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2 Solucion Parcial

2.1) $x(t) = |6 \operatorname{sen}(3t + \pi/4)|^2 = 36 \operatorname{sen}^2(3t + \pi/4)$

Sabemos que: $\operatorname{Sen}^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$

$\Rightarrow 36\left[\frac{1}{2} - \frac{1}{2}\cos(6t + \pi/2)\right]$

$\Rightarrow x(t) = 18 - 18\cos(6t + \pi/2)$

Sabemos que: $\cos(\theta + \pi/2) = -\operatorname{Sen}(\theta)$

$x(t) = 18 + 18 \operatorname{Sen}(6t)$

Forma trigonometrica:

$x(t) = a_0 + \sum_{n=-N}^{N} a_n \cos(n\omega_0 t) + b_n \operatorname{Sen}(n\omega_0 t)$

Como $x(t)$ es una funcion seno, esta presenta simetria impar: $x(t) = -x(-t)$

Por ende:

$a_n = 0$

$x(t) = 18 + 18 \operatorname{sen}(6t) = a_0 + \sum_{n=-N}^{N} b_n \operatorname{Sen}(n\omega_0 t)$

$a_0 = C_0 = \frac{1}{t_f - t_i}\int_{t_i}^{t_f} x(t)\, dt$

$$a_0 = \frac{1}{\pi-(-\pi)} \int_{-\pi}^{\pi} 18 + 18\,sen(6t)\,dt$$

$u = 6t$

$\frac{du}{6} = dt$

$$= \frac{18}{2\pi}\left[\int_{-\pi}^{\pi} dt + \int_{-\pi}^{\pi} sen(6t)\,dt\right.$$

$$= \frac{18}{2\pi}\left[t \Big|_{-\pi}^{\pi} - \frac{1}{6}\cos(6t)\Big|_{-\pi}^{\pi}\right.$$

$$= \frac{18}{2\pi}\left[2\pi - \frac{1}{6}\underbrace{\cancel{\left(\cos(6\pi) - \cos(-6\pi)\right)}}_{0}\right]$$

$a_0 = 18$ //

Para b_n:

$$b_n : \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,sen(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\int_{-\pi}^{\pi} (18 + 18\,sen(6t))\,sen(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\left[\int_{-\pi}^{\pi} 18\,sen(n\omega_0 t)\,dt + \int_{-\pi}^{\pi} 18\,sen(6t)\,sen(n\omega_0 t)\,dt\right]$$

Sabemos que: $\quad sen(\theta)\,sen(\alpha) : \frac{\cos(\theta-\alpha) - \cos(\theta+\alpha)}{2}$

$\omega_0 = \frac{2\pi}{T} \qquad T = 2\pi \qquad \omega_0 = 1\ rad/s$

$$= \frac{2}{2\pi}\left[\int_{-\pi}^{\pi} 18\,\mathrm{sen}(nt)\,dt + \int_{-\pi}^{\pi} \frac{18\cos(6t-nt)}{2}\,dt - \int_{-\pi}^{\pi} \frac{18\cos(6t+nt)}{2}\,dt\right]$$

$$u = nt \quad \frac{du}{n} = dt \qquad \begin{cases} u = (6-n)t \\ \frac{du}{6-n} = dt \end{cases} \qquad \begin{cases} u = (6+n)t \\ \frac{du}{6+n} = dt \end{cases}$$

$$= \frac{36}{2\pi n}\left[-\cos(nt)\Big|_{-\pi}^{\pi}\right] + \frac{36}{4\pi(6-n)}\left[\mathrm{Sen}(6-n)t\Big|_{-\pi}^{\pi}\right] - \frac{36}{4\pi(6+n)}\left[\mathrm{Sen}(6+n)t\Big|_{-\pi}^{\pi}\right]$$

$$= \cancel{\frac{36}{2\pi n}\Big[\underbrace{-\cos(n\pi) + \cos(-n\pi)}_{0}\Big]} + \frac{18}{2\pi}\cdot\left[\frac{\mathrm{Sen}(6-n)\pi - \mathrm{Sen}(6-n)-\pi}{(6-n)}\right]$$

$$- \frac{18}{2\pi}\cdot\left[\frac{\mathrm{Sen}(6+n)\pi - \mathrm{Sen}(6+\pi)-\pi}{(6+n)}\right]$$

$$b_n = 18\cdot\frac{\mathrm{Sen}((6-n)\pi) - \mathrm{Sen}((6-n)-\pi)}{2\pi(6-n)} - 18\cdot\frac{\mathrm{Sen}((6+n)\pi) - \mathrm{Sen}((6+n)-\pi)}{2\pi(6+n)} \quad //$$

Para $n \neq 6$, b . No obstante, para n 6 debemos calcula el limite y aplicamos la indeterminación 0/0

$$b_6 = 18\lim_{n\to 6} \frac{\frac{d}{dn}\left(\mathrm{Sen}((6-n)\pi) - \mathrm{Sen}((6-n)-\pi)\right)}{\frac{d}{dn}\left(2\pi(6-n)\right)}$$

$$b_6 = \frac{\cos((6-n)\pi)(-\pi) - \cos((6-n)-\pi)(\pi)}{-2\pi}$$

$$b_6 = 18\left(\frac{\cos(0)(-\pi) - \cos(0)(\pi)}{-2\pi}\right)$$

$$b_6 = 18 \quad //$$

En conclusión :

$$a_n = \begin{cases} 18 & n=0 \\ 0 & \forall n / \{0\} \end{cases}$$

$$b_n = \begin{cases} 18 & n=6 \\ -18 & n=-6 \\ 0 & \forall n \setminus \{6,-6\} \end{cases}$$

Forma Exponencial

$C_0 = a_0 = 18$

$C_n = \dfrac{a_n - jb_n}{2}$

$C_{-6} = j9$

$C_6 = \dfrac{0 - j18}{2} = \dfrac{-j18}{2} = -j9$

Se obtiene que:

$$C_n = \begin{cases} 18 & n=0 \\ -9j & n=6 \\ 9j & n=-6 \\ 0 & \forall \setminus \{0,6,-6\} \end{cases}$$

Para continuar con la señal

$$x(t) = \sum_{n=-N}^{N} C_n e^{jnt}$$

$$x(t) = C_{-6} e^{-j6t} + C_0 e^{0} + C_6 e^{j6t}$$

$$= [9j(\cos(6t) - 9j^2 sen(6t)) + 18 - 9j\cos(6t) - 9j^2 sen(6t)]$$

$$x(t) = 18 + 18\, sen(6t)$$

El error relativo se calcula:

$$E_x[\%] = \left[1 - \frac{1}{P_x} \sum_{n=-N}^{N} |C_n|^2\right] 100\%$$

$$P_x = \frac{1}{T}\int_{t_i}^{t_f} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} |18 + 18\,\text{sen}(6t)|^2\, dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 18^2 + (2)(18)(18\,\text{sen}(6t)) + 18^2\,\text{sen}^2(6t)\, dt$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} 324\,dt + \int_{-\pi}^{\pi} 648\,\text{sen}\,6t\,dt + 18^2\int_{-\pi}^{\pi} \frac{1}{2} - \frac{\cos(12t)}{2}\,dt\right.$$

$$= \frac{1}{2\pi}\left[324t\Big|_{-\pi}^{\pi} + \cancel{\frac{648}{6}(-\cos(6t))}\Big|_{-\pi}^{\pi} + \frac{18^2 t}{2}\Big|_{-\pi}^{\pi} - \frac{18^2}{24}\text{sen}(12t)\Big|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[648\pi + 0 + 324\pi - 0\right]$$

$$P_x = 486$$

$$E_x = \left[1 - \frac{(-9)^2 + (18)^2 + (9)^2}{486}\right] \times 100\% = 0\%$$

2.2)

$C(t) = Ac \, Cos(2\pi Fc t), \quad Ac, fc \in \mathbb{R} \qquad y(t) = \left(1 + \frac{m(t)}{Ac}\right) C(t)$

$F\{C(t)\} + F\left\{\frac{m(t) C(t)}{Ac}\right\}$

$F\{Ac \, Cos(2\pi Fc t)\} = Ac \cdot F\left\{\frac{e^{j2\pi Fct} + e^{-j2\pi Fct}}{2}\right\}$

$Ac \cdot \left[F\left\{\frac{e^{j2\pi Fct}}{2}\right\} + F\left\{\frac{e^{-j2\pi Fct}}{2}\right\} \right]$

$\frac{Ac}{2} [2\pi \delta(\omega - 2\pi Fc) + 2\pi\delta(\omega + 2\pi Fc)]$

$Ac \pi \delta(\omega - 2\pi Fc) + Ac \pi \delta(\omega + 2\pi Fc) \rightarrow C(\omega)$

$C(\omega) = Ac \pi \delta [(\omega - 2\pi Fc) + (\omega + 2\pi Fc)]$

$F\left\{\frac{m(t)(Ac \, Cos(2\pi Fct))}{Ac}\right\} = F\{Cos(2\pi Fct) m(t)\} = F\left\{\frac{m(t) e^{j2\pi Fct}}{2}\right\} + F\left\{\frac{m(t) e^{-j2\pi Fct}}{2}\right\}$

$\frac{M(\omega - 2\pi Fct)}{2} + \frac{M(\omega + 2\pi Fct)}{2} = \frac{1}{2} M[(\omega - 2\pi Fct) + (\omega + 2\pi Fct)]$

$y(\omega) = Ac \pi \delta [(\omega - 2\pi Fct) + (\omega + 2\pi Fct)] + \frac{1}{2} M[(\omega - 2\pi Fct) + (\omega + 2\pi Fct)]$