Distributed Optimization for Networks of Battery Energy Storage Systems in Energy Communities with Shared Energy Incentives

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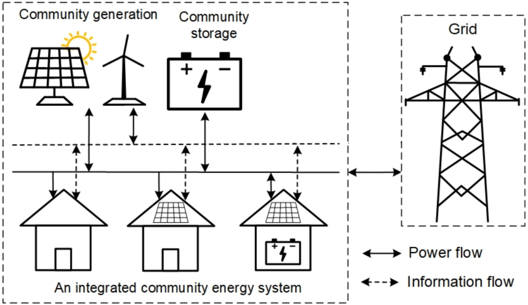
Outline

1. The problem of interest
2. Distributed optimization problem formulation
3. Results and discussion

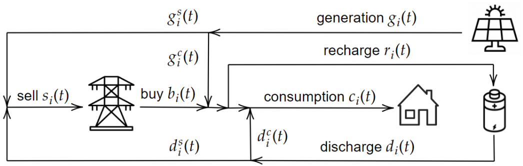
Outline

Cooperative energy management in renewable energy communities (CER)

- **The set-up**: Each member of the community may:
  - Consume energy
  - Produce energy (e.g., solar panels)
  - Store energy (e.g., battery energy systems)
- **The objective**: cooperation among members to minimize the cost by maximizing the shared energy
- **The strategy**: control the charge/discharge behavior of the batteries
- **The challenge**: absence of global information of the community generation and the only exploitation of local information exchanged between neighbors



Energy flow model of a member within the energy community



Model of the (dis)charging behavior of the battery:

$$e_i^{\mathrm{MAX}} \frac{d}{dt} \varepsilon_i(t) = \eta_i r_i(t) - d_i(t), \quad (1)$$

where:

- $e_i^{\mathrm{MAX}} \in \mathbb{R}_{\geq 0}$ is the maximum energy capacity.
- $\varepsilon_i(t) \in [0, 1]$ is state of charge (SoC).
- $\eta_i \in [0, 1]$ is the efficiency

Given a continuous-time signal $x(t) \in \mathbb{R}$ with $t \in \mathbb{R}$ and a sampling time $\Delta \in \mathbb{N}_+$, we denote by $t_k = \Delta k$ with $k \in \mathbb{N}$ the discrete times at which the signal is sampled, yielding the discrete time signal $x(t_k) \in \mathbb{R}$. We also denote by $[x]_k^T$, where $k, T \in \mathbb{N}$ the vector collecting T samples of the continuous time signal starting from t_k and use the slender notation $\boldsymbol{x}$ when clear from the context:

$$\boldsymbol{x} = [x]_k^T = [x(t_k), \cdots, x(t_{k+T-1})]^\top. \quad (2)$$

The concept of shared energy (Italian regulation)

Definition

The shared energy is the minimum between the energy fed into the network and the energy consumed by the community members in a given time period window of time $W = \Upsilon\Delta$ with $\Upsilon \in \mathbb{N}$:

$$E_{sh}(\boldsymbol{b}, \boldsymbol{s}, \Upsilon) = \min\left\{\sum_{i\in\mathcal{V}} g(\boldsymbol{b}_i, \Upsilon), \sum_{i\in\mathcal{V}} g(\boldsymbol{s}_i, \Upsilon)\right\} \in \mathbb{R}^{\lceil k,\Upsilon \rceil},$$

where, given the horizon $H = h\Upsilon\Delta$ with $h \in \mathbb{N}$, the function g is defined as follows:

$$g(\boldsymbol{x}, \Upsilon) = \Delta \begin{bmatrix} \mathbf{1}^\top [x]_k^{\Upsilon - \mathrm{mod}(k,\Upsilon)} \\ I_{h-1} \otimes \mathbf{1}_\Upsilon^\top [x]_{\lceil (k+1)/\Upsilon \rceil \Upsilon}^{(h-1)\Upsilon} \\ \mathbf{1}^\top [x]_{(\lceil k/\Upsilon \rceil + h - 1)\Upsilon}^{\mathrm{mod}(k,\Upsilon)} \end{bmatrix}.$$

Problem of interest

In the scenario of an energy community operating under an incentive scheme based on the self-consumption realized by the whole community, **the objective is to** minimize the costs for the whole community by maximizing the shared energy over the horizon H.

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Optimization problem formulation: objective function and constraints

The objective function we aim to minimize is

$$f(\boldsymbol{v}) = p_e^\top \sum_{i\in\mathcal{V}} g(\boldsymbol{b}_i, \Upsilon) - p_{sh}^\top E_{sh}(\boldsymbol{b}, \boldsymbol{s}, \Upsilon), \qquad \boldsymbol{v} = \left[\boldsymbol{v}_1^\top, \cdots, \boldsymbol{v}_n^\top\right]^\top. \quad \text{and} \quad \boldsymbol{v}_i = \left[\boldsymbol{r}_i^\top, \boldsymbol{d}_i^\top, \boldsymbol{d}_i^{c\top}, \boldsymbol{g}_i^{c\top}\right]^\top.$$

The local constraints are:

$$\begin{array}{ccccc}
\mathbf{0} & \leq & \boldsymbol{r}_i & \leq & r_i^{\text{MAX}}\mathbf{1}, \\
\mathbf{0} & \leq & \boldsymbol{d}_i & \leq & d_i^{\text{MAX}}\mathbf{1}, \\
\mathbf{0} & \leq & \boldsymbol{d}_i/d_i^{\text{MAX}} + \boldsymbol{r}_i/r_i^{\text{MAX}} & \leq & \mathbf{1}, \\
\mathbf{0} & \leq & \boldsymbol{d}_i^c & \leq & \boldsymbol{d}_i, \\
\mathbf{0} & \leq & \boldsymbol{g}_i^c & \leq & \boldsymbol{g}_i, \\
\mathbf{0} & \leq & \boldsymbol{b}_i & \leq & b_i^{\text{MAX}}\mathbf{1}, \\
\mathbf{0} & \leq & \boldsymbol{s}_i & \leq & s_i^{\text{MAX}}\mathbf{1},
\end{array}$$

together with those related to the (dis)charge dynamics of the battery:

$$\varepsilon_i^{\text{MIN}}\mathbf{1} \leq D^{-1}\left[\frac{e_i^{\text{MAX}}}{\Delta}\left(\eta_i \boldsymbol{r}_i - \boldsymbol{d}_i\right) + \boldsymbol{e}_1 \varepsilon_i(t_{k-1})\right] \leq \varepsilon_i^{\text{MAX}}\mathbf{1}, \quad \text{where} \quad \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

Optimization problem formulation: LP transformation

We compactly write the optimization problem as follows:

$$\begin{aligned}\min_{\boldsymbol{v},\boldsymbol{\theta}} \quad & p_e^\top \sum_{i\in\mathcal{V}} g(\boldsymbol{b}_i,\Upsilon) - p_{sh}^\top E_{sh}(\boldsymbol{b},\boldsymbol{s},\Upsilon),\\ \text{s.t.} \quad & \text{Local constraints} \quad \forall i\in\mathcal{V}.\end{aligned}$$

By using the standard trick $z=\min\{x,y\} \Rightarrow z\le x$ and $z\le y$, we obtain an LP formulation:

$$\begin{aligned}\min_{\boldsymbol{v},\boldsymbol{\theta}} \quad & p_e^\top \sum_{i\in\mathcal{V}} g(\boldsymbol{b}_i,\Upsilon) - p_{sh}^\top \boldsymbol{\theta},\\ \text{s.t.} \quad & \text{Local constraints} \quad \forall i\in\mathcal{V},\\ & \boldsymbol{\theta} - \textstyle\sum_{i\in\mathcal{V}} g(\boldsymbol{b}_i,\Upsilon) \le \mathbf{0},\\ & \boldsymbol{\theta} - \textstyle\sum_{i\in\mathcal{V}} g(\boldsymbol{s}_i,\Upsilon) \le \mathbf{0}.\end{aligned}$$

A distributed formulation: the objective function

The term $\boldsymbol{\theta}$ associated with the shared energy is replaced by introducing local variables $\boldsymbol{\vartheta}_i$ representing a fraction of the shared energy, i.e., $\boldsymbol{\theta} = \sum_{i=1}^{n} \boldsymbol{\vartheta}_i$, yielding:

$$f(\boldsymbol{v}, \boldsymbol{\theta}) = \sum_{i \in \mathcal{V}} f_i(\boldsymbol{v}_i, \boldsymbol{\vartheta}_i), \quad \text{where} \tag{3a}$$

$$f_i(\boldsymbol{v}_i, \boldsymbol{\vartheta}_i) = p_e^\top g(\boldsymbol{b}_i, \Upsilon) - p_{sh}^\top \boldsymbol{\vartheta}_i. \tag{3b}$$

By noticing that variables $\boldsymbol{v}_i$ satisfy box constraints of the kind $\boldsymbol{v}_i^{\mathrm{MIN}} \leq \boldsymbol{v}_i \leq \boldsymbol{v}_i^{\mathrm{MAX}}$, denoting $\overline{\boldsymbol{v}}_i = \frac{1}{2}(\boldsymbol{v}_i^{\mathrm{MIN}} + \boldsymbol{v}_i^{\mathrm{MAX}})$ we force strong convexity by regularizing the local objective functions as follows:

$$\widetilde{f}_i(\boldsymbol{v}_i, \boldsymbol{\vartheta}_i) = f_i(\boldsymbol{v}_i, \boldsymbol{\vartheta}_i) + \sigma \|\boldsymbol{v}_i - \overline{\boldsymbol{v}}_i\|_2^2 + \varsigma \|\boldsymbol{\vartheta}_i\|_2^2, \quad \sigma, \varsigma \in \mathbb{R}_{\geq 0},$$

A distributed formulation: the constraints

As a last step, we introduce local variables

$$\boldsymbol{\alpha}_i \geq 0, \quad \boldsymbol{\beta}_i \geq 0, \qquad \forall i \in \mathcal{V},$$

to transform the inequality constraints into equality constraints:

$$\begin{aligned}
\min_{\{\boldsymbol{v}_i,\boldsymbol{\vartheta}_i,\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i\}_{i\in\mathcal{V}}} \quad & \sum_{i\in\mathcal{V}} \widetilde{f}_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i), \\
\text{s.t.} \quad & \text{Local. constraint} \quad \forall i\in\mathcal{V}, && (4) \\
& \textstyle\sum_{i\in\mathcal{V}}(\boldsymbol{\vartheta}_i - g(\boldsymbol{b}_i,\Upsilon) + \boldsymbol{\alpha}_i) = \mathbf{0}, && (5) \\
& \textstyle\sum_{i\in\mathcal{V}}(\boldsymbol{\vartheta}_i - g(\boldsymbol{s}_i,\Upsilon) + \boldsymbol{\beta}_i) = \mathbf{0}. && (6)
\end{aligned}$$

Algorithm 1 DC-ADMM applied to the distributed optimization problem to minimize costs of an energy community

Require: Arbitrary initial values $\boldsymbol{v}_i(0), \boldsymbol{\vartheta}_i(0), \boldsymbol{\alpha}_i(0), \boldsymbol{\beta}_i(0), \boldsymbol{p}_i(0)$ for $i \in \mathcal{V}$ and the parameter $\rho > 0$

1: **for** k=1,2,3,… (until a stopping criterion is satisfied) **do**

2: **for each prosumer** $i \in \mathcal{V}$ **(in parallel) do**

$$\begin{bmatrix} \boldsymbol{v}_i(k) \\ \boldsymbol{\vartheta}_i(k) \\ \boldsymbol{\alpha}_i(k) \\ \boldsymbol{\beta}_i(k) \end{bmatrix} = \underset{\boldsymbol{v}_i,\boldsymbol{\vartheta}_i,\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i}{\operatorname{argmin}} \left\{ \widetilde{f}_i(\boldsymbol{v}_i,\boldsymbol{\vartheta}_i) + \frac{\rho}{4|\mathcal{N}_i|} \left\| \begin{bmatrix} (\boldsymbol{\vartheta}_i - g(\boldsymbol{b}_i,\Upsilon) + \boldsymbol{\alpha}_i)/\rho \\ (\boldsymbol{\vartheta}_i - g(\boldsymbol{s}_i,\Upsilon) + \boldsymbol{\beta}_i)/\rho \end{bmatrix} - \frac{1}{\rho}\boldsymbol{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} (\boldsymbol{y}_i(k-1) + \boldsymbol{y}_j(k-1)) \right\|_2^2 \right\}$$

$$\text{s.t.} \qquad \text{Loc. const.} \qquad \forall i \in \mathcal{V},$$

$$\boldsymbol{y}_i(k) = \frac{1}{2|\mathcal{N}_i|} \left(\begin{bmatrix} (\boldsymbol{\vartheta}_i(k) - g(\boldsymbol{b}_i(k),\Upsilon) + \boldsymbol{\alpha}_i(k))/\rho \\ (\boldsymbol{\vartheta}_i(k) - g(\boldsymbol{s}_i(k),\Upsilon) + \boldsymbol{\beta}_i(k))/\rho \end{bmatrix} - \frac{1}{\rho}\boldsymbol{p}_i(k-1) + \sum_{j\in\mathcal{N}_i} (\boldsymbol{y}_i(k-1) + \boldsymbol{y}_j(k-1)) \right)$$

$$\boldsymbol{p}_i(k) = \boldsymbol{p}_i(k-1) + \rho \sum_{j\in\mathcal{N}_i} (y_i(k) - y_j(k))$$

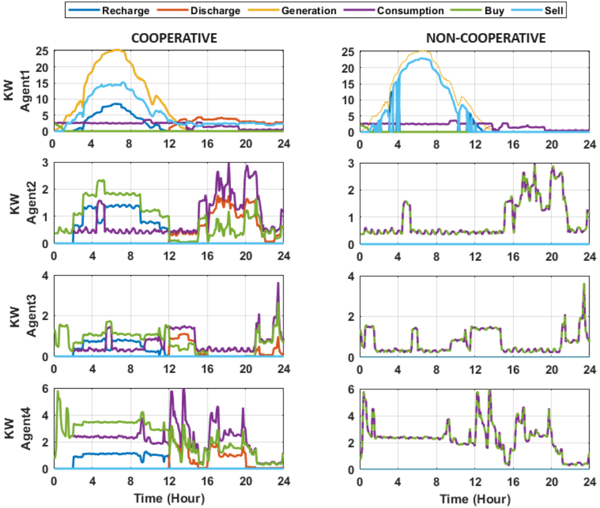
3: **end for**

4: **end for**

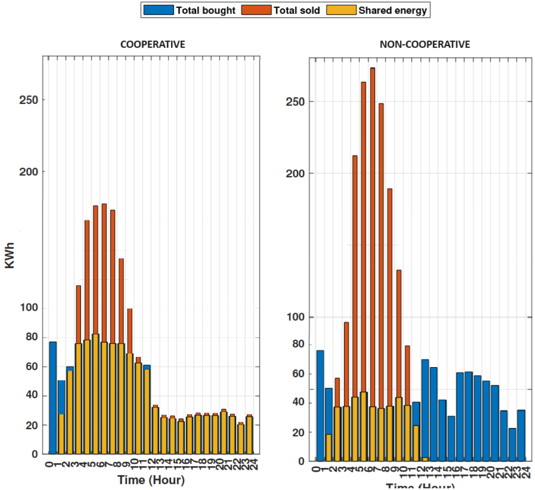
T.H. Chang, M. Hong, and X. Wang, "*Multi-agent distributed optimization via inexact consensus ADMM*", IEEE Transactions on Signal Processing, vol. 63, no. 2, pp. 482–497, 2014.

Outline

Results and discussion: profiles of consumption, generation and storage



Results and discussion: shared energy



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Thank you for your attention!

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