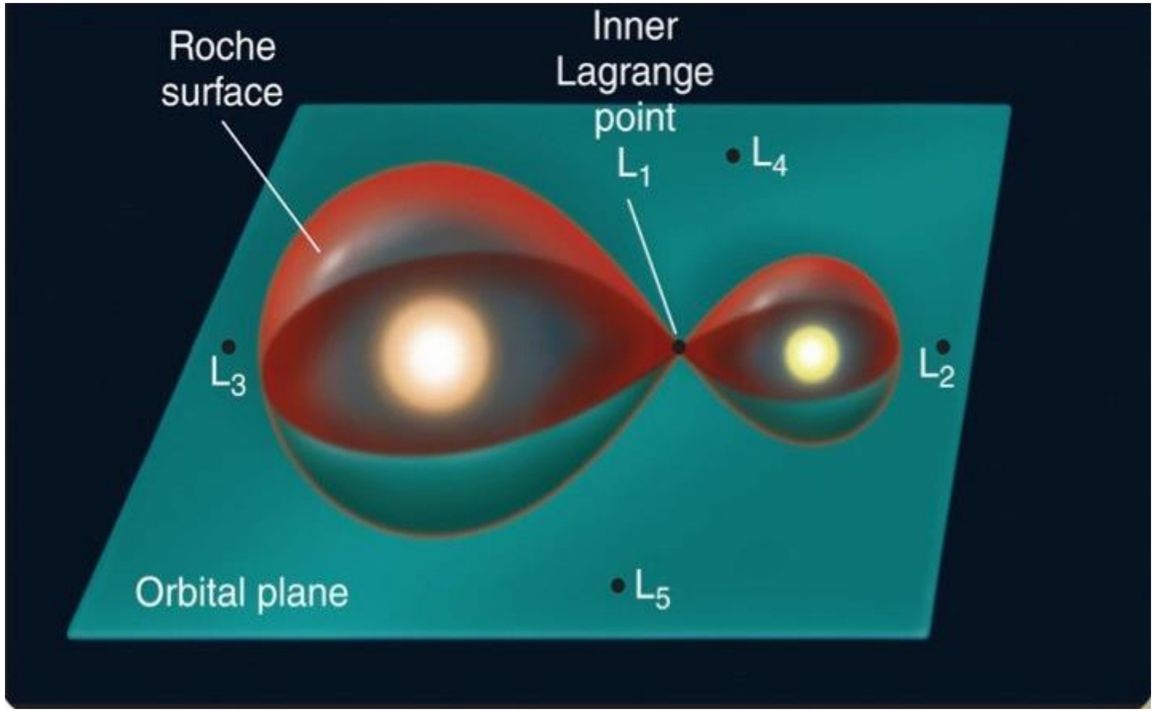

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Roche lobe overflow

↓

Mass transfer

↙ ↘

Stable

Unstable

Mass transfer via Roche Lobe Overflow

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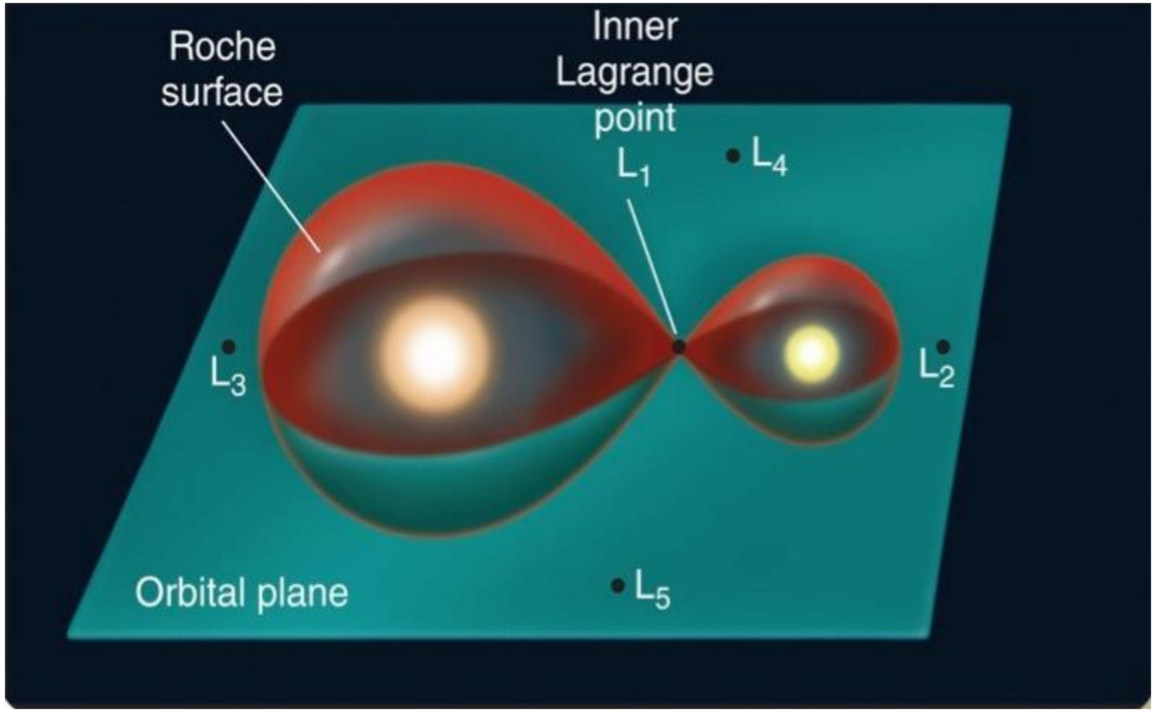

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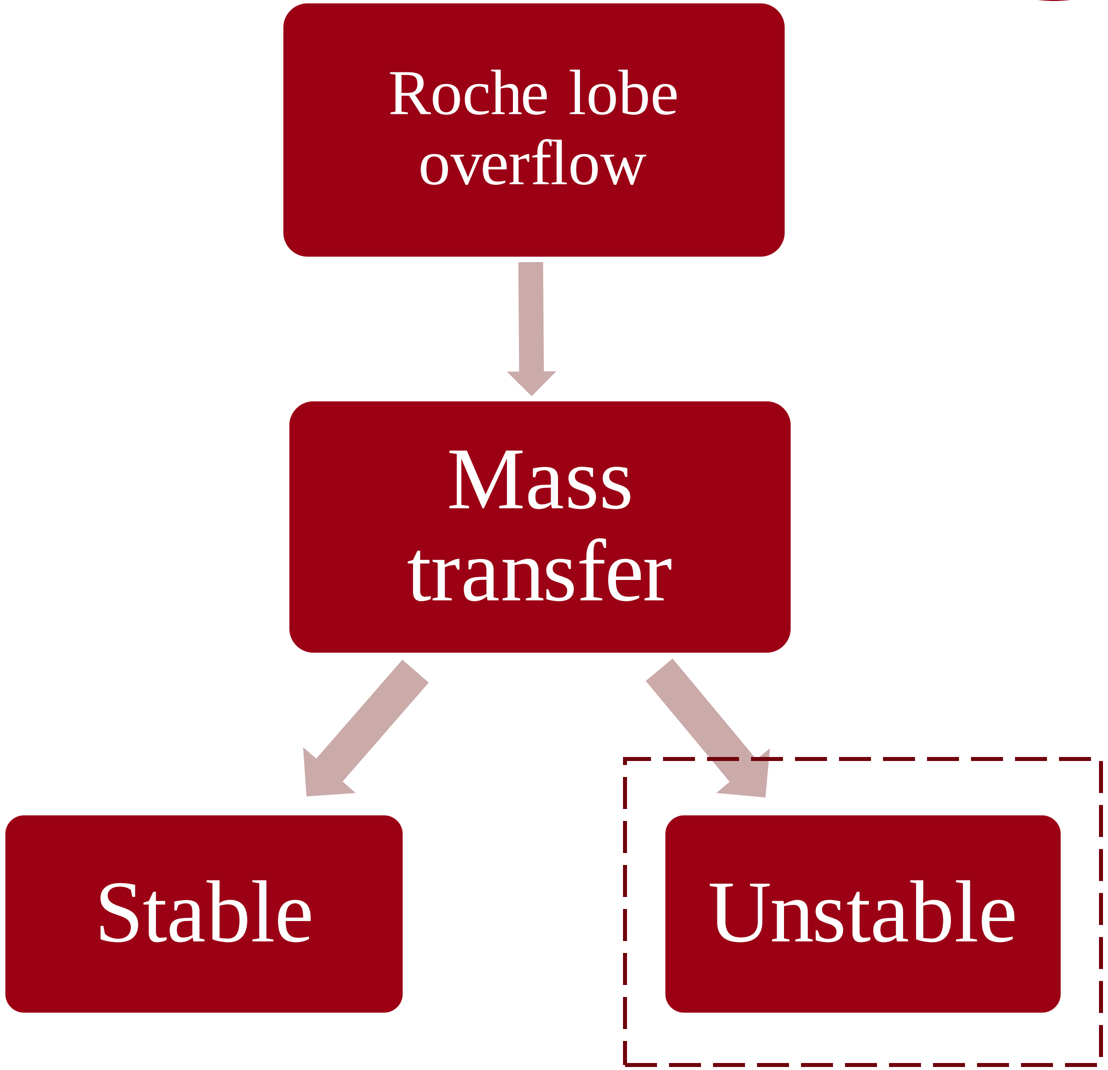
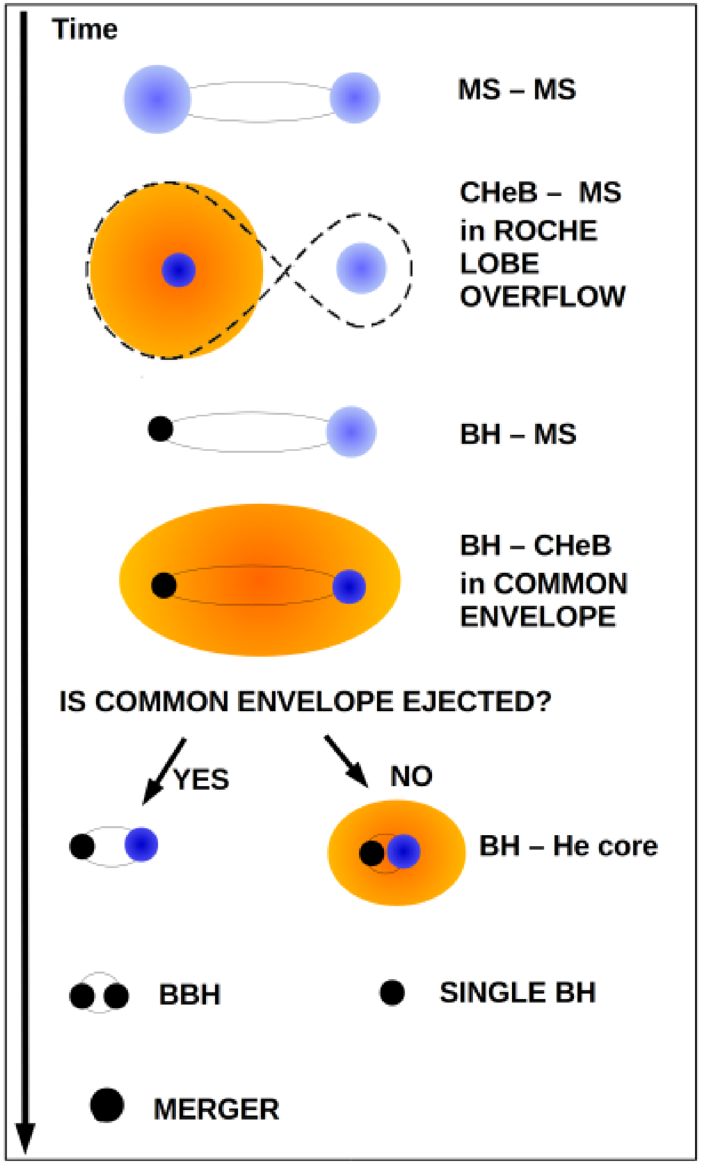


Unstable mass transfer



Credits: Mapelli (2021)

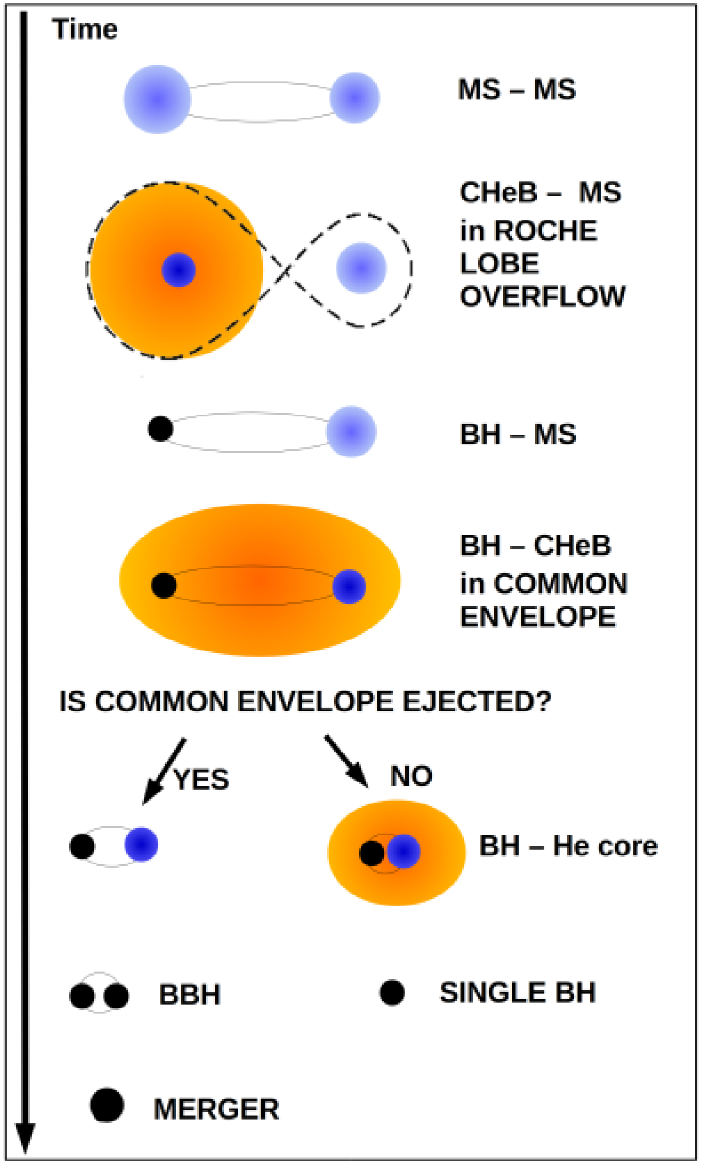
→ Binary system with stars in main sequence

→ Roche lobe overflow

→ Black hole - main sequence star

→ Common envelope phase

Unstable mass transfer



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→ Binary system with stars in main sequence

→ Roche lobe overflow

→ Black hole - main sequence star

→ Common envelope phase

→
- Yes: Binary Black Hole and merger
- No: two components merge without gravitational waves event

The α formalism

The **α formalism** describes the **energy balance** during the common envelope phase.

$$\Delta E = \alpha(E_{b,f} - E_{b,i}) = \alpha \frac{G m_{c,1} m_{c,2}}{2} \left(\frac{1}{a_f} - \frac{1}{a_i} \right)$$

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Encodes the **efficiency** of the conversion between **gravitational energy** and **internal energy** of the envelope.

Dataset

- **Dataset**: set of **simulated binary black holes formations**.

	Binary ID	ZAMS Mass 1 [Msun]	ZAMS Mass 2 [Msun]	Black Hole Mass 1 [Msun]	Black Hole Mass 2 [Msun]	Formation Delay Time [Myrs]	Semi Major Axis [Rsun]	Orbital Eccentricity	Formation by Common Envelope	Alpha	Metallicity	Q	Q_BH
1878045	1_1339891	27.795000	27.720300	4.737200	4.619000	11795.084000	11.461000	0.174990	True	3.000000	0.001600	1.002695	1.025590
1589326	1_1272522	26.738100	20.396400	5.877000	11.986900	915.528200	9.288200	0.114790	True	3.000000	0.000400	1.310923	0.490285
835409	1_1434130	25.979100	25.630200	5.908600	5.208300	8001.689900	17.491000	0.617170	True	1.000000	0.000400	1.013613	1.134458
1091111	4_1581057	21.064600	18.994800	3.024900	7.702200	185.318000	4.539100	0.347010	True	1.000000	0.001200	1.108967	0.392732
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1339656	1_537277	43.895600	25.066100	16.281300	19.313400	145.338200	9.888200	0.014047	True	3.000000	0.000200	1.751194	0.843005
2022989	1_1307248	24.482000	17.403100	4.418400	9.325700	861.019600	7.704200	0.209460	True	5.000000	0.000200	1.406761	0.473787
952697	4_1973207	66.332400	48.536300	26.816900	33.535400	173.236400	15.426000	0.019395	True	1.000000	0.000400	1.366655	0.799659
1712477	4_1088638	36.890900	22.139700	13.769600	16.231300	8287.779300	24.353000	0.016666	True	3.000000	0.000400	1.666278	0.848336

Dataset

▪Features

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$$Q = \frac{ZAMS\ Mass\ 1}{ZAMS\ Mass\ 2} \qquad BHQ = \frac{Black\ Hole\ Mass\ 1}{Black\ Hole\ Mass\ 2}$$

these features were added for **physical** and **computational** reasons

Dataset

- **Label**: value we want to predict

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Dataset imbalance

Imbalanced dataset:

- 86 % Common envelope
- 14 % Stable mass transfer

Solution: under-sampling to obtain 50/50 dataset

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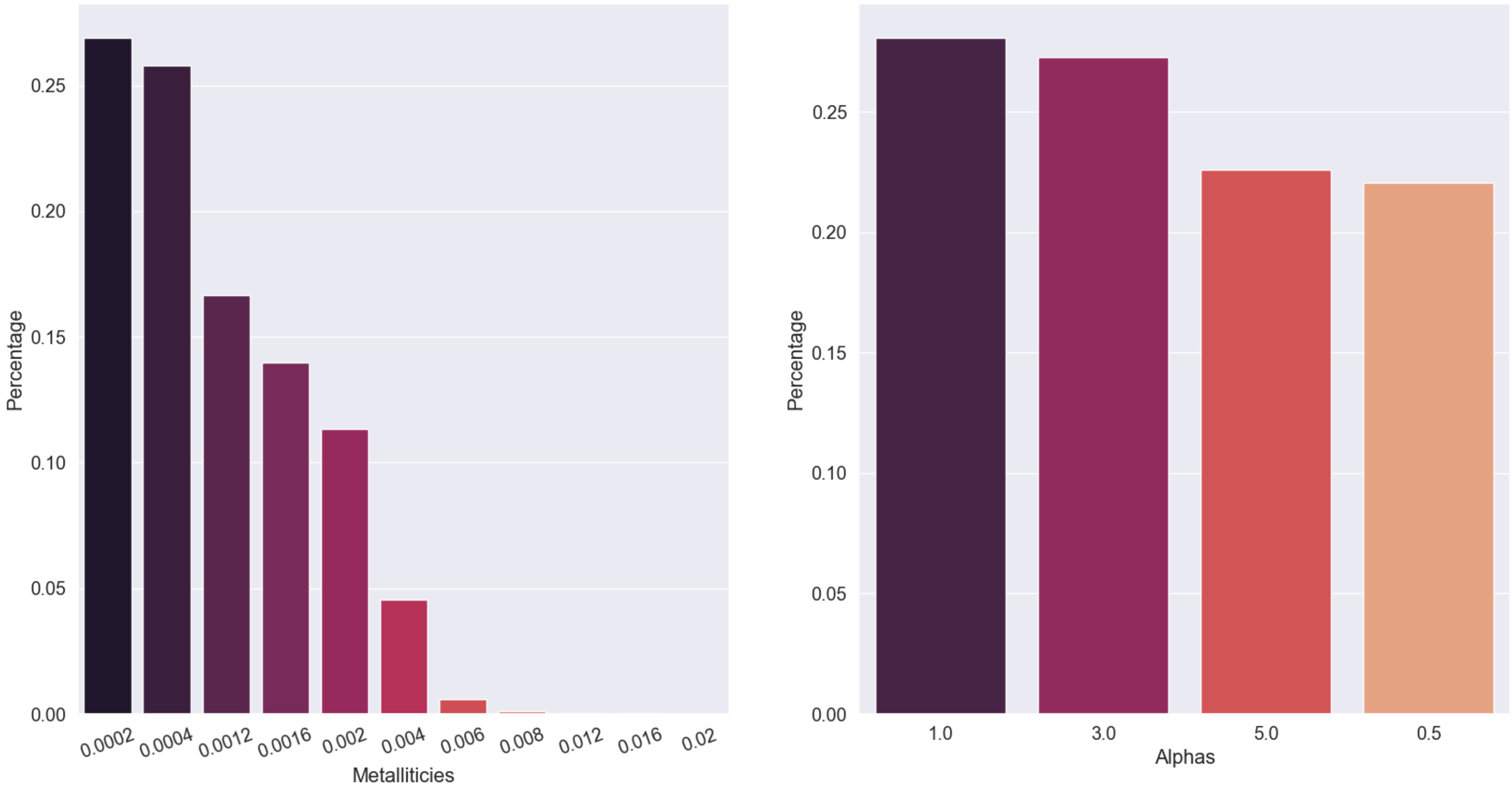
Imbalanced dataset:

- 86 % Common envelope
- 14 % Stable mass transfer

Solution: under-sampling to obtain 50/50 dataset

- Alphas well distributed
- Mainly low metallicities → merger is more likely



Alphas and metallicity percentages

Removing the outliers

High stds and extreme outliers makes it difficult to cluster our data

	count	mean	std	min	1%	50%	99%	max
Orbital Eccentricity	692008.0	0.074763	0.139523	0.000015	0.007946	0.01602	0.686288	1.0
Semi Major Axis [Rsun]	692008.0	26.397353	607.786604	0.664170	3.389100	19.46700	53.129650	329100.0

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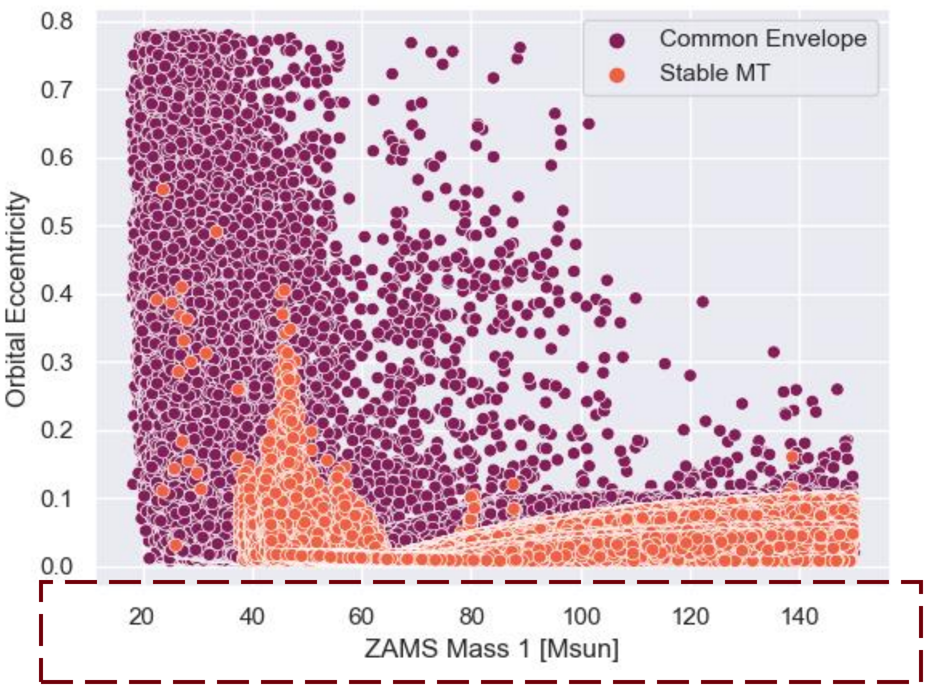
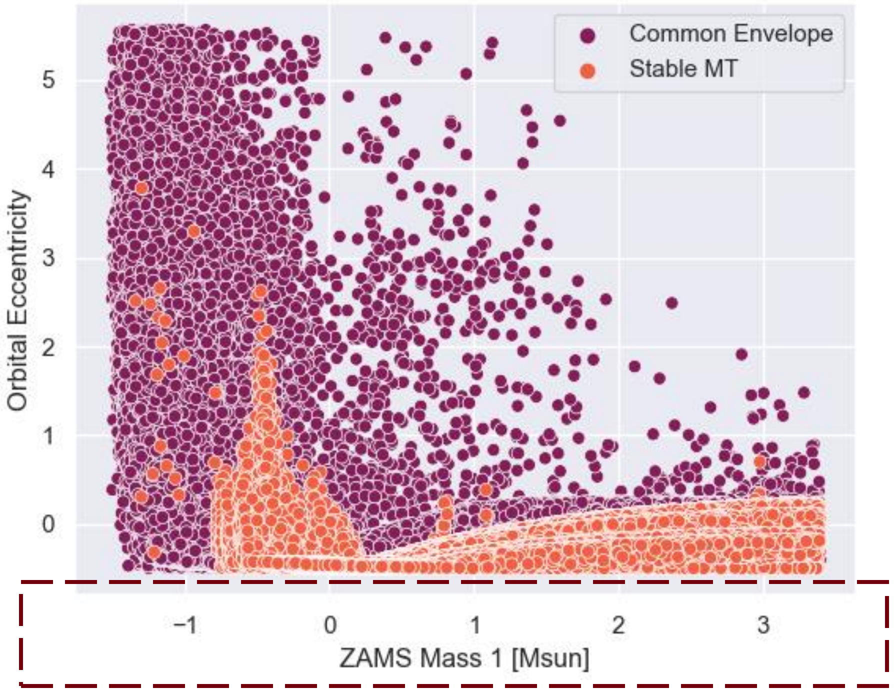
Filtering the extreme 1%

	count	mean	std	min	1%	50%	99%	max
Orbital Eccentricity	680471.0	0.071232	0.127306	0.00800	0.008286	0.016089	0.61662	0.77999
Semi Major Axis [Rsun]	680471.0	20.767367	12.262600	0.66417	3.412000	19.321000	52.60300	88.30800

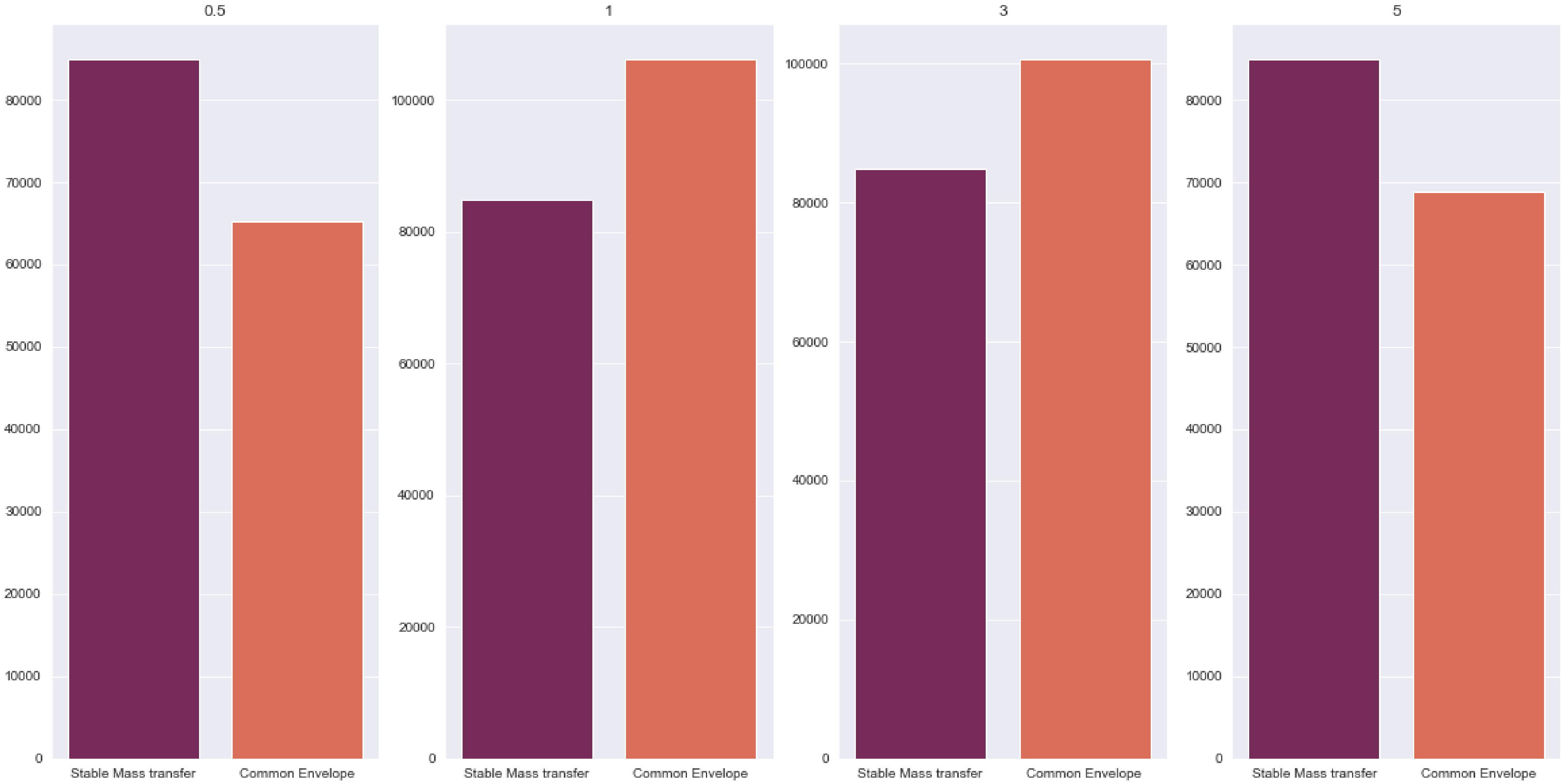
Normalization





Normalization is important for
distance-based algorithms

Alpha



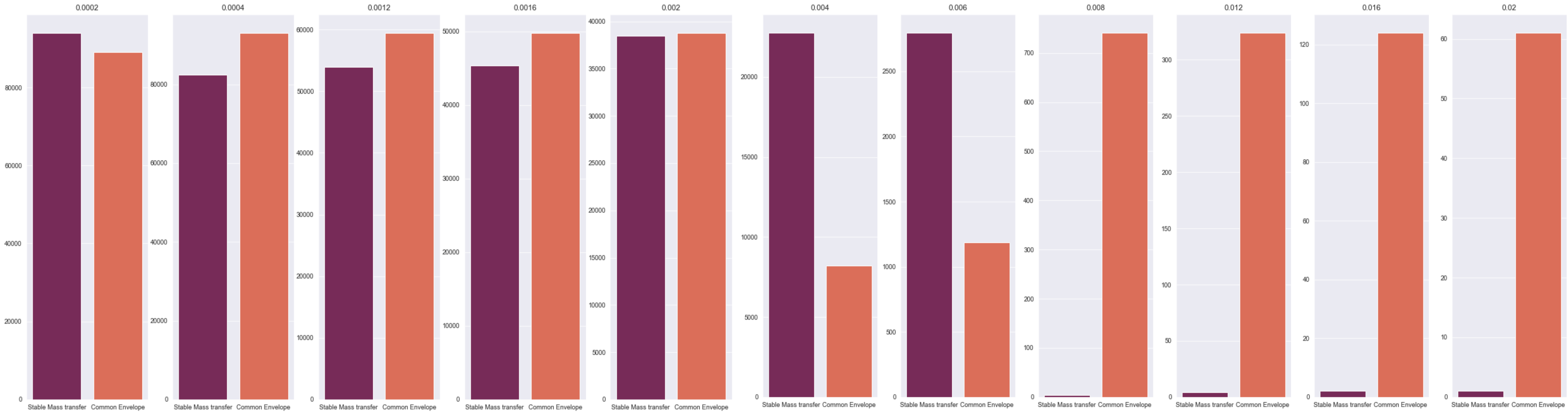
Alpha distribution for different labels

Alpha parameters are **well distributed** between the two evolution cases

Metallicities

The **metallicities** are **imbalanced**

For **higher** metallicities only Common envelope



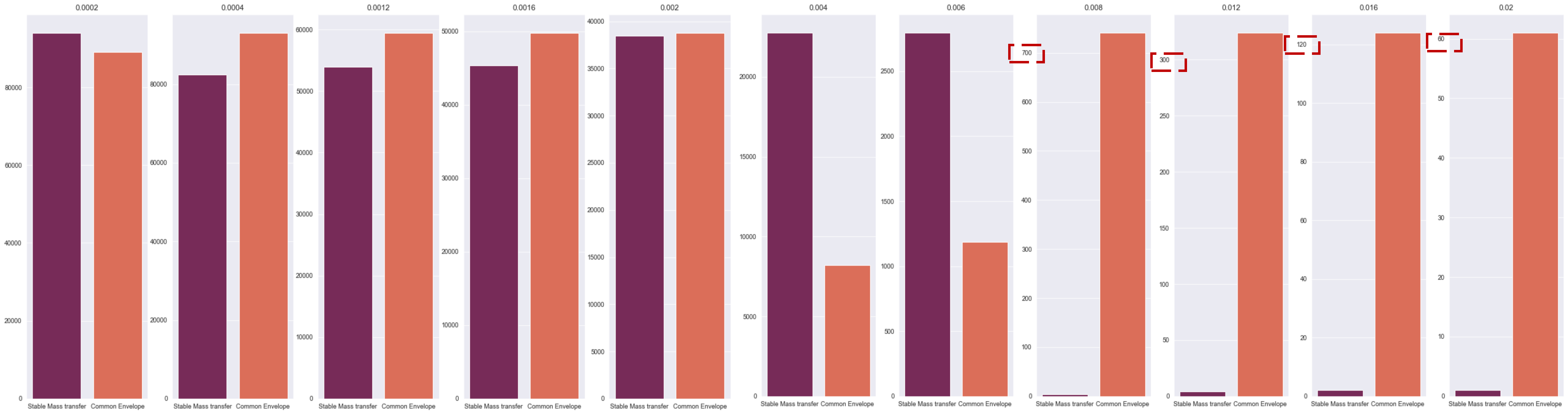
Metallicity distribution for different labels

Metallicities

The **metallicities** are **imbalanced**

For **higher** metallicities only Common envelope

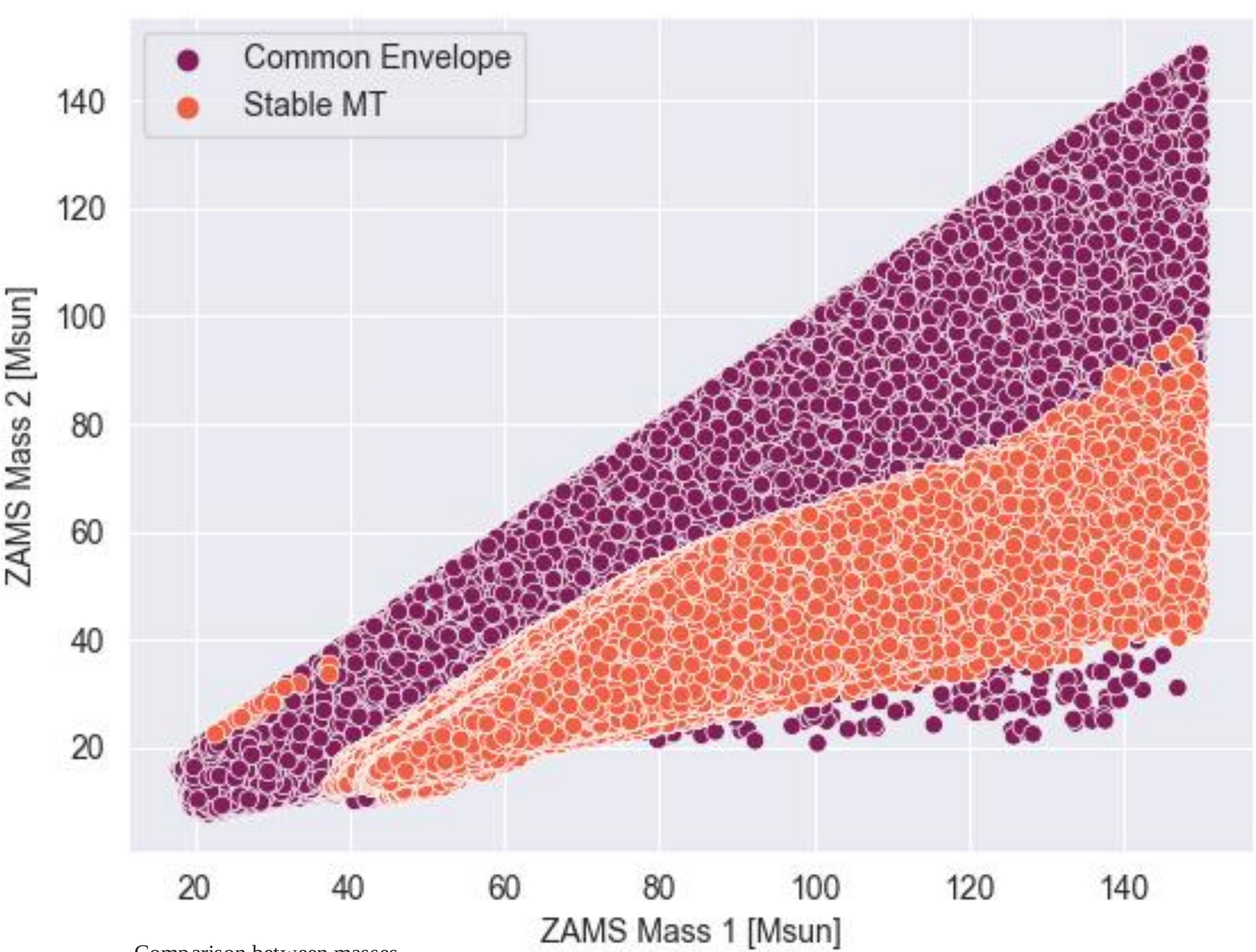
Small number of instances, **negligible**



Metallicity distribution for different labels

Masses considerations

ZAMS Mass 1 > ZAMS Mass2
by **construction**



Comparison between masses

Masses considerations

ZAMS Mass 1 > ZAMS Mass2 by **construction**

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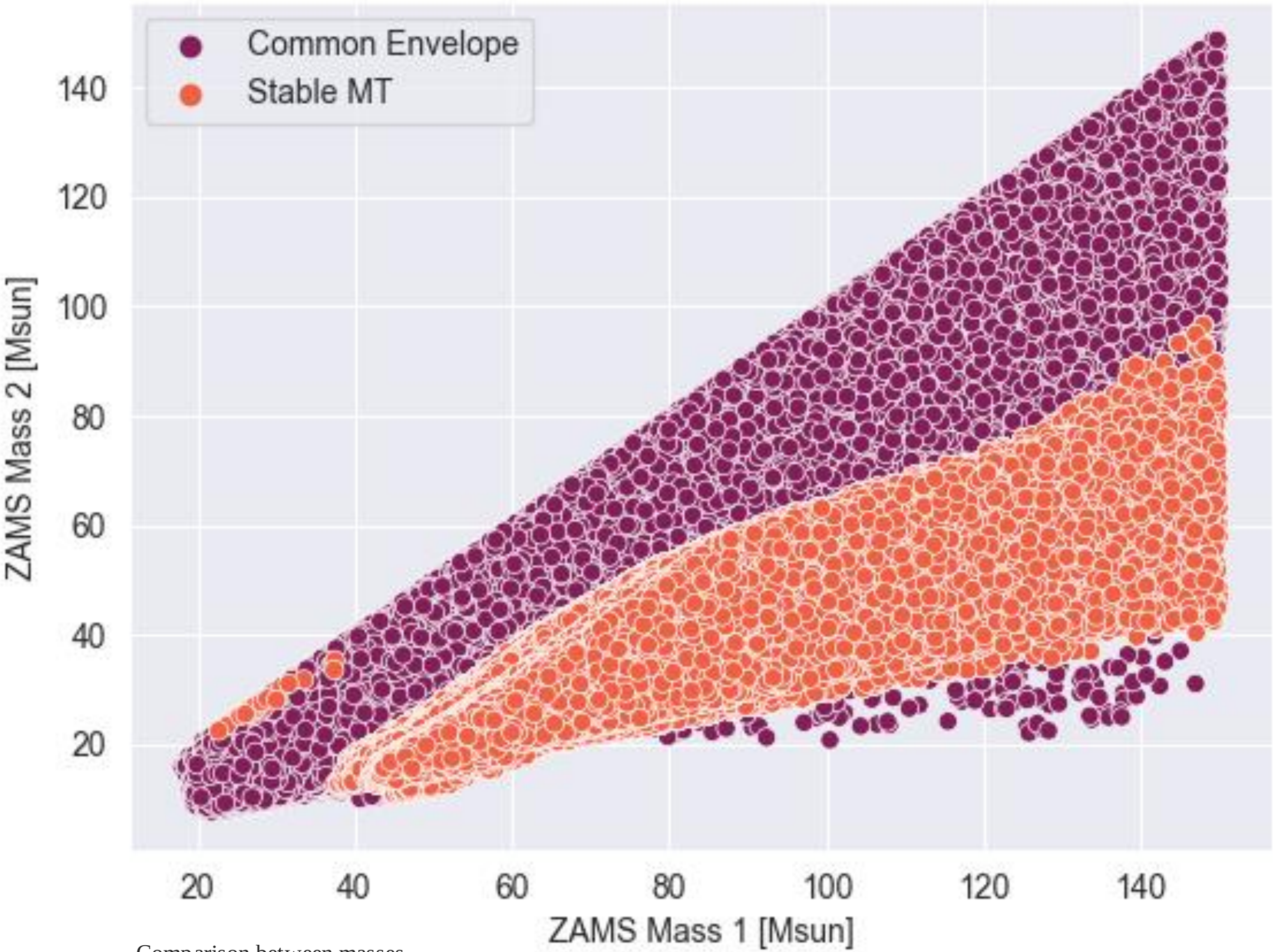
Introduces a **bias**



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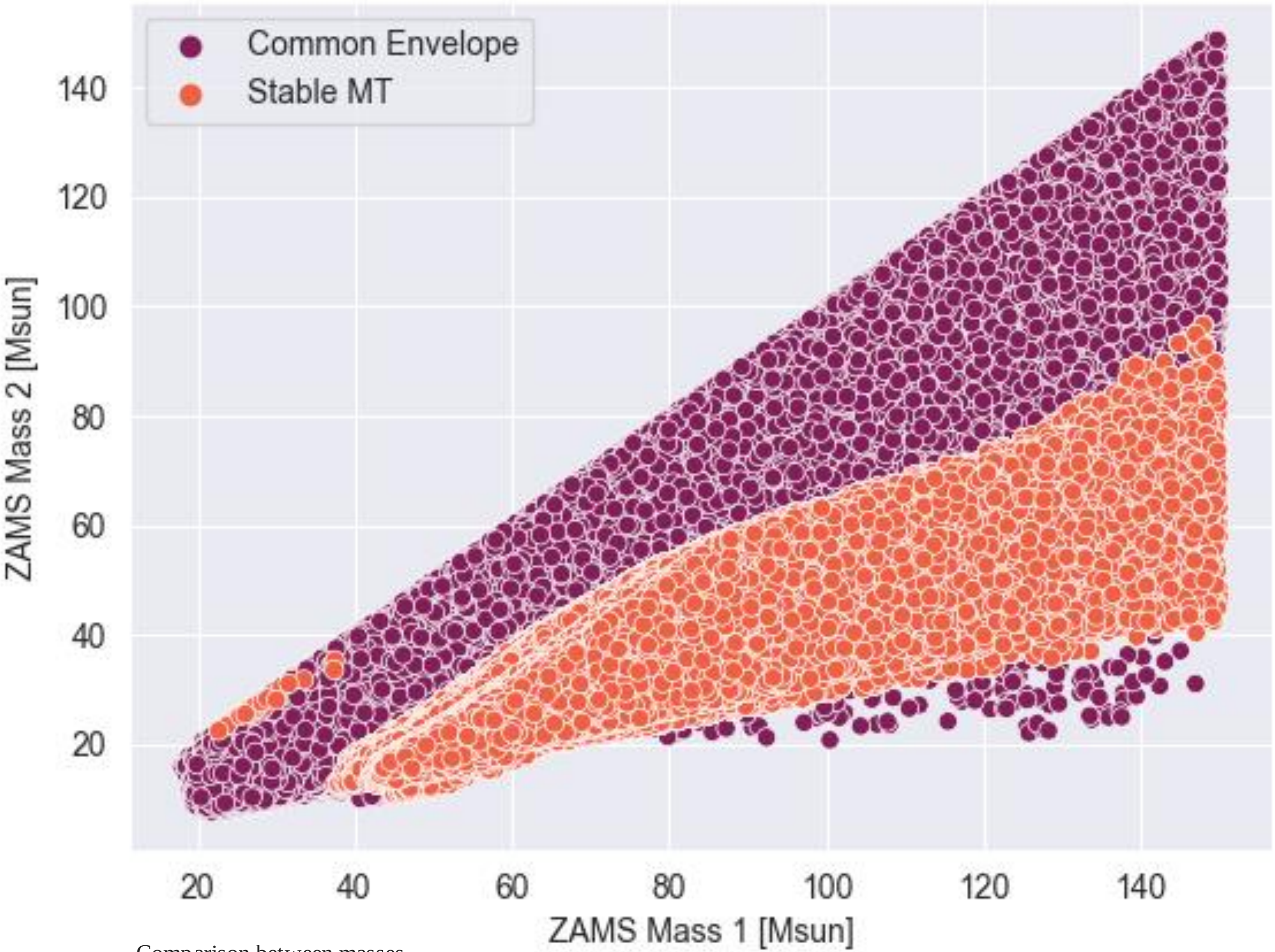
Masses considerations

ZAMS Mass 1 > ZAMS Mass2 by **construction**

Introduces a **bias**

Problems with Machine Learning algorithms

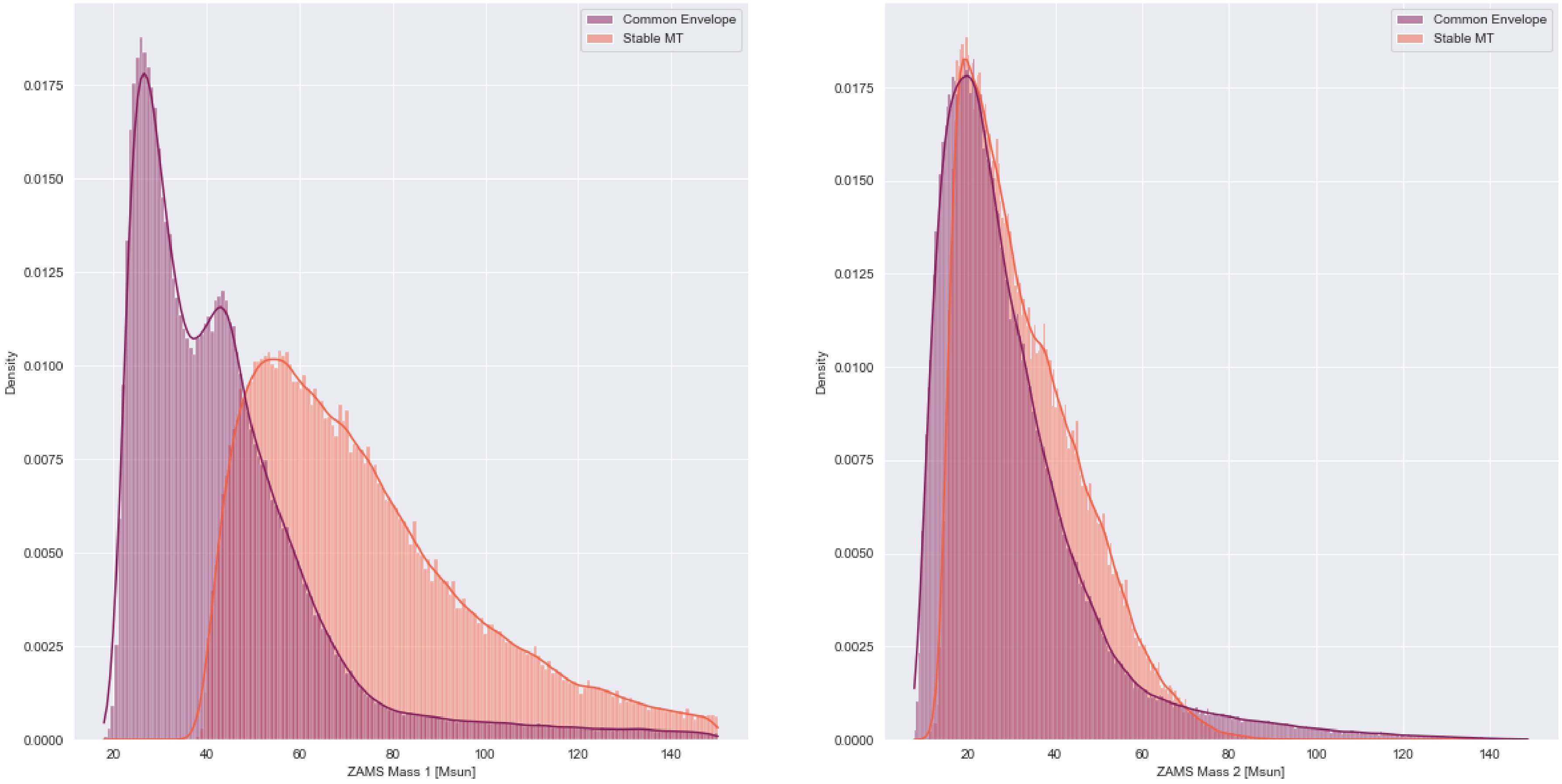


Comparison between masses

Masses considerations

Primary mass
- Clear pattern
- CE – SMT boundary at around 40[Msun]

Secondary mass:
- Overlapping distributions



Probability density of events for Mass 1 and Mass 2

Masses considerations

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- Clear pattern
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Secondary mass:

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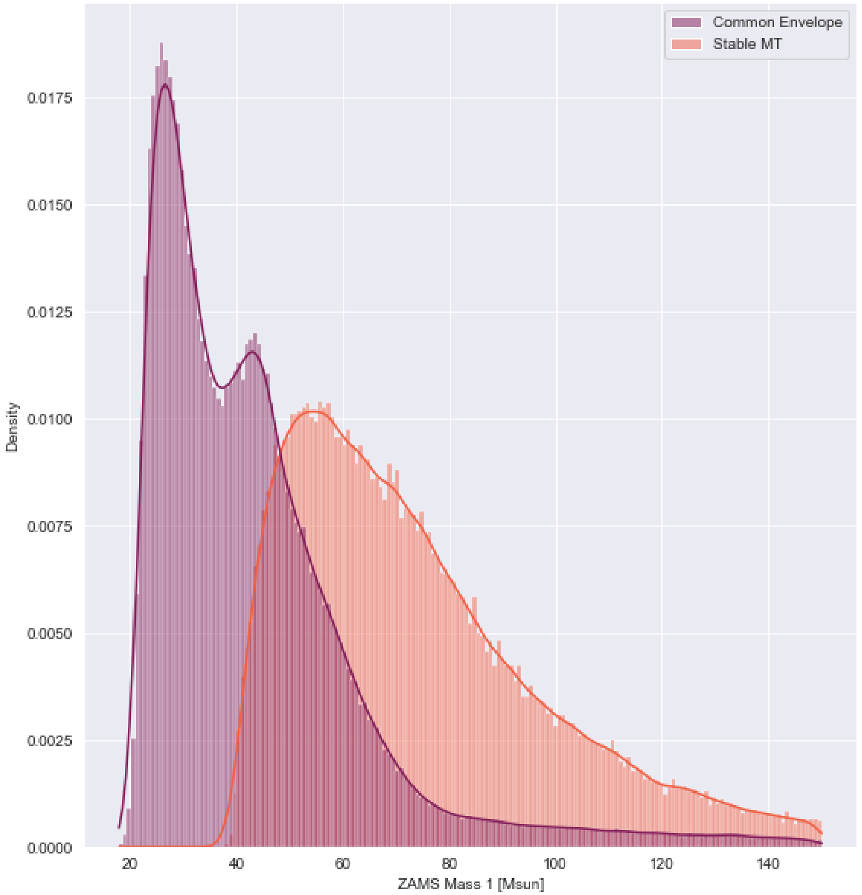
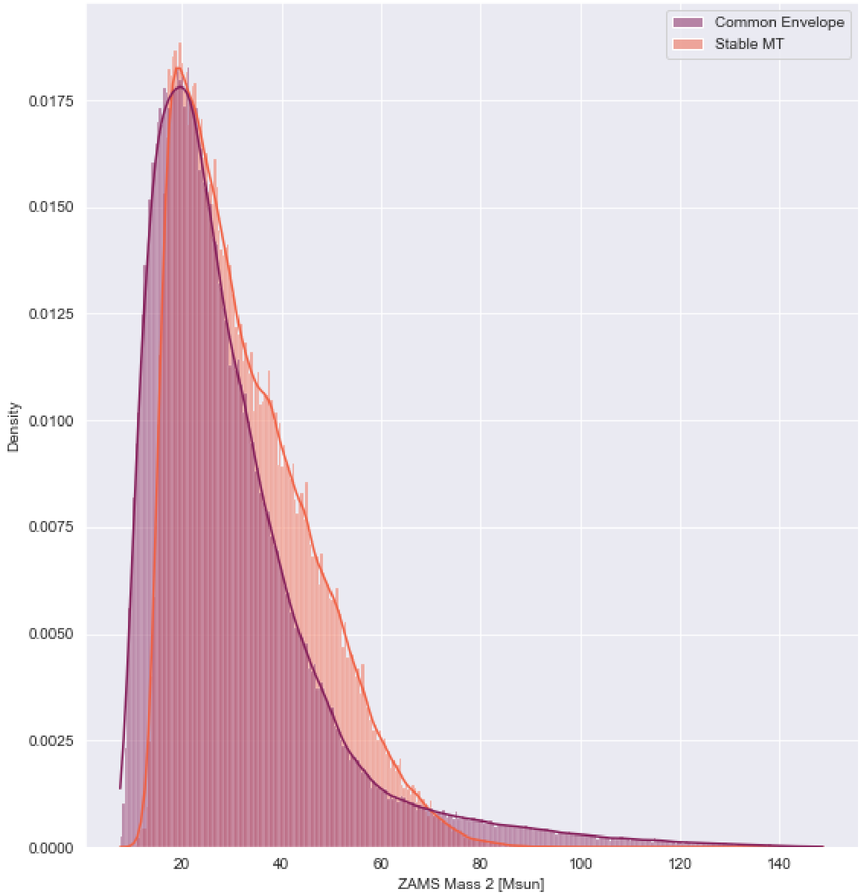
Problems with **machine learning** algorithms



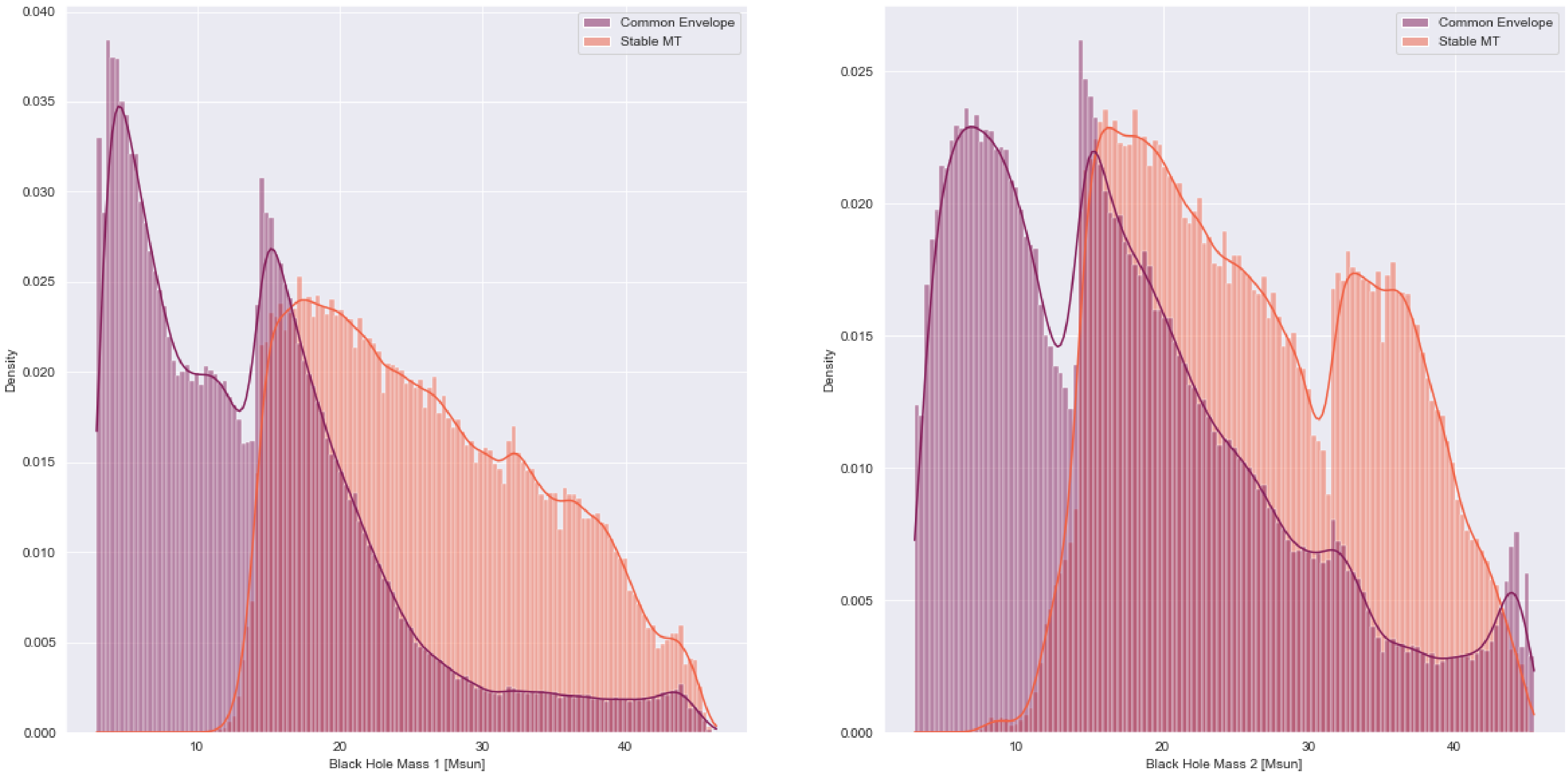


Probability density of events for Mass 1 and Mass 2

BBH masses considerations

Binary black holes masses

Same pattern of primary ZAMS mass



Probability density of events for BBH Mass 1 and BBH Mass 2

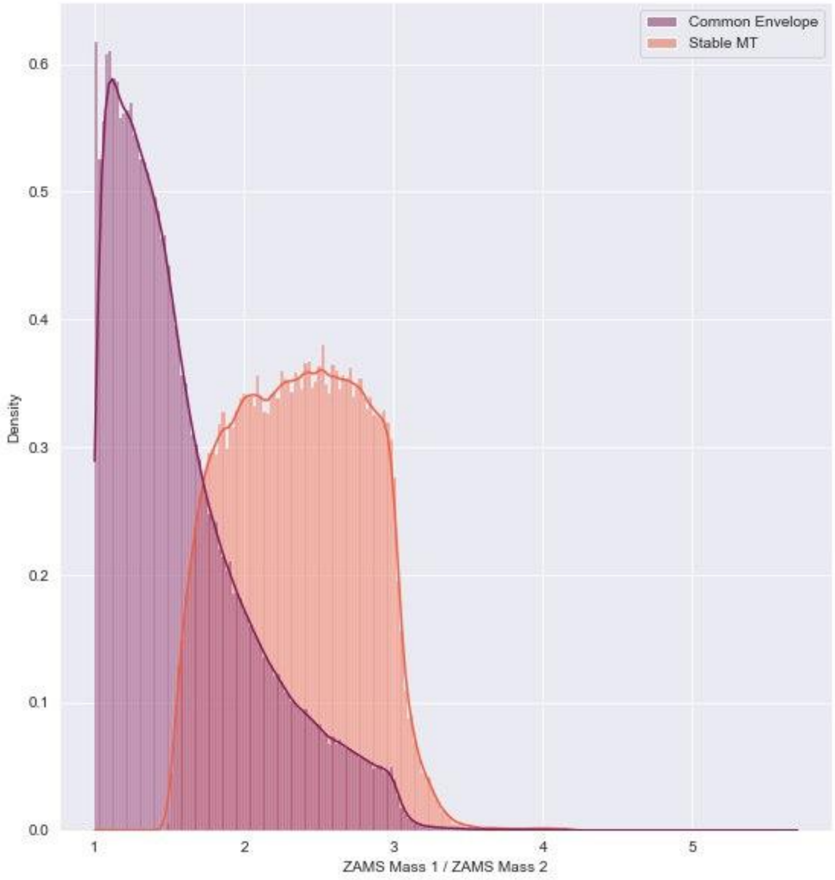
Q and BHQ

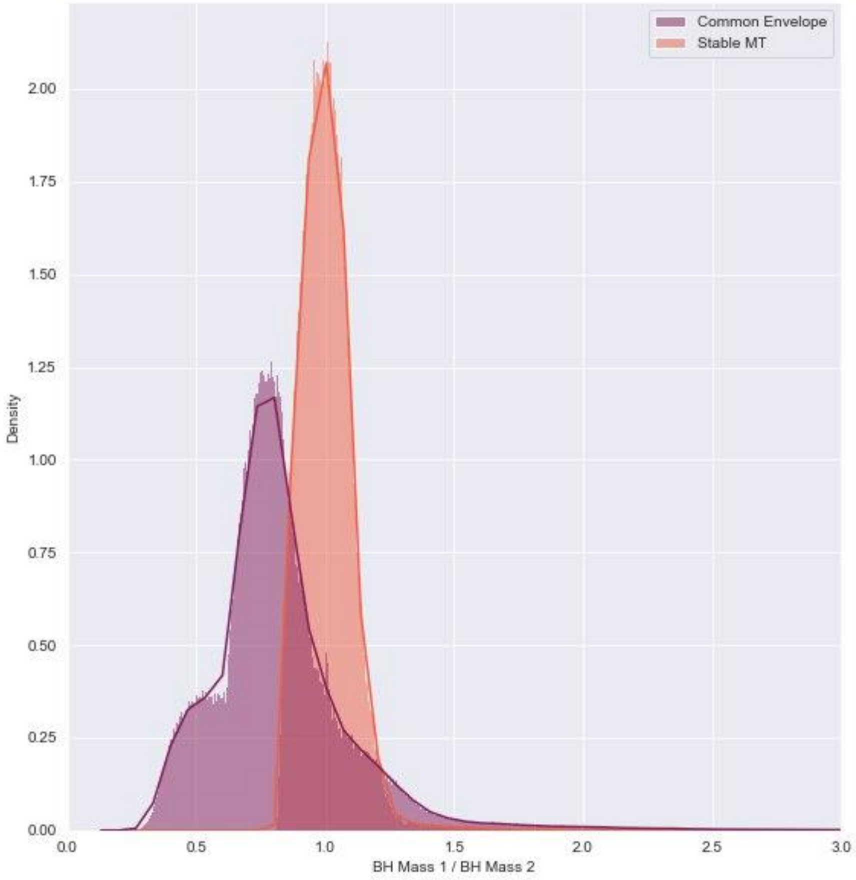
$$Q = \frac{ZAMS\ Mass\ 1}{ZAMS\ Mass\ 2}$$

$$BHQ = \frac{Black\ Hole\ Mass\ 1}{Black\ Hole\ Mass\ 2}$$

these features were added for **physical** and **computational** reasons

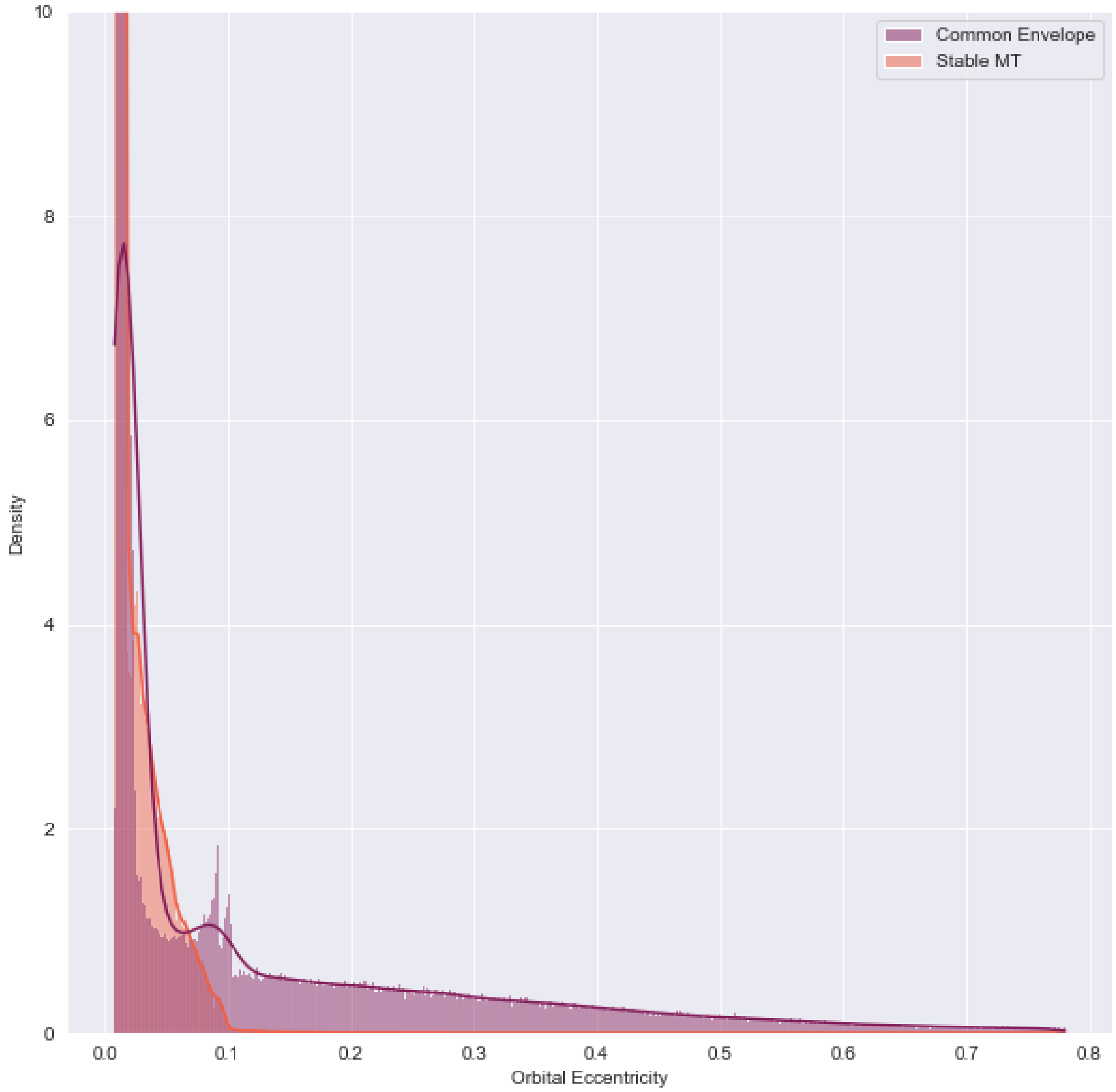
Q and BHQ show a good separation between the two classes and a small std





Probability density of events for BBH Mass 1 and BBH Mass 2

Orbital Eccentricity



Probability density of events for orbital eccentricity

Low eccentricity →
- Common Envelope
- Stable MT

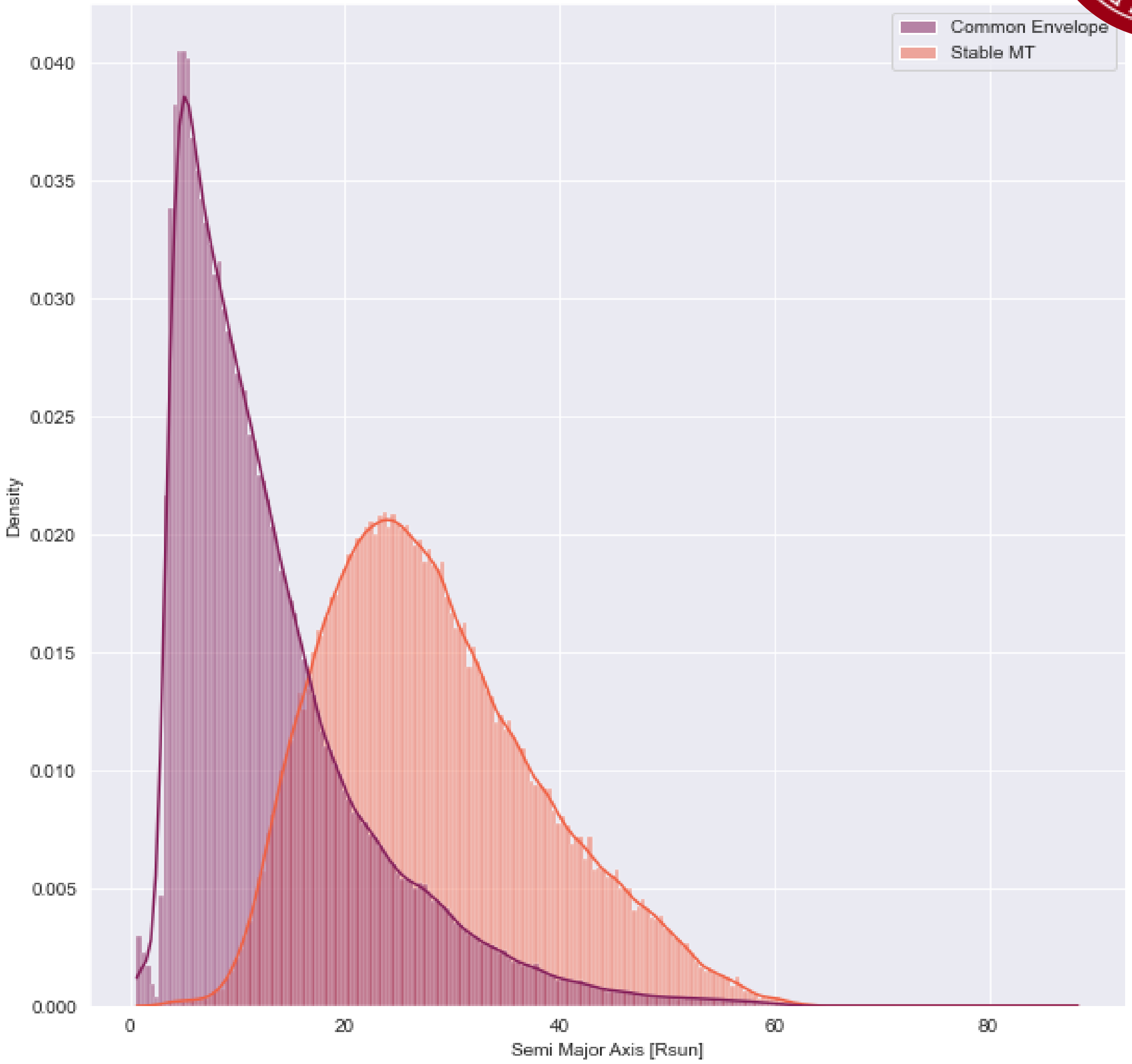
High eccentricity → Common Envelope

Semi Major Axis

Low orbital separation

↓

Common envelope more likely



Probability density of events for Semi Major Axis

Dataset

- **Expectations:** from dataset visualization

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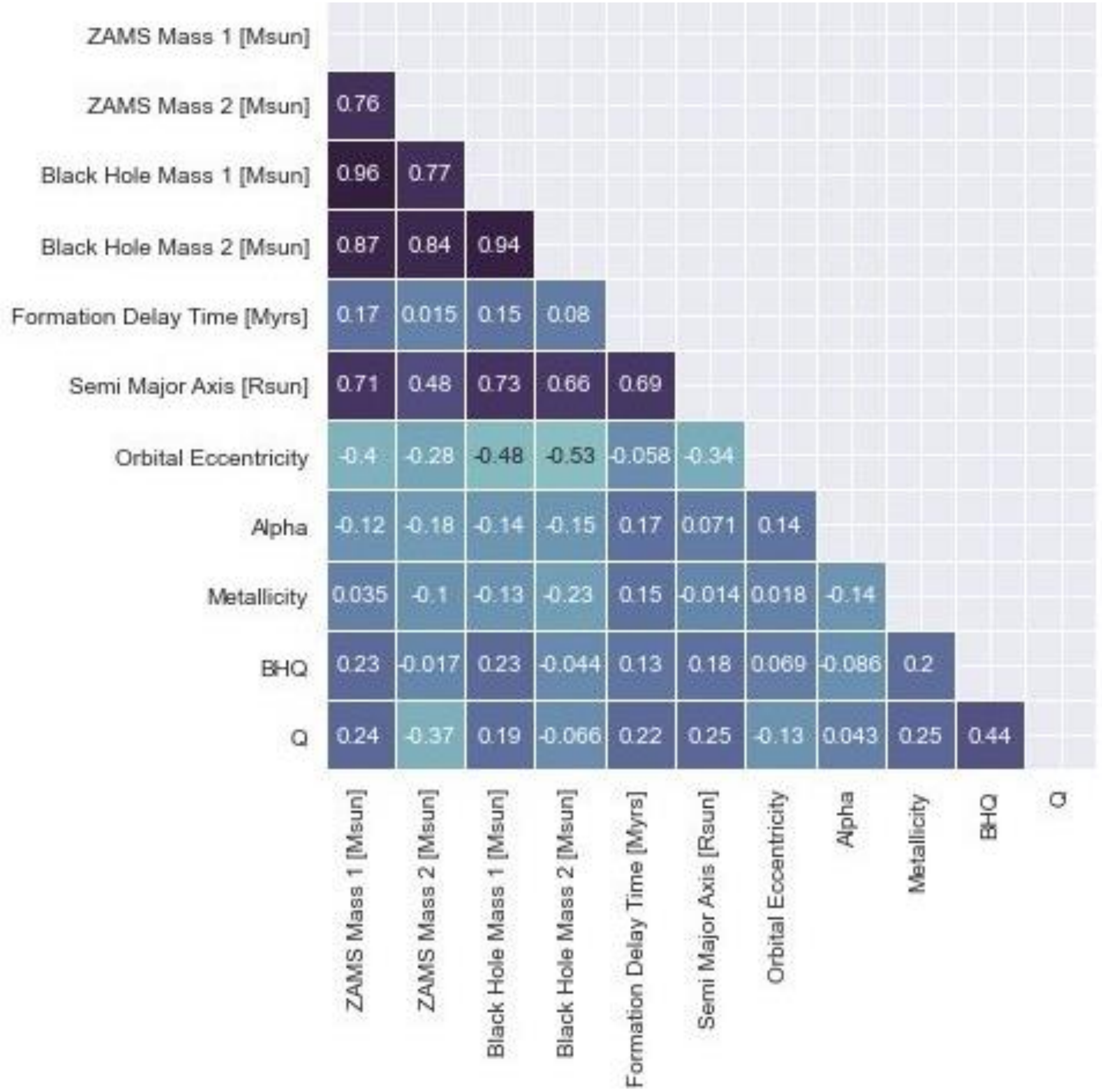
Correlation matrix



Feature correlation matrix

Masses are **highly correlated**

This could **negatively affect** prediction of the feature importances

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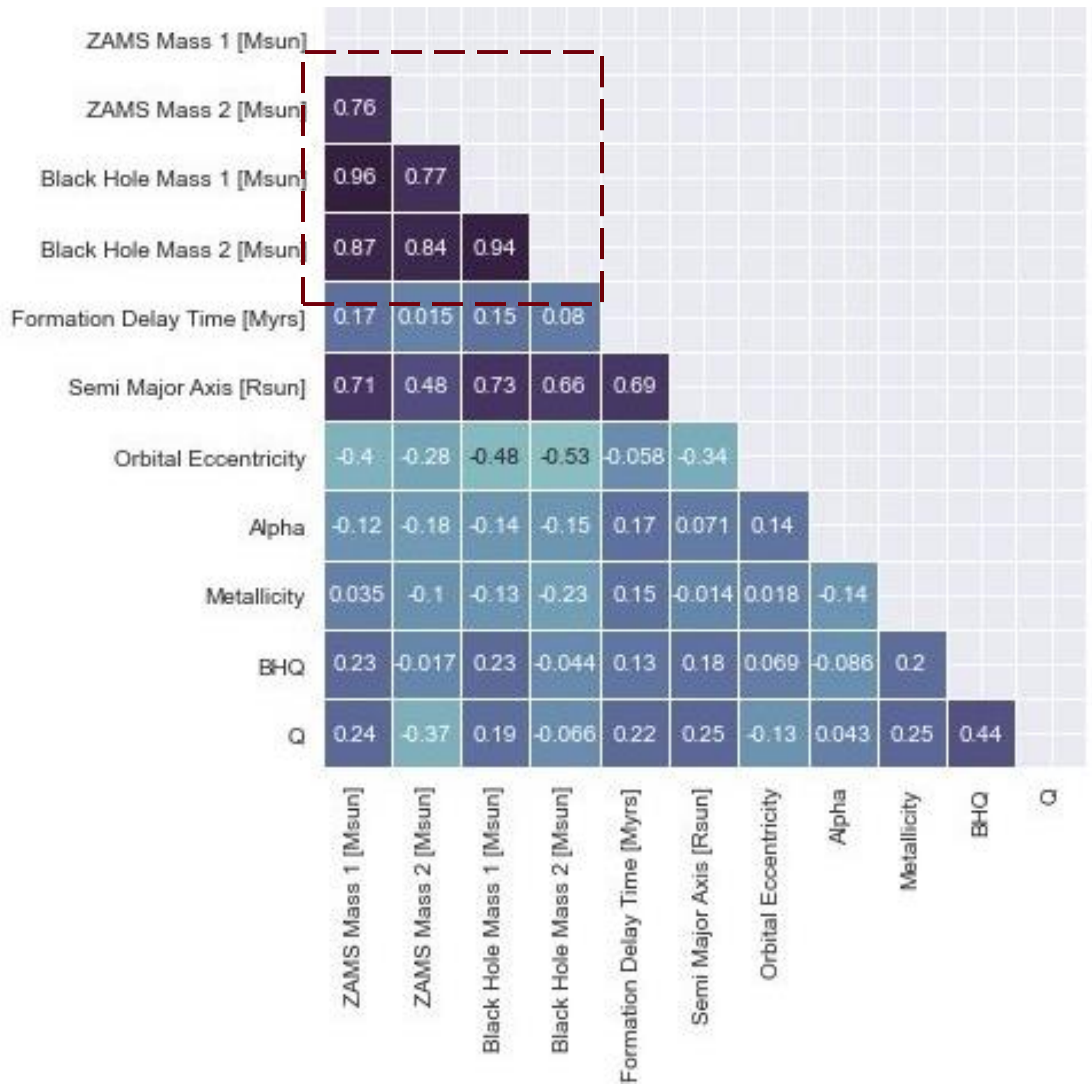

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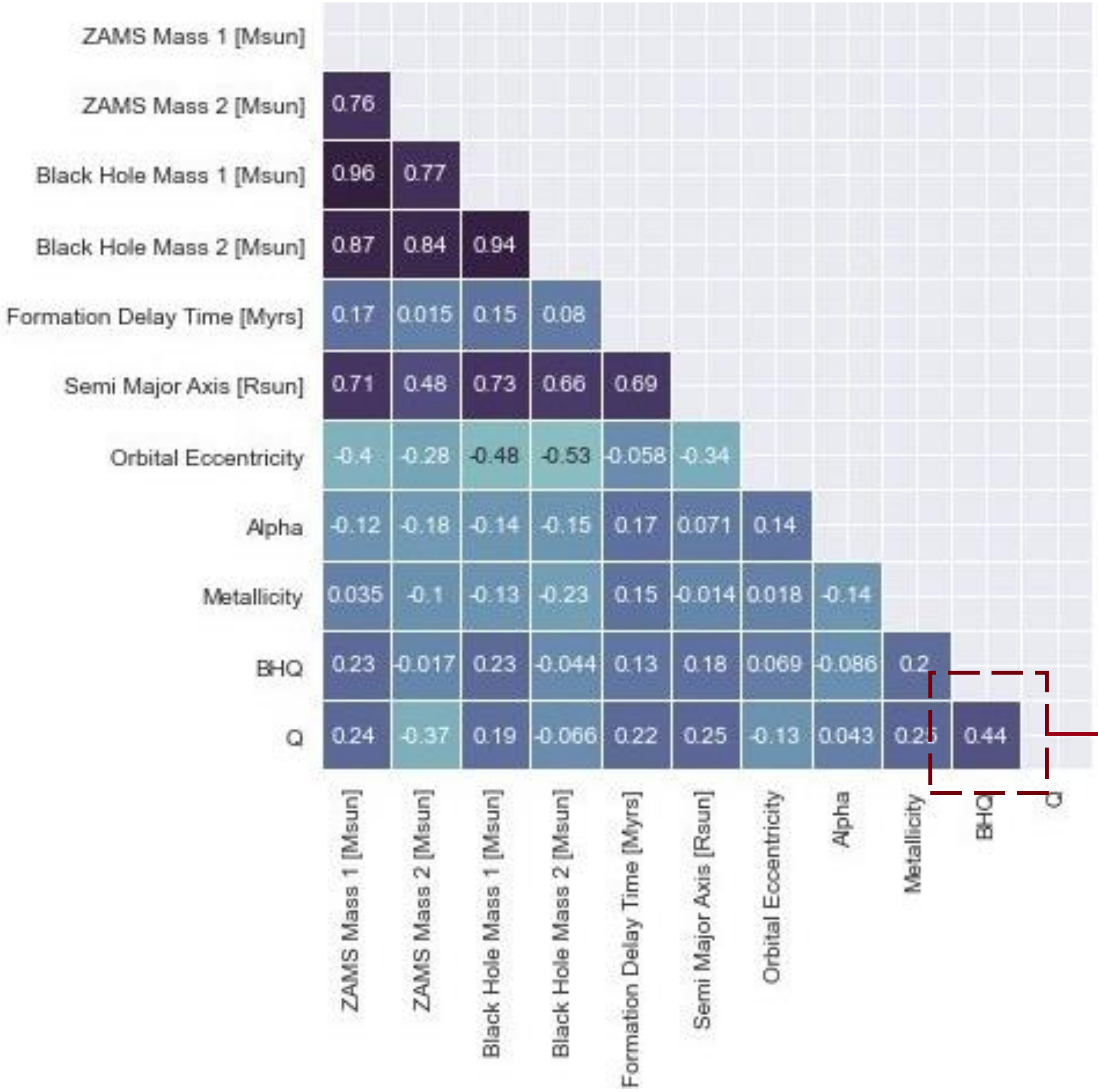

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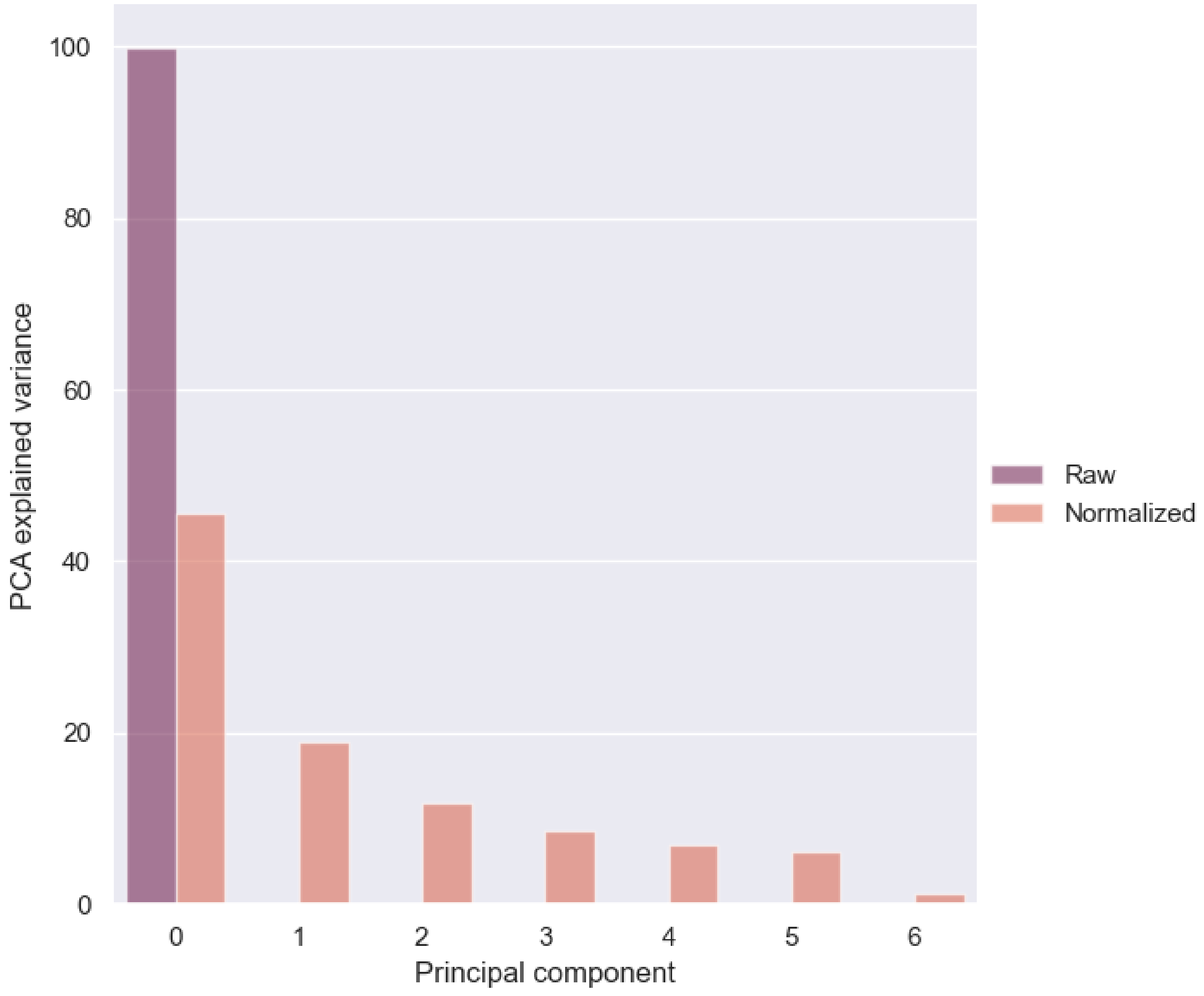
This could **negatively affect** prediction of the feature importances

$$Q = \frac{ZAMZ\ Mass\ 1}{ZAMZ\ Mass\ 2} \qquad BHQ = \frac{Black\ Hole\ Mass\ 1}{Black\ Hole\ Mass\ 2}$$

these features were added for **physical** and **computational** reasons

Principal Component Analysis



Without normalization:

1 PC ~ 99% variance

With normalization:

variance is distributed across more PC

Feature importance

Model in-built method:

- Weight for Linear SVM
- Medium Difference in Impurity for RFC

Model agnostic methods:

- Permutation importance
- Feature-dropping

Mean Decrease in Impurity for RFC

Mean Decrease Impurity (MDI): total decrease in node impurity, weighted by the probability of reaching that node (approximated by the proportion of samples reaching that node), averaged over all trees of the ensemble

Disadvantage:

- Biased toward high cardinality and continuous features

Permutation feature importance doesn't suffer from this

Feature permutation

Feature permutation: decrease in a model score when a single **feature is randomly shuffled**

Disadvantages:

- Misleading values on strongly correlated features
- Creates new instances that can be physically impossible

Feature-dropping could be a good fallback

Feature-dropping

Feature-dropping: change in model score when a single **feature is removed**

Every time the model is retrained

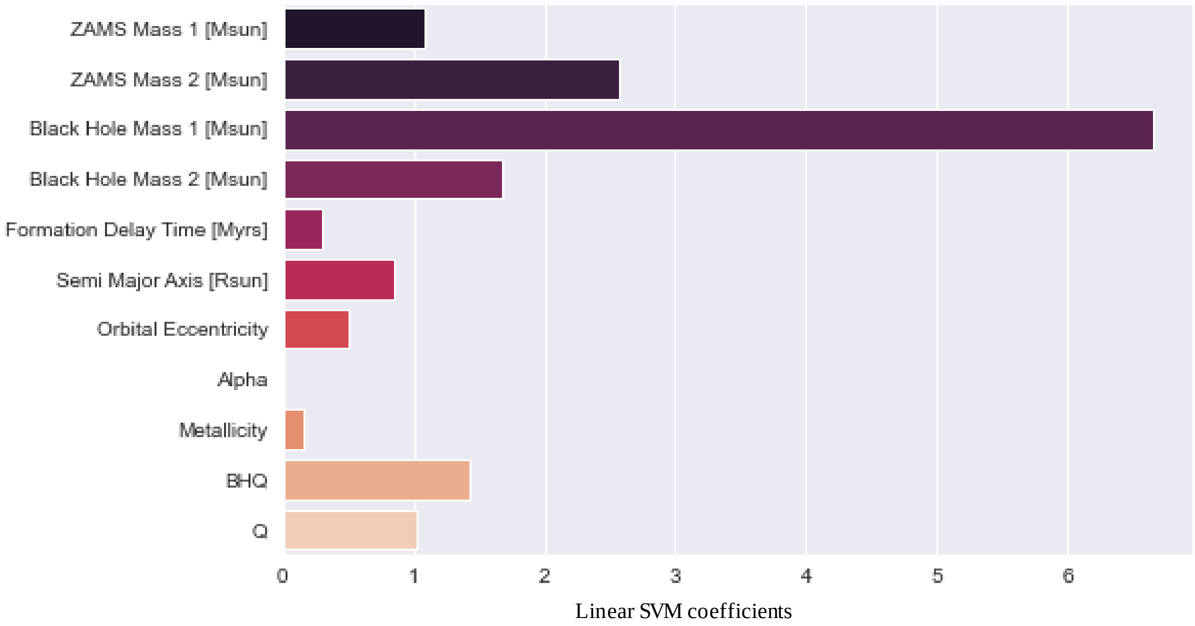
Disadvantage:

- Long calculation time
- Larger algorithm complexity

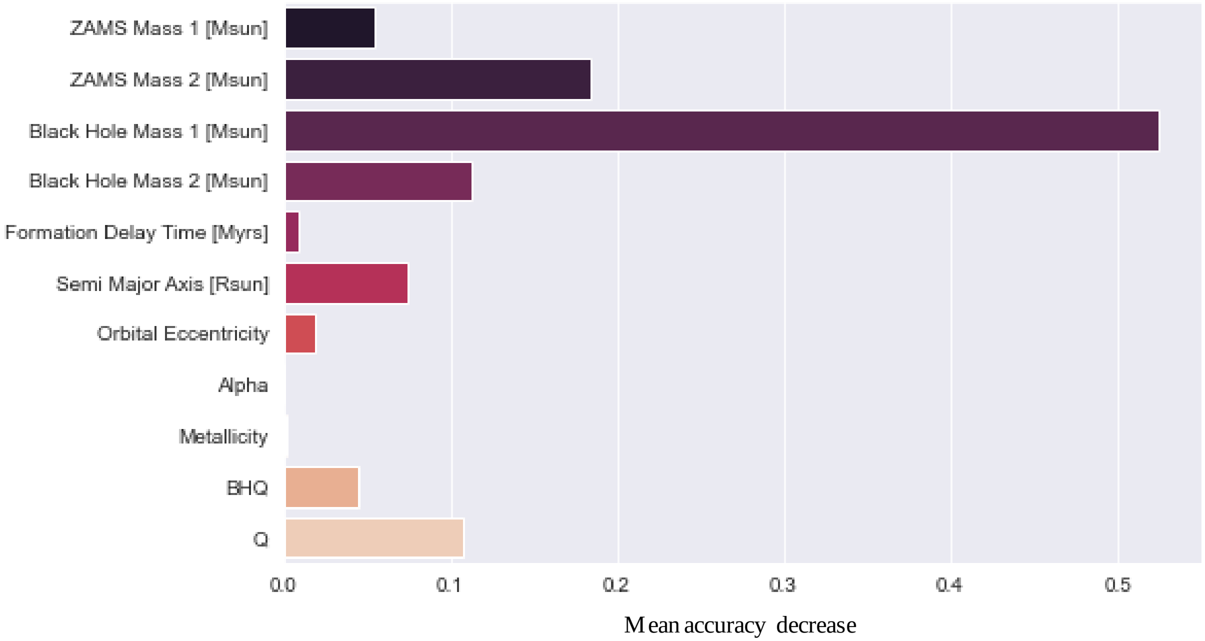
Linear SVM

Accuracy

SVM coefficients



Feature permutation

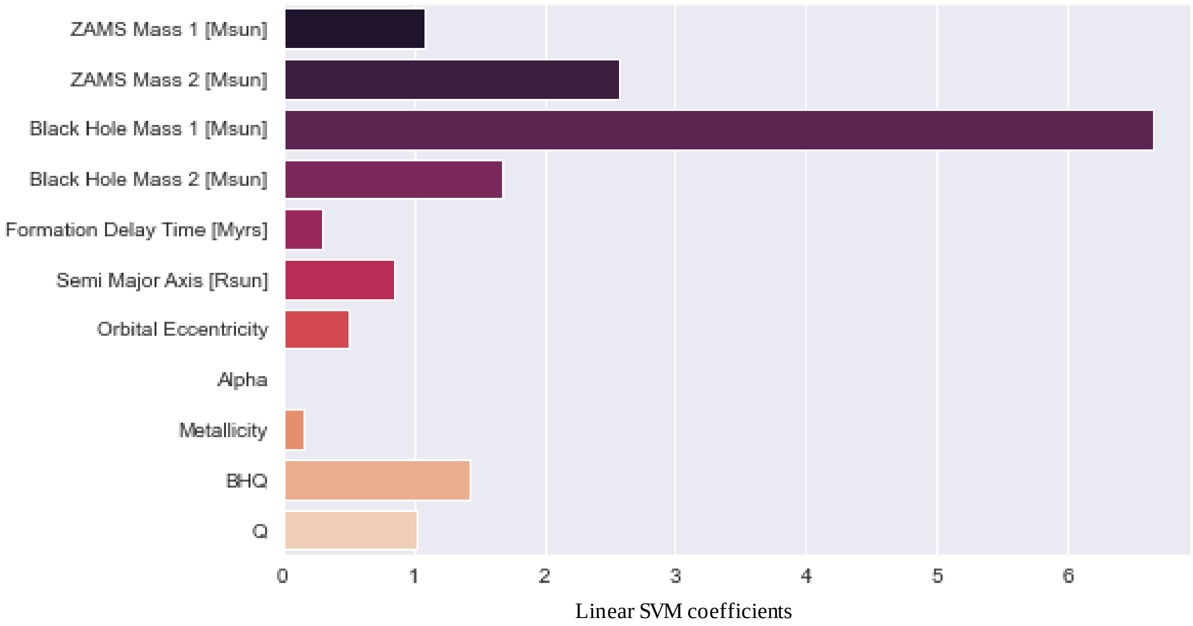


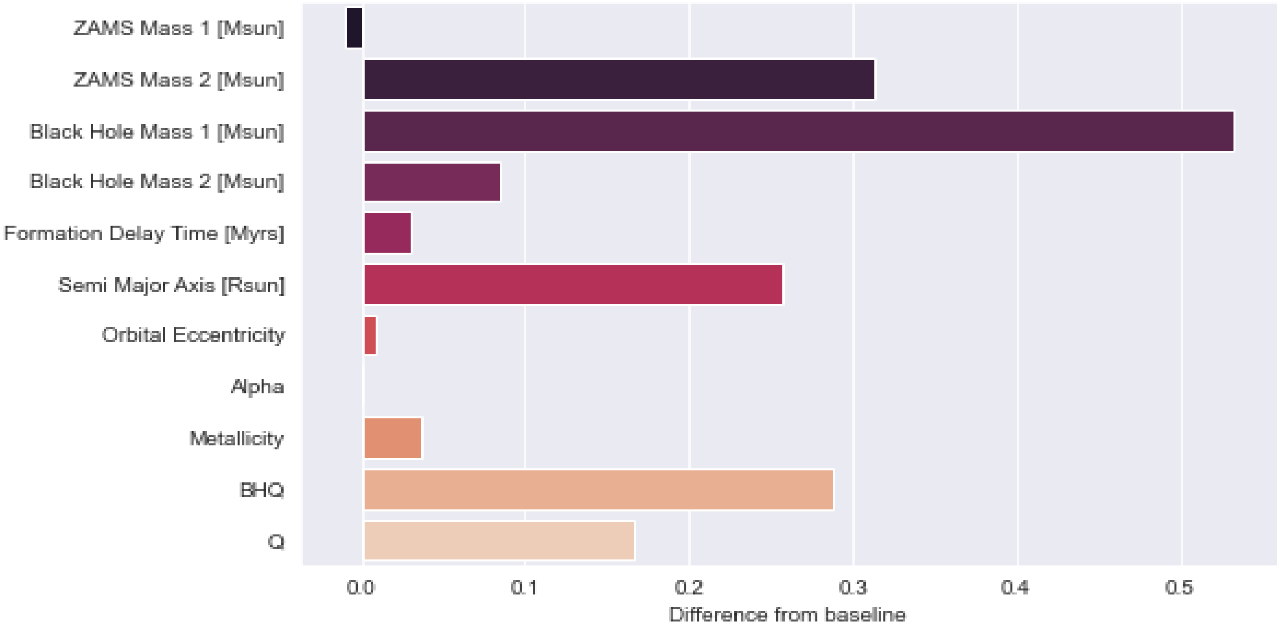
Linear SVM

Accuracy

SVM coefficients

Feature-dropping

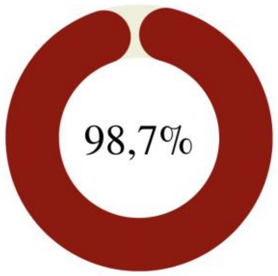




Linear SVM: ranking

Ranking	Linear SVM	Feature permutation	Feature-dropping
1	BH Mass 1	BH Mass 1	BH Mass 1
2	ZAMS Mass 2	ZAMS Mass 2	ZAMS Mass 2
3	BH Mass 2	BH Mass 2	BHQ
10	Metallicity	Metallicity	Orbital eccentricity
11	Alpha	Alpha	Alpha

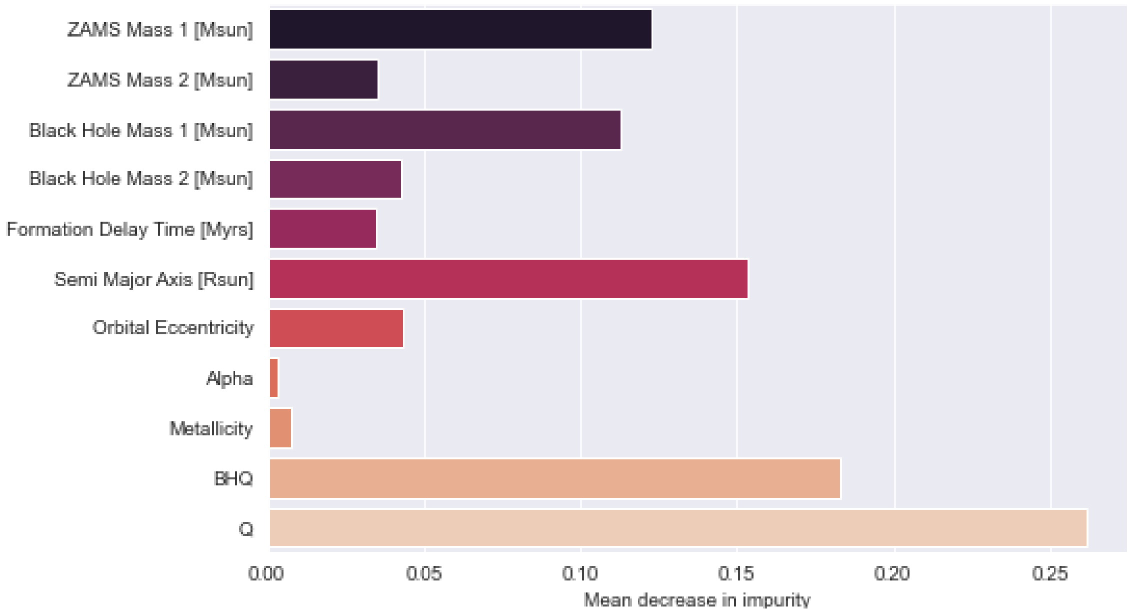
Random Forest

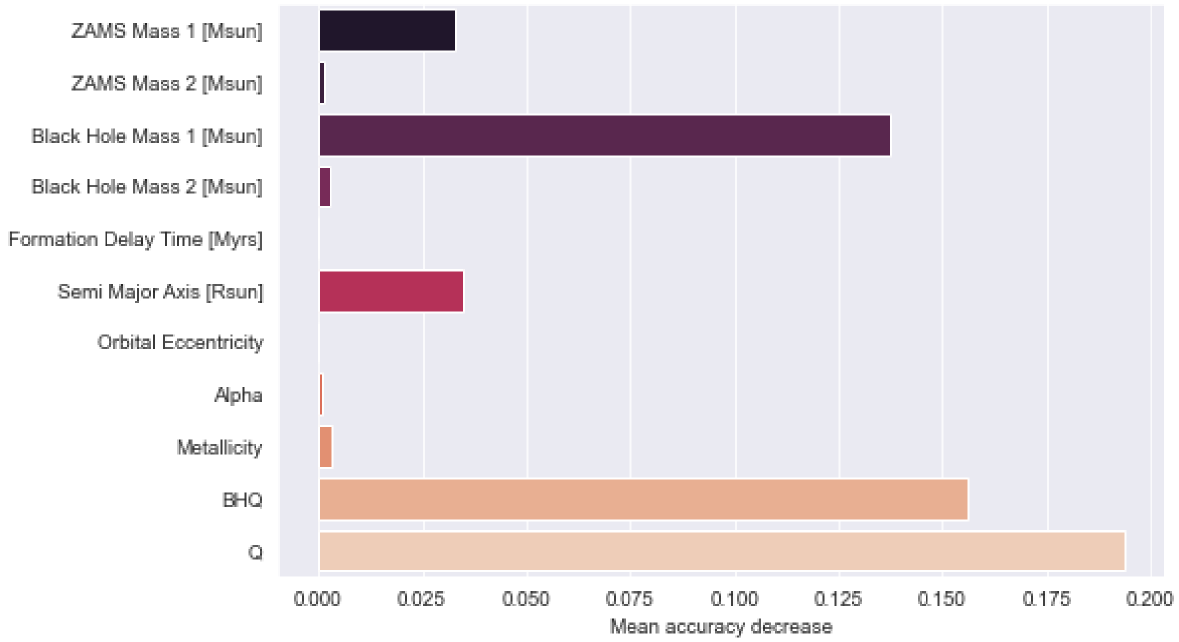


Accuracy

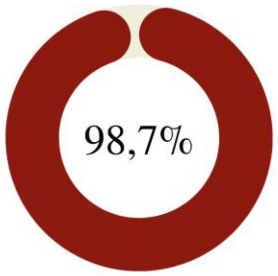
Mean Decrease in Impurity

Feature permutation



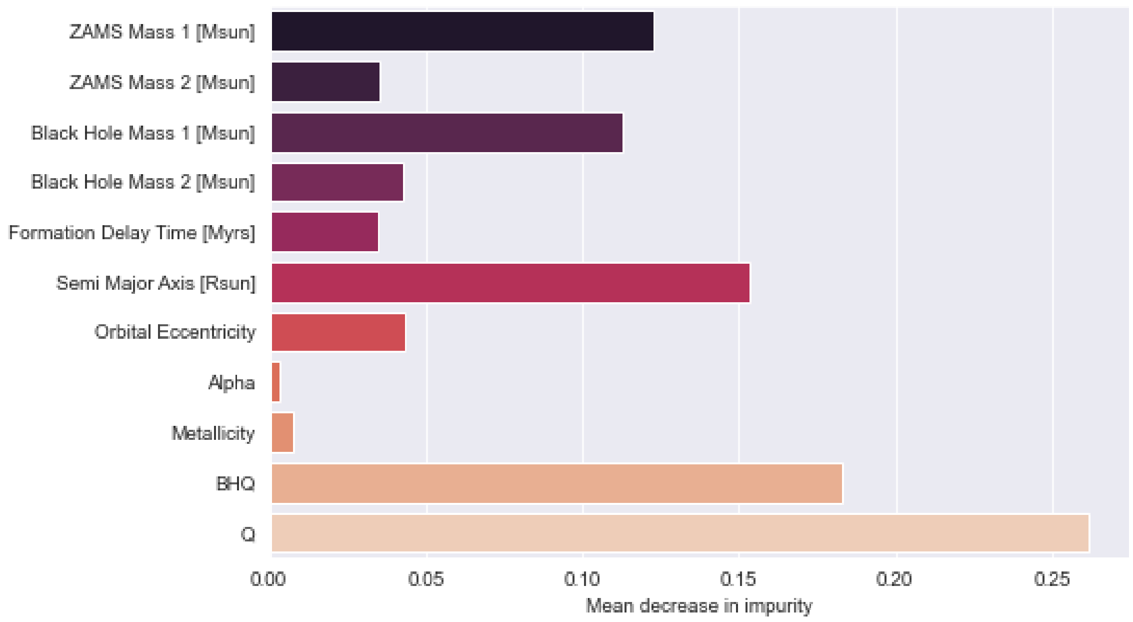


Random Forest

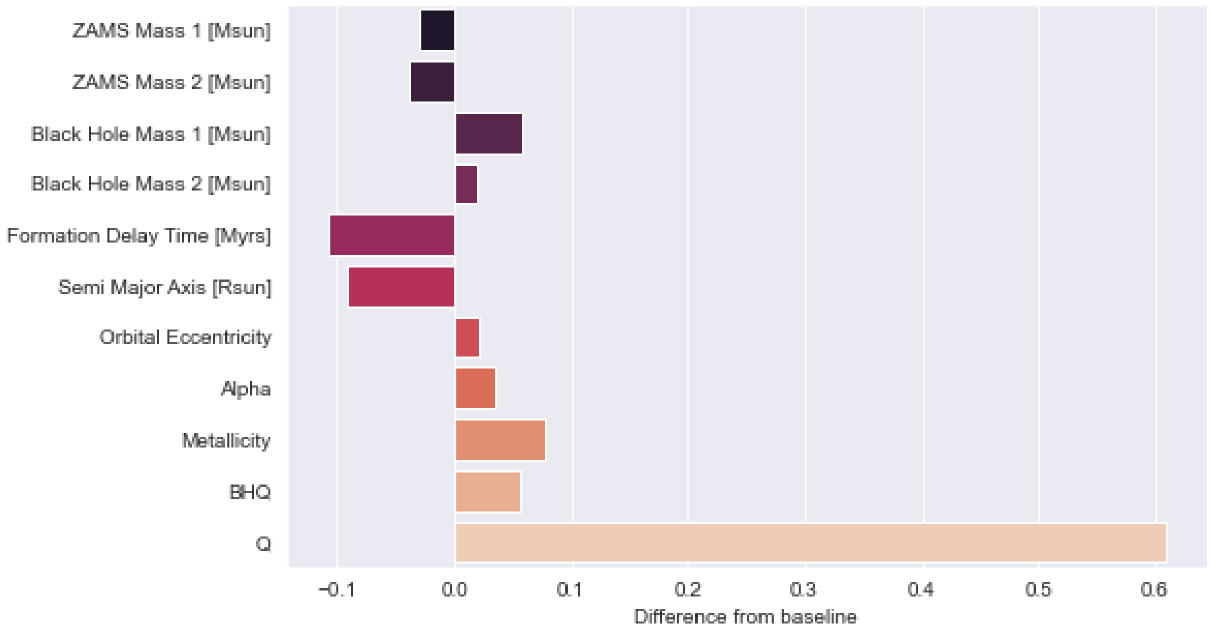


Accuracy

Mean Decrease in Impurity



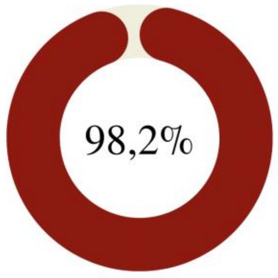
Feature-dropping



Random Forest: ranking

Ranking	MDI	Feature permutation	Feature-dropping
1	Q	Q	x
2	BHQ	BHQ	x
3	Semi-major axis	BH Mass 1	x
10	Metallicity	Formation delay time	x
11	Alpha	Orbital eccentricity	x

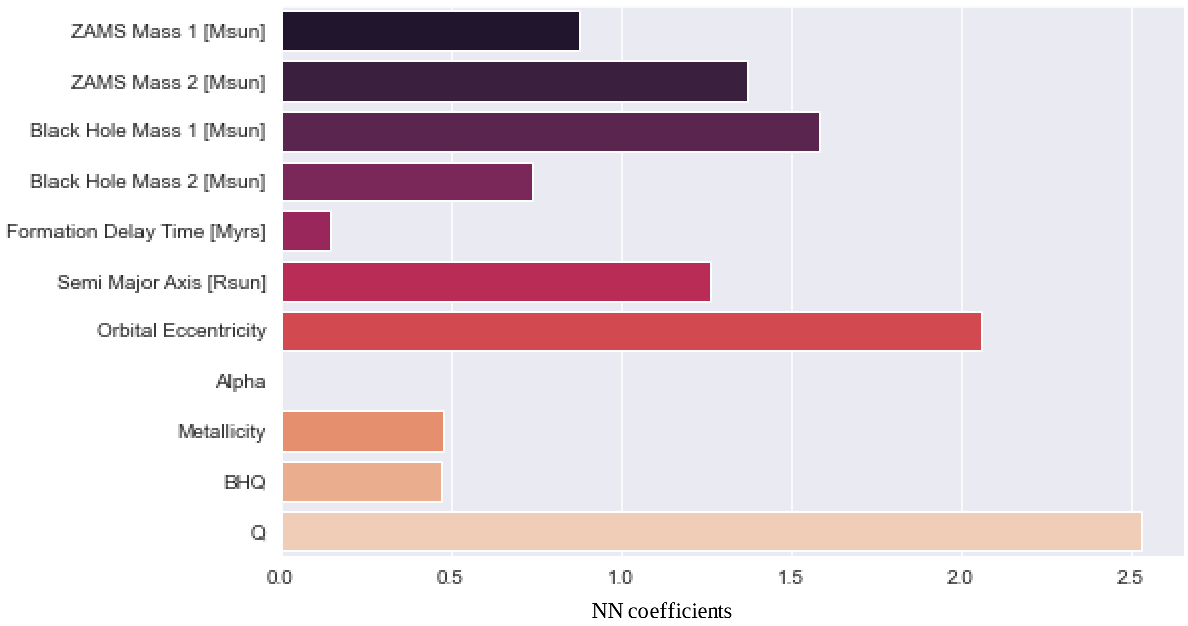
Neural Network

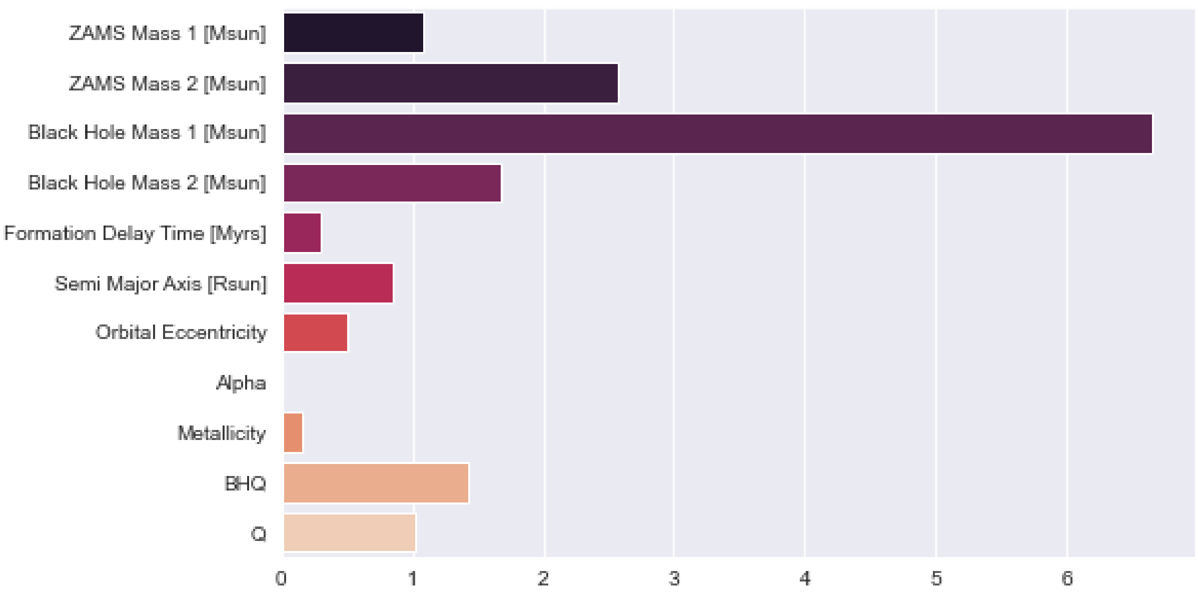


Accuracy

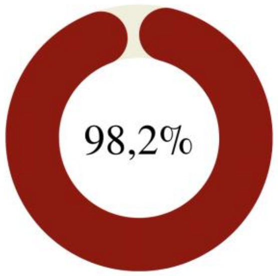
NN coefficients

Linear SVM coefficients





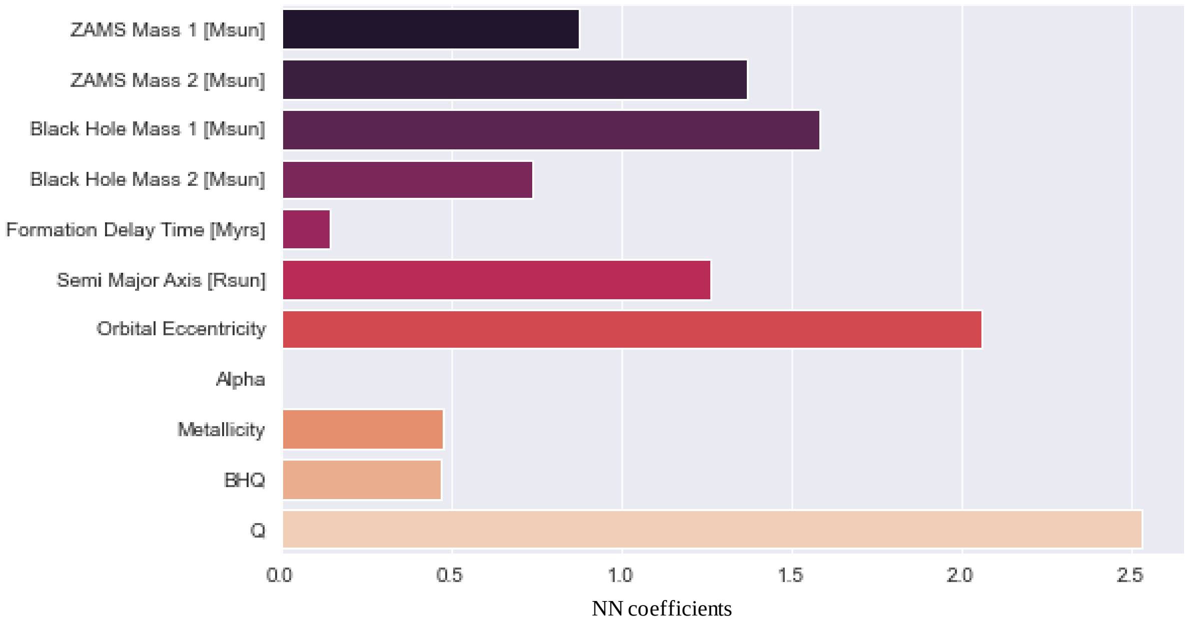
Neural Network

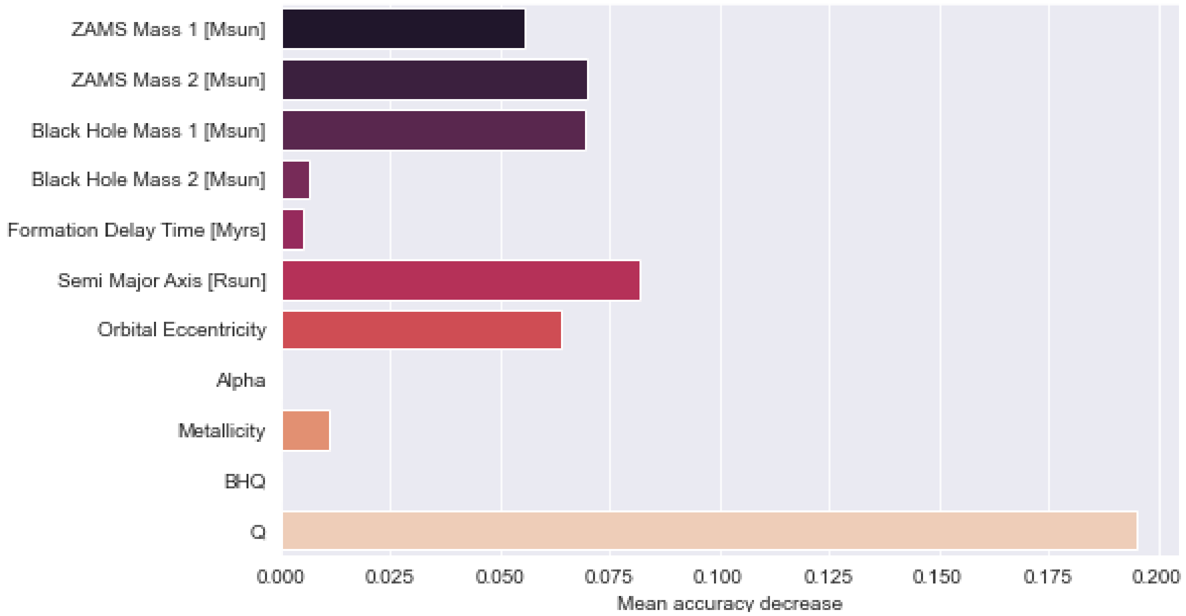


Accuracy

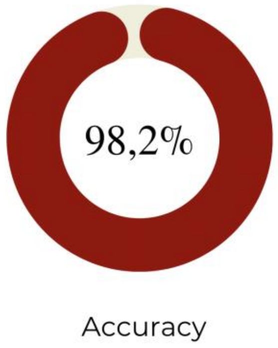
NN coefficients

Feature-permutation



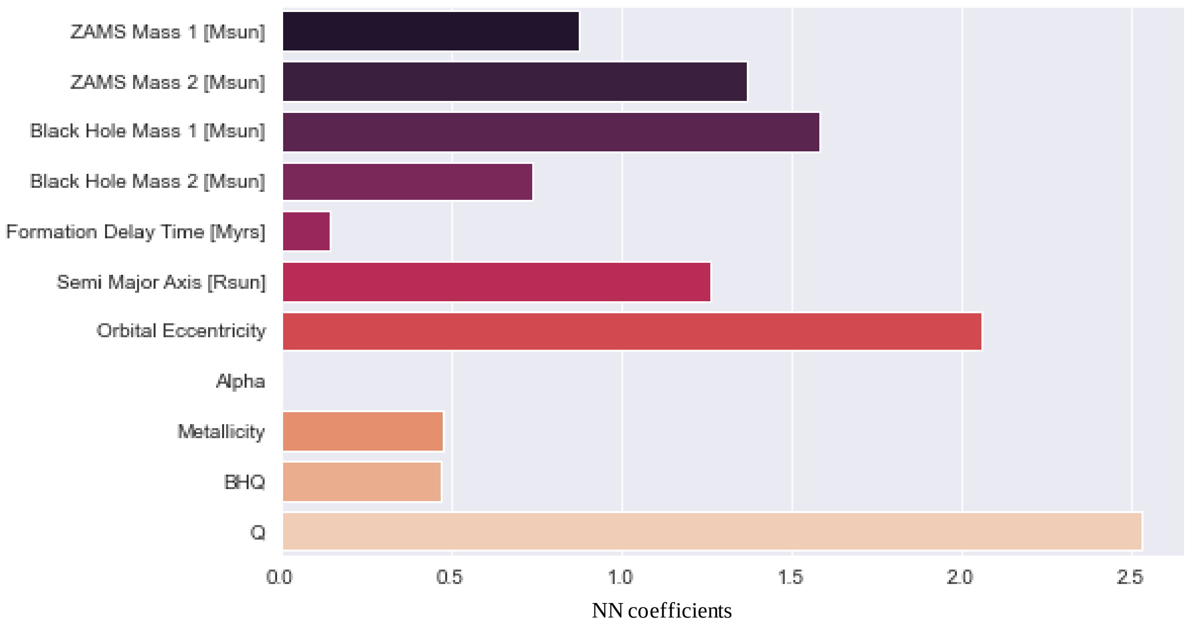


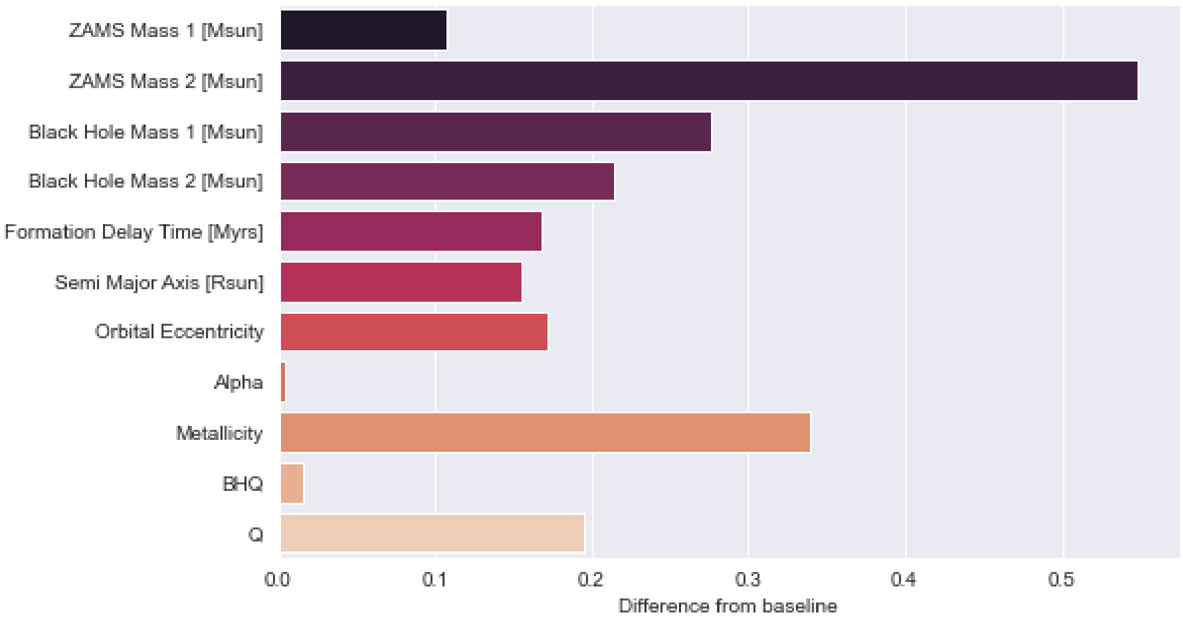
Neural Network



NN coefficients

Feature-dropping





Neural Network: ranking

Ranking	NN coefficients	Feature permutation	Feature-dropping
1	Q	Q	ZAMS Mass 2
2	Orbital eccentricity	Semi-major axis	Metallicity
3	BH Mass 1	ZAMS Mass 2	BH Mass 1
10	Formation delay time	BHQ	BHQ
11	Alpha	Alpha	Alpha

Conclusion

- Goal: Infer what are the most important features for determining the evolution of binary stars into binary black holes

- Dataset preprocess:
  - Balance the possible evolution paths
  - Removal of the outliers
  - Normalization

- Data evaluation → qualitative observation of 1:1 feature plots :
  - ZAMS Masses
  - BBH masses
  - Q and BHQ values
  - Orbital eccentricity
  - Semi-major axis

→ Expected important features

Conclusions

- Machine learning algorithm:
  - Built-in vs custom methods:
    - MDI (built-in)
    - Feature permutation (custom)
    - Feature dropping (custom)
  - Models:
    - Linear SVM
    - Random Forest
    - Neural Network
- Best model → Random Forest, it resembles expectations

- Most important features → Masses (alone or ratios) and semi-major axis