Commodity Inflation Factor (Working Draft)

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November 2024

1 Introduction

2 Factor Construction

First begin with a series of n rates $r_{i,t}$ where i correspond for some tenor. The series of rates $[r_{1,t}, r_{2,t}, ..., r_{i,t}]$ can be decomposed into $k < n$ factors via principal component analysis. This is somewhat trivial and commonplace with interest-rate trading. The fitted values can be computed for the term structure $\boldsymbol{R}$. First decompose the rates via PCA.

$$\begin{aligned} C &= \frac{\boldsymbol{R}^\top \boldsymbol{R}}{n-1} && (1) \\ &= \boldsymbol{W \Lambda W}^\top && (2) \end{aligned}$$

Then apply some $k < i$ components back to the data

$$\boldsymbol{R}' = \boldsymbol{R}\boldsymbol{W}_k \tag{3}$$

For the most part rate-based term structures can be decomposed into 3 principal components. The first principal component is level, the second is slope, and the third is convexity. Now using OLS, build out the *characteristic* function using a rolling regression. In this case the rolling regression is done on a single-asset single-principal component basis.

For some PC_i $i \leq 3$ and for a commodity future j and for some lookback window $\tau < T$

$$C_{t,\tau} = \beta \cdot PC_{i,t,\tau} + \alpha \tag{4}$$

In this case we set $\tau = 30$ which is somewhat arbitrary. Once this is done a matrix of rolling lagged βs exist per each principal component

For some PC_i $i \leq 3$ a matrix of βs ($\boldsymbol{\beta}$) can be made where each column is a commodity future's rolling βs and each row is for some time t in this case the βs must be lagged.

For PC_i with j commodity futures with

$$\hat{\boldsymbol{\beta}}_{(i,\tau),t-1 \times j} = \begin{bmatrix} \hat{\beta}_{i,t-1,1} & \hat{\beta}_{i,t-1,2} & \cdots & \hat{\beta}_{i,t-1,j} \\ \hat{\beta}_{i,t,1} & \hat{\beta}_{i,t,2} & \cdots & \hat{\beta}_{i,t,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_{i,T-i,1} & \hat{\beta}_{i,T-i,2} & \cdots & \hat{\beta}_{i,T-1,j} \end{bmatrix} \tag{5}$$

Now per each time step t or any time step $t + t'$ for any $t' \in \mathbb{N}$ decile the βs into long short legs. In this case daily deciling is quite expensive while monthly deciling performs better. The methodology within the code uses month end data for the next day which isn't a fixed $t + t'$. In this case per each decile use exponential weighting. Using exponential weighting gives more weight to securities that isolate "more" of the factor exposure.

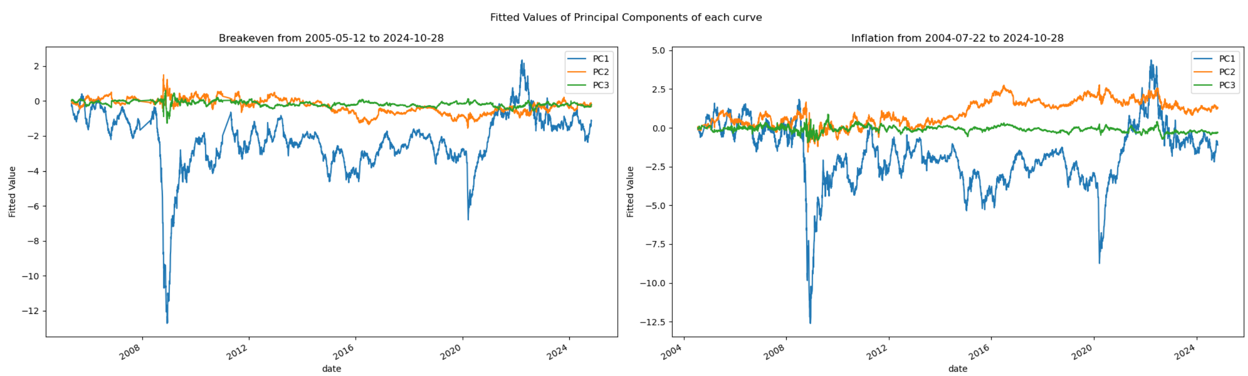
The methodology listed above is for creating a β-based factor. While that was the original approach it proved to not fully capture the "inflation component" within commodities. Other methods are used but are still based on the OLS regression of the commodities with the principal components of each curve.

3 Data and Pre-Processing

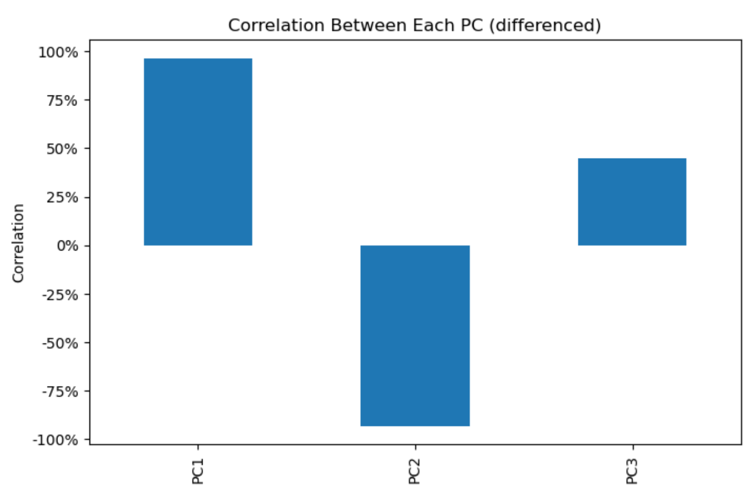
In this case the term structure chosen is the Treasury Breakeven Inflation curve and the Inflation Swap Curve. These were chosen for a couple of reasons, the first is there aren't many inflation numbers that get published daily. Inflation publications are also backward-looking since they are a measurement of the previous period's inflation, while treasury breakevens and inflation swaps are forward-looking. Both suffer from similar problems, the main being liquidity, although they are used as just input to the models.

Its difficult to parse out a *daily-forward-looking-inflation*. Although regressing against one inflation swap or one treasury breakeven may work the PCA decomposes all of the rates across the curve to extract a *global* rate. The level of the rate doesn't have much significance since all PCs are differenced, which is why it makes it an ideal candidate as a regressor.

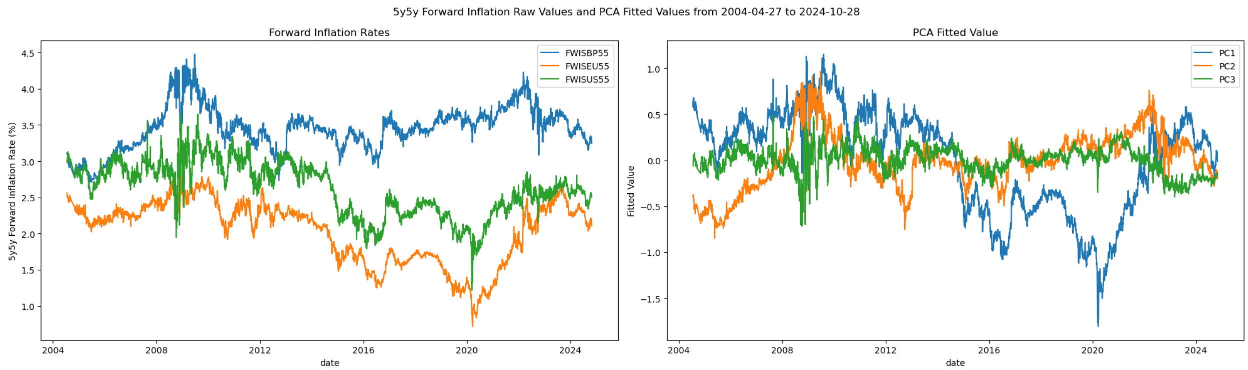
Below are the fitted values of each curve via PCA



The PC fitted values are highly correlated, below is the plot of the correlation



Another candidate for inflation is the 5y5y forward inflation swaps which are commonly used. In this case they are not collected across the curve but rather for each tradable market. In this case these inflation measures are done for different countries, specifically for the US, UK, and EU. Below is the 5y5y forward inflation and their PCs.



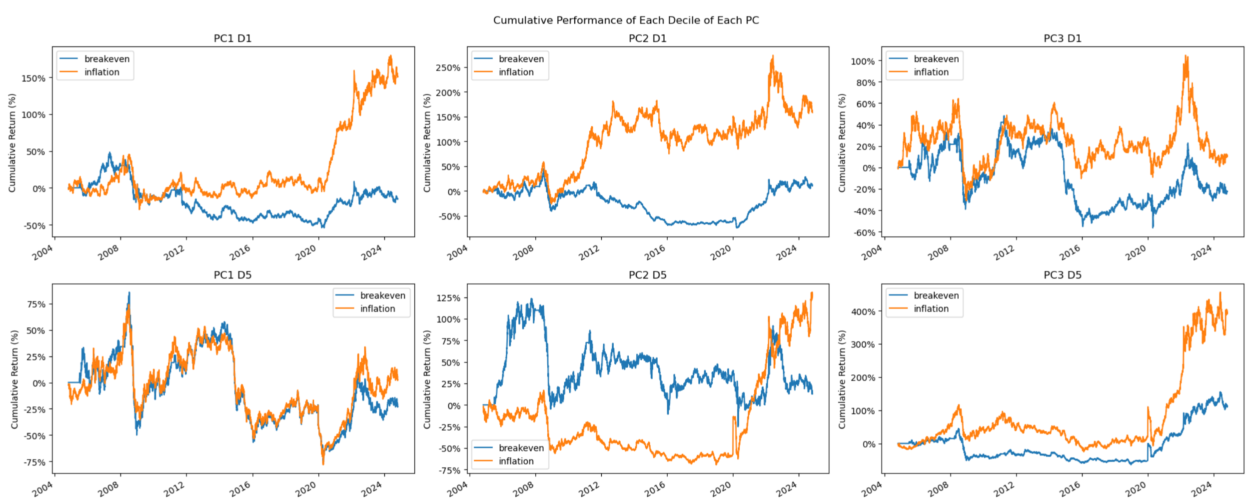
4 Factor Results

The factor results for the most part, admittedly; have not proven useful. From a trading standpoint they are not suitable. There is a chance that they can be useful as a risk management tool, but the factors don't generate enough sharpe to be worth trading. They could be used to for factor risk budgeting. Another problem that presents itself if these factors are for trading is that its not completely clear which leg to be long or short. In this case both ways are tested (long high beta, short low beta). Within this PCA OLS framework there are a series of attempts to try and find a factor worth trading. Below are a list of them

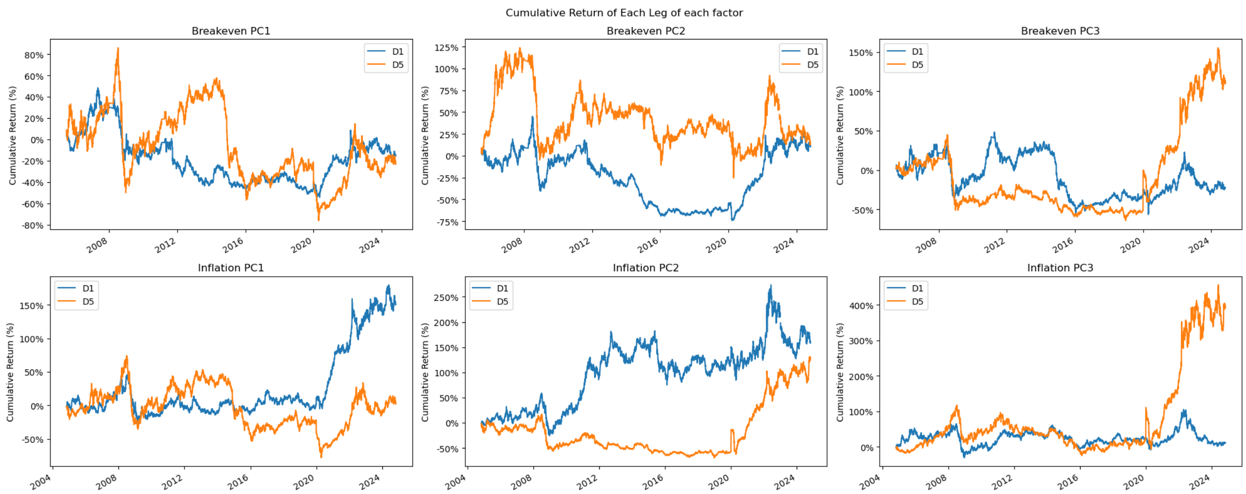
- Quartiled β factor of each PCA component for each curve
- Quartiled β factor of each PCA component for the 5y5y forward inflation
- Spread in Quartiled β of each PCA component for each curve

4.1 Quartiled Beta Factor

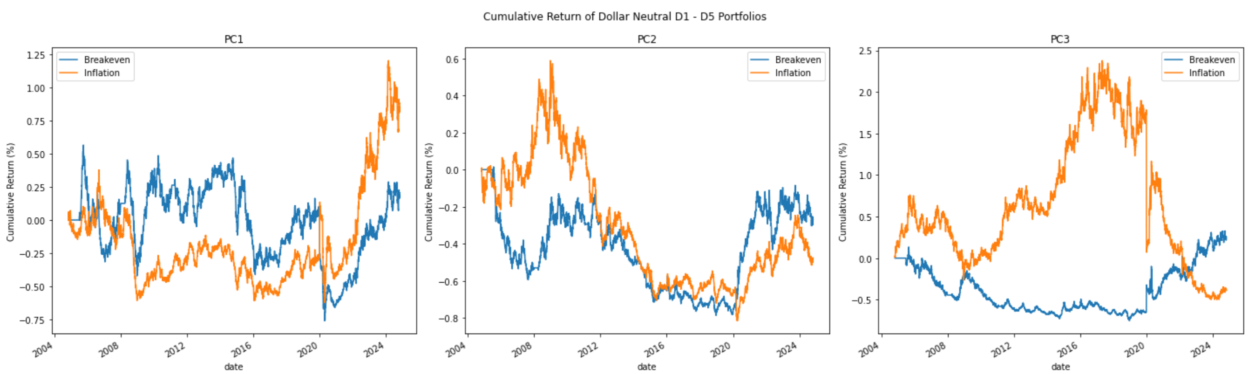
The quartiled beta factor is done by regressing the principal components of the raw rates of the breakeven and inflation swap curve. Below is a plot of each quartile leg with exponential weighting rebalanced daily, but quartiled-securities *re-picked* monthly.



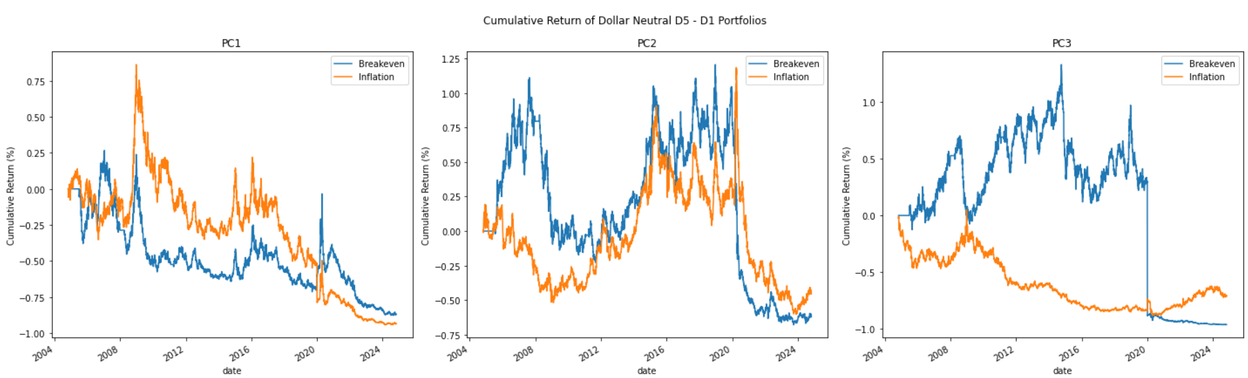
From this its not likely that any of the legs could be traded by themselves. Comparing each quartile *leg-by-leg* per each PC.



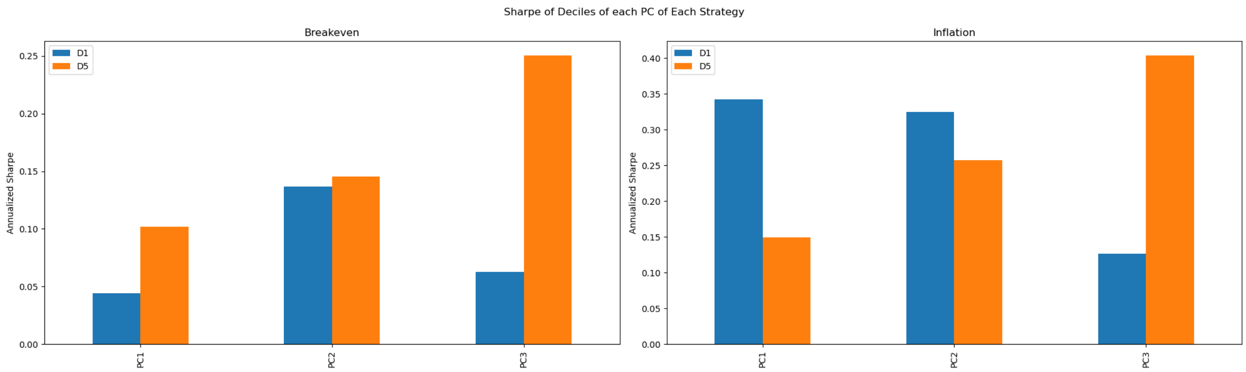
In this case build out dollar neutral L/S pairs of each quartile spread ($D5 - D1$ & $D1 - D5$)



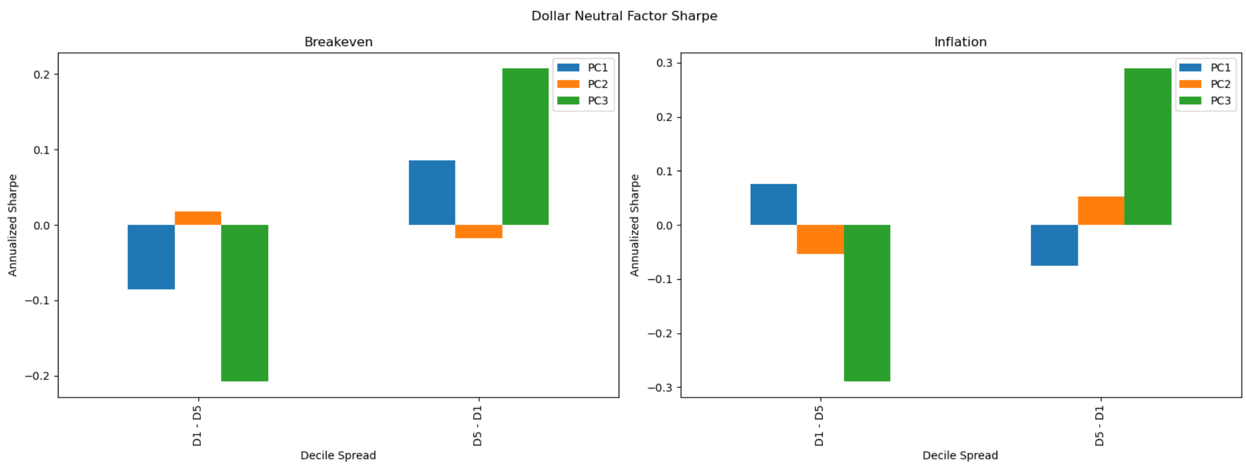
And its *inverse* $D5 - D1$



The sharpes for the individual quartiles below, which are pretty weak and not tradable.



Its corresponding sharpes for the dollar neutral positions



4.2 Quartiled 5y5y Forward Inflation Beta Factor