Distributed Optimization for Networks of Battery Energy Storage Systems in Energy Communities with Shared Energy Incentives

Mohammed A. M. Messilem, Diego Deplano, Mauro Franceschelli, Elio Usai, Ruggero Carli

Abstract— **In this paper we propose a distributed optimization framework for Italian energy communities, characterized by the possibility of receiving a monetary state incentive based on the amount of "shared energy" consumed. Renewable energy produced by members of the community can be "shared", i.e., other members of the community not physically connected to renewable energy sources can consume energy at a reduced cost due to state incentives according to specific rules. In our model we assume that users may have access to either, neither or both, battery energy storage systems (BESS) and renewable energy sources. We propose a cooperative distributed optimization framework which is the basis for distributed predictive control of a network of BESSs that changes the hourly amount of shared energy consumed in the community. The proposed approach can be used to preserve privacy of user consumption data. We provide numerical results indicating that the total cost of energy for a community is significantly reduced by the proposed approach and that cooperation among the BESS owned by the members of the energy community is beneficial.**

I. Introduction

The increasing need for energy and rapid expansion of electricity consumption pose significant challenges for current power systems. Moreover, there is a growing imperative to reduce reliance on fossil fuels, leading to heightened interest in renewable energy sources [1].

According to the study reported by US energy information administration (EIA), the worldwide energy consumption will grow by 56 % from 2010 to 2040. The objectives in the 2030 UN's Agenda for sustainable development, which focus on ensuring access to affordable, sustainable, and modern energy, are in harmony with the worldwide endeavor to combat climate change, advance clean energy, and realize sustainable development goals [2]. This encourage the establishment of a green power system where users are increasingly adopting distributed renewable energy generators to satisfy their local demand. This approach can efficiently decrease both the carbon dioxide emissions associated with conventional fossil fuel-based power plants and the transmission losses incurred when supplying power to distant users. Smart grids offer the infrastructure and technologies necessary to incorporate renewable energy sources (RESs), such as solar and wind power, into the grid. Nevertheless, the intermittent and stochastic nature of renewable energy sources can result in imbalances between supply and demand, leading to fluctuations in power system frequency and voltage [3]. But smart grid can forecast their output and coordinate their integration with conventional generation to ensure a stable and efficient power supply. Additionally, smart grids enable the integration of energy storage system (ESS), including batteries, pumped hydro, and flywheels. Implementing ESS presents a practical solution to mitigate power fluctuations in renewable energy generation and enhance system reliability [4]. These storage solutions assist in capturing excess electricity during periods of low demand or high renewable energy generation and release it during peak demand periods. Furthermore, energy storage can be effectively utilized through shared usage among multiple energy consumers with varying demand patterns. Small-scale energy cooperatives offer the opportunity to share generated electricity among building owners [5]. This arrangement can yield financial benefits for each building owner. Additionally, the need for backup capacity and the issue of overproduction are mitigated with small scale-energy cooperative in place [6]. The increased capacity of shared energy concepts facilitates greater opportunities for energy charging and discharging. This shared energy setup enables simultaneous charging and discharging for different consumers, thereby enhancing the overall utilization of the resource. By participating in the shared energy, consumers gain access to energy charged by others, serving as an additional energy source that aids in reducing electricity expenses. To ensure the effective implementation of shared energy concept systems in residential communities with appropriate configurations, it is crucial to conduct comprehensive analysis and investigation of operations and controls. This includes estimating the anticipated benefits that can be realized through the utilization of shared energy.

Currently, many researchers are actively pursuing diverse initiatives to advocate for polycentric and distributed energy supply concepts, aiming to facilitate a more efficient transition in energy systems [7], [8], [9]. Consequently, numerous studies have been conducted employing various techniques and models to assess the tangible advantages of shared energy in contrast to individual energy systems [10]. For instance, [11] utilized battery and solar PV simulation models to assess both solar and economic metrics for individual and shared energy storage scenarios, while in [12], [13] proposed multi-agent demand-side response control schemes

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for networks of thermostatically controlled loads. In [14] a novel robust framework for the day-ahead energy scheduling of a residential microgrid comprising interconnected smart users is proposed. Arghandeh et al. [15] introduced a market-based optimization algorithm, which was solved using multi-objective, gradient-based heuristic optimization approach. this algorithm aims to generate an optimal schedule for community energy storage, considering both real-time and day-ahead dimensions.

Model predictive control (MPC) is being considered as a promising control strategy for efficiently managing users resources. It is particularly well suited for system with constraints and dynamics behaviour, making it an effective approach for managing resources in various applications [16], [17], [18]. Economic MPC stands as a well-established methodology for effectively managing both the demand and supply aspects of energy systems [19]. In MPC, the optimization problem needs to be solved at each time instant, which underscores the importance of designing efficient online solvers. As the number of agents in the network increases, the complexity of the optimization problem grows. To tackle this issue, distributed optimization methods have been developed [20], [21], [22]. In addition to computational efficiency, distributed optimization methods in MPC also addresses concerns regarding the privacy of individual agents within the network. This paper presents a perspective on the notion of energy sharing with a smart grid in a community comprising various prosumers and assets, achieved through the deployment of an innovative energy management system.

The **main contribution** of this work can be summarized as follows:

- A formulation of the cooperative energy cost optimization problem for energy communities with shared energy incentives and BESS (see also [23] for a formulation that includes TCLs).
- A regularized version of the proposed optimization problem that can be solved in distributed fashion despite the coupling effects of shared energy with the DC-ADMM algorithm [24], which preserves the privacy of user data consumption.
- Numerical simulations which compare distributed and centralized cooperative approaches with a non-cooperative one. The results show a clear cost benefit for the energy community achieved by our proposed optimization framework.

II. Notation and model of the energy community

The set of real and integer numbers are denoted by $\mathbb{R}$ and $\mathbb{Z}$, while $\mathbb{R}_{\geq 0}$, $\mathbb{N}$ and $\mathbb{R}_+$,$\mathbb{N}_+$ denote their restriction to nonnegative and positive entries, respectively. Matrices are denoted by uppercase letters and vectors by bold lowercase letters, whose entries are denoted by lowercase, nonbold symbols. For instance, $\boldsymbol{x} = [x_1, \dots, x_n]^\top$ denotes a vector of $n \in \mathbb{N}_+$ entries $x_i \in \mathbb{R}$ with $i = 1, \dots, n$, and $M = \{m_{ij}\}$ denotes a square matrix of dimension $n \in \mathbb{N}_+$ with entries $m_{ij} \in \mathbb{R}$ with $i, j = 1, \dots, n$. Moreover, the identity matrix is denoted by I_n while $\mathbf{1}_n$ denotes a vector of ones of dimension $n \in \mathbb{N}_+$. When clear from the context, the subscript is omitted.

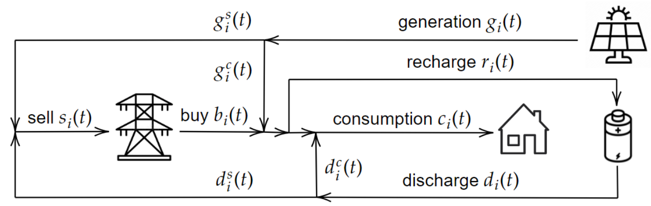

Fig. 1: Diagram illustrating the energy flow model of a prosumer within an energy community.

A. Signals and sampling

Given a continuous-time signal $x(t) \in \mathbb{R}$ with $t \in \mathbb{R}$ and a sampling time $\Delta \in \mathbb{N}_+$, we denote by $t_k = \Delta k$ with $k \in \mathbb{N}$ the discrete times at which the signal is sampled, yielding the discrete time signal $x(t_k) \in \mathbb{R}$. We also denote by $[x]_k^T$, where $k, T \in \mathbb{N}$ the vector collecting T samples of the continuous time signal starting from t_k, namely

$$[x]_k^T = [x(t_k), \cdots, x(t_{k+T-1})]^\top. \tag{1}$$

B. Networks and Graphs

We consider networks of $n \in \mathbb{N}_+$ interconnected agents and describe the pattern of interactions by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, n\}$ is the set of nodes modeling the agents, and $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$ is the set of edges modeling the point-to-point interactions. The interactions among the agents are assumed to be bidirectional, and therefore the graph is *undirected*, i.e., if $(i, j) \in \mathcal{E}$ then $(j, i) \in \mathcal{E}$. An undirected graph $\mathcal{G}$ is said to be *connected* if between any pair of nodes $i, j \in \mathcal{V}$ there exists a path. Nodes $i, j \in \mathcal{V}$ are said to be *neighbors* if there exists an edge between them, i.e., $(i, j) \in \mathcal{E}$. The set of neighbors of the i-th node is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in E\}$. We consider graphs without self-loops, i.e., $i \notin \mathcal{N}_i$.

C. Model of the energy community

We consider energy communities consisting of $n \in \mathbb{N}$ prosumers, interconnected according to an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which may, other than consuming power, also produce power thanks to a renewable generator (RG), and store energy thanks to a Battery Energy Storage System (BESS). We now describe the energy flow model at the prosumer $i \in \mathcal{V}$ level depicted in Fig. 1. The energy production due to the RG is denoted by $g_i(t) \in \mathbb{R}_{\geq 0}$, which can be divided into two components $g_i^c(t)$ and $g_i^s(t)$, representing consumed (by the user) and sold (to the grid) portions of generation, respectively,

$$g_i(t) = g_i^c(t) + g_i^s(t). \tag{2}$$

The energy utilized by the prosumer to recharge and discharge the BESS are denoted by $r_i(t) \in \mathbb{R}_{\geq 0}$ and $d_i(t) \in \mathbb{R}_{\geq 0}$, respectively. We model the charging/discharging behavior of the BESS as follows [25, Section III-A]

$$e_i^{\text{MAX}} \frac{d}{dt} \varepsilon_i(t) = \eta_i r_i(t) - d_i(t), \tag{3}$$

where:

- $e_i^{\text{MAX}} \in \mathbb{R}_{\geq 0}$ is the maximum energy storage capacity.
- $\varepsilon_i(t) = e_i(t)/e_i^{\text{MAX}} \in [0, 1]$ is state of charge (SoC).
- $\eta_i \in [0, 1]$ is the round trip efficiency, i.e., the ratio between the energy supplied to the storage system and the energy retrieved from it.

The BESS's discharging power $d_i(t) \in \mathbb{R}_{\geq 0}$ consists of two components $d_i^c(t) \in \mathbb{R}_{\geq 0}$ and $d_i^s(t) \in \mathbb{R}_{\geq 0}$, representing consumed (by the user) and sold (to the grid) portions of discharge power, respectively,

$$d_i(t) = d_i^c(t) + d_i^s(t). \tag{4}$$

Finally, the energy withdrawn from (bought) and injected into (sold) the grid are denoted by $b_i(t) \in \mathbb{R}$ and $s_i(t) \in \mathbb{R}$, respectively, satisfying the following:

$$\begin{aligned} b_i(t) &= r_i(t) + c_i(t) - d_i^c(t) - g_i^c(t), \\ s_i(t) &= d_i^s(t) + g_i^s(t). \end{aligned} \tag{5}$$

III. Optimization problem formulation

In what follows we will use $\Delta \in \mathbb{R}_{\geq 0}$ to denote the sampling time of the continuous-time signals in play, and $H = T\Delta \in \mathbb{R}_{\geq 0}$ for the optimization horizon, where $T \in \mathbb{N}_+$ denotes the number of samples within the horizon. Accordingly, we denote by $t_k = k\Delta$ with $k \in \mathbb{N}$ the discrete times at which samples are taken. To make the presentation more straightforward, we are going to drop the subscript k and the superscript T in eq. (1) and use the slender notation $\boldsymbol{x}$ instead:

$$\boldsymbol{x} := [x]_k^T = [x(t_k), \cdots, x(t_{k+T-1})]^\top \in \mathbb{R}^T.$$

A. Decision variables and local constraints

We assume that the consumption of the prosumers cannot be modified to guarantee the highest comfort. Therefore, $\boldsymbol{c}_i \in \mathbb{R}^T$ will not be considered as a decision variable but, instead, as a fixed parameter representing the expected consumption of the user in the optimization horizon. The same holds for the energy produced $\boldsymbol{g}_i \in \mathbb{R}^T$ by the RG. Consequently, we are going to consider as decision variables the recharge $\boldsymbol{r}_i \in \mathbb{R}^T$ and discharge $\boldsymbol{d}_i \in \mathbb{R}^T$ of the battery, as well as the portions of the discharge $\boldsymbol{d}_i^c \in \mathbb{R}^T$ and of the generation $\boldsymbol{g}_i^c \in \mathbb{R}^T$ that are consumed by the prosumer, which must satisfy the following constraints:

$$\begin{array}{ccccc} \mathbf{0} & \leq & \boldsymbol{r}_i & \leq & r_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & \boldsymbol{d}_i & \leq & d_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & \boldsymbol{d}_i^c & \leq & \boldsymbol{d}_i, \\ \mathbf{0} & \leq & \boldsymbol{g}_i^c & \leq & \boldsymbol{g}_i, \\ \mathbf{0} & \leq & \boldsymbol{d}_i/d_i^{\text{MAX}} + \boldsymbol{r}_i/r_i^{\text{MAX}} & \leq & \mathbf{1}, \\ \mathbf{0} & \leq & \overbrace{\boldsymbol{r}_i + \boldsymbol{c}_i - \boldsymbol{d}_i^c - \boldsymbol{g}_i^c}^{\boldsymbol{b}_i} & \leq & b_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & \underbrace{\boldsymbol{d}_i^s + \boldsymbol{g}_i^s}_{\boldsymbol{s}_i} & \leq & s_i^{\text{MAX}}\mathbf{1}, \end{array} \tag{6}$$

where $r_i^{\text{MAX}} \in \mathbb{R}_{\geq 0}$ and $d_i^{\text{MAX}} \in \mathbb{R}_{\geq 0}$ represent the maximum allowable recharge and discharge power of each BESS, respectively.

In residential loads, the total energy transferred from and to the grid is limited due to the installed infrastructure. Thus, according to eq. (5), we include the following constraints:

$$\begin{array}{ccccc} \mathbf{0} & \leq & \overbrace{\boldsymbol{r}_i + \boldsymbol{c}_i - \boldsymbol{d}_i^c - \boldsymbol{g}_i^c}^{\boldsymbol{b}_i} & \leq & b_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & \underbrace{\boldsymbol{d}_i^s + \boldsymbol{g}_i^s}_{\boldsymbol{s}_i} & \leq & s_i^{\text{MAX}}\mathbf{1}, \end{array} \tag{7}$$

where $b_i^{\text{MAX}} \in \mathbb{R}_{\geq 0}$ and $s_i^{\text{MAX}} \in \mathbb{R}_{\geq 0}$ are the upper bounds of the residential load power transfer from and to the grid, respectively.

The SoC of each BESSs must remain within the bounds stated by their manufacturing companies,

$$\varepsilon_i^{\text{MIN}}\mathbf{1} \leq \boldsymbol{\varepsilon}_i \leq \varepsilon_i^{\text{MAX}}\mathbf{1}, \tag{8}$$

where $\varepsilon_i^{\text{MIN}} \in [0, 0.5]$ and $\varepsilon_i^{\text{MAX}} \in [0.5, 1]$ represent the minimum and the maximum allowable SoC, respectively. The dynamics of the SoC in eq. (3) translates into the following set of constraints:

$$\frac{e_i^{\text{MAX}}}{\Delta}\left(D\boldsymbol{\varepsilon}_i - \boldsymbol{e}_1\varepsilon_i(t_{k-1})\right) = \eta_i\boldsymbol{r}_i - \boldsymbol{d}_i, \tag{9}$$

where $\boldsymbol{e}_1 \in \{0, 1\}^T$ and $\mathbf{D}_\varepsilon \in \mathbb{R}^{T\times T}$ are given by

$$\boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

Injecting eq. (9) into eq. (8) yields

$$\varepsilon_i^{\text{MIN}}\mathbf{1} \leq D^{-1}\left[\frac{e_i^{\text{MAX}}}{\Delta}(\eta_i\boldsymbol{r}_i - \boldsymbol{d}_i) + \boldsymbol{e}_1\varepsilon_i(t_{k-1})\right] \leq \varepsilon_i^{\text{MAX}}\mathbf{1}. \tag{10}$$

We now introduce the concept of "*shared energy*", defined as the minimum, over windows $W = \Upsilon\Delta$ of length $\Upsilon \in \mathbb{N}$, between the energy injected into the grid and the energy withdrawn from the grid by the users within that window. We will always assume that the optimization horizon $H = T\Delta$ contains a positive integer number $h \in \mathbb{N}_+$ of pricing windows W, namely

$$\lfloor T/\Upsilon \rfloor = h \geq 1.$$

To formalize this concept, we define a function that sums the samples of vector $\boldsymbol{x} = [x]_k^T$ within windows of length Υ as follows

$$g(\boldsymbol{x}, \Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top [x]_k^{\Upsilon - \text{mod}(k,\Upsilon)} \\ I_{h-1} \otimes \mathbf{1}_\Upsilon^\top [x]_{\lceil (k+1)/\Upsilon \rceil \Upsilon}^{(h-1)\Upsilon} \\ \mathbf{1}^\top [x]_{(\lceil k/\Upsilon \rceil + h - 1)\Upsilon}^{\text{mod}(k,\Upsilon)} \end{bmatrix},$$

where the first row is a scalar corresponding to the sum of the first time window in the vector $\boldsymbol{x} = [x]_k^T$, the second row is a vector corresponding to the sum of the vector $\boldsymbol{x} = [x]_k^T$ over the $h - 1$ time windows in the middle. the third row is a scalar corresponding to sum of $\boldsymbol{x} = [x]_k^T$ over the last time window.

Note that if $\mathrm{mod}(k,\Upsilon)=0$, the last element in the matrix is intended to be an empty vector, while for all other values $\mathrm{mod}(k,\Upsilon)\in[1,\Upsilon-1]$ contains at least one element. Thus, E_{sh} either contains h elements or $h+1$ elements, thus its dimension is $\lceil k,\Upsilon\rceil$.

This allows us to formally define shared energy as follows:

$$E_{sh}(\boldsymbol{b},\boldsymbol{s},\Upsilon)=\min\left\{\sum_{i\in\mathcal{V}}g(\boldsymbol{b}_i,\Upsilon),\sum_{i\in\mathcal{V}}g(\boldsymbol{s}_i,\Upsilon)\right\}\in\mathbb{R}^{\lceil k,\Upsilon\rceil}.$$

where $\boldsymbol{b}=[\boldsymbol{b}_1^\top,\dots,\boldsymbol{b}_n^\top]^\top$ and $\boldsymbol{s}=[\boldsymbol{s}_1^\top,\dots,\boldsymbol{s}_n^\top]^\top$.

B. Centralized and distributed LP Problem Formulations

The problem of interest in this work is that of minimizing the costs for the whole energy community by exploiting the incentives provided by the Italian state for the amount of energy that is shared within the community. Indeed, according to the Italian GME (Gestore dei Mercati Elettrici), prosumers will receive an economic reward that is proportional to the shared energy, thus constituting a reduced cost for the energy. We will denote by $p_e\in\mathbb{R}_{\geq 0}^{\lceil k,\Upsilon\rceil}$ the price of the energy and by $p_{sh}\in\mathbb{R}_{\geq 0}^{\lceil k,\Upsilon\rceil}$ the economic reward given by the state for the shared energy, where each element corresponding to a different pricing window. Let us stack all decision variables into

$$\boldsymbol{v}_i=\left[\boldsymbol{r}_i^\top,\boldsymbol{d}_i^\top,\boldsymbol{d}_i^{c\top},\boldsymbol{g}_i^{c\top}\right]^\top,\quad \boldsymbol{v}=\left[\boldsymbol{v}_1^\top,\cdots,\boldsymbol{v}_n^\top\right]^\top.$$

Thus, the objective function we aim to minimize is

$$f(\boldsymbol{v})=p_e^\top\sum_{i=1}^n g(\boldsymbol{b}_i,\Upsilon)-p_{sh}^\top E_{sh}(\boldsymbol{b},\boldsymbol{s},\Upsilon),\qquad(11)$$

Thus, the centralized optimization problem of interest in this manuscript reads as:

$$\begin{aligned}\min_{\boldsymbol{v}}\quad & f(\boldsymbol{v}),\\ \text{s.t.}\quad & \text{Loc. const. (6),(7),(10)}\quad\forall i\in\mathcal{V}.\end{aligned}\qquad(12)$$

Although Problem (12) has linear inequality constraints, it has a nonlinear objective function due to the shared energy term, which is the (component-wise) minimum between the energy bought and sold in a given time window. We seek a linear equivalent formulation of the objective function by noticing that the minimum of two values $x,y\in\mathbb{R}$ is always lesser or equal than both values, i.e.,

$$z=\min\{x,y\}\quad\Rightarrow\quad\begin{cases}z\leq x,\\ z\leq y.\end{cases}\qquad(13)$$

Therefore, one can substitute the nonlinear term "$\min\{x,y\}$" by introducing a new variable z and by adding the constraints as in (13). In so doing, we denote by $\boldsymbol{\theta}\in\mathbb{R}^{\lceil k,\Upsilon\rceil}$ the new set of variables replacing the nonlinear term of the shared energy $E_{sh}(\boldsymbol{b},\boldsymbol{s},\Upsilon)$ yielding the new objective function

$$f(\boldsymbol{v},\boldsymbol{\theta})=p_e^\top\sum_{i\in\mathcal{V}}g(\boldsymbol{b}_i,\Upsilon)-p_{sh}^\top\boldsymbol{\theta},\qquad(14)$$

and construct an equivalent linear optimization problem:

$$\begin{aligned}\min_{\boldsymbol{v},\boldsymbol{\theta}}\quad & f(\boldsymbol{v},\boldsymbol{\theta}),\\ \text{s.t.}\quad & \text{Loc. const. (6),(7),(10)}\quad\forall i\in\mathcal{V},\\ & \boldsymbol{\theta}-\textstyle\sum_{i\in\mathcal{V}}g(\boldsymbol{b}_i,\Upsilon)\leq\mathbf{0},\\ & \boldsymbol{\theta}-\textstyle\sum_{i\in\mathcal{V}}g(\boldsymbol{s}_i,\Upsilon)\leq\mathbf{0}.\end{aligned}\qquad(15)$$

Given the inherently distributed nature of the scenario at hand, it is pertinent to investigate whether the linear problem proposed in equation (12) can be reformulated as a distributed optimization problem. It is unrealistic to assume the presence of a central entity with full knowledge of the expected consumption and generation of all prosumers in the community, along with the authority to manipulate their battery behavior at will. A more plausible assumption is that each prosumer is aware of its own expected consumption/generation profile and is equipped with a local automatic optimizer to manage its battery's on/off profile, with the objective of cost savings.

We note that the objective function in eq. (14) consists of two terms. The first one, associated with the energy withdrawn from the network, is already in a distributed form because it is expressed as the sum over the prosumers. Instead, the second term, associated with the shared energy within the community, must be replaced by introducing local variables $\boldsymbol{\vartheta}_i$ representing a fraction of the original global variable $\boldsymbol{\theta}$, i.e., $\boldsymbol{\theta}=\sum_{i=1}^n\boldsymbol{\vartheta}_i$. The objective function can be written in a fully decoupled way as follows

$$f(\boldsymbol{v},\boldsymbol{\theta})=\sum_{i\in\mathcal{V}}f_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i),\quad\text{where}\qquad(16a)$$

$$f_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i)=p_e^\top g(\boldsymbol{b}_i,\Upsilon)-p_{sh}^\top\boldsymbol{\vartheta}_i.\qquad(16b)$$

The problem now involves a cost function that is the sum of separable strictly convex functions subject to local inequality constraints in eq. (18a) and coupled equality constraints in eq. (18b)-(18c), and can be effectively tackled using various protocols established in contemporary literature. In this manuscript, we employ the Distributed Consensus Alternating Direction Method of Multipliers (DC-ADMM) algorithm developed in [24], which emerges as a prominent choice due to its robustness and efficiency in addressing similar optimization challenges. However, since the objective in eq. (16) is simply linear in the decision variables and, in turn, the optimization problem is simply convex (and not strongly convex), it suffers from the well-known duality-gap problem, which refers to a situation in which the optimal values of the primal problem and its corresponding dual problem do not match. For a more detailed discussion of this issue when dealing with LP problems we refer the reader to [26], [27]. Thus motivated, we approximate the objective function by adding two regularization terms to make the problem strongly convex. By noticing that variables $\boldsymbol{v}_i$ satisfy box constraints of the kind $\boldsymbol{v}_i^{\text{MIN}}\leq\boldsymbol{v}_i\leq\boldsymbol{v}_i^{\text{MAX}}$, denoting $\bar{\boldsymbol{v}}_i=\frac{1}{2}(\boldsymbol{v}_i^{\text{MIN}}+\boldsymbol{v}_i^{\text{MAX}})$ we consider the regularized local objective functions

$$\widetilde{f}_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i)=f_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i)+\sigma\|\boldsymbol{v}_i-\bar{\boldsymbol{v}}_i\|_2^2+\varsigma\|\boldsymbol{\vartheta}_i\|_2^2,\quad\sigma,\varsigma\in\mathbb{R}_{\geq 0},$$

Algorithm 1 DC-ADMM applied to the distributed optimization problem in (18) to minimize costs of an energy community

Require: Arbitrary initial values $\boldsymbol{v}_i(0), \boldsymbol{\vartheta}_i(0), \boldsymbol{\alpha}_i(0), \boldsymbol{\beta}_i(0), \boldsymbol{p}_i(0)$ for $i \in \mathcal{V}$ and the parameter $\rho > 0$

1: **for** k=1,2,3,... (until a stopping criterion is satisfied) **do**

2: **for each prosumer** $i \in \mathcal{V}$ **(in parallel) do**

$$\begin{bmatrix}\boldsymbol{v}_i(k)\\ \boldsymbol{\vartheta}_i(k)\\ \boldsymbol{\alpha}_i(k)\\ \boldsymbol{\beta}_i(k)\end{bmatrix} = \underset{\boldsymbol{v}_i,\boldsymbol{\vartheta}_i,\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i}{\operatorname{argmin}} \left\{ \widetilde{f}_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i) + \frac{\rho}{4|\mathcal{N}_i|} \left\| \begin{bmatrix}(\boldsymbol{\vartheta}_i - g(\boldsymbol{b}_i,\Upsilon)+\boldsymbol{\alpha}_i)/\rho\\ (\boldsymbol{\vartheta}_i - g(\boldsymbol{s}_i,\Upsilon)+\boldsymbol{\beta}_i)/\rho\end{bmatrix} - \frac{1}{\rho}\boldsymbol{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} (\boldsymbol{y}_i(k-1)+\boldsymbol{y}_j(k-1)) \right\|_2^2 \right\}$$

$$\text{s.t.} \qquad \text{Loc. const. (6),(7),(10),(17)} \quad \forall i \in \mathcal{V},$$

$$\boldsymbol{y}_i(k) = \frac{1}{2|\mathcal{N}_i|} \left(\begin{bmatrix}(\boldsymbol{\vartheta}_i(k) - g(\boldsymbol{b}_i(k),\Upsilon)+\boldsymbol{\alpha}_i(k))/\rho\\ (\boldsymbol{\vartheta}_i(k) - g(\boldsymbol{s}_i(k),\Upsilon)+\boldsymbol{\beta}_i(k))/\rho\end{bmatrix} - \frac{1}{\rho}\boldsymbol{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} \big(\boldsymbol{y}_i(k-1)+\boldsymbol{y}_j(k-1)\big) \right)$$

$$\boldsymbol{p}_i(k) = \boldsymbol{p}_i(k-1) + \rho \sum_{j\in\mathcal{N}_i} (y_i(k) - y_j(k))$$

3: **end for**

4: **end for**

where σ and ς are two positive real numbers weighting the effects of the regularization terms. As a last step, we introduce local variables

$$\boldsymbol{\alpha}_i \geq 0, \quad \boldsymbol{\beta}_i \geq 0, \qquad \forall i \in \mathcal{V}, \tag{17}$$

in order to transform the inequality constraints into equality constraints. Thus, the distributed optimization problem becomes:

$$\begin{aligned} \min_{\{\boldsymbol{v}_i,\boldsymbol{\vartheta}_i,\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i\}_{i\in\mathcal{V}}} \quad & \sum_{i\in\mathcal{V}} \widetilde{f}_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i), \\ \text{s.t.} \quad & \text{Loc. const. (6),(7),(10),(17)} \quad \forall i \in \mathcal{V}, && \text{(18a)} \\ & \textstyle\sum_{i\in\mathcal{V}} (\boldsymbol{\vartheta}_i - g(\boldsymbol{b}_i,\Upsilon) + \boldsymbol{\alpha}_i) = \mathbf{0}, && \text{(18b)} \\ & \textstyle\sum_{i\in\mathcal{V}} (\boldsymbol{\vartheta}_i - g(\boldsymbol{s}_i,\Upsilon) + \boldsymbol{\beta}_i) = \mathbf{0}. && \text{(18c)} \end{aligned}$$

For the sake of completeness, we detail the implementation in Algorithm 1 for the problem in (18) and conclude the section with some important remarks:

(i) each node $i \in \mathcal{V}$, besides $\boldsymbol{v}_i, \boldsymbol{\vartheta}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i$ stores in memory the auxiliary variables $\boldsymbol{y}_i, \boldsymbol{p}_i$;

(ii) each node $i \in \mathcal{V}$ transmits only $\boldsymbol{y}_i(k)$ it to all its neighbors $j \in \mathcal{N}_i$, while all other variables are not exchanged among agents;

(iii) Algorithm 1 involves two update steps between which there is a transmission step: first the agents update variables $\boldsymbol{v}_i, \boldsymbol{\vartheta}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \boldsymbol{y}_i$, then transmit to their neighbors the variable $\boldsymbol{y}_i$, finally update the variable $\boldsymbol{p}_i$.

(iv) the real number $\rho \geq 0$ is a free design parameter tuning the performance (i.e., convergence rate) of the algorithm. Nevertheless, it is known that ADMM-based algorithms (such as Algorithm 1) converges to the optimal solution of (18) for any choice of $\rho \geq 0$ (cfr. [28], [22]).

(v) for $\sigma \to 0$ and $\varsigma \to 0$, the optimal solution of the distributed regularized problem in eq. (18) converges to an optimal solution of the distributed problem in eq. (15).

IV. Numerical simulations

In this section, we consider a scenario of a small energy community consisting of $n = 4$ prosumers, where all are equipped with an energy storage system but only prosumer 1 has energy production capabilities; the expected energy production and consumption profiles are taken from IEEE-PES (Power and Energy Society[1]). The results of the simulations are shown in Figs. 2-3, which displays (on the left) the results obtained by solving the distributed problem in eq. (18) via the DC-ADMM Algorithm (as detailed in Algorithm 1); (in the middle) the results obtained by solving the centralized optimization problem in eq. (12); (on the right) the results obtained by assuming no cooperation among the agents, which only try to minimize their costs by minimizing their local cost (i.e., the objective function in (16b) without the variable $\boldsymbol{\vartheta}_i$). In what follows we discuss our main findings.

As already described, prosumer 1 is the only agent that has generation. Notice from Fig. 2 that agent 1 starts to sell energy when its generation is higher than its consumption. From Fig. 2 it is evident that, when cooperating, the agents charge their batteries only when there is some energy generation (in the community), maximizing the shared energy as shown in Fig.-3. In particular, considering the centralized solution, the energy generated by the first agent ends up being fully used to charge the batteries of the agents, in a way that allows to fulfil the future expected consumptions of the whole community. This is not exactly true for the solution provided by the distributed algorithm, whose solution forces the prosumers to buy energy from the grid also after the 12-th hour. thus fully contributing to the shared energy. Finally, the solution resulting by applying a selfish behavior does not entails recharging of the batteries, i.e., it does not take advantage of the incentives for the shared energy. In the non cooperative method, from Fig.(2), agent one charges its battery with an amount of energy sufficient for

[1]https://ieee-pes.org/

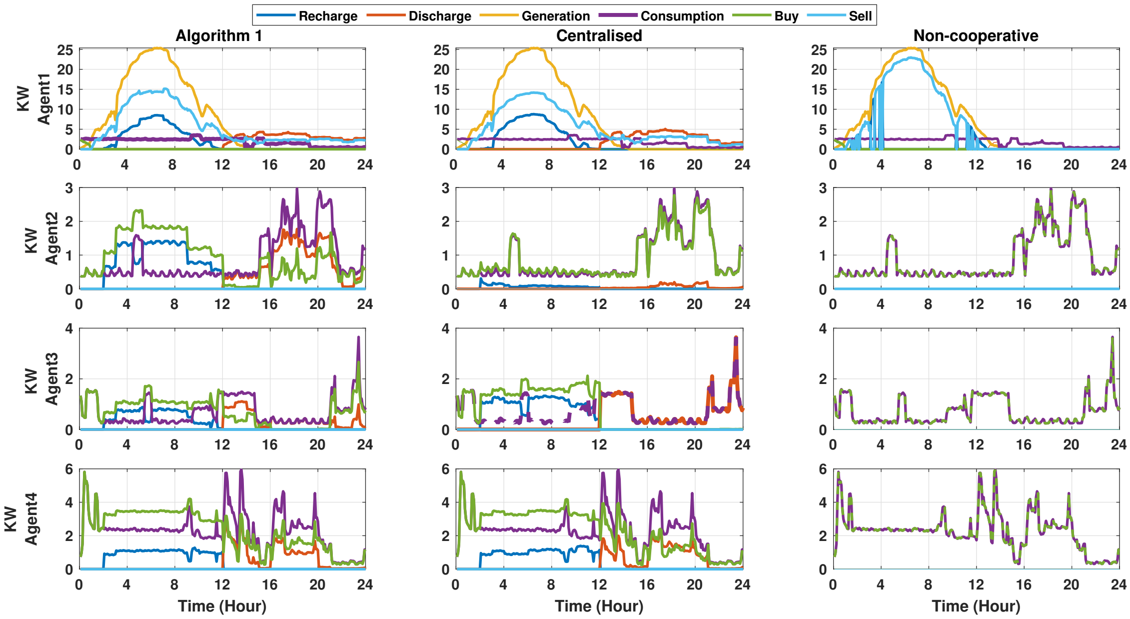

Fig. 2: Comparison between the temporal evolution of recharge $\boldsymbol{r}_i(t)$, discharge $\boldsymbol{d}_i(t)$, generation $\boldsymbol{g}_i(t)$, consumption $\boldsymbol{c}_i(t)$ and the corresponding energy bought from the grid $\boldsymbol{b}_i(t)$ and sold to the grid $\boldsymbol{s}_i(t)$ obtained by solving the centralized problem in eq. (12) (middle), the distributed regularized problem in (15) (left), and without cooperative (right).

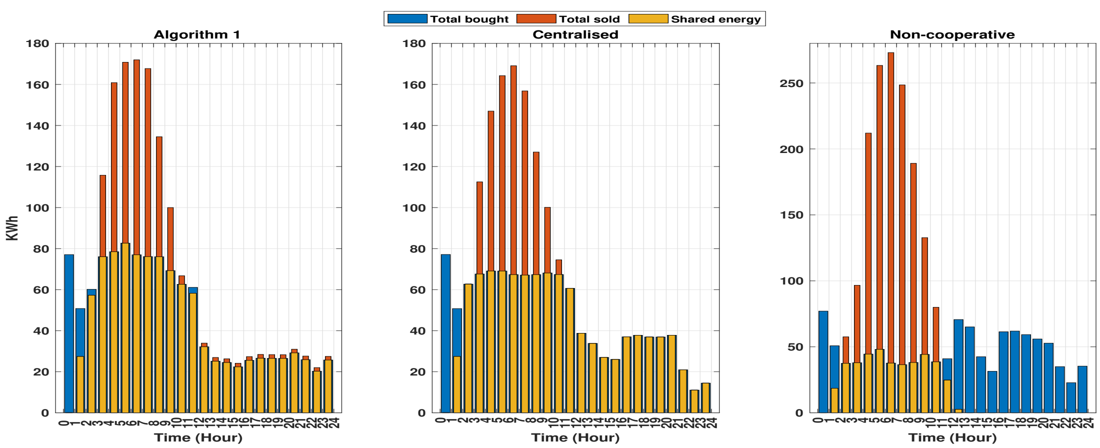

Fig. 3: The total energy bought and sold by agents in each hour and the corresponding shared energy

it anticipated future consumption, while the other agent does not use their batteries to store energy for the future consumption, and instead buy the needed energy directly from the grid. This indicates the lack of any collaborative energy sharing arraignment between prosumers. We now compare the expected costs and the shared energy when considering the solution obtained by solving the centralized optimization problem in eq. (12) and the solution obtained in a distributed way by the agents implementing Algorithm 1. In the first case, the expected cost and the shared energy for the energy community is about € 146.7 and 1052.23 KWh, respectively, while in the second case is about € 147.7 and 1051.73 KWh, which are very close as expected. Finally, we also simulate the case in which the agents optimize their own costs without cooperating with other agents (i.e., eq. (16b) without considering the variable $\boldsymbol{\vartheta}_i$ associated with the concept of shared energy), which yields a total cost for the energy community equal to € 172.81, which amounts to a 17% higher costs, and shared energy equal to 409.15 KWh In energy community systems, employing distributed algorithms offers significant advantages over centralised approaches. Notably, distributed algorithm enhance privacy by allowing data to remain localized, minimizing the risk of sensitive information exposure. Moreover, they mitigate the complexity associated with solving optimization problems by distributing computational load across nodes, thus facilitating efficient and scalable solutions.

V. Conclusion

This study presents the outcomes of deploying an advanced energy management system within a cluster of prosumers, aimed at reducing the operational energy expenses. Conducted through a distributed optimization using DC-ADMM strategy to optimize the cost savings of energy community. Based on the weather forecasts and forthcoming electricity market prices, the distributed cost optimization assesses the most efficient electricity flow across every system components at each time interval. This includes optimizing electricity exchange within the smart grid among the community prosumers and with the centralised grid. The approach can yield financial benefits of 14.53% using the shared energy concept compared to the concept that neglects the energy sharing. we will explore the potential of leveraging consensus-based algorithms for distributed optimization and online learning [29], [30], [31], [32], [22], [33] could be exploited to infer global information useful for the energy community to maximize the shared energy through the coordination of batteries and loads [23].

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