A Heuristic Algorithm to Optimize Execution Time of Multi-Robot Path

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Nanyang Technological University & University of Cagliari

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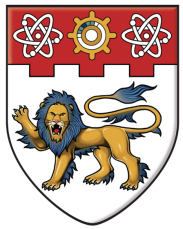

NANYANG TECHNOLOGICAL UNIVERSITY

Outline

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State identification problem for discrete event systems

Problem

Given a trajectory solution for a motion planning problem, how can we optimize it in terms of execution time?

A way to do it for systems modeled by timed automata is to rearrange events in the trajectory in order to maximize parrallelism.

- Preserving priorities:
  the aim is to establish the earliest firing time for each event in the trajectory.

- Changing priorities:
  the aim is to identify sub-trajectories requiring common resources and change their priorities over such a resources.

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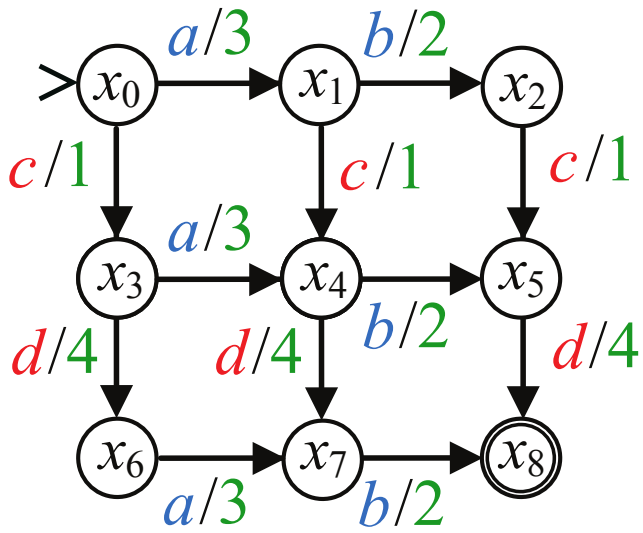
Example

Consider this timed automaton $\mathcal{G}$ with events of machine 1 and machine 2 and their time transitions. Where a, c require the same resource.



Given $t = abcd \in L_m(\mathcal{G})$

Preserving priorities

output trajectory $t_1 = acbd \in L_m(\mathcal{G})$

Changing priorities

output trajectory $t_2 = cdab \in L_m(\mathcal{G})$

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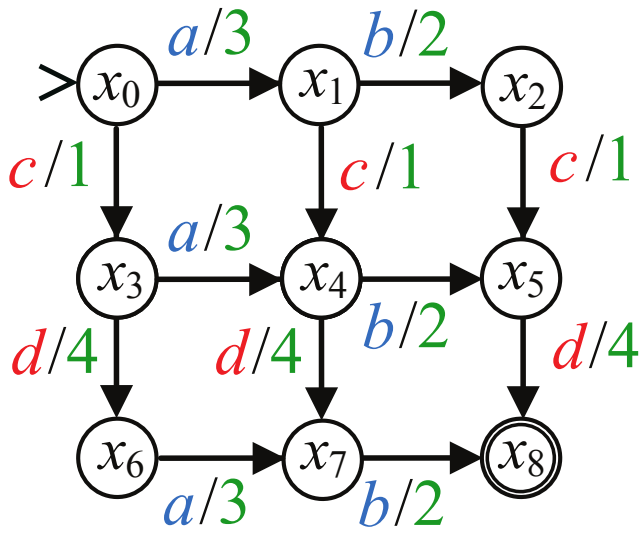

Given $t = abcd \in L_m(\mathcal{G})$

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output trajectory $t_1 = a\boldsymbol{c}bd \in L_m(\mathcal{G})$

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output trajectory $t_2 = \boldsymbol{c}\boldsymbol{d}ab \in L_m(\mathcal{G})$

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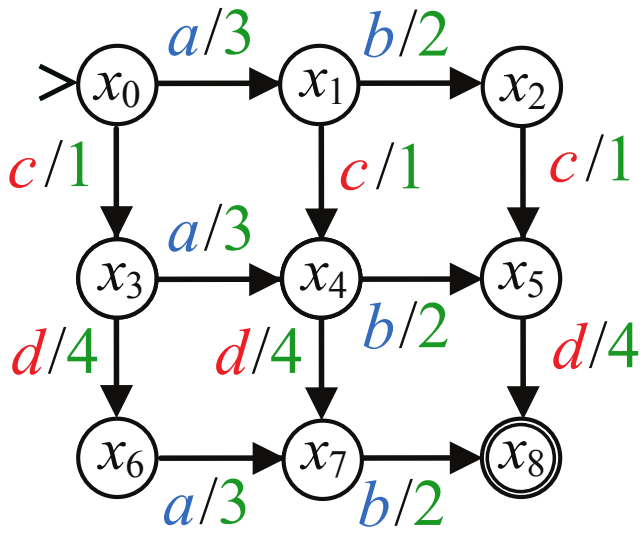

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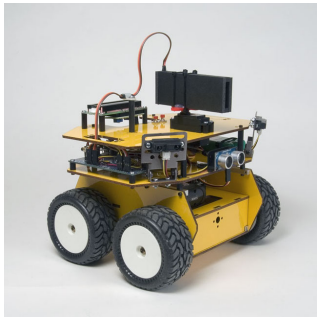
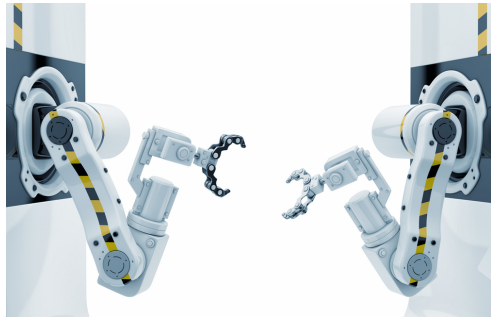
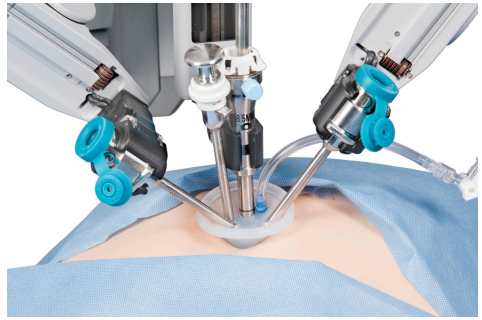
Applications



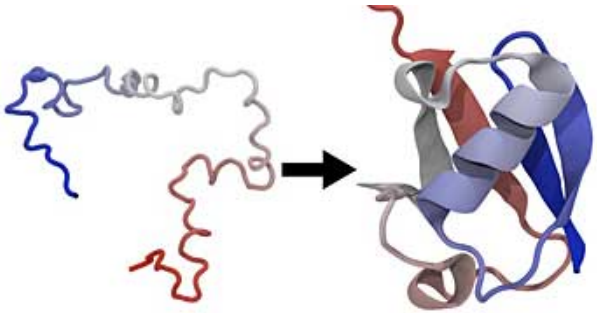
(navigation)



(manufacturing)



(surgery)



(protein folding)

(videogames)

(graphic animation)

Field of interest: Multi-Robot Motion-Planning

Question: How can we find a trajectory which directs each robot from its current position to its destination such that the total execution time is small (but not necessarily optimal)?

Main problems:

- State space grows exponentially with # of robots being coordinate and dimensions of the environment (state space explosion);
- Algorithms to compute optimal trajectory: PSPACE-Hard;
- Algorithms to compute sub-optimal trajectory: far from the optimum.

Answer: Implementing an algorithm which efficiently improves a given trajectory by merging preserving and changing priorities techniques.

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- Description of the Multi-Robot Motion-Planning problem in a DES framework
- Execution time of the solution
- A preserving priorities approach to optimize a trajectory.
- The novel changing priorities approach subject of this work to improve a trajectory.
- Some experimental results to evaluate effectiveness the presented approach.

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Environment

The environment where a robot operates can be partitioned into a grid, where:

- **White squares** denote accessible positions;
- **Black squares** denote inaccessible positions;
- **Thin sides** denote an available transition;
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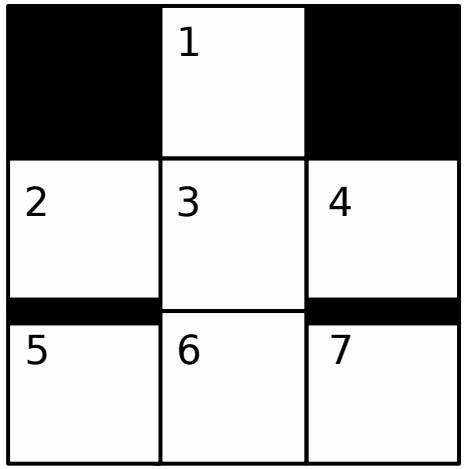

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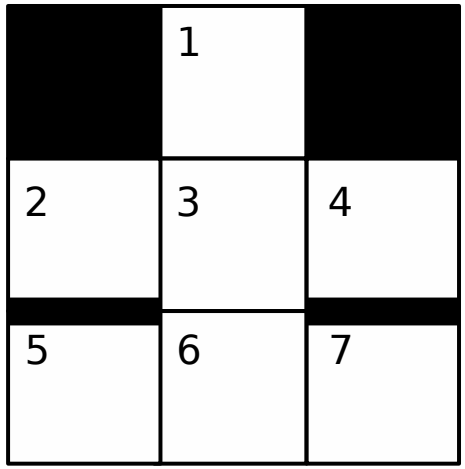

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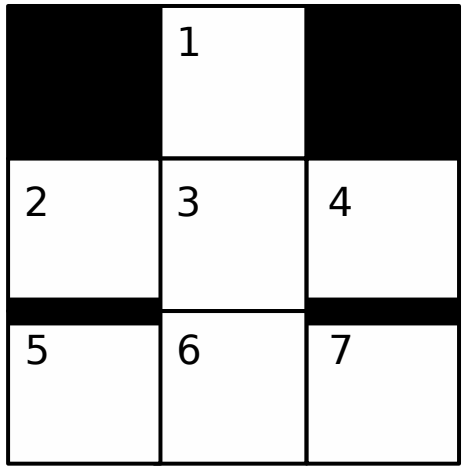

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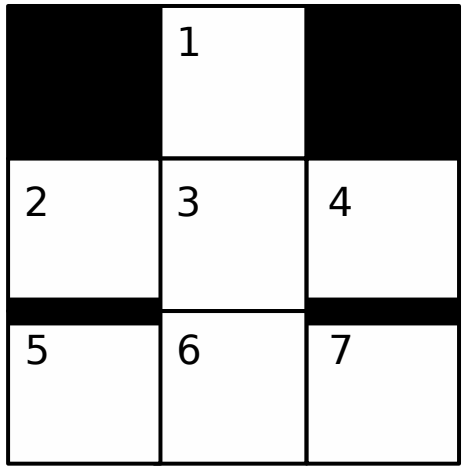

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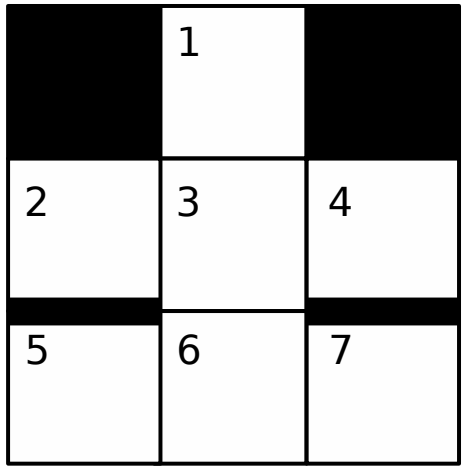

Automaton representation

For each robot we define an automaton $G = (X, \Sigma, \delta, x_0, x_m)$ with

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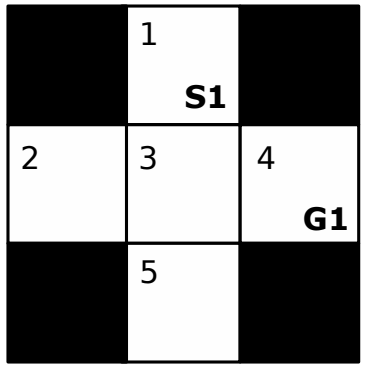

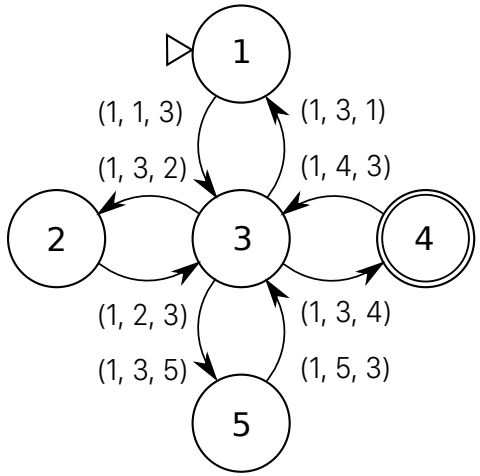

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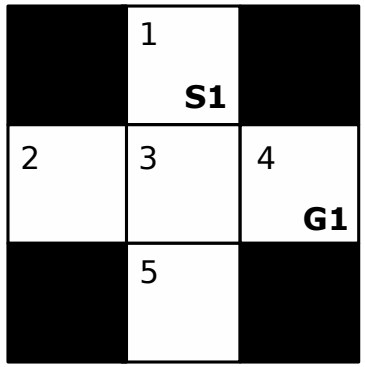

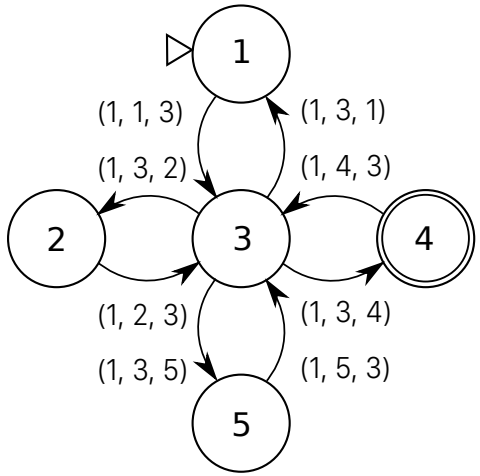

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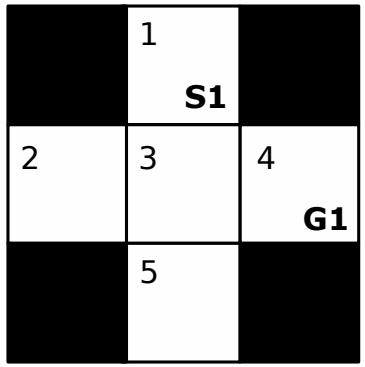

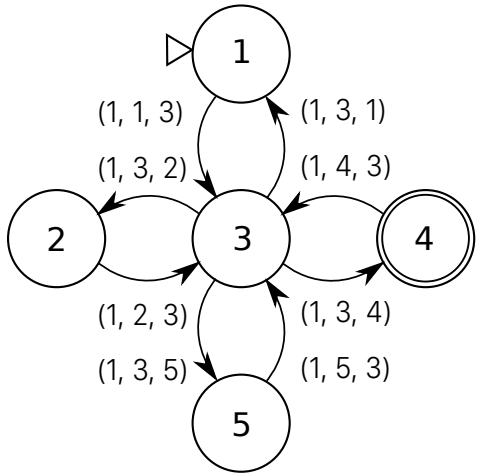

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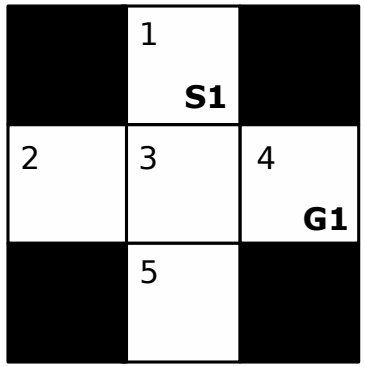

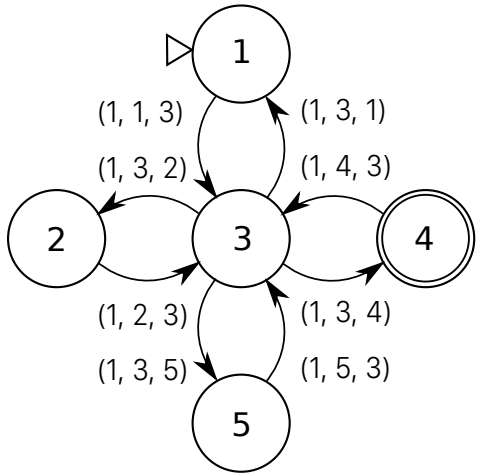

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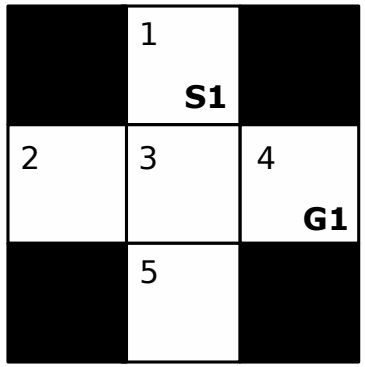

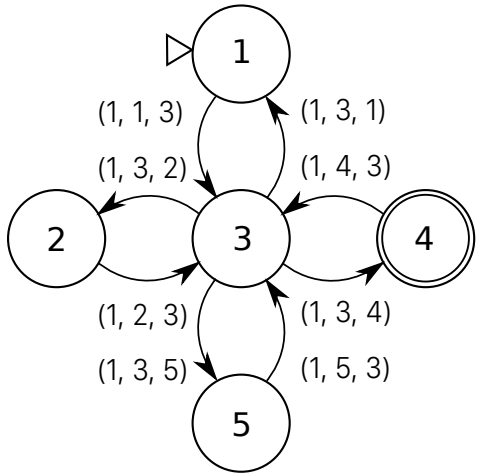

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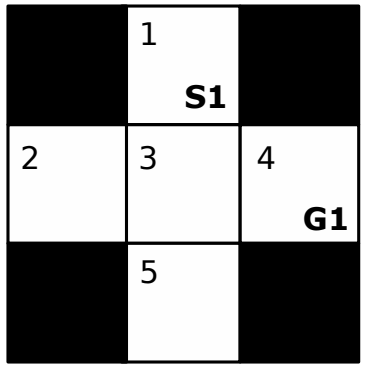
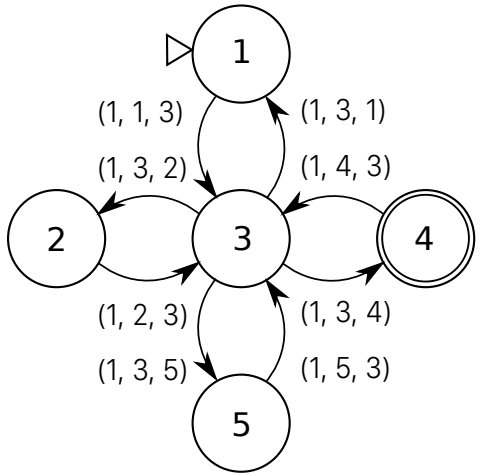
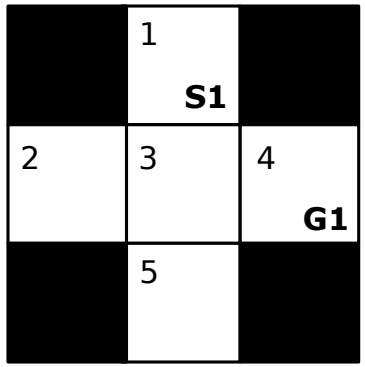


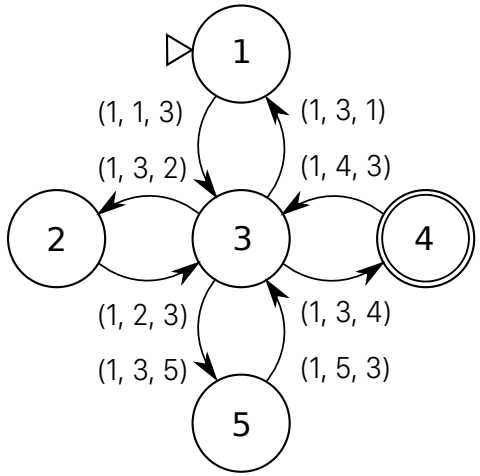


Automaton representation

Example of $G = (X, \Sigma, \delta, x_0, x_m)$ automaton of the robot 1

- $X = \{1, 2, 3, 4, 5\}$;
- $\Sigma = \{(1,3,2), (1,3,1), (1,4,3), (1,2,3), (1,3,5), (1,3,4), (1,5,3)\}$;
- $\delta : X \times \Sigma \to X$;
- $x_0 = 1$;
- $x_m = 4$.

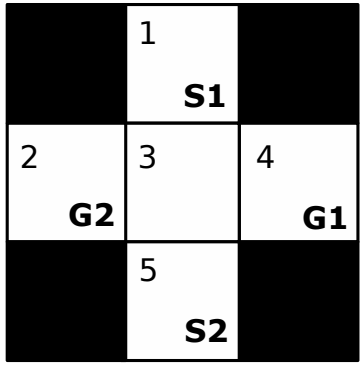




Solution

Defined $G = G_1 \times \cdots G_n$ the product of the n robot automata in the environment, a string $s \in L_m(G)$ is a valid solution to the problem – we call it trajectory – if it directs each robot from it current position to its destination without collisions.

Example: $t = (1,1,3)(1,3,4)(2,5,3)(2,3,2)$ is a valid trajectory while $t' = (1,1,3)(2,5,3)(1,3,4)(2,3,2)$ is not because robots 1 and 2 collide in position 3.



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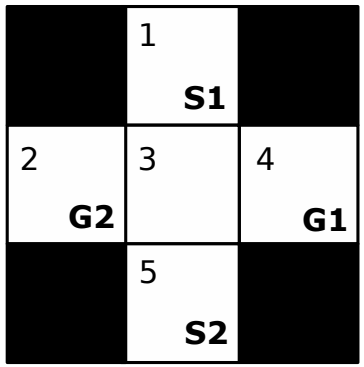

Outline

Execution time

We now introduce the concept of the time in the system. When a string s is executed the events must be fired sequentially based on the order in the string. Therefore, each event $\sigma_k \in s$ has:

1. A starting time t_k;
2. An execution time $f(\sigma_k)$;
3. A completion time $t_k + f(\sigma_k)$.

A time stamp $\rho = [t_k \in \mathbb{R}_0^+ | k \in \{1, \cdots, |s|\}]$ of the string s with respect to G which establishes the starting time of each event is a nondecreasing list of nonnegative real numbers.
A time stamp is legal if will fire each event avoiding simultaneous execution of mutually exclusive events.
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- All event time executions are equal to 1;
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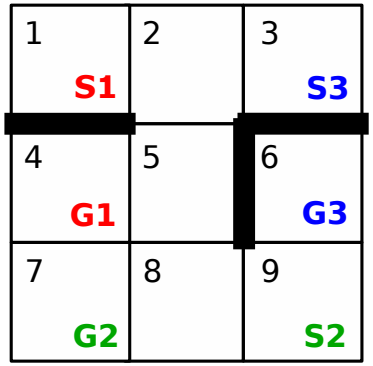

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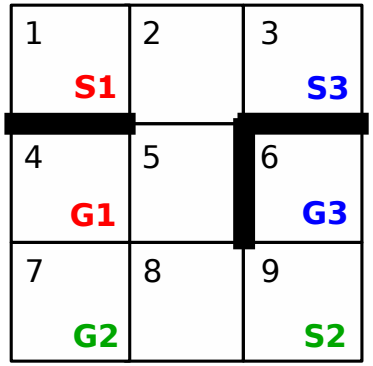

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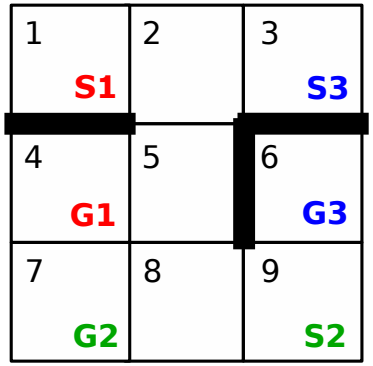

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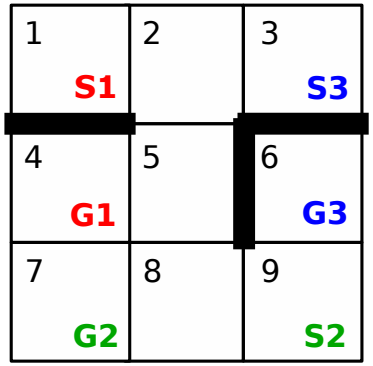

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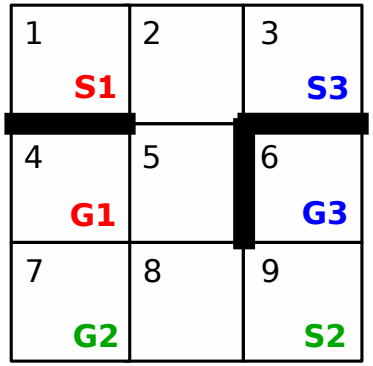

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Mutual exclusiveness and resources

Definition 2

Two events σ_1, σ_2, with t_1, t_2 as their starting times and $f(\sigma_1), f(\sigma_2)$ as their execution times are mutually exclusive if it is not allowed that $t_1 \in [t_2, t_2 + f(\sigma_2)]$ or $t_2 \in [t_1, t_1 + f(\sigma_2)]$, i.e. if one event is under execution the other cannot be fired.

Definition 3

A resource $r = \{\sigma | \sigma \in \Sigma\}$ is a set of mutually exclusive events within the same alphabet Σ. □

Definition 4

A set of resources $R = \{r | r \subseteq \Sigma\}$ is a finite set of resources where for every pair of events $\sigma, \sigma' \in \Sigma$ there exists a resource $r \in R$ such that $\sigma, \sigma' \in r$ if and only if σ and σ' are mutually exclusive. □

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Problem as a timed automaton

We define a novel kind of timed automaton by coupling a positive real number, the execution time, with each event in the alphabet of the composite automaton G and including the concept of mutual exclusive events in the definition.
We construct the timed automaton $\mathcal{G} = (G, f, R)$ with

- $G = G_1 \times \cdots G_n$ is the product of the n robot automata in the environment;
- $f : \Sigma = \Sigma_1 \cup \cdots \Sigma_n \to \mathbb{R}$ is a weight function which assigns a weight to all events equal to the execution time of the event;
- R is a set of resources such that it contains:
  1. **Robot-resources**: one for each robot such that all events of the robot is in that resource.
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Rearranging events - Preserving Priorities

- Preserving priorities in rearranging events within a trajectory means that if a resource is used first by event e_1 and then from e_2 it is not allowed to move e_2 before e_1. Let's see an example:
- All event time executions are equal to 1;
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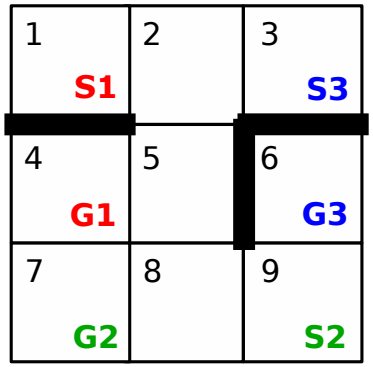

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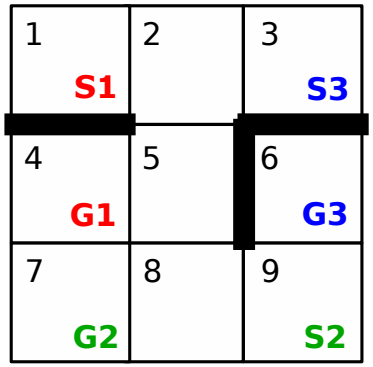

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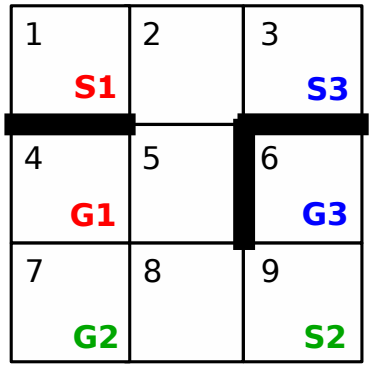

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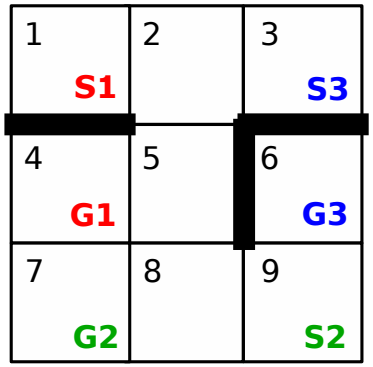

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Rearranging events - Preserving Priorities

- Preserving priorities in rearranging events within a trajectory means that if a resource is used first by event e_1 and then from e_2 it is not allowed to move e_2 before e_1. Let's see an example:
- All event time executions are equal to 1;
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- The output trajectory is $t_p \in L_m(G)$ such that
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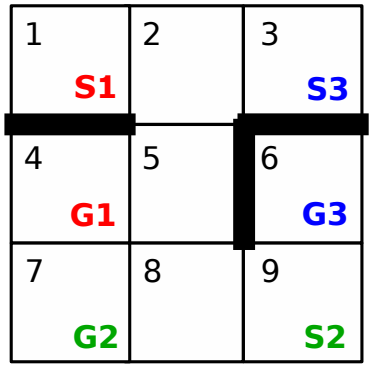

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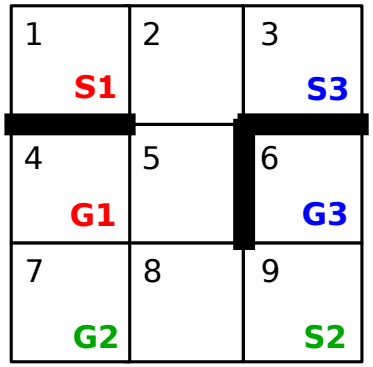

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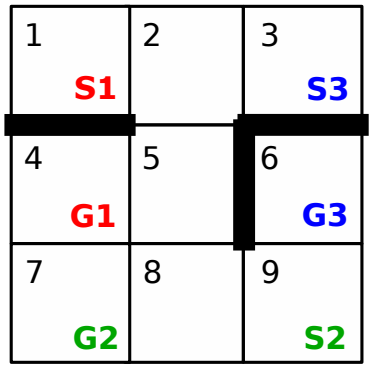

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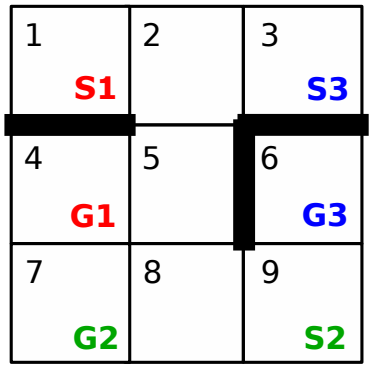

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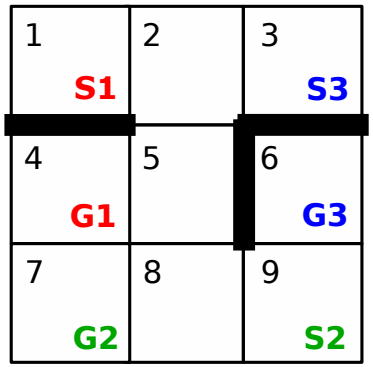

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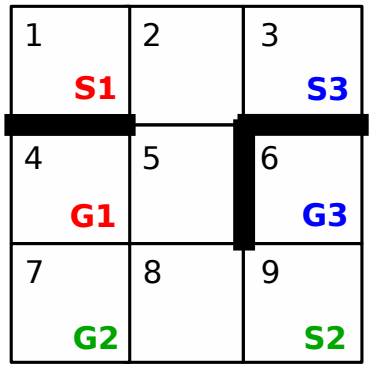

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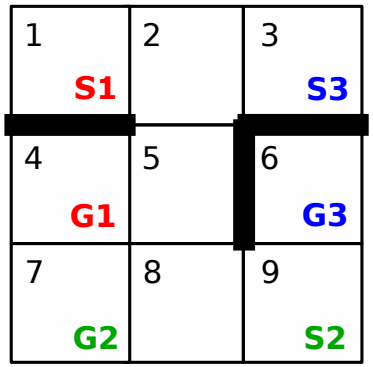

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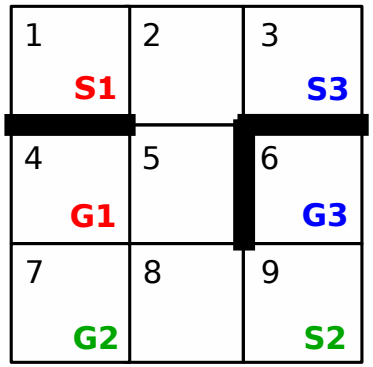

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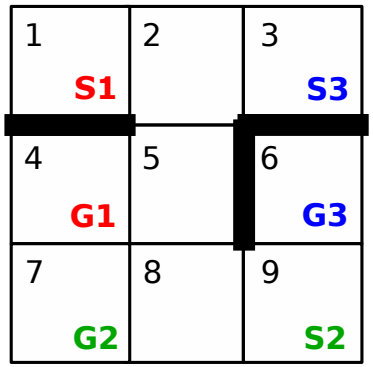

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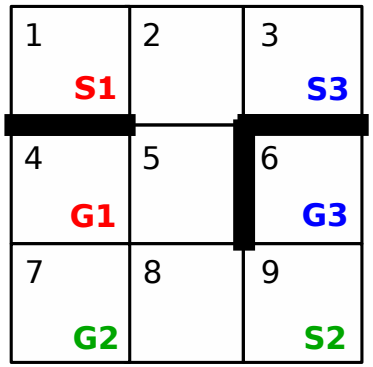

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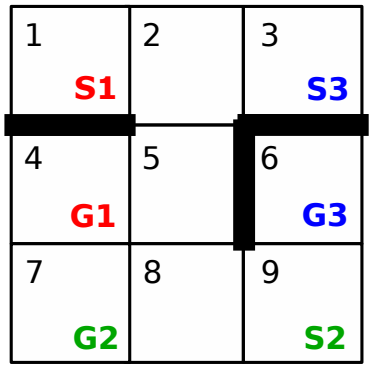

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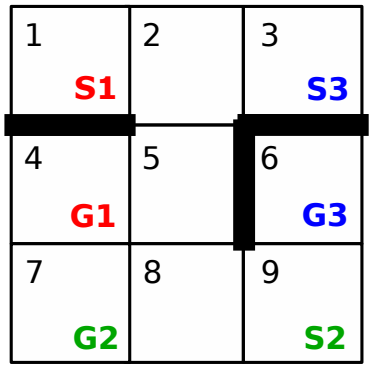

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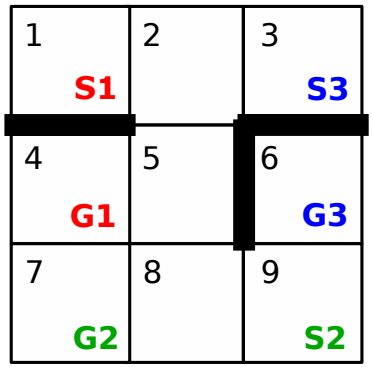

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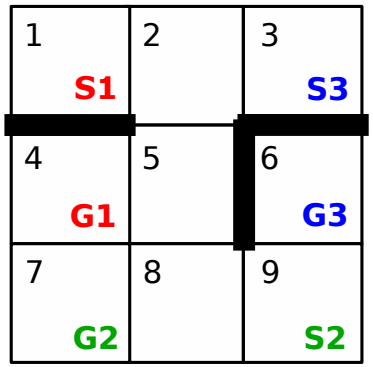

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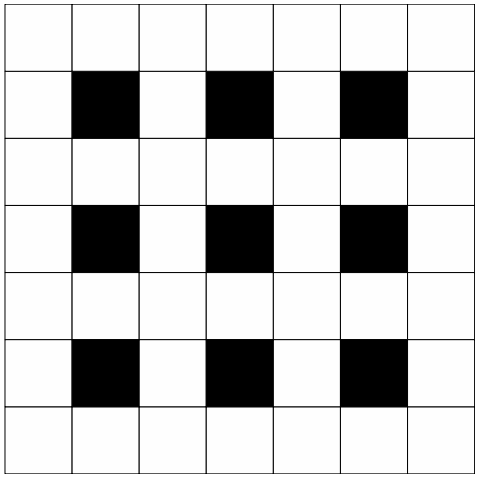
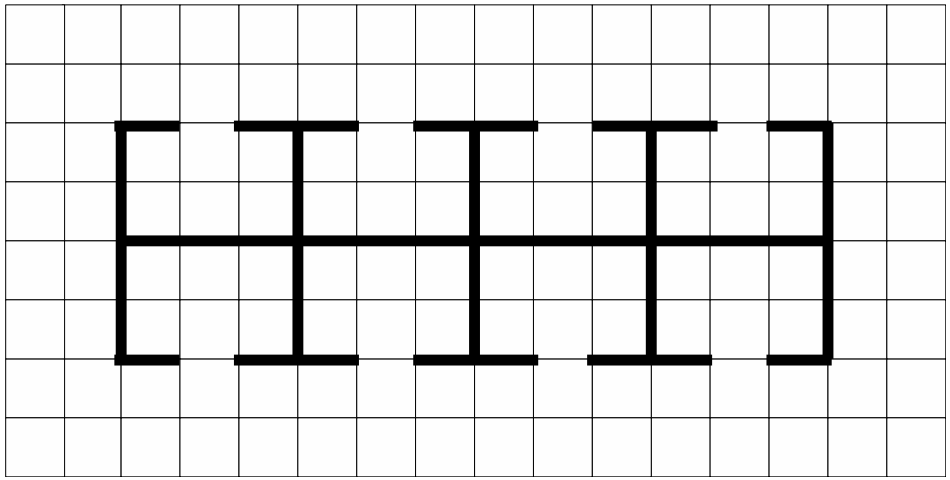
Outline

Evnvironments

Grid



Cyclic Corridor



Test description

- Random start and goal position from amongst all the possible free location in the environment, such that:
  1. Two robot can not share the same start position;
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  3. A robot can have another robot's start position as its goal position;
  4. A robot can have the same start and goal position.
- The grid was also expanded to dimensions 11 and 15.
- For each environment we run 100 different experiments and made the average of all results.
- For each experiment we calculated a lower bound (LB) for the minimum execution time of the optimal solution in the following manner:
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  1. Calculate the shortest path for each individual robot assuming that there are no other robots to get in its way;
  2. Find the robot with the longest journey and select it's duration as our LB.

Test description

- Random start and goal position from amongst all the possible free location in the environment, such that:
  1. Two robot can not share the same start position;
  2. Two robot can not share the same goal position;
  3. A robot can have another robot's start position as its goal position;
  4. A robot can have the same start and goal position.
- The grid was also expanded to dimensions 11 and 15.
- For each environment we run 100 different experiments and made the average of all results.
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For each experiment four types of test were carried out, for purposes of presentation we will refer to these cases by an acronym:

1. First we found a solution by applying an optimization algorithm based on priorities preservation previously developed from our group: we refer to this case by acronym "c" which stands for "Compress".
2. Second we applied the novel algorithm proposed in this work to the result of tests c; we refer to this case by acronym "se" which stands for "Swap at the End".
3. Third we found a new solution by applying the novel algorithm while computing the trajectory: we refer to this case by acronym "sw" which stands for "Swap While".
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Grid results

	Unit	Grid 7x7				Grid 11x11				Grid 15x15			
ROBOTS		4	8	12	16	9	19	28	38	17	35	52	70
CONGESTION	%	10	20	30	40	10	20	30	40	10	20	30	40
$S_c \equiv S_{se}$	%	100	100	100	91	100	100	100	73	100	100	98	38
S_{sw}	%	100	100	100	91	100	100	100	74	100	100	98	40
S_{bs}	%	100	100	100	94	100	100	100	85	100	100	98	51
T_c	s	0.05	0.26	0.71	1.26	0.5	3.46	10.28	17.77	2.88	23.51	69.65	103.81
T_{se}	s	0.06	0.29	0.75	1.29	0.61	3.76	10.71	17.91	3.6	25.49	71.74	104.25
T_{sw}	s	0.07	0.36	0.94	1.61	0.9	5.8	15.54	26.03	7.62	51.23	125.23	179.5
T_{bs}	s	0.13	0.65	1.69	2.89	1.5	9.56	26.25	43.94	11.22	76.72	196.97	283.75
μ_c		7.89	9.57	12.82	20.8	13.55	16.94	23.84	42.49	20.06	25.04	35.66	63.03
μ_{se}		7.89	9.44	12.12	19.32	13.52	16.51	22.27	37.9	19.86	24.32	33.43	55.92
μ_{sw}		7.89	9.44	12.07	18.98	13.52	16.41	21.89	36.34	19.82	23.91	31.91	51.85
μ_{bs}		7.89	9.44	12.04	18.94	13.52	16.4	21.66	35.99	19.82	23.79	31.41	52.55
μ_c/LBE		1.04	1.13	1.44	2.24	1.03	1.13	1.53	2.67	1.02	1.19	1.62	2.77
μ_{se}/LBE		1.04	1.11	1.36	2.08	1.03	1.1	1.43	2.39	1.01	1.16	1.52	2.45
μ_{sw}/LBE		1.04	1.11	1.36	2.07	1.03	1.1	1.41	2.28	1.01	1.14	1.45	2.32
μ_{bs}/LBE		1.04	1.11	1.35	2.05	1.03	1.1	1.39	2.26	1.01	1.13	1.43	2.33
I_{se}	%	0	12.98	17.83	12.56	8.95	22.13	19.08	17.25	45.01	18.15	16.42	17.71
I_{sw}	%	0	12.98	19.01	13.92	8.95	27.71	23.86	23.75	54.46	29.11	27.63	25.09
I_{bs}	%	0	12.98	19.91	15.23	8.95	28.18	26.72	24.9	54.46	31.91	31.29	24.58

Cyclic corridor results

	Unit	Results							
Robots		5	10	15	20	25	30	35	40
Congestion	%	3	7	11	15	19	23	27	31
$S_c \equiv S_{se}$	%	100	100	100	100	100	100	100	96
S_{sw}	%	100	100	100	100	100	100	100	96
S_{bs}	%	100	100	100	100	100	100	100	98
T_c	s	0.26	1.42	4.23	9.44	17.01	27.78	39.28	51.24
T_{se}	s	0.36	1.72	4.77	10.25	18.02	29.03	40.51	52.33
T_{sw}	s	0.5	2.67	7.37	15.55	27.04	42.53	58.68	75.01
T_{bs}	s	0.86	4.39	12.14	25.8	45.06	71.56	99.2	127.34
μ_c		18.03	20.3	22.48	25.2	28.27	33.14	37.98	48.07
μ_{se}		17.95	20.08	22.08	24.51	27.49	31.99	36.4	44.93
μ_{sw}		17.95	20.02	21.91	24.05	26.57	31.13	35.42	43.83
μ_{bs}		17.95	20.02	21.89	24.02	26.45	30.78	34.81	42.95
μ_c/LBE		1.01	1.04	1.12	1.24	1.36	1.59	1.8	2.22
μ_{se}/LBE		1.01	1.03	1.1	1.21	1.32	1.53	1.73	2.08
μ_{sw}/LBE		1.01	1.03	1.09	1.18	1.28	1.49	1.68	2.03
μ_{bs}/LBE		1.01	1.03	1.09	1.18	1.27	1.47	1.65	1.99
I_{se}	%	34.01	25.26	16.91	14.3	10.2	9.39	9.32	11.94
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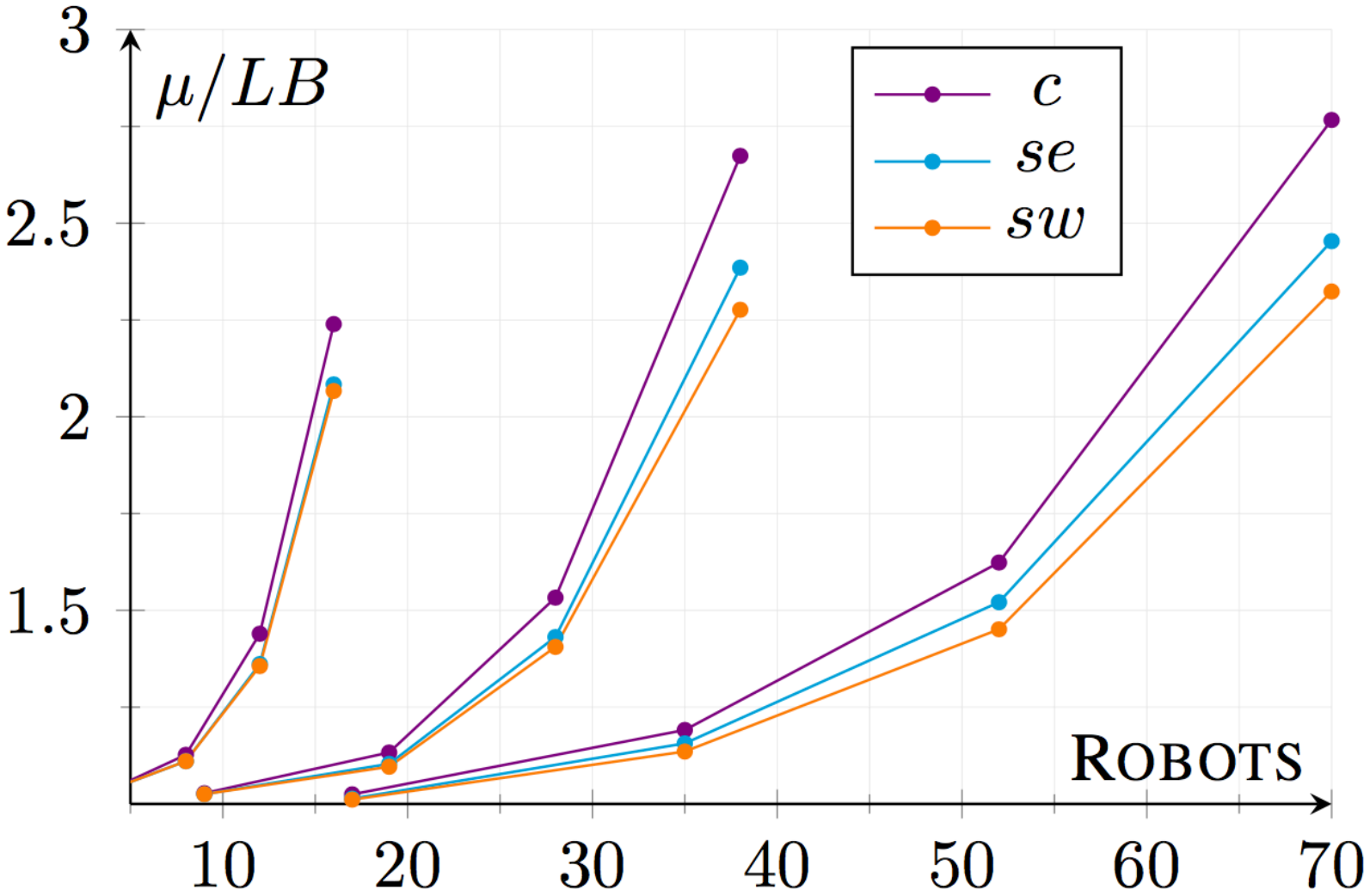
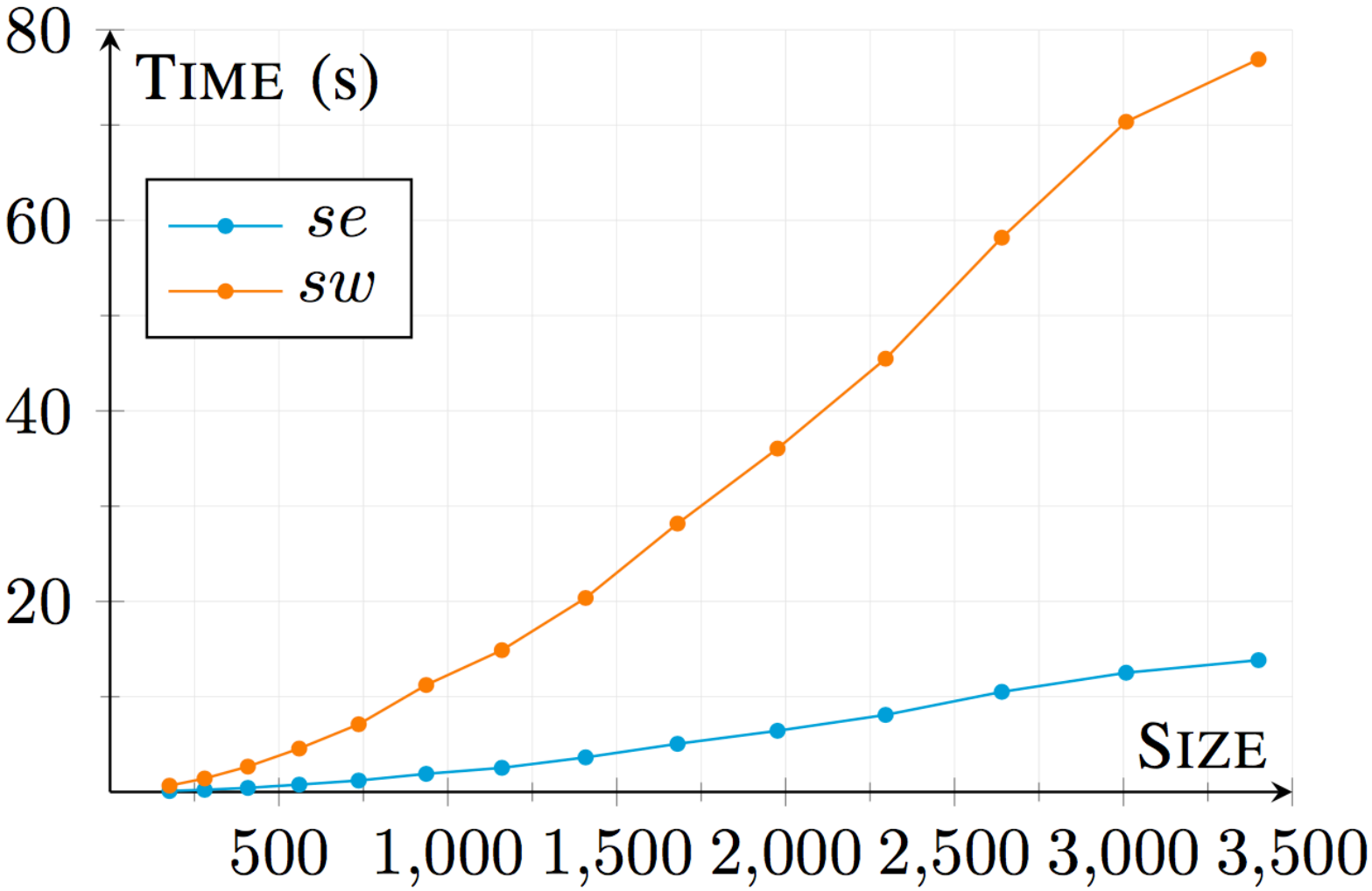
Grid improvements



Time required



Outline

Conclusions

- In this work we proposed a heuristic algorithm to improve solutions to the multi-robot path-planning problem implemented in a DES context using timed discrete event automata.
- The algorithm was tested on a variety of problems and it was shown that its use in any planning strategy is useful in terms of:
  1. **Execution time**: it was reduced up to 45%;
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Future works

Future works will aim at

- A smart choosing of the sub-trajectories to be prioritized;
- Planning strategies to avoid cases in which applying the swap strategy a solution is not found;
- Testing of more complicated environments, such as:
  1. The presence of places in which a vehicle cannot stop;
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Thank you for the attention