Assignment 3: The Two-Body problem(s*)

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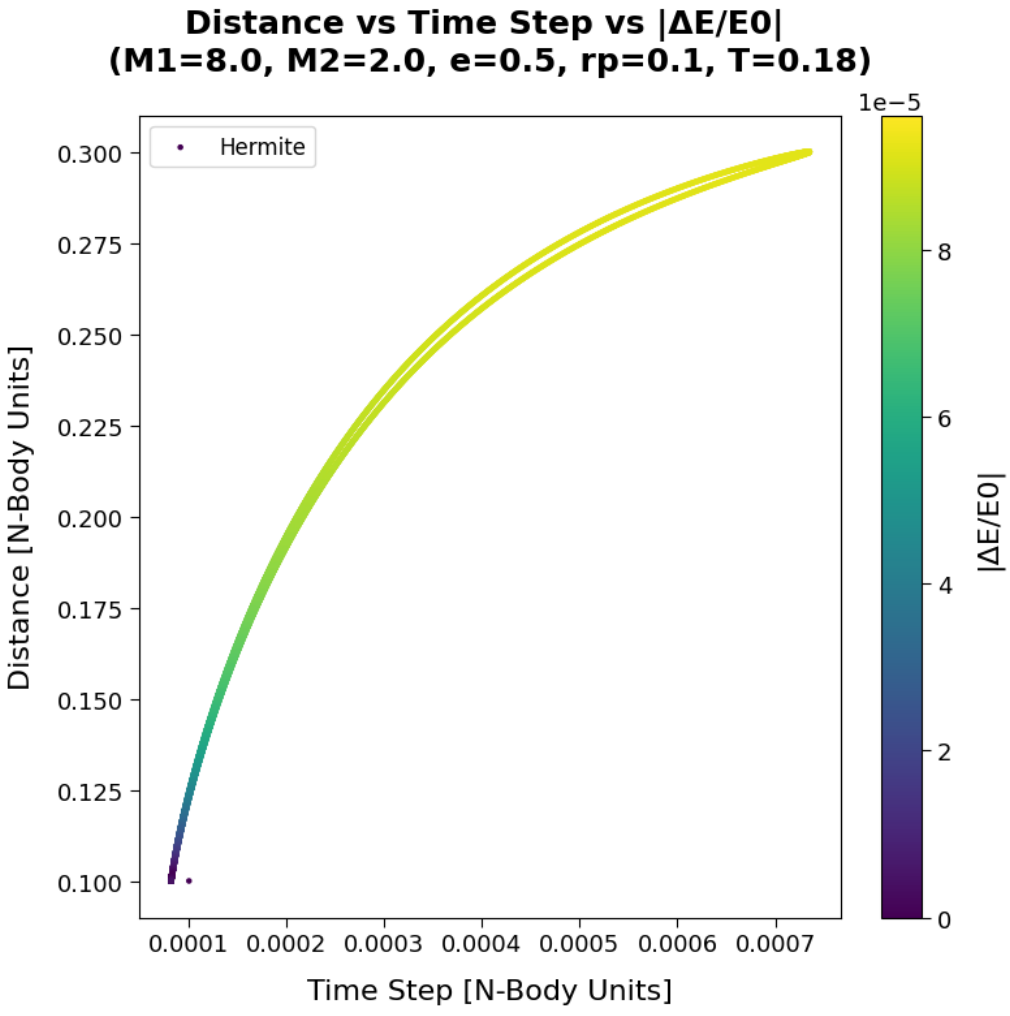
- **C) "Let's stay together, Circular orbit" + D) "Come closer, darling!, Eccentric orbits"**
  Scripts description:
  - `ass3_all.py` : script used for the integration of the systems without the adaptive timestep and softening;
  - `ass3_all_tstep.py` : script used for the integration of the systems with the adaptive timestep and softening;
  - `ass3_all_plots.py` : script used to produce the plots from which we selected those submitted.
  Note: In the analysis of the tests performed we will consider the TSUNAMI integrator as our ground truth.
  These are the setting used in the tests:
  - $e = 0.0$, $rp = 0.1$
  - $e = 0.0$, $rp = 1.0$
  - $e = 0.5$, $rp = 0.1$
  - $e = 0.5$, $rp = 1.0$
  We choose the following timesteps: 0.00001, 0.0001, 0.001.
  - $e = 0.9$, $rp = 0.1$
  - $e = 0.99$, $rp = 0.1$
  With these value of the eccentricity we opted for using only a value of $rp = 0.1$ and timesteps of 0.0001, 0.001, 0.01 due to runtime and resources reasons. Nevertheless, these initial conditions can lead to valid conclusions.

Regarding the simulation without the adaptive timestep, we can conclude that all the integrators performed according to theoretical expectations and the choice of the above mentioned timesteps helped evolving the systems in the correct way. Symplecticity is maintained by the integrators that in theory are (modified Euler and Leapfrog (or Verlet)). While the others stay non-symplectic.
Overall the 4^{th} order Runge-Kutta integrator, with its four force estimations, is the closest one to the TSUNAMI integrator.
For systems with $e = 0.0$, $e = 0.5$, the choice of $h = 0.00001$, $h = 0.0001$ is effective. $h = 0.001$ is already a more borderline case, and in systems with extreme eccentricity it makes correct integration of the equations difficult. Similarly, $h = 0.01$ is also totally ineffective.

The use of adaptive timestep and softening greatly aids systems integration. There remain some cases where, at the same timestep with other integrators, integration is not effective. This is because we used a single configuration: $\eta = 0.01$ for the adaptive timestep, softening of 10^{-4}; so probably for certain integrators a finetuning of the parameters had to be performed to achieve the best performance.
In general, on average, there was a 90% reduction in integration steps, still completing the evolution for the predetermined time of $10 \cdot T_{period}$.



From the theory we expect adaptive timesteps, indeed self-adjusting: small timesteps when the two bodies are very close and closer integrations of the system state are needed for the orbital evolution to be the correct one; larger timesteps when the two bodies are far apart and the mutual interaction is less preponderant.
That is what we can see in this graph, for illustrative use. On the x-axis we have the timestep values, on the y-axis we have the value of the Euclidean distance between the two bodies, and the color indicates the relative energy error, on a scale of 10^{-4}.
When the distance between the two bodies is small, the timestep is small; as the distance increases, the timestep also increases.
With small timesteps, evolution maintains an error on the energy small, while at larger distances the integrator "can afford" to make somewhat larger errors on the energy, while still integrating the orbits correctly when the two bodies are further apart.