Commodity Inflation Factor

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November 2024

1 Introduction

2 Factor Construction

First begin with a series of n rates $r_{i,t}$ where i correspond for some tenor. The series of rates $[r_{1,t}, r_{2,t}, ..., r_{i,t}]$ can be decomposed into $k < n$ factors via principal component analysis. This is somewhat trivial and commonplace with interest-rate trading. The fitted values can be computed for the term structure $\boldsymbol{R}$. First decompose the rates via PCA.

$$C = \frac{\boldsymbol{R}^\top \boldsymbol{R}}{n-1} \tag{1}$$

$$= \boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^\top \tag{2}$$

Then apply some $k < i$ components back to the data

$$\boldsymbol{R}' = \boldsymbol{R}\boldsymbol{W}_k \tag{3}$$

For the most part most rate-based term structures can be decomposed into 3 principal components. The first principal component is level, the second is slope, and the third is convexity. Now using OLS we can build out the *characteristic* function using a rolling regression. In this case the rolling regression is done a single-asset single-principal component, and the β is used as the characteristic value.

For some PC_i $i \leq 3$ and for a commodity future j and for some lookback window $\tau < T$

$$C_{t,\tau} = \beta \cdot PC_{i,t,\tau} + \alpha \tag{4}$$

In this case we set $\tau = 30$ which is somewhat arbitrary. Once this is done a matrix of rolling lagged βs exist per each principal component

For some PC_i $i \leq 3$ a matrix of βs ($\boldsymbol{\beta}$) can be made where each column is a commodity future's rolling βs and each row is for some time t in this case the βs must be lagged.

For PC_i with j commodity futures with

$$\hat{\boldsymbol{\beta}}_{(i,\tau),t-1\times j} = \begin{bmatrix} \hat{\beta}_{i,t-1,1} & \hat{\beta}_{i,t-1,2} & \cdots & \hat{\beta}_{i,t-1,j} \\ \hat{\beta}_{i,t,1} & \hat{\beta}_{i,t,2} & \cdots & \hat{\beta}_{i,t,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{i,T-i,1} & \hat{\beta}_{i,T-i,2} & \cdots & \hat{\beta}_{i,T-1,j} \end{bmatrix} \tag{5}$$

Now per each time step t or any time step $t + t'$ for any $t' \in \mathbb{N}$ decile the βs into long short legs. In this case daily deciling is quite expensive while monthly deciling performs better. The methodology within the code uses month end data for the next day which isn't a fixed $t + t'$. In this case per each decile use exponential weighting. Using exponential weighting gives more weight to securities that isolate "more" of the factor exposure.

The methodology listed above is for creating a β-based factor. While that was the original approach it proved to not fully capture the "inflation component" within commodities. Other methods are used but are still based on the OLS regression of the commodities with the principal components of each curve.

3 Factor Results

3.1 Deciling Beta Factor