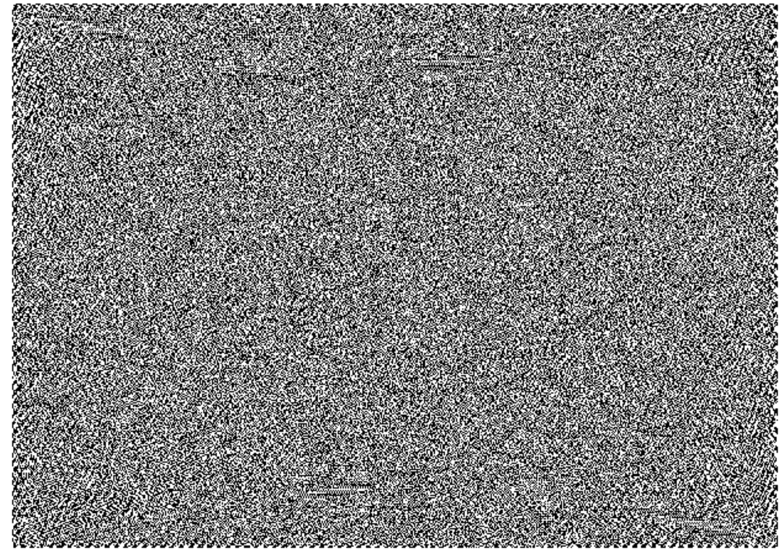
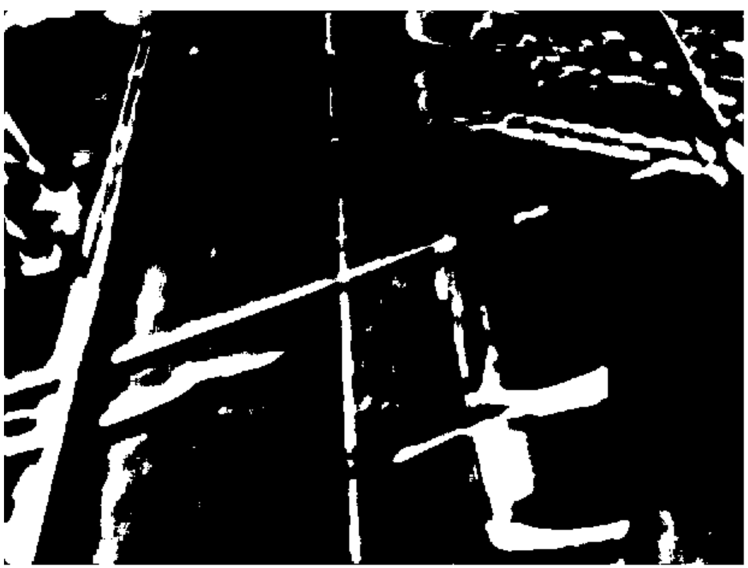
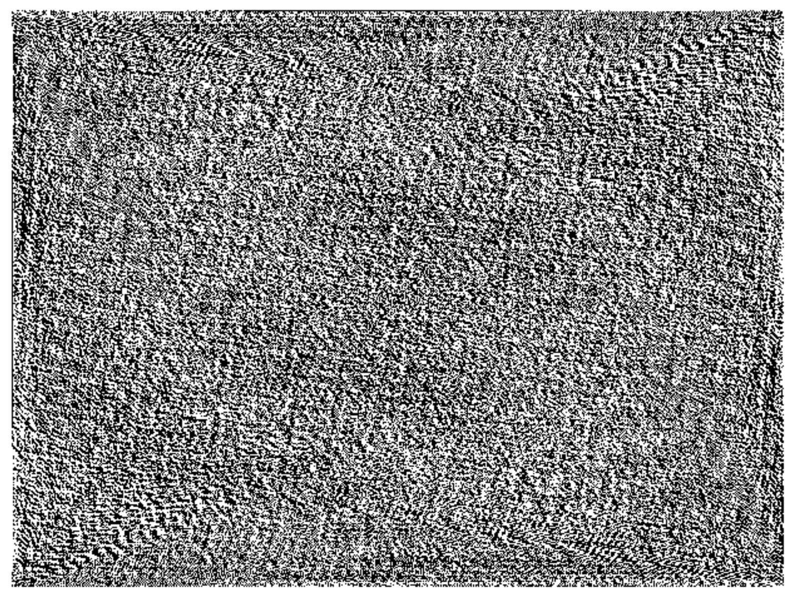
# Transformada de Fourier en imágenes

# T. F. en imágenes







# Identidad de Euler

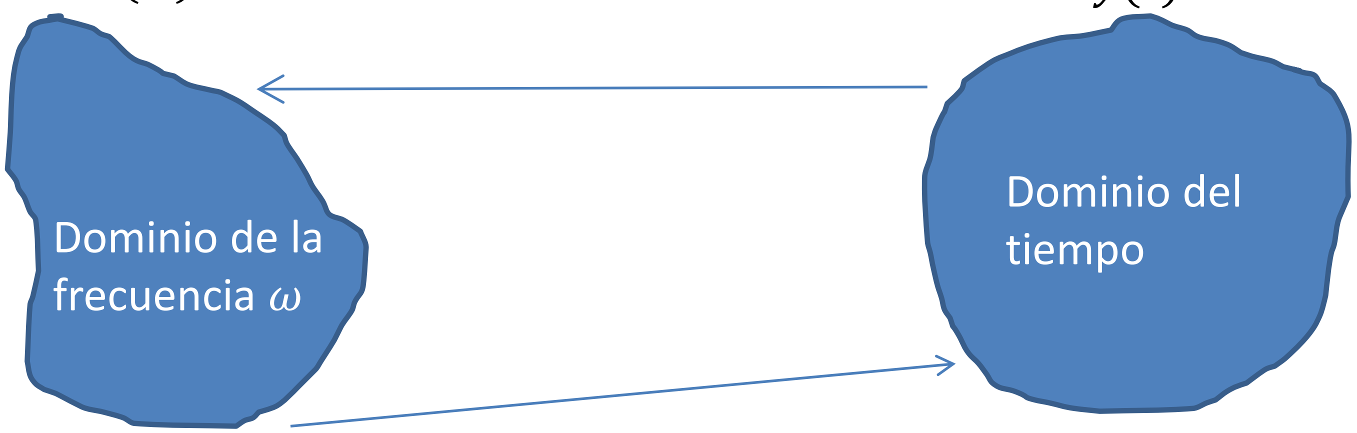
$$e^{ix} = \cos x + i\ \sin x$$

La exponencial puede expresarse en términos del coseno y seno imaginario.

# Definición de la T. F. unidimensional

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

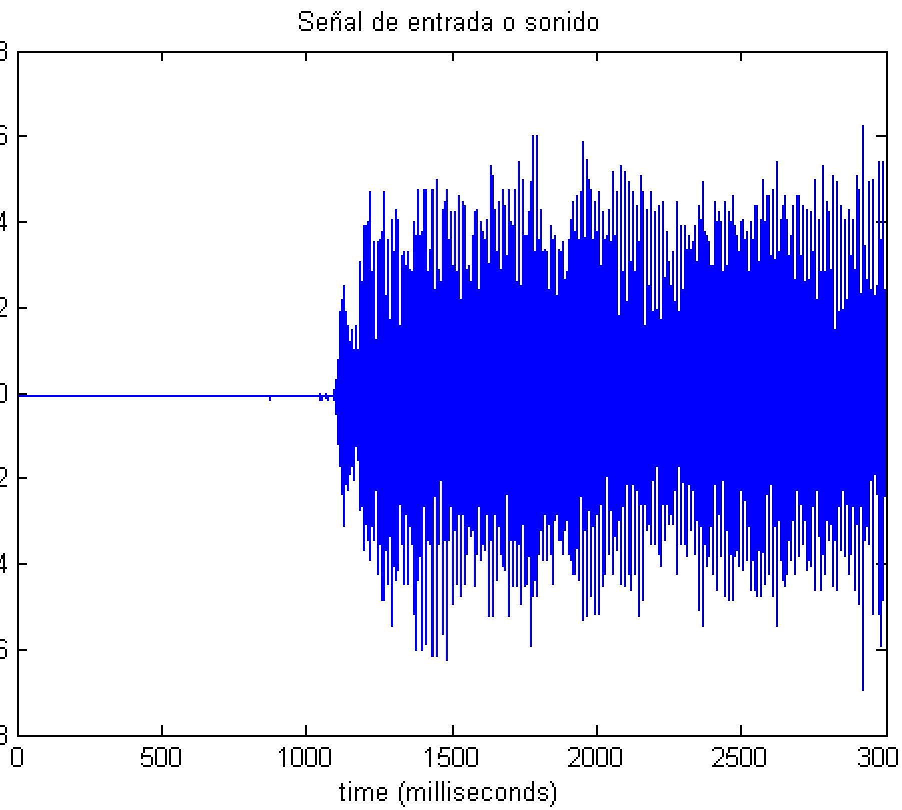
$Y(\omega)$ $y(t)$

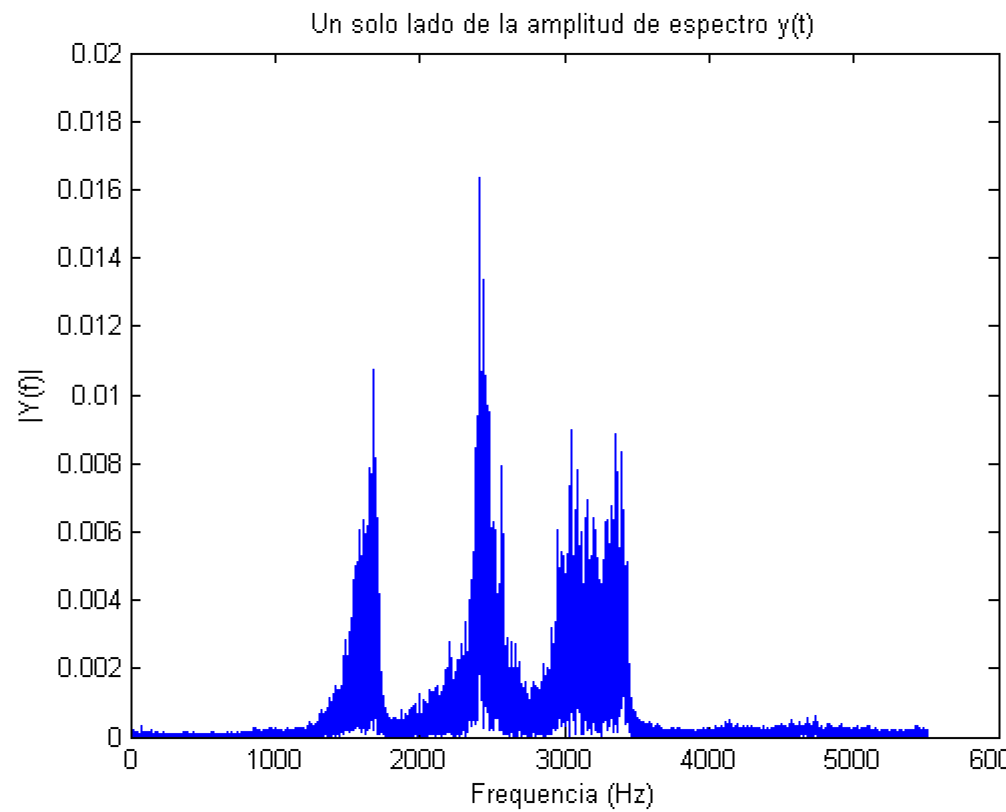



$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega) e^{j\omega t} d\omega$$

$\boldsymbol{\omega = 2\pi f}$

# Transformada de Fourier de una señal unidimensional



# Código en Matlab

```
[y,Fs]=wavread('bird_caw1.wav');

[L,M]=size(y);
figure(1)
plot(Fs*t(1:3000),y(1:3000))
title('Señal de entrada o sonido')
xlabel('time (milliseconds)')


NFFT = 2^nextpow2(L); % Next power of 2 from
length of y
Y = fft(y,NFFT)/L;
f = Fs/2*linspace(0,1,NFFT/2+1);

% Plot single-sided amplitude spectrum.
figure(2)
plot(f,2*abs(Y(1:NFFT/2+1)))
title('Un solo lado de la amplitud de espectro y(t)')
xlabel('Frequencia (Hz)')
ylabel('|Y(f)|')
```

# DFT- Discret Fourier Transform

Transformada de Fourier continua

$$\mathrm{F}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Transformada de Fourier Discreta 1D

$$F_k = \sum_{n=0}^{N-1} f_n e^{-\frac{2\pi i}{N} kn}$$

Transformada de Fourier Discreta 2D

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

Transformada inversa de Fourier discreta 1D

$$f = \frac{1}{N} \sum_{k=0}^{N-1} F_k e^{\frac{2\pi i}{N} kn}$$

Transformada inversa de Fourier discreta 2D

$$\mathrm{f}(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} f(u, v) e^{2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

# DFT de una matriz de 3x3

$$\begin{array}{cc} & \begin{array}{ccc} & x & \end{array} \\ y & \begin{bmatrix} a & b & c \\ d & l & f \\ g & h & m \end{bmatrix} \end{array}$$

$$F(1,0) = \sum_{x=0}^{3-1} \sum_{y=0}^{3-1} f(x,y) e^{-2\pi i\left(\frac{1(x)}{3} + \frac{0(y)}{3}\right)} = \sum_{x=0}^{2} \sum_{y=0}^{2} \ f(x,y) e^{-2\pi i\left(\frac{1(x)}{3}\right)}$$

$$\left[ae^{-2\pi i\left(\frac{1(0)}{3}\right)} + de^{-2\pi i\left(\frac{1(0)}{3}\right)} + ge^{-2\pi i\left(\frac{1(0)}{3}\right)}\right] = [a + d + g]e^{-2\pi i\left(\frac{1(0)}{3}\right)}$$

$$\left[be^{-2\pi i\left(\frac{1(1)}{3}\right)} + le^{-2\pi i\left(\frac{1(1)}{3}\right)} + he^{-2\pi i\left(\frac{1(1)}{3}\right)}\right] = [b + e + h]e^{-2\pi i\left(\frac{1(1)}{3}\right)}$$

$$\left[ce^{-2\pi i\left(\frac{1(2)}{3}\right)} + fe^{-2\pi i\left(\frac{1(2)}{3}\right)} + ie^{-2\pi i\left(\frac{1(2)}{3}\right)}\right] = [c + f + i]e^{-2\pi i\left(\frac{1(2)}{3}\right)}$$

F(1,0)= $[a + d + g]e^{-2\pi i\left(\frac{1(0)}{3}\right)} + [b + l + h]e^{-2\pi i\left(\frac{1(1)}{3}\right)} + [c + f + m]e^{-2\pi i\left(\frac{1(2)}{3}\right)}$

# DFT

F(2,0)= $[a+d+g]e^{-2\pi i\left(\frac{2(0)}{3}\right)}+[b+l+h]e^{-2\pi i\left(\frac{2(1)}{3}\right)}+[c+f+m]e^{-2\pi i\left(\frac{2(2)}{3}\right)}$

F(0,0)= $[a+d+g]e^{-2\pi i\left(\frac{0(0)}{3}\right)}+[b+l+h]e^{-2\pi i\left(\frac{0(1)}{3}\right)}+[c+f+m]e^{-2\pi i\left(\frac{0(2)}{3}\right)}$

$$F(1,1)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-2\pi i\left(\frac{1(x)}{3}+\frac{1(y)}{3}\right)}$$

$$F(0,1)=ae^{-2\pi i\left(\frac{0(0)}{3}+\frac{1(0)}{3}\right)}+\mathrm{d}e^{-2\pi i\left(\frac{0(0)}{3}+\frac{1(1)}{3}\right)}+ge^{-2\pi i\left(\frac{0(0)}{3}+\frac{1(2)}{3}\right)}+$$

$$\mathrm{b}e^{-2\pi i\left(\frac{0(1)}{3}+\frac{1(0)}{3}\right)}+le^{-2\pi i\left(\frac{0(1)}{3}+\frac{1(1)}{3}\right)}+he^{-2\pi i\left(\frac{0(1)}{3}+\frac{1(2)}{3}\right)}+$$

$$ce^{-2\pi i\left(\frac{0(2)}{3}+\frac{1(0)}{3}\right)}+fe^{-2\pi i\left(\frac{0(2)}{3}+\frac{1(1)}{3}\right)}+me^{-2\pi i\left(\frac{0(2)}{3}+\frac{1(2)}{3}\right)}$$

# DFT

$$F(1,1) = ae^{-2\pi i\left(\frac{1(0)}{3}+\frac{1(0)}{3}\right)} + \mathrm{d}e^{-2\pi i\left(\frac{1(0)}{3}+\frac{1(1)}{3}\right)} + ge^{-2\pi i\left(\frac{1(0)}{3}+\frac{1(2)}{3}\right)} +$$

$$\mathrm{b}e^{-2\pi i\left(\frac{1(1)}{3}+\frac{1(0)}{3}\right)} + le^{-2\pi i\left(\frac{1(1)}{3}+\frac{1(1)}{3}\right)} + he^{-2\pi i\left(\frac{1(1)}{3}+\frac{1(2)}{3}\right)} +$$

$$ce^{-2\pi i\left(\frac{1(2)}{3}+\frac{1(0)}{3}\right)} + fe^{-2\pi i\left(\frac{1(2)}{3}+\frac{1(1)}{3}\right)} + me^{-2\pi i\left(\frac{1(2)}{3}+\frac{1(2)}{3}\right)}$$

$$F(2,1) = ae^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(0)}{3}\right)} + \mathrm{d}e^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(1)}{3}\right)} + ge^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(2)}{3}\right)} +$$

$$\mathrm{b}e^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(0)}{3}\right)} + le^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(1)}{3}\right)} + he^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(2)}{3}\right)} +$$

$$ce^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(0)}{3}\right)} + fe^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(1)}{3}\right)} + me^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(2)}{3}\right)}$$

# DFT

$$F(2,0) = ae^{-2\pi i\left(\frac{2(0)}{3}+\frac{0(0)}{3}\right)} + \mathrm{d}e^{-2\pi i\left(\frac{2(0)}{3}+\frac{0(1)}{3}\right)} + ge^{-2\pi i\left(\frac{2(0)}{3}+\frac{0(2)}{3}\right)} +$$

$$\mathrm{b}e^{-2\pi i\left(\frac{2(1)}{3}+\frac{0(0)}{3}\right)} + le^{-2\pi i\left(\frac{2(1)}{3}+\frac{0(1)}{3}\right)} + he^{-2\pi i\left(\frac{2(1)}{3}+\frac{0(2)}{3}\right)} +$$

$$ce^{-2\pi i\left(\frac{2(2)}{3}+\frac{0(0)}{3}\right)} + fe^{-2\pi i\left(\frac{2(2)}{3}+\frac{0(1)}{3}\right)} + me^{-2\pi i\left(\frac{2(2)}{3}+\frac{0(2)}{3}\right)}$$

$$F(2,1) = ae^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(0)}{3}\right)} + \mathrm{d}e^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(1)}{3}\right)} + ge^{-2\pi i\left(\frac{2(0)}{3}+\frac{1(2)}{3}\right)} +$$

$$\mathrm{b}e^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(0)}{3}\right)} + le^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(1)}{3}\right)} + he^{-2\pi i\left(\frac{2(1)}{3}+\frac{1(2)}{3}\right)} +$$

$$ce^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(0)}{3}\right)} + fe^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(1)}{3}\right)} + me^{-2\pi i\left(\frac{2(2)}{3}+\frac{1(2)}{3}\right)}$$

$$F(2,2) = ae^{-2\pi i\left(\frac{2(0)}{3}+\frac{2(0)}{3}\right)} + \mathrm{d}e^{-2\pi i\left(\frac{2(0)}{3}+\frac{2(1)}{3}\right)} + ge^{-2\pi i\left(\frac{2(0)}{3}+\frac{2(2)}{3}\right)} +$$

$$\mathrm{b}e^{-2\pi i\left(\frac{2(1)}{3}+\frac{2(0)}{3}\right)} + le^{-2\pi i\left(\frac{2(1)}{3}+\frac{2(1)}{3}\right)} + he^{-2\pi i\left(\frac{2(1)}{3}+\frac{2(2)}{3}\right)} +$$

$$ce^{-2\pi i\left(\frac{2(2)}{3}+\frac{2(0)}{3}\right)} + fe^{-2\pi i\left(\frac{2(2)}{3}+\frac{2(1)}{3}\right)} + me^{-2\pi i\left(\frac{2(2)}{3}+\frac{2(2)}{3}\right)}$$

# Links de librerías de visión

http://www.csse.uwa.edu.au/~pk/research/matlabfns/

http://petercorke.com/Toolbox_software.html