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Online Coordination of BESS and Thermostatically Control Loads for Shared Energy Optimization in Energy Communities

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***Abstract*— This paper introduces a mixed-integer linear programming (MILP) optimization scheme aimed at cost optimization in Energy Communities through the coordination of prosumers' Battery Energy Storage Systems (BESS) and Thermostatically Controlled Loads (TCLs). Inspired by recent guidelines from the Italian Gestore dei Mercati Energetici, we first introduce the concept of shared energy. Shared energy is defined as the minimum value, over a specified time window, between the renewable energy injected into the grid and the total energy consumed by the community. Subsequently, we formulate the coordination problem as a MILP optimization, where the degrees of freedom include the TCLs' status and the BESS's State-of-Charge. The effectiveness and performance of the proposed formulation are evaluated through numerical simulations, demonstrating the advantages of the coordinated scheme over uncoordinated ones.**

***Index Terms*— Energy Communities, Shared Energy, Battery Energy Storage Systems, Thermostatically Control Loads, Network Coordination.**

I. Introduction

An energy community can be defined as an arrangement of entities (called "agents") connected to the same power network. An agent can be a regular user of the network (i.e. a consumer) or, in a wider and more general setting, it can produce, store and sell energy to the network. From this perspective, an agent is commonly called prosumer. In particular, a renewable energy community is an energy community with agents or prosumers equipped with renewable generation capabilities [1].

In a renewable energy community, the behaviour or activity of each agent can be different. On one hand, they can participate in an uncoordinated fashion, only pursuing an individual benefit (i.e. reducing consumption, store or eventually sell surplus power). On the other hand, a more interesting possibility is that every agent in the community operates in a coordinated fashion. In this sense, a common objective is considered and a global benefit is preferred and pursued [2], [3].

In a community with such a coordinated operation scenario, it arises the concept of "shared energy" among agents of the same community [4], [5], according to the incentive scheme which has been adopted in Italy for renewable energy communities since 2020 [6], [7]. The main idea is basically that each agent can take advantage of the renewable energy generated by any other prosumer. Formally, the shared energy is defined as the minimum, over a given time window, between the energy injected into the grid and the energy consumed by the agents.

Given a renewable energy community, obtaining an accurate coordination of each prosumer behavior is specially important [8]. In particular, minimizing costs while maximizing the shared energy within the community is particularly interesting as it yields the maximal economic reward for the whole energy community. The possibility of shifting in time the load consumption is a useful degree of freedom to further reduce the costs, and the coordination of thermostatically controlled loads has already proven to be of great potential impact importance in this regard and also on power peak reduction [9], [10], [11], [12], [13]. It is worth stressing that centralized and decentralized strategies also need to address security, resiliency, and privacy concerns [14]. In this framework, this paper presents a centralized optimization proposal, intended to reduce the overall energy cost for the community over a given time window. This objective is achieved by coordinating the thermostatically controlled loads and energy storage and usage, considering costs and maximizing the shared energy, which represents a reward for the community.

In the next section, the mathematical model for each prosumer of the energy community is presented. Also, the main decision variables and constraints are defined. Lastly, the objective function is set and the resulting optimization problem is stated. In Section III a Mixed-Integer Linear Problem (MILP) formulation of the problem is derived and in Section IV some numerical results are presented. Finally, in Section V concluding remarks and perspectives for future extensions of the presented results are given.

II. Model of the Energy Community

The general energy community consists of $n \in \mathbb{N}$ interconnected prosumers which may, as mentioned before, consume power, produce power thanks to a renewable generator (RG), and store energy thanks to a Battery Energy Storage System (BESS) (Fig. 1). Thus, the state $x_i(t) \in \mathbb{R}^3$ of each prosumer $i = 1, \ldots, n$ is given by

$$x_i(t) = \begin{bmatrix} c_i(t) \\ g_i(t) \\ e_i(t) \end{bmatrix},$$

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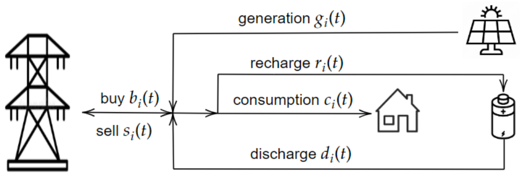

Fig. 1: Model of energy flow of a prosumer in an energy community

where

- $c_i(t)$ denotes the power consumption (kW);
- $g_i(t)$ denotes the power generation (kW);
- $e_i(t)$ denotes the energy storage (kWh).

Consumption: The power consumption of each prosumer $c_i(t)$ could be divided into two components $c_i^c(t)$ and $c_i^n(t)$, representing controllable and non-controllable portions of consumption, respectively,

$$c_i(t) = c_i^c(t) + c_i^n(t), \tag{1}$$

The controllable portion of power consumption is assumed to be a Thermostatically Controlled Load (TCL), i.e.:

$$c_i^c(t) = P_i^{\text{TCL}}\delta_i(t), \tag{2}$$

where P_i^{TCL} is the power consumption of the TCL and $\delta_i(t)$ is a boolean variable to set the TCL on and off. A simple first order model for a generic TCL considered for each prosumer is as follows [15]:

$$C_i^{\text{TCL}}\frac{d}{dt}\Theta_i(t) = \zeta_i P_i^{\text{TCL}}\delta_i(t) - R_i^{\text{TCL}}\left(\Theta_i(t) - \Theta_i^{\text{AMB}}\right), \tag{3}$$

where $\Theta_i(t)$ and Θ_i^{AMB} are the TCL and ambient temperature, C_i^{TCL} and R_i^{TCL} are the thermal capacity and resistance, respectively. Lastly ζ_i is the TCL efficiency.

Generation: The RG's generated power is denoted by $g_i(t)$, and it is considered as non-controllable.

Storage: The model for the BESS is drawn from [16, Section III-A] and reads as

$$e_i^{\text{MAX}}\frac{d}{dt}\varepsilon_i(t) = \eta_i r_i(t) - d_i(t), \tag{4}$$

where $i = 1, \ldots, n$ denotes different BESSs in the network. The other variables are introduced next:

- e_i^{MAX} represents the maximum energy storage capacity.
- $\varepsilon_i(t) = e_i(t)/e_i^{\text{MAX}} \in [0, 1]$ represents state of charge (SoC).
- $r_i(t)$, $d_i(t)$ denotes the recharching/discharging power; we assume that if $r_i(t) > 0$ then $d_i(t) = 0$ and vice versa.
- $\eta_i \in [0, 1]$ represents the round trip efficiency (RTE), i.e., the ratio between the energy supplied to the storage system and the energy retrieved from it.

A. *Sampling and horizon time window*

Consider a sampling time Δ measured in seconds [s] and a receding horizon time window $H = T\Delta$, where $T \in \mathbb{N}$ denote the number of samples within the horizon. For instance, if $\Delta = 60$ then Δ is one minute, if $H = 24 \cdot 3600$ then H is one day, and consequently one has $T = 1440$ samples within the horizon. Thus, we consider discretized time steps $t_k = k\Delta$ with $k \in \mathbb{N}$. With this notation, given a quantity $q_i(t) \in \mathbb{R}$, we define by $q_i(k, T) \in \mathbb{R}^T$ the samples of $q_i(t)$ from t_k to t_{k+T-1},

$$q_i(k,T) = \begin{bmatrix} q_i(t_k) \\ q_i(t_{k+1}) \\ \vdots \\ q_i(t_{k+T-2}) \\ q_i(t_{k+T-1}) \end{bmatrix}, \quad \text{with} \quad t_k = k\Delta.$$

B. *Local Constraints*

Decision Variables: The decision variables $r_i(k,T)$, $d_i(k,T)$, $\rho_i(k,T)$ of each prosumer are subject to the next bounds

$$\begin{array}{ccccc} \mathbf{0} & \leq & r_i(k,T) & \leq & r_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & d_i(k,T) & \leq & d_i^{\text{MAX}}\mathbf{1}. \end{array} \tag{5}$$

$$\rho_i(k,T) = r_i(k,T) - d_i(k,T). \tag{6}$$

where $\rho_i(k,T)$ is an extra variable of the minimization problem, representing the exchange power to charge or discharge the battery and r_i^{MAX}, d_i^{MAX} are the maximum allowable recharge and discharge power of each BESS, respectively.

Consumption: Let us denote by b_i and s_i the total power transferred from and to the grid, respectively. We have:

$$f_i(k,T) = c_i(k,T) - g_i(k,T) + \rho_i(k,T),$$

$$\begin{array}{l} b_i(k,T) = \max\{f_i(k,T), \mathbf{0}\}, \\ s_i(k,T) = \max\{-f_i(k,T), \mathbf{0}\}. \end{array} \tag{7}$$

In residential loads, these powers are limited due to the installed infrastructure, thus we include the following constraints:

$$\begin{array}{ccccc} \mathbf{0} & \leq & b_i(k,T) & \leq & b_i^{\text{MAX}}\mathbf{1}, \\ \mathbf{0} & \leq & s_i(k,T) & \leq & s_i^{\text{MAX}}\mathbf{1}, \end{array} \tag{8}$$

where b_i^{MAX} and s_i^{MAX} are the upper bounds of the residential load power transfer from and to the grid, respectively. Moreover, letting $\hat{c}_i(k,T)$ the estimation of the non-controllable power consumption of the user, we include the following constraint

$$c_i^n(k,T) \quad = \quad \hat{c}_i(k,T). \tag{9}$$

Clearly, to make the above constraints feasible, it must hold that $\hat{c}_i(k,T) \leq \bar{b}_i^{\text{MAX}}\mathbf{1}$.

Regarding the TCLs, the temperature must remain within the values set up by each prosumer:

$$\Theta_i^{\text{MIN}}\mathbf{1} \leq \Theta_i(k,T) \leq \Theta_i^{\text{MAX}}\mathbf{1}, \tag{10}$$

where Θ_i^{MIN} and Θ_i^{MAX} represent the minimum and maximum allowable temperature of each TCL, respectively. The linear model of the physical relation in eq. (3) can be approximated with these set of constraints:

$$\begin{aligned} \mathbf{D_i^{\Theta}}\Theta_i(k,T) - \mathbf{e}_1 e^{-\alpha_i\Delta}\Theta_i(t_{k-1}) &= \left(1 - e^{-\alpha_i\Delta}\right)\Theta_i^{\text{AMB}} \\ &+ \left(1 - e^{-\alpha_i\Delta}\right)\zeta_i R_i^{\text{TCL}} P_i^{\text{TCL}}\delta_i(k,T) \end{aligned} \tag{11}$$

where $\mathbf{e}_1 \in \mathbb{R}^T$ is a vector with all the elements equal to 0, except of the first element that is equal to 1. On the other hand $\alpha_i = 1/R_i^{\text{TCL}} C_i^{\text{TCL}}$ and the matrix $\mathbf{D_i^\Theta} \in \mathbb{R}^{T\times T}$ is given by

$$\mathbf{D_i^\Theta} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -e^{-\alpha_i \Delta} & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -e^{-\alpha_i \Delta} & 1 \end{bmatrix}.$$

We note that $\Theta_i(t_{k-1})$ is the TCL temperature at time t_{k-1}, i.e. the initial temperature at k time window.

Generation: We consider that the estimation $\hat{g}_i(k,T)$ of the power generated by the RG is satisfactorily precise, which yields to the following constraint

$$g_i(k,T) \quad = \quad \hat{g}_i(k,T). \tag{12}$$

Storage: The SoC of each BESSs must remain in the bound stated by their manufacturing companies, thus

$$\varepsilon_i^{\text{MIN}} \mathbf{1} \le \varepsilon_i(k,T) \le \varepsilon_i^{\text{MAX}} \mathbf{1}, \tag{13}$$

where $\varepsilon_i^{\text{MIN}}$ and $\varepsilon_i^{\text{MAX}}$ represent the minimum and the maximum allowable SoC, respectively. The relation between SoC and the power limits must follow the logic that the maximum amount of power flowing from/to the battery is directly tied to the battery's level of charge. More precisely, the maximum recharge power decreases as the battery's charge level increases, up to 0 when the battery is fully charged, and the maximum discharge power decreases as the battery's charge level decreases, up to 0 when the battery is fully discharged. We model this relation through the following set of constraints

$$\begin{aligned} r_i(k,T) &\le m_i^r \cdot (\varepsilon_i^{\text{MAX}} \mathbf{1} - \varepsilon_i(k,T)), \\ d_i(k,T) &\le m_i^d \cdot (\varepsilon_i(k,T) - \varepsilon_i^{\text{MIN}} \mathbf{1}), \end{aligned} \tag{14}$$

where $m_i^r, m_i^d \ge 0$ are the slopes of the power limits on recharge and discharge, respectively. Figure 2 shows the feasible region delimited by these constraints.

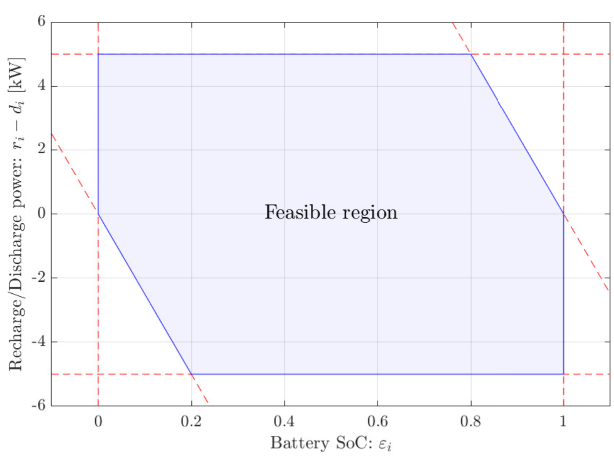

Fig. 2: Feasible region described by the kinetic battery model.

We now approximate the physical relation in eq. (4) between the SoC and the decision variables with the following set of constraints:

$$\frac{e_i^{\text{MAX}}}{\Delta} \left(\mathbf{D}_\varepsilon \varepsilon_i(k,T) - \mathbf{e_1} \varepsilon_i(t_{k-1}) \right) = \eta_i r_i(k,T) - d_i(k,T), \tag{15}$$

where $\mathbf{e_1}$ stands for the canonical vector with 1 in the first element, i.e. $\mathbf{e_1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^\top \in \mathbb{R}^T$ and the matrix $\mathbf{D}_\varepsilon \in \mathbb{R}^{T\times T}$ is given by

$$\mathbf{D}_\varepsilon = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

Note that $\varepsilon_i(t_{k-1})$ stands for the state of charge at time t_{k-1}. This is necessary to enable the approximated computation of the SoC's derivative at time t_k.

Coupling constraints A large number of BESSs increases the potential short-term load variations of residential customers, posing a potential threat to the stability of the power distribution infrastructure. Specifically, to prevent overloads in high voltage substations, the following coupling constraints need to be satisfied:

$$\begin{aligned} \sum_{i=1}^n [\rho_i(k,T) + c_i(k,T) - g_i(k,T)] &\ge -L^{\text{MAX}} \mathbf{1}, \\ \sum_{i=1}^n [\rho_i(k,T) + c_i(k,T) - g_i(k,T)] &\le L^{\text{MAX}} \mathbf{1}, \end{aligned} \tag{16}$$

where L^{MAX} is the maximum allowable power transfer at the substation.

C. Shared energy

We now introduce the concept of 'shared energy', defined as the minimum, over a specified window of time $W = \Upsilon\Delta$ with $\Upsilon \in \mathbb{N}$, between the energy injected into the grid and the energy withdrawn from the grid by the users. These two quantities over a window of time $W = \Upsilon\Delta$ at time t_k are given by

$$S_i(k,\Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top s_i(k, \Upsilon - \text{mod}(k,\Upsilon)) \\ \mathbf{1}^\top s_i(\lceil k/\Upsilon \rceil \Upsilon, \text{mod}(k,\Upsilon)) \end{bmatrix},$$

$$B_i(k,\Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top b_i(k, \Upsilon - \text{mod}(k,\Upsilon)) \\ \mathbf{1}^\top b_i(\lceil k/\Upsilon \rceil \Upsilon, \text{mod}(k,\Upsilon)) \end{bmatrix}.$$

Note that if k corresponds to an instant of time where a window W starts, i.e., $\text{mod}(k,\Upsilon) = 0$, then $S_i(k,\Upsilon) = \Delta \mathbf{1}_\Upsilon^\top s_i(k,\Upsilon) \in \mathbb{R}$. If instead, k falls within a time window W, i.e., $\text{mod}(k,\Upsilon) \neq 0$, then $S_i(k,\Upsilon) \in \mathbb{R}^2$ has two components, one summing the energy in the remaining part of the current window W, and one summing the energy in a fraction of the next window W. The same holds for $B_i(k,\Upsilon)$.

We can now extend this concept over the whole horizon time window $H = T\Delta$ such that it contains a positive integer number h of pricing windows W, namely

$$T/\Upsilon = h > 1.$$

Then, we can also define

$$S_i(k,h,\Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top s_i(k, \Upsilon - \mathrm{mod}(k,\Upsilon)) \\ (I_{h-1} \otimes \mathbf{1}_\Upsilon^\top) s_i(\lceil (k+1)/\Upsilon \rceil \Upsilon, (h-1)\Upsilon) \\ \mathbf{1}^\top s_i((\lceil k/\Upsilon \rceil + h - 1)\Upsilon, \mathrm{mod}(k,\Upsilon)) \end{bmatrix},$$
$$B_i(k,h,\Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top b_i(k, \Upsilon - \mathrm{mod}(k,\Upsilon)) \\ (I_{h-1} \otimes \mathbf{1}_\Upsilon^\top) b_i(\lceil (k+1)/\Upsilon \rceil \Upsilon, (h-1)\Upsilon) \\ \mathbf{1}^\top b_i((\lceil k/\Upsilon \rceil + h - 1)\Upsilon, \mathrm{mod}(k,\Upsilon)) \end{bmatrix}. \quad (17)$$

A graphical example of the construction of these two vectors is presented in Figure 3. Each element is basically the area under the curve $f_i(k,T)$ within the corresponding pricing window W. Note that if $\mathrm{mod}(k,\Upsilon) = 0$, the last element becomes zero, but can be greater than zero for any other value for $\mathrm{mod}(k,\Upsilon) \in [1; \Upsilon - 1]$. For this is that both vectors require $h+1$ elements.

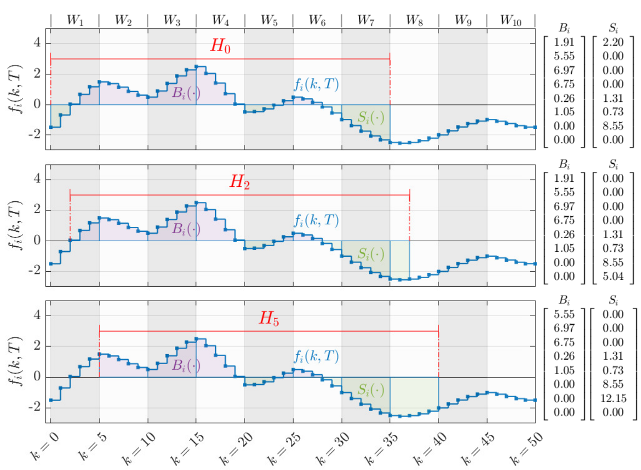

Fig. 3: Energy computation example, for different k values ($h = 7$, $\Upsilon = 5$, $T = 35$)

With this notation, we can formally define the shared energy as follows

$$E_{sh}(k,h,\Upsilon) = \min \left\{ \sum_{i=1}^n S_i(k,h,\Upsilon), \sum_{i=1}^n B_i(k,h,\Upsilon) \right\}, \quad (18)$$

where the min operation has to be intended component-wise.

D. Objective Function

Consider a generic discrete time $t_k = k\Delta$ where Δ is the sampling time and let $H = T\Delta$ be the horizon time window. Assume that the reward for the shared energy is determined over periods of time $W = \Upsilon\Delta$ and assume that the energy price is determined over the same periods, such that

$$T/\Upsilon = h > 1.$$

where $h \in \mathbb{N}$. We define now the vector $v_i(k,T) \in \mathbb{R}^{4T}$ of main decision variables for each user as

$$v_i(k,T) = \begin{bmatrix} r_i(k,T) \\ d_i(k,T) \\ \rho_i(k,T) \\ \delta_i(k,T) \end{bmatrix}, \quad (19)$$

and the vector of decision variables for the community as:

$$v(k,T) = \begin{bmatrix} v_1(k,T) \\ v_2(k,T) \\ \vdots \\ v_n(k,T) \end{bmatrix} \in \mathbb{R}^{4nT}. \quad (20)$$

In order to maximize the shared energy, the objective function can be formlated as:

$$J(v(k,T)) = p_e^\top \sum_{i=1}^n B_i(k,h,\Upsilon) - \\ - p_{sh}^\top \underbrace{\min \left\{ \sum_{i=1}^n S_i(k,h,\Upsilon), \sum_{i=1}^n B_i(k,h,\Upsilon) \right\}}_{E_{sh}(k,h,\Upsilon)} \quad (21)$$

and the following minimization problem has to be solved:

$$\min_{v(k,T)} \quad J(v(k,T)) \\ \text{subj. to} \quad \text{eqs.}(5),(6),(7),(8),(10), \\ (11),(13),(14),(15),(16) \quad (22)$$

where

- $B_i(k,h,\Upsilon)$ is the energy bought by the i-th prosumer;
- p_e is the energy price in €/kWh in each time window;
- $E_{sh}(k,h,\Upsilon)$ is the shared energy in the community;
- p_{sh} is the economic reward in €/kWh for the shared energy in each time window;

III. Mixed-Integer Linear Programming

The objective function presented in eq. 21 it is nonlinear and non smooth, due to the presence of the $\min(\cdot)$ function for the computation of the shared energy and the computation of the powers $b_i(k,T)$ and $s_i(k,T)$ in eq. 7. These non linearity can be overcome by rearranging the problem through the incorporation of new decision variables.

Firstly, the $\min(\cdot)$ associated to the shared energy computation can be solved considering $E_{sh}(k,h,\Upsilon)$ a set of $h+1$ decision variables and considering these couple of constraints:

$$-\sum_{i=1}^n S_i(k,h,\Upsilon) + E_{sh}(k,h,\Upsilon) \leq \mathbf{0}, \quad (23)$$
$$-\sum_{i=1}^n B_i(k,h,\Upsilon) + E_{sh}(k,h,\Upsilon) \leq \mathbf{0}, \quad (24)$$

Secondly, consider the energy computation in eq. (17). This vectors can be computed as:

$$B_i(k,h,\Upsilon) = \Delta \cdot A_E(k) \cdot b_i(k,T), \quad (25)$$

where $A_E(k) \in \mathbb{R}^{(h+1)\times T}$ is a matrix computed as follows:

$$A_E(k) = A_h \cdot L_{T+\Upsilon}^{\mathrm{mod}(k,\Upsilon)} \cdot I_{(T+\Upsilon)\times T} \quad (26)$$

and $L_{T+\Upsilon} \in \mathbb{R}^{(T+\Upsilon)\times(T+\Upsilon)}$ is a lower shift matrix, i.e. $L_{ij} = \delta_{i,j+1}$ with $\delta_{i,j}$ the Kronecker delta. On the other hand:

$$A_h = I_{h+1} \otimes \mathbf{1}_\Upsilon^\top \in \mathbb{R}^{(h+1)\times T} \quad (27)$$

Finally $I_{(T+\Upsilon)\times T}$ is a non square matrix as follows:

$$I_{(T+\Upsilon)\times T}=\begin{bmatrix} I_T \\ \mathbf{0}_{\Upsilon\times T}\end{bmatrix}. \tag{28}$$

It is important to notice that there are Υ different matrices $A_E(k)$. Then, considering that:

$$s_i(k,T)=b_i(k,T)-f_i(k,T), \tag{29}$$

and then:

$$S_i(k,h,\Upsilon)=\Delta\cdot A_E(k)\cdot\left(b_i(k,T)-f_i(k,T)\right), \tag{30}$$

Now, incorporating $b_i(k,T)$ as a decision variable and given that $b_i(k,T)=\max\left(f_i(k,T),\mathbf{0}\right)$, these couple of constraints should be incorporated:

$$b_i(k,T) \geq f_i(k,T), \tag{31}$$

$$b_i(k,T) \geq \mathbf{0}, \tag{32}$$

although the second one it is also considered as a lower bound of $b_i(k,T)$ in eq. (8).

Now, redefining the the decision variables vector as:

$$\nu(k,T)=\begin{bmatrix}\nu_1(k,T)\\ \nu_2(k,T)\\ \vdots \\ \nu_n(k,T)\end{bmatrix}\in\mathbb{R}^{5nT}. \tag{33}$$

where:

$$\nu_i(k,T)=\begin{bmatrix} r_i(k,T)\\ d_i(k,T)\\ \rho_i(k,T)\\ \delta_i(k,T)\\ b_i(k,T)\end{bmatrix}\in\mathbb{R}^{5T}. \tag{34}$$

Then, the MILP problem results:

$$\min_{\nu,\ E_{sh}} \quad p_e^\top\cdot\Delta\cdot A_E(k)\cdot\sum_{i=1}^{n} b_i(k,T)- \\ -p_{sh}^\top\cdot E_{sh}(k,h,\Upsilon) \tag{35}$$

subject to variables bounds and all the linear inequality constraints in eqs. (5), (8), (10), (11), (13), (14), (15), (16), (23), (24), (31) and the equality constraint in eq. (6).

IV. Numerical Simulations

In this section, simulations results are presented, showing the MILP problem minimization in a small set of prosumers. The example does not pretend to be representative of a real practical configuration, but solve the problem under a general variable load and generation powers situation. The obtained results allows to conceptualise the benefits of sharing energy within a community arrangement.

For the simulations, a six prosumers community is considered ($n=6$). Each prosumer consume energy from the network and has a BESS and a room temperature control as a TCL. The receding horizon window H is set to be of half day, with a sample time Δ of 15 minutes. This configures the problem with $T=48$, $h=12$ and $\Upsilon=4$. All the constants and parameters values for each prosumer are shown in Table (I). All these values are generic and intended to evaluate the minimization problem resolution, not a specific practical representation. In this same sense, a price of 0.15€/kWh from 6pm to 6am, while a price of 0.195€/kWh from 6am to 6pm are considered. On the other hand, a constant reward at each hour of 0.07€/kWh is configured.

Parameter	Value	Parameter	Value
r_i^{MAX}	5kW	d_i^{MAX}	5kW
b_i^{MAX}	20kW	s_i^{MAX}	20kW
Θ_i^{MAX}	20°C	Θ_i^{MIN}	18°C
R_i^{TCL}	83.33°C/kW	C_i^{TCL}	300kWs/°C
P_i^{TCL}	0.2kW	ζ_i	0.8
$\varepsilon_i^{\text{MAX}}$	0.9	$\varepsilon_i^{\text{MIN}}$	0.1
m_i^r	50kW	m_i^d	50kW
e_i^{MAX}	20kWh	η_i	0.8

TABLE I: Constants and parameters of for simulation

In Figure 4, the prosumers non-controllable consumption and generation powers forecasting are depicted. These profiles are considered to be generated for a higher level process and are not subject of analysis in this preliminary work.

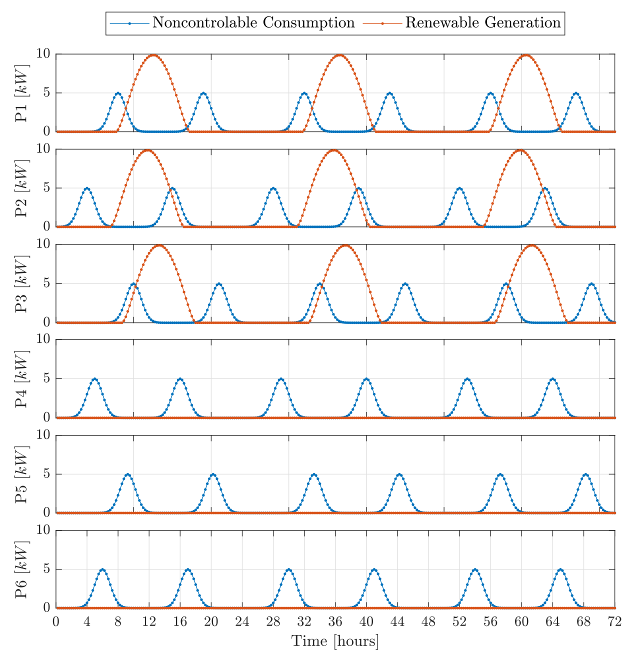

Fig. 4: Prosumers consumption (blue) and generation (orange)

The optimization problem is solved at each k, adjusting each initial prosumer SoC (ε_i) and room temperature (Θ_i) at each step. The MILP problem is solved using the MATLAB™ function `intlinprog`. Figure 5 presents the optimization results for the power bought by each prosumer (b_i) and the recharge (r_i) and discharge (d_i) powers.

It is important to note that in general, each prosumer recharge the battery at times were some renewable power is been generated or, if not available, from time instants where energy cost is lower. Conversely, the batteries are discharge at moments were no renewable power generations is available or energy cost is higher. This is consistent with the objective

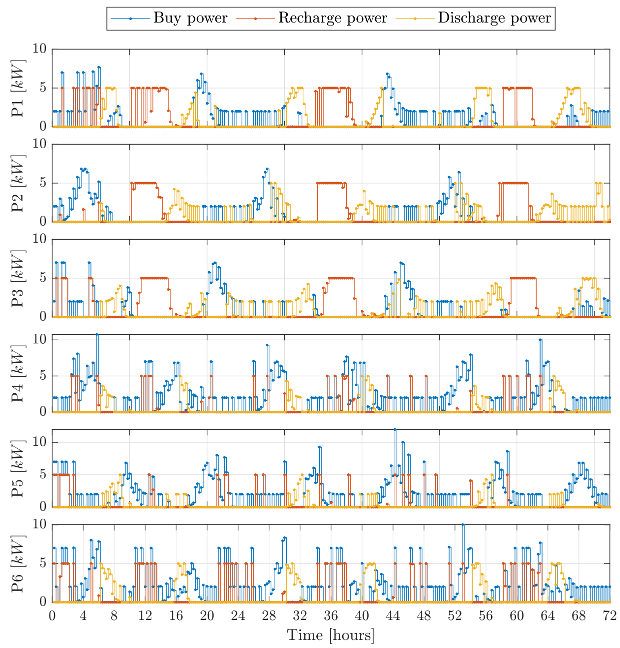

Fig. 5: Prosumers buy (b_i), recharge (r_i) and discharge (d_i) powers

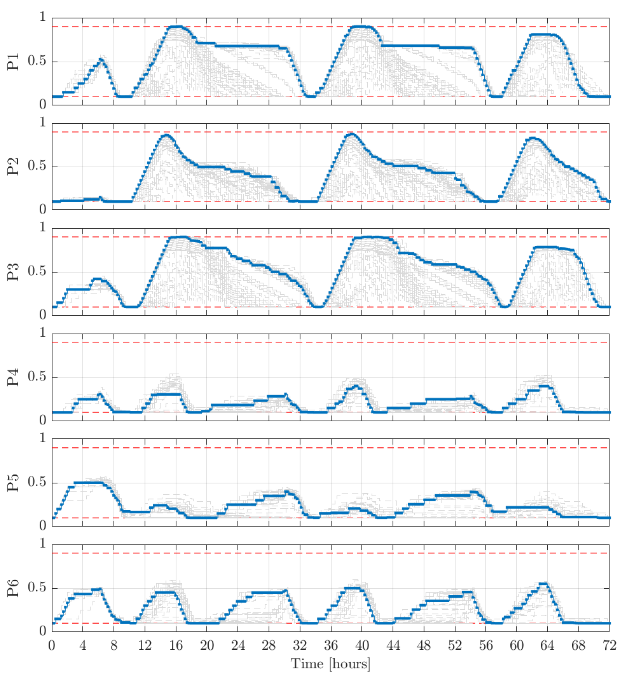

Fig. 6: Prosumers batteries SoC ($\varepsilon_i(k,T)$)

of reducing the self consumption from the network and simultaneously "sharing energy" within the community to reduce the overall cost of the energy. This is particularly notorious in the last three agents, that do not generate power. These agents recharge their batteries only at times when the other three agents generate energy, i.e. taking advantage of the shared energy.

Following the recharge and discharge profiles defined by the optimization, Figure 6 presents the SoC of each prosumer battery. The gray or shadowed curves are partial results of the optimization process, In this sense, it can be noted again how the prosumers take advantage of the shared energy by recharging the batteries while an excess of renewable energy is available or while the energy cost is lower. Also, considering the SoC upper and lower limits constrains, it can be seen that is fulfilled during all the simulation.

On the other hand, the boolean variable that govern the TCLs is depicted in Figure 7. The figure demonstrates that, when renewable energy is available, the TCL are turn on more frequently in order to better leverage this resource.

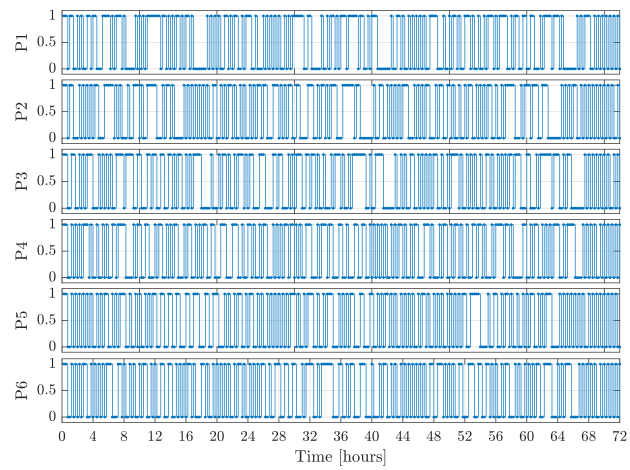

Fig. 7: Prosumers TCL boolean variables ($\delta_i(k,T)$)

Finally, in Figure 8 the TCLs temperatures are shown. These curves show that the temperature is incremented while renewable energy is available. In the case of the first three prosumers, this is more marked, mainly due to the fact that the optimization problem tend to minimize the bought power by each prosumer. In the case of the last three prosumers, while this behaviour is also present, in general tend to keep the temperature as low as possible, reducing also the bought power. In the same way that the SoC, the temperature of each TCL is controlled in a way that it is always within the minimum and maximum settled values.

In order to compare the advantages of the optimization and sharing energy through a community, the overall cost during the three day simulation is computed. In this sense, the computed overall cost using the proposed optimization is 92.30€. On the other hand, in a non-collaborative scheme, were no reward is given for sharing energy, the overall cost in the same consumption and generations scenario, results 100.27€. This means a reduction on the energy cost by maximizing the shared energy, of about 8%. It is interesting to note that, in case of not using batteries to store the surplus energy, the energy cost becomes 124.79€, representing, compared to the proposed collaborative scheme, about a 35% of energy cost increment.

V. Conclusions and Future Directions

This article address the problem of minimizing the energy costs for renewable energy communities consisting of

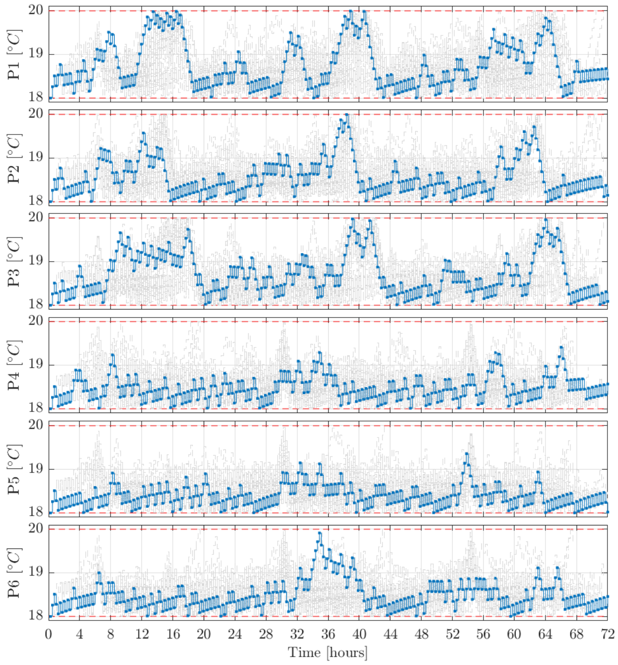

Fig. 8: Prosumers TCL temperatures ($\Theta_i(k,T)$)

multiple agents that generate, store, and consume energy. Both storage and power consumption were assumed to be partially controllable, specifically by allowing the recharge of the battery directly from the grid and by allowing the control of TCLs in an on/off scheme. Numerical simulations for a small community of six agents corroborate the validity of the formulation, allowing to reduce the energy costs when an economic reward for the shared energy was provided. Agents benefitted from the active exploitation of the varying energy costs throughout the day by storing cheap energy for use during costly hours. TCLs behaved similarly to batteries, storing thermal energy during power surpluses. Future works will focus on the distributed formulation of the problem presented in this manuscript, as in our preliminary work [17] where a simpler scenario without TCLs is considered. In particular, we will investigate how consensus-based algorithms for distributed optimization and online learning [18], [19], [20], [21], [22], [23] could be exploited to infer global information useful for the energy community to maximize the shared energy through coordination of batteries and loads.

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