HW3_diegoek2

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1 STATS 542: Homework 3

Diego Kleiman (diegoek2)

Due: Monday 11:59 PM CT, Feb 23rd

1.1 About HW3

In the first question, we will use a simulation study to confirm the theoretical analysis we developed during the lecture. In the second question, we will practice several linear model selection techniques such as AIC, BIC, and best subset selection. However, some difficulties are at the data processing part, in which we use the Bitcoin data from Kaggle. This is essentially a time-series data, and we use the information in previous days to predict the price in a future day. Make sure that you process the data correctly to fit this task.

1.2 Question 1 [40 Points] A Simulation Study

Let's use a simulation study to confirm the bias-variance trade-off of linear regressions. Consider the following model.

$$Y = \sum_{j}^{p} 0.9^j \times X_j + \epsilon$$

All the covariates and the error term follow i.i.d. standard Gaussian distribution. The true model involves all the variables; however, variables with larger indexes do not contribute significantly to the variation. Hence, there could be a benefit using a smaller subset for prediction purposes. Let's confirm that with a simulation study. The study essentially repeats the following steps 200 times and obtain the averaged results:

- Generate 300 training data (both covariates and outcomes) with $p = 100$, and generate another 300 outcomes as the testing data Y using the same covariate value.
- Consider using only the first j variables to fit the linear regression. Let j ranges from 1 to 100. Calculate and record the corresponding prediction error.
- For each j value, we also have the theoretical analysis of the testing error based on the lecture. In that analysis, we have the formula of both the Bias^2 and variance. Plug-in the simulated data to calculate the Bias^2 and use the theoretical value for the variance.

After finishing all simulation runs, plot your results using the `number of variables` as the x-axis, and the following 4 lines:

- The averaged prediction error based on your 200 simulation runs
- The averaged Bias^2 based on your 200 simulation runs
- The theoretical variance
- The sum of Bias^2 + variance + Irreducible Error

Does your simulation result match our theoretical analysis? Comment on your findings.

[1]:
```python
import numpy as np
from matplotlib import pyplot as plt
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
```

[2]:
```python
plt.style.use('seaborn-poster')
plt.style.use('seaborn-darkgrid')
```

[3]:
```python
# Initilize arrays/asign variables

beta = np.asarray([0.9**j for j in range(1, 101)])
prediction_errors = np.zeros(100)
bias_terms = np.zeros(100)
trials = 200

# Repeat 200 times
for _ in range(trials):

    X = np.random.normal(0, 1, size=(300, 100))
    e_train = np.random.normal(0, 1, size=(300))
    e_test = np.random.normal(0, 1, size=(300))

    X_beta = np.matmul(X, beta)
    Y_train = X_beta + e_train
    Y_test = X_beta + e_test

    for j in range(100):

        # Fit model using first j covariates
        X_temp = X[:, :(j+1)]
        model = LinearRegression(fit_intercept=False).fit(X_temp, Y_train)

        # Predict on test data
        Y_hat = model.predict(X_temp)
        residuals = (Y_test-Y_hat)
        error = np.sum(residuals**2)

        # Compute hat matrix
```

```
        H = np.matmul(np.matmul(X_temp, np.linalg.inv(np.matmul(X_temp.T,␣
↪X_temp))), X_temp.T)

        # Populate data vectors
        prediction_errors[j] += error/300
        bias_terms[j] += np.sum((X_beta - np.matmul(H, X_beta))**2)/300

# Divide by number of trials
prediction_errors /= trials
bias_terms /= trials
```

Theoretical values:

$$Variance = \frac{p \cdot \sigma^2}{n}$$

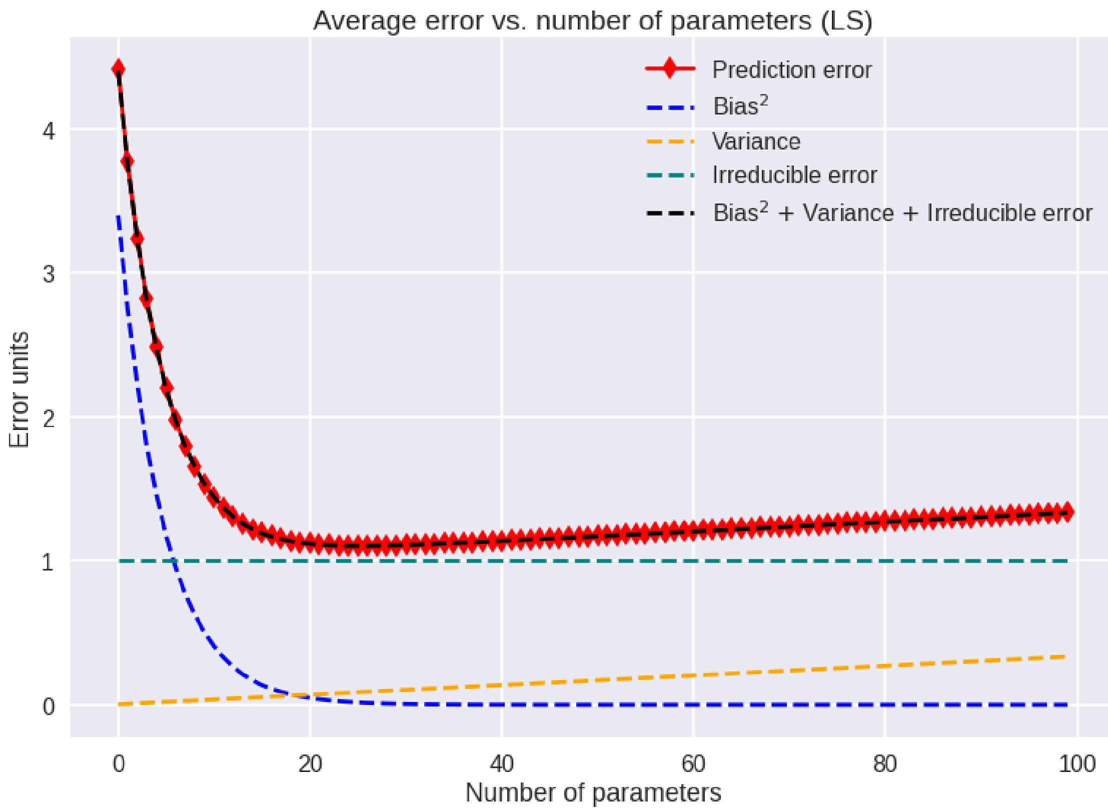
$$Irreducible error = \sigma^2$$

[4]:

```
# Theoretical values
sigma = 1
n = 300
variance = np.arange(1, 101)*(sigma**2)/n
irr_error = np.ones(100)*(sigma**2)
```

[5]:

```
plt.plot(prediction_errors, 'd-', c='red')
plt.plot(bias_terms, '--', c='blue')
plt.plot(variance, '--', c='orange')
plt.plot(irr_error, '--', c='teal')
plt.plot(bias_terms + variance + irr_error, '--', c='black')
plt.title("Average error vs. number of parameters (LS)")
plt.legend(["Prediction error", "Bias$^2$", "Variance", "Irreducible error",␣
↪"Bias$^2$ $+$ Variance $+$ Irreducible error"])
plt.xlabel("Number of parameters")
plt.ylabel("Error units")
plt.plot()
plt.show()
plt.close()
```



Does your simulation result match our theoretical analysis? Comment on your findings.

Answer:

My simulation clearly matches our theoretical analysis. This is clear from the plot because the sum of the three error terms (Bias^2 + Variance + Irreducible error) shows excellent agreement with the test error from the simulations.

1.3 Question 2 [60 Points] Bitcoin price prediction

For this question, we will use the Bitcoin data provided on the course website. The data were posted originally on Kaggle (link). Make sure that you read relevant information from the Kaggle website. Our data is the `bitcoin_dataset.csv` file. You should use a training/testing split such that your training data is constructed using only information up to 12/31/2016, and your testing data is constructed using only information starting from 01/01/2017. The goal of our analysis is to predict the `btc_market_price`. Since this is longitudinal data, we will use the information from previous days to predict the market price at a future day. In particular, on each calendar day (say, day 1), we use the information from three days onward (days 1, 2, and 3) to predict the market price on the 7th day.

Hence you need to first reconstruct the data properly to fit this purpose. This is mainly to put the outcome (of day 7) and the covariates (of the previous days) into the same row. Note that for this question, you may face issues such as missing data, categorical predictors, outliers, scaling issue, computational issue, and maybe others. Use your best judgment to deal with them. There is no general ''best answer". Hence the grading will be based on whether you provided reasoning for your decision and whether you carried out the analysis correctly.

1.3.1 [15 Points] Data Construction

Data pre-processing is usually the most time-consuming and difficult part of an analysis. We will use this example as a practice. Construct your data appropriately such that further analysis can be performed. Make sure that you consider the following:

- The data is appropriate for our analysis goal: each row contains the outcome on the seventh day and the covariates of the first three days
- Missing data is addressed (you can remove variable, remove observation, impute values or propose your own method)
- Process each single covariate/outcome by considering centering/scaling/transformation and/or removing outliers

For each of the above tasks, make sure that you **clearly document your choice**. In the end, provide a summary table/figure of your data. You can consider using boxplots, quantiles, histogram, or any method that is easy for readers to understand. You are required to pick one at least one method to present.

Answer:

I am following these steps: 1. Convert `Date` column to datetime type. This is just to create the train/test split. 1. Split data in training and testing sets. (Question requirement.) 1. Rearrange data by placing all covariates from days n to $n+3$ in the same row and set `btc_market_price` of day $n+6$ as the outcome of said row for $n \in [1, N-6]$ where N is the total number of days for which we have data in each dataset (train and test). (Question requirement.) 1. Drop rows where a column is missing. (This is a data processing decision that I consider appropriate since very few values were missing.) 1. Stardardize all covariates and outcome. I use stardardization because I want the data to be centered and scaled. I will use RobustScaler from sklearn as it is robust against outliers because it uses the median and interquartile range instead of the mean and standard deviation.

[6]:
```
import pandas as pd
bitcoin_raw_data = pd.read_csv('bitcoin.csv') # Load data
```

[7]:
```
bitcoin_raw_data.Date = pd.to_datetime(bitcoin_raw_data.Date) # Convert to␣
↪datetime format
```

[8]:
```
# Split test and train data
bitcoin_raw_train = bitcoin_raw_data[bitcoin_raw_data.Date <= '2016-12-31'] #␣
↪Before or on 12/31/2016
bitcoin_raw_test = bitcoin_raw_data[bitcoin_raw_data.Date >= '2017-01-01'] # On␣
↪or after 01/01/2017
bitcoin_raw_test = bitcoin_raw_test.reset_index(drop=True) # Make sure index␣
↪starts at zero
```

[9]:
```
# Get new (unique) column names for each column in design matrix
temp = bitcoin_raw_train.loc[:2, bitcoin_raw_train.columns != 'Date']
temp.index = temp.index + 1
temp2 = temp.stack()
temp2.index = temp2.index.map('{0[1]}_{0[0]}'.format)
new_column_names = temp2.to_frame().T.columns.to_list()
```

[10]:
```
# Convert to numpy to rearrange data
bitcoin_raw_train_numpy = bitcoin_raw_train.to_numpy()
bitcoin_raw_test_numpy = bitcoin_raw_test.to_numpy()

# Drop "Date" column (they are ordered by index anyway)
bitcoin_raw_train_numpy = bitcoin_raw_train_numpy[:, 1:]
bitcoin_raw_test_numpy = bitcoin_raw_test_numpy[:, 1:]

# Build training dataset
N = bitcoin_raw_train.shape[0]
train_set = np.empty((N-6, 69)) # N-6 samples and 69 columns
train_outcomes = np.empty((N-6))
for n in range(N-6):
    row = bitcoin_raw_train_numpy[n:n+3].reshape(1, 69)
    train_set[n, :] = row
    train_outcomes[n] = bitcoin_raw_train_numpy[n+6, 0]

# Build testing dataset
N = bitcoin_raw_test.shape[0]
test_set = np.empty((N-6, 69)) # N-5 samples and 69 columns
test_outcomes = np.empty((N-6))
for n in range(N-6):
    row = bitcoin_raw_test_numpy[n:n+3].reshape(1, 69)
    test_set[n, :] = row
    test_outcomes[n] = bitcoin_raw_test_numpy[n+6, 0]
```

[11]:
```
train_set = pd.DataFrame(np.append(train_set, train_outcomes.
 ↪reshape(train_outcomes.shape[0], 1), axis=1), columns=new_column_names +␣
 ↪['outcome'])
```

[12]:
```
train_set
```

[12]:
```
      btc_market_price_1  btc_total_bitcoins_1  btc_market_cap_1  \
0               0.000000             2110700.0      0.000000e+00
1               0.000000             2120200.0      0.000000e+00
2               0.000000             2127600.0      0.000000e+00
3               0.000000             2136100.0      0.000000e+00
4               0.000000             2144750.0      0.000000e+00
…                    …                    …                 …
2493          824.218938            16056275.0      1.326698e+10
```

```
2494          860.599875            16058262.5      1.381557e+10
2495          901.318238            16060162.5      1.450361e+10
2496          891.612612            16061875.0      1.436526e+10
2497          886.900375            16063937.5      1.437240e+10

      btc_trade_volume_1  btc_blocks_size_1  btc_avg_block_size_1  \
0           0.000000e+00           0.000000              0.000216
1           0.000000e+00           0.000000              0.000282
2           0.000000e+00           0.000000              0.000227
3           0.000000e+00           0.000000              0.000319
4           0.000000e+00           0.000000              0.000223
…                    …                  …                   …
2493        4.485404e+07       94932.381938              0.963966
2494        5.417773e+07       95078.635893              0.919836
2495        7.112861e+07       95213.610311              0.887990
2496        3.663265e+07       95330.730173              0.854890
2497        3.141146e+07       95438.863856              0.647507

      btc_n_orphaned_blocks_1  btc_n_transactions_per_block_1  \
0                         0.0                        1.000000
1                         0.0                        1.000000
2                         0.0                        1.000000
3                         0.0                        1.000000
4                         0.0                        1.000000
…                         …                             …
2493                      0.0                     2048.283019
2494                      0.0                     1978.893082
2495                      0.0                     2006.559211
2496                      0.0                     1757.810219
2497                      2.0                     1195.293413

      btc_median_confirmation_time_1  btc_hash_rate_1  …  \
0                           0.000000     3.153929e-05  …
1                           0.000000     3.571305e-05  …
2                           0.000000     2.781859e-05  …
3                           0.000000     3.195378e-05  …
4                           0.000000     3.251768e-05  …
…                               …               …   …
2493                        8.783333     2.451435e+06  …
2494                        8.583333     2.451435e+06  …
2495                       12.775000     2.343510e+06  …
2496                       16.325000     2.112243e+06  …
2497                       10.300000     2.574778e+06  …

      btc_cost_per_transaction_3  btc_n_unique_addresses_3  \
0                       0.000000                     150.0
1                       0.000000                     176.0
```

```
2                           0.000000                        176.0
3                           0.000000                        165.0
4                           0.000000                        187.0
…                                …                          …
2493                        5.923338                     522516.0
2494                        6.670388                     420206.0
2495                        9.637165                     353857.0
2496                        6.872294                     448557.0
2497                        6.455267                     509115.0

      btc_n_transactions_3  btc_n_transactions_total_3  \
0                    150.0                     42959.0
1                    176.0                     43135.0
2                    176.0                     43311.0
3                    165.0                     43476.0
4                    187.0                     43663.0
…                      …                         …
2493              304997.0                 181735754.0
2494              240820.0                 181976574.0
2495              199614.0                 182176188.0
2496              244914.0                 182421102.0
2497              296546.0                 182717648.0

      btc_n_transactions_excluding_popular_3  \
0                                      150.0
1                                      176.0
2                                      176.0
3                                      165.0
4                                      187.0
…                                        …
2493                                237189.0
2494                                195740.0
2495                                241536.0
2496                                292846.0
2497                                301503.0

      btc_n_transactions_excluding_chains_longer_than_100_3  \
0                                                 150.0
1                                                 176.0
2                                                 176.0
3                                                 165.0
4                                                 187.0
…                                                   …
2493                                           205183.0
2494                                           177864.0
2495                                           148365.0
2496                                           172836.0
```

```
2497                                         199187.0

      btc_output_volume_3  btc_estimated_transaction_volume_3  \
0            8.100000e+03                          700.000000
1            2.934900e+04                        13162.000000
2            9.101000e+03                          450.000000
3            1.339900e+04                         5250.000000
4            1.030000e+04                         1000.000000
…                     …                                   …
2493         2.633796e+06                       431035.319296
2494         1.526634e+06                       223986.762894
2495         1.105165e+06                       232569.579907
2496         1.513037e+06                       270013.079151
2497         2.242010e+06                       345270.853758

      btc_estimated_transaction_volume_usd_3     outcome
0                               0.000000e+00    0.000000
1                               0.000000e+00    0.000000
2                               0.000000e+00    0.000000
3                               0.000000e+00    0.000000
4                               0.000000e+00    0.000000
…                                        …           …
2493                            3.892594e+08  930.376000
2494                            2.003270e+08  967.480375
2495                            2.080800e+08  963.381625
2496                            2.439460e+08  952.156375
2497                            3.224864e+08  959.879875

[2498 rows x 70 columns]
```

[13]:
```
test_set = pd.DataFrame(np.append(test_set, test_outcomes.reshape(test_outcomes.
 ↪shape[0], 1), axis=1), columns=new_column_names + ['outcome'])
```

[14]:
```
test_set
```

[14]:
```
     btc_market_price_1  btc_total_bitcoins_1  btc_market_cap_1  \
0            997.729875            16077350.0      1.614487e+10
1           1015.977112            16079337.5      1.644112e+10
2           1023.141875            16081387.5      1.660403e+10
3           1126.763338            16083300.0      1.827867e+10
4            994.674875            16085050.0      1.602071e+10
..                  …                    …                …
405         8319.876566            16857300.0      1.402507e+11
406         8343.455000            16859187.5      1.406639e+11
407         8811.343333            16861262.5      1.485704e+11
408         8597.767500            16863312.5      1.449868e+11
409         9334.633333            16865550.0      1.574337e+11
```

```
     btc_trade_volume_1  btc_blocks_size_1  btc_avg_block_size_1  \
0          3.399191e+07       96345.452397              0.698540
1          5.759703e+07       96496.686192              0.945211
2          4.155099e+07       96645.364249              0.906574
3          1.431343e+08       96797.229978              0.986141
4          2.084983e+08       96935.017543              0.984197
..                  …                  …                   …
405        9.185402e+08      156085.952197              1.034792
406        7.963917e+08      156237.996989              1.006919
407        8.366237e+08      156410.851393              1.041292
408        1.884544e+08      156580.256203              1.032956
409        9.151655e+08      156762.090638              1.015835

     btc_n_orphaned_blocks_1  btc_n_transactions_per_block_1  \
0                        0.0                     1157.064103
1                        1.0                     1835.512500
2                        0.0                     1839.414634
3                        1.0                     2143.922078
4                        0.0                     2060.721429
..                       …                            …
405                      0.0                     1271.209790
406                      0.0                      974.887417
407                      0.0                     1126.012048
408                      0.0                     1115.756098
409                      0.0                     1097.301676

     btc_median_confirmation_time_1  btc_hash_rate_1  …  \
0                         10.850000     2.463611e+06  …
1                          9.816667     2.526780e+06  …
2                         12.350000     2.589950e+06  …
3                         10.700000     2.432026e+06  …
4                         10.916667     2.210933e+06  …
..                              …               …   …
405                        9.450000     2.043482e+07  …
406                        7.533333     2.157803e+07  …
407                       10.200000     2.372154e+07  …
408                       11.050000     2.343574e+07  …
409                       10.225000     2.557925e+07  …

     btc_cost_per_transaction_3  btc_n_unique_addresses_3  \
0                      7.418693                  515023.0
1                      7.300058                  545635.0
2                      6.438563                  495909.0
3                      5.471699                  562749.0
4                      5.842191                  503340.0
..                          …                       …
```

```
405                 101.114203                437380.0
406                  99.544224                414749.0
407                 109.366318                450367.0
408                  85.662879                475029.0
409                 109.005818                422415.0

     btc_n_transactions_3  btc_n_transactions_total_3  \
0                301664.0                 184640586.0
1                330164.0                 184970750.0
2                288501.0                 185259251.0
3                346405.0                 185605656.0
4                284719.0                 185890375.0
..                    …                         …
405              186918.0                 299092204.0
406              182984.0                 299275188.0
407              196417.0                 299471605.0
408              198183.0                 299669788.0
409              187738.0                 299857526.0

     btc_n_transactions_excluding_popular_3  \
0                                  326689.0
1                                  284086.0
2                                  284086.0
3                                  342362.0
4                                  280726.0
..                                      …
405                                183345.0
406                                179616.0
407                                192621.0
408                                194351.0
409                                180685.0

     btc_n_transactions_excluding_chains_longer_than_100_3  \
0                                             200407.0
1                                             215669.0
2                                             193322.0
3                                             233379.0
4                                             187066.0
..                                                 …
405                                           133062.0
406                                           128687.0
407                                           140594.0
408                                           139075.0
409                                           126413.0

     btc_output_volume_3  btc_estimated_transaction_volume_3  \
0           1.950525e+06                       329964.393518
```

```
1           2.503952e+06                       386434.819254
2           2.998215e+06                       526684.339439
3           3.100652e+06                       483798.943372
4           2.162213e+06                       344066.519218
..                   …                                  …
405         1.674742e+06                       181029.654089
406         1.315763e+06                       146929.345775
407         1.557017e+06                       148097.503082
408         1.418774e+06                       129180.361091
409         1.102896e+06                       106729.749001

     btc_estimated_transaction_volume_usd_3       outcome
0                              3.406882e+08    896.830375
1                              4.391832e+08    908.149037
2                              5.245776e+08    894.180250
3                              4.284523e+08    906.056914
4                              3.102585e+08    785.223737
..                                      …            …
405                            1.595114e+09  10127.161667
406                            1.263264e+09  10841.991667
407                            1.382436e+09  10503.298333
408                            1.288852e+09  11110.965000
409                            1.080869e+09  11390.391667

[410 rows x 70 columns]
```

[15]:
```
# Check for missing data
train_set.isnull().values.any()
```

[15]:
```
True
```

Since missing data were found in train_set, we will drop them.

[16]:
```
train_set = train_set.dropna()
```

[17]:
```
train_set
```

[17]:
```
      btc_market_price_1  btc_total_bitcoins_1  btc_market_cap_1  \
0               0.000000             2110700.0      0.000000e+00
1               0.000000             2120200.0      0.000000e+00
2               0.000000             2127600.0      0.000000e+00
3               0.000000             2136100.0      0.000000e+00
4               0.000000             2144750.0      0.000000e+00
…                    …                     …                 …
2493          824.218938            16056275.0      1.326698e+10
2494          860.599875            16058262.5      1.381557e+10
2495          901.318238            16060162.5      1.450361e+10
```

```
2496        891.612612         16061875.0      1.436526e+10
2497        886.900375         16063937.5      1.437240e+10

      btc_trade_volume_1  btc_blocks_size_1  btc_avg_block_size_1  \
0           0.000000e+00           0.000000              0.000216
1           0.000000e+00           0.000000              0.000282
2           0.000000e+00           0.000000              0.000227
3           0.000000e+00           0.000000              0.000319
4           0.000000e+00           0.000000              0.000223
…                    …                  …                     …
2493        4.485404e+07       94932.381938              0.963966
2494        5.417773e+07       95078.635893              0.919836
2495        7.112861e+07       95213.610311              0.887990
2496        3.663265e+07       95330.730173              0.854890
2497        3.141146e+07       95438.863856              0.647507

      btc_n_orphaned_blocks_1  btc_n_transactions_per_block_1  \
0                         0.0                        1.000000
1                         0.0                        1.000000
2                         0.0                        1.000000
3                         0.0                        1.000000
4                         0.0                        1.000000
…                          …                              …
2493                      0.0                     2048.283019
2494                      0.0                     1978.893082
2495                      0.0                     2006.559211
2496                      0.0                     1757.810219
2497                      2.0                     1195.293413

      btc_median_confirmation_time_1  btc_hash_rate_1  …  \
0                           0.000000     3.153929e-05  …
1                           0.000000     3.571305e-05  …
2                           0.000000     2.781859e-05  …
3                           0.000000     3.195378e-05  …
4                           0.000000     3.251768e-05  …
…                                …              …  …
2493                        8.783333     2.451435e+06  …
2494                        8.583333     2.451435e+06  …
2495                       12.775000     2.343510e+06  …
2496                       16.325000     2.112243e+06  …
2497                       10.300000     2.574778e+06  …

      btc_cost_per_transaction_3  btc_n_unique_addresses_3  \
0                       0.000000                     150.0
1                       0.000000                     176.0
2                       0.000000                     176.0
3                       0.000000                     165.0
```

```
4                       0.000000                  187.0
…                          …                       …
2493                    5.923338               522516.0
2494                    6.670388               420206.0
2495                    9.637165               353857.0
2496                    6.872294               448557.0
2497                    6.455267               509115.0

      btc_n_transactions_3  btc_n_transactions_total_3  \
0                    150.0                     42959.0
1                    176.0                     43135.0
2                    176.0                     43311.0
3                    165.0                     43476.0
4                    187.0                     43663.0
…                     …                          …
2493              304997.0                 181735754.0
2494              240820.0                 181976574.0
2495              199614.0                 182176188.0
2496              244914.0                 182421102.0
2497              296546.0                 182717648.0

      btc_n_transactions_excluding_popular_3  \
0                                      150.0
1                                      176.0
2                                      176.0
3                                      165.0
4                                      187.0
…                                       …
2493                                237189.0
2494                                195740.0
2495                                241536.0
2496                                292846.0
2497                                301503.0

      btc_n_transactions_excluding_chains_longer_than_100_3  \
0                                                     150.0
1                                                     176.0
2                                                     176.0
3                                                     165.0
4                                                     187.0
…                                                      …
2493                                               205183.0
2494                                               177864.0
2495                                               148365.0
2496                                               172836.0
2497                                               199187.0
```

```
      btc_output_volume_3  btc_estimated_transaction_volume_3  \
0            8.100000e+03                          700.000000
1            2.934900e+04                        13162.000000
2            9.101000e+03                          450.000000
3            1.339900e+04                         5250.000000
4            1.030000e+04                         1000.000000
…                   …                                   …
2493         2.633796e+06                       431035.319296
2494         1.526634e+06                       223986.762894
2495         1.105165e+06                       232569.579907
2496         1.513037e+06                       270013.079151
2497         2.242010e+06                       345270.853758

      btc_estimated_transaction_volume_usd_3     outcome
0                                 0.000000e+00    0.000000
1                                 0.000000e+00    0.000000
2                                 0.000000e+00    0.000000
3                                 0.000000e+00    0.000000
4                                 0.000000e+00    0.000000
…                                          …        …
2493                              3.892594e+08  930.376000
2494                              2.003270e+08  967.480375
2495                              2.080800e+08  963.381625
2496                              2.439460e+08  952.156375
2497                              3.224864e+08  959.879875

[2449 rows x 70 columns]
```

[18]:
```
test_set.isnull().values.any()
```

[18]:
```
False
```

Since we don't have any missing data in the test data, we don't have to drop any rows.

[19]:
```
# Stardardizing columns
# Columns will be stardardized to make sure they follow a centered Gaussian␣
↪distribution
from sklearn.preprocessing import RobustScaler

scaler = RobustScaler().fit(train_set)
train_set = scaler.transform(train_set)
test_set = scaler.transform(test_set)
```

1.3.2 [15 Points] Model Selection Criterion

Use AIC and BIC criteria to select the best model and report the result from each of them. Use the forward selection for AIC and backward selection for BIC. Report the following two error quantities from **both training and testing data**.

- The mean squared error: $n^{-1} \sum_i (Y_i - \widehat{Y}_i)^2$
- The proportion of explained variation (R^2): $1 - \frac{n^{-1} \sum_i (Y_i - \widehat{Y}_i)^2}{n^{-1} \sum_i (Y_i - \overline{Y}_i)^2}$

Since these quantities can be affected by scaling and transformation, make sure that you **state any modifications applied to the outcome variable**. Compare the training data errors and testing data errors, which model works better? Provide a summary of your results.

Note: I have applied RobustScaler on the outcome variable.

[20]:
```
from statsmodels.regression.linear_model import OLS
from statsmodels.tools import add_constant # Will use to add intercept term
```

Select best model based on AIC using forward selection.

[21]:
```
total_parameters = train_set.shape[1] - 1
remaining_parameters = np.asarray(range(total_parameters)).astype(int)
selected_parameters = np.asarray([]).astype(int) # We start with null model

y = train_set[:, -1]

best_AIC_prev = np.inf
best_AIC = 1e21
best_parameter_AIC = 1e20 # Initialize value to some big number
current_AIC = 0

while (remaining_parameters.size > 0) and (best_AIC < best_AIC_prev):

    best_parameter_AIC = 1e20 # Initialize value to some big number

    # Check which parameter is better to add
    for i in remaining_parameters:
        partial_train_set = train_set[:, np.append(selected_parameters, i)] #␣
↪Add parameter i
        model = OLS(y, add_constant(partial_train_set)).fit()
        current_AIC = model.aic # Retrieve AIC for model
        if (current_AIC < best_parameter_AIC):
            best_parameter = i
            best_parameter_AIC = current_AIC

    # Check if model is better overall
    if (best_parameter_AIC < best_AIC):
        # Add new parameter to list of selected parameters
        selected_parameters = np.append(selected_parameters, best_parameter)
        # Remove parameter from remaining parameters
        remaining_parameters = remaining_parameters[remaining_parameters !=␣
↪best_parameter]
```

```
    # Update score so far
    best_AIC_prev = best_AIC
    best_AIC = best_parameter_AIC
```

[22]:
```
# Lock-in best model from forward selection
best_model_forward = OLS(y, add_constant(train_set[:, selected_parameters])).
↪fit()
selected_parameters_forward = selected_parameters
```

Features that were selected in forward selection:

[23]:
```
print(np.asarray(new_column_names)[selected_parameters])
```

```
['btc_market_price_3' 'btc_estimated_transaction_volume_usd_3'
 'btc_estimated_transaction_volume_usd_2' 'btc_cost_per_transaction_3'
 'btc_miners_revenue_3' 'btc_n_transactions_per_block_3'
 'btc_avg_block_size_3' 'btc_trade_volume_3' 'btc_n_transactions_total_1'
 'btc_blocks_size_2' 'btc_total_bitcoins_1' 'btc_miners_revenue_1'
 'btc_n_transactions_2' 'btc_n_transactions_per_block_2'
 'btc_n_transactions_excluding_popular_3' 'btc_market_cap_1'
 'btc_estimated_transaction_volume_usd_1' 'btc_n_transactions_1'
 'btc_hash_rate_2' 'btc_transaction_fees_3' 'btc_blocks_size_3'
 'btc_n_transactions_3' 'btc_median_confirmation_time_3'
 'btc_n_unique_addresses_2' 'btc_transaction_fees_1'
 'btc_n_orphaned_blocks_1' 'btc_difficulty_3' 'btc_hash_rate_3'
 'btc_n_transactions_total_2']
```

Select best model based on BIC using backward selection.

[24]:
```
total_parameters = train_set.shape[1] - 1
selected_parameters = np.asarray(range(total_parameters)).astype(int) # We␣
↪start with full model

y = train_set[:, -1]

best_BIC_prev = np.inf
best_BIC = 1e21 # Initialize value to some big number
best_parameter_BIC = 1e20 # Initialize value to some big number
current_BIC = 0

while (selected_parameters.size > 0) and (best_BIC < best_BIC_prev):

    best_parameter_BIC = 1e20 # Initialize value to some big number

    # Check which parameter is better to remove
    for i in selected_parameters:
        parameters = selected_parameters[selected_parameters != i] # Remove␣
↪parameter i
```

```
        partial_train_set = train_set[:, parameters]
        model = OLS(y, add_constant(partial_train_set)).fit()
        current_BIC = model.bic # Retrieve BIC for model
        if (current_BIC < best_parameter_BIC):
            best_parameter_to_remove = i
            best_parameter_BIC = current_BIC

    # Check if model is better overall
    if (best_parameter_BIC < best_BIC):
        # Remove parameter from selected parameters
        selected_parameters = selected_parameters[selected_parameters !=␣
↪best_parameter_to_remove]

    # Update score so far
    best_BIC_prev = best_BIC
    best_BIC = best_parameter_BIC
```

[25]:
```
# Lock-in best model from backward selection
best_model_backward = OLS(y, add_constant(train_set[:, selected_parameters])).
↪fit()
selected_parameters_backward = selected_parameters
```

Features that were selected in backward selection:

[26]:
```
print(np.asarray(new_column_names)[selected_parameters])
```

```
['btc_market_cap_1' 'btc_miners_revenue_1' 'btc_n_transactions_total_1'
 'btc_blocks_size_2' 'btc_n_transactions_2'
 'btc_estimated_transaction_volume_usd_2' 'btc_market_price_3'
 'btc_total_bitcoins_3' 'btc_trade_volume_3'
 'btc_n_transactions_per_block_3' 'btc_hash_rate_3' 'btc_difficulty_3'
 'btc_miners_revenue_3' 'btc_cost_per_transaction_3'
 'btc_n_transactions_3' 'btc_n_transactions_total_3'
 'btc_estimated_transaction_volume_usd_3']
```

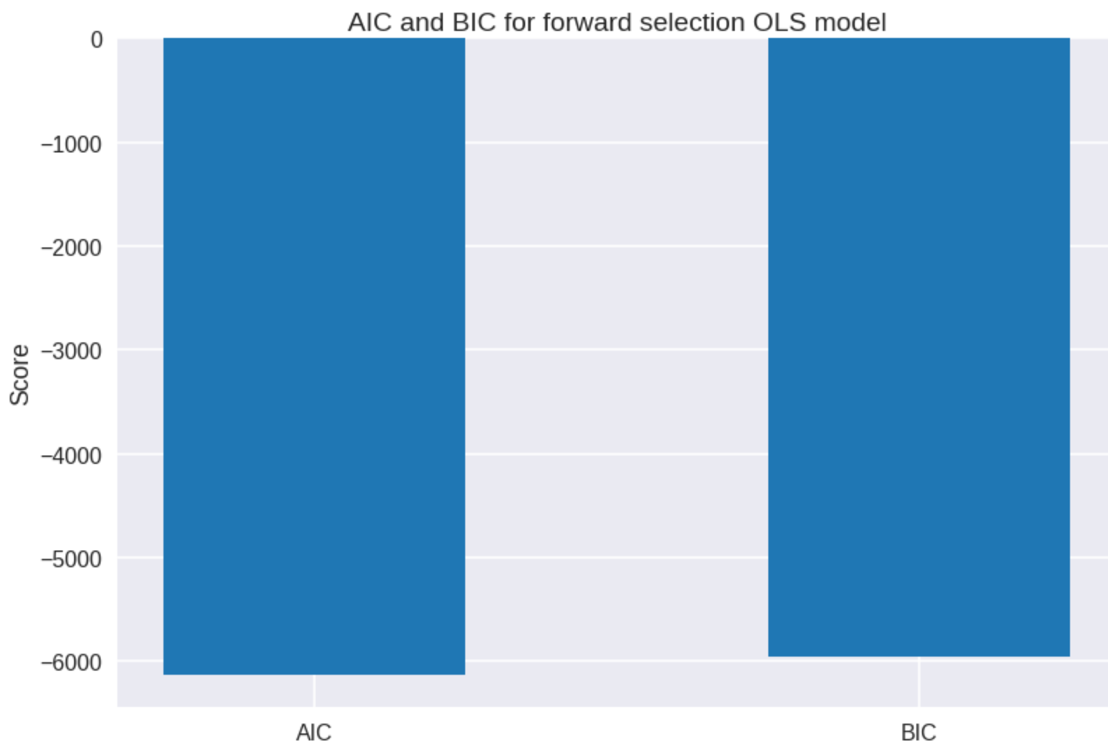
1.3.3 Error reporting

Errors from AIC model (forward selection)

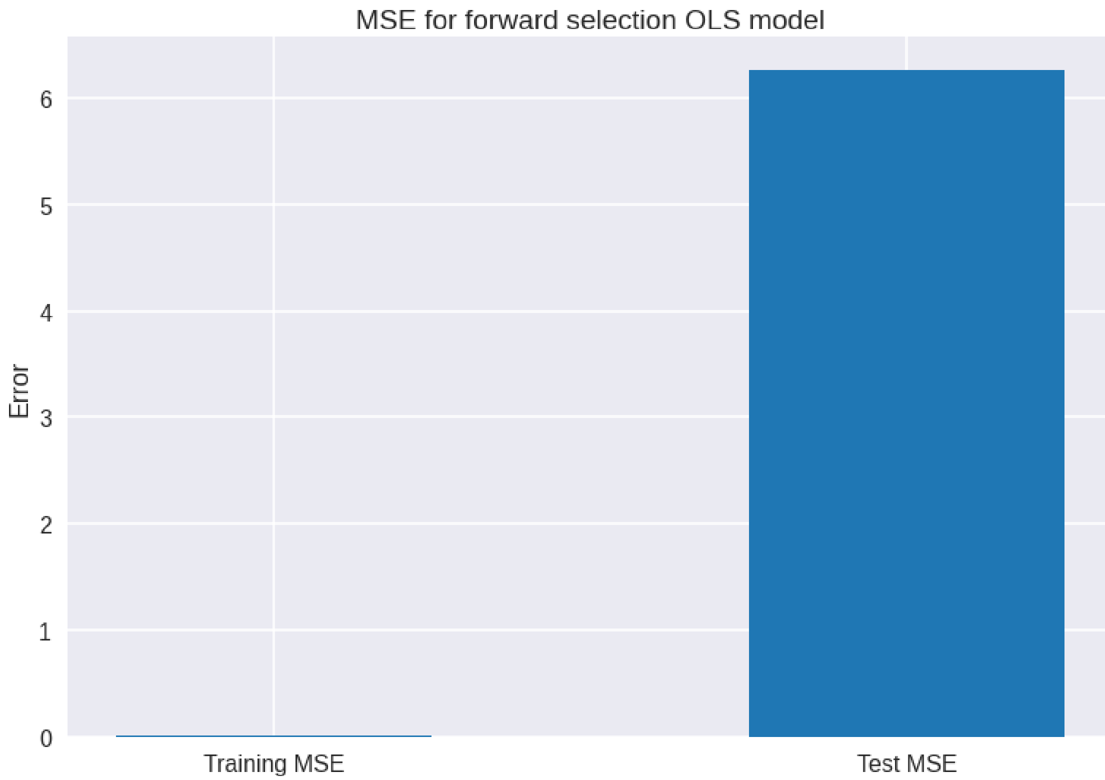
[27]:
```
AIC_forward = best_model_forward.aic
BIC_forward = best_model_forward.bic
MSE_forward_train = np.mean((train_set[:, -1] - best_model_forward.
↪predict(add_constant(train_set[:, selected_parameters_forward])))**2)
MSE_forward_test = np.mean((test_set[:, -1] - best_model_forward.
↪predict(add_constant(test_set[:, selected_parameters_forward])))**2)
R2_forward_train = best_model_forward.rsquared
```

```
R2_forward_test = 1 - (MSE_forward_test/np.mean((test_set[:, -1] - np.
 ↪mean(test_set[:, -1]))**2))
```

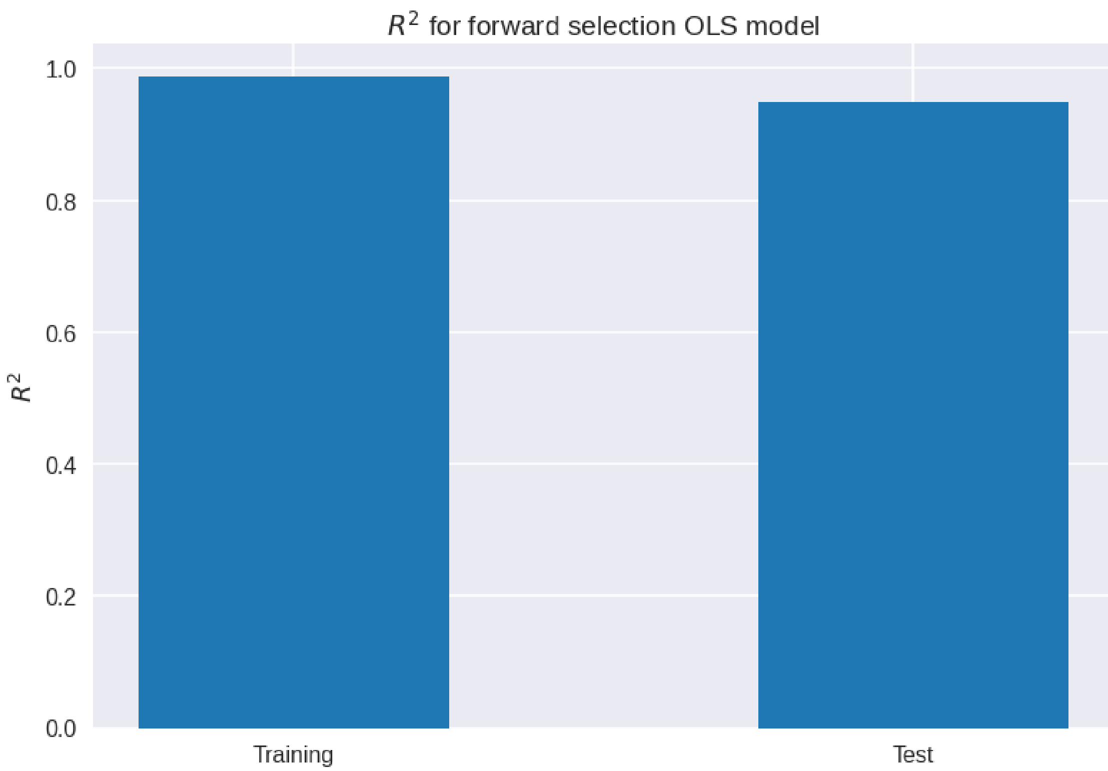
[28]:
```
x = [0.5, 1.5]
plt.bar(x, [AIC_forward, BIC_forward], width=0.5)
plt.xticks(x, labels=["AIC", "BIC"])
plt.ylabel("Score")
plt.title("AIC and BIC for forward selection OLS model")
plt.show()
plt.close()
```



[29]:
```
x = [0.5, 1.5]
plt.bar(x, [MSE_forward_train, MSE_forward_test], width=0.5)
plt.xticks(x, labels=["Training MSE", "Test MSE"])
plt.ylabel("Error")
plt.title("MSE for forward selection OLS model")
plt.show()
plt.close()
```



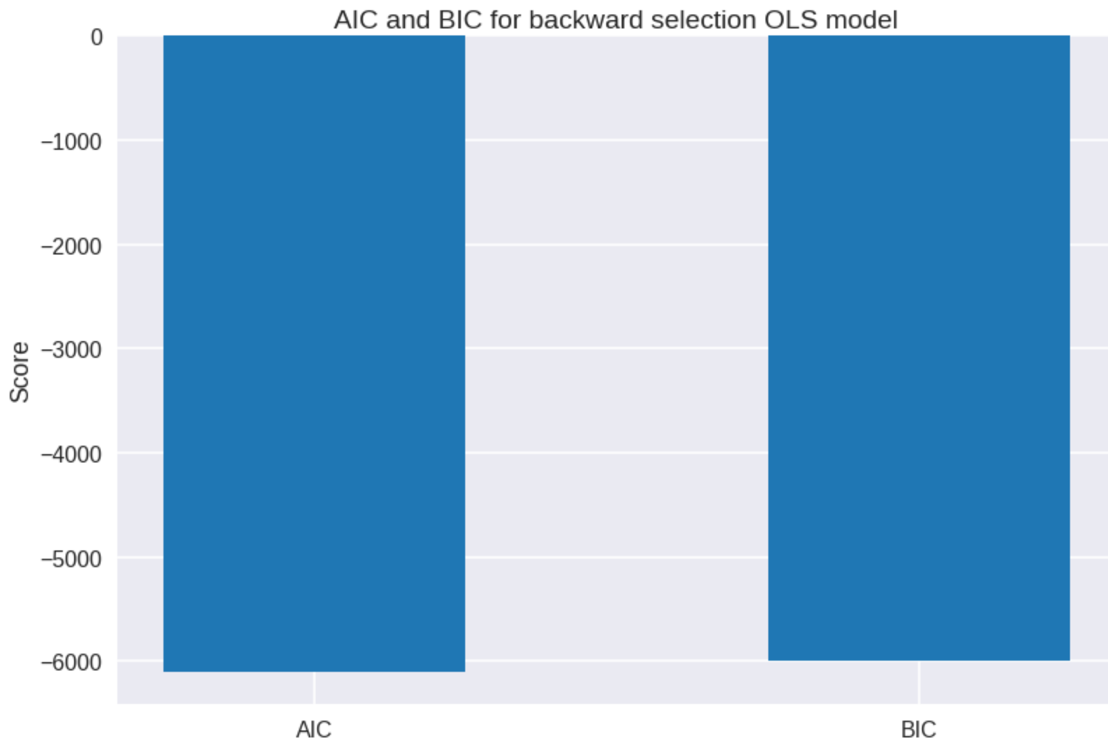
[30]:
```
x = [0.5, 1.5]
plt.bar(x, [R2_forward_train, R2_forward_test], width=0.5)
plt.xticks(x, labels=["Training", "Test"])
plt.ylabel("$R^2$")
plt.title("$R^2$ for forward selection OLS model")
plt.show()
plt.close()
```



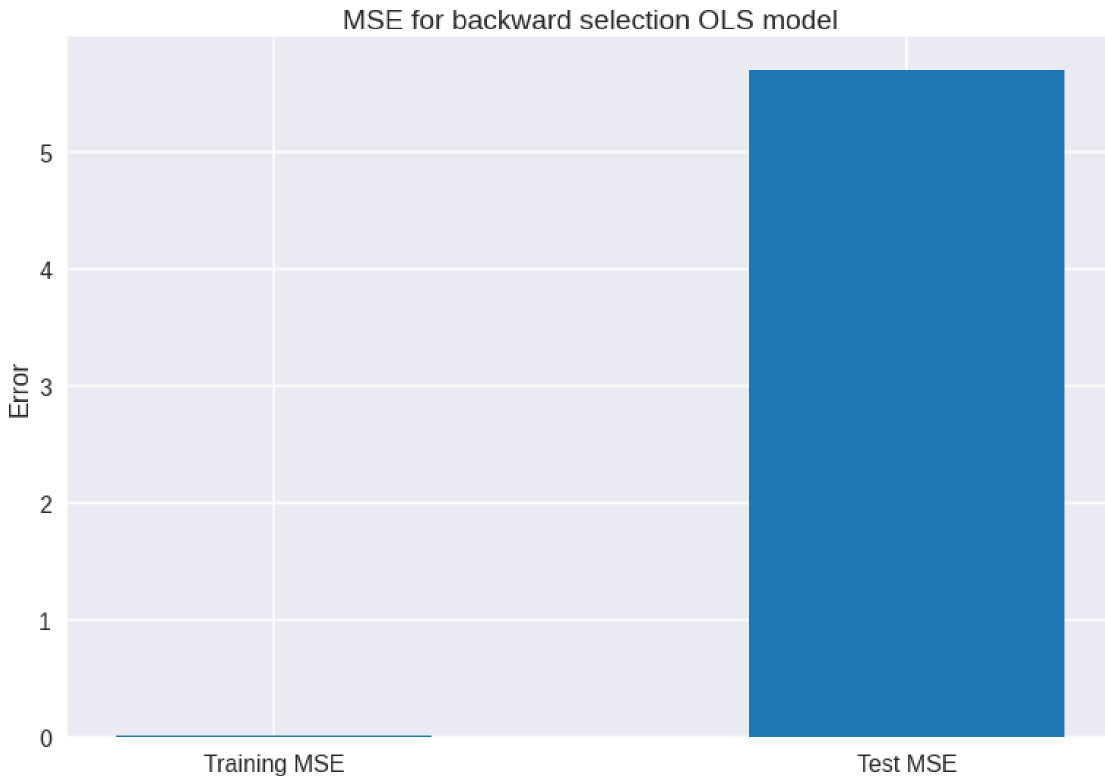
Errors from BIC model (backward selection)

[31]:
```
AIC_backward = best_model_backward.aic
BIC_backward = best_model_backward.bic
MSE_backward_train = np.mean((train_set[:, -1] - best_model_backward.
 ↪predict(add_constant(train_set[:, selected_parameters_backward])))**2)
MSE_backward_test = np.mean((test_set[:, -1] - best_model_backward.
 ↪predict(add_constant(test_set[:, selected_parameters_backward])))**2)
R2_backward_train = best_model_backward.rsquared
R2_backward_test = 1 - (MSE_backward_test/np.mean((test_set[:, -1] - np.
 ↪mean(test_set[:, -1]))**2))
```

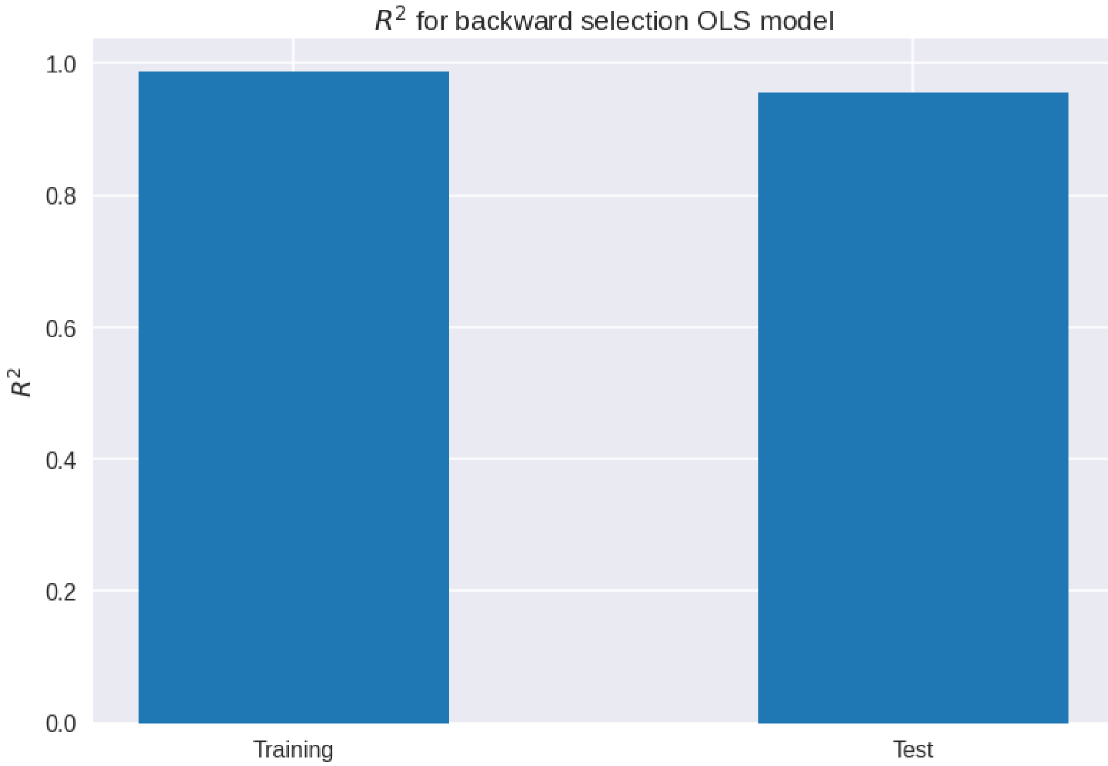
[32]:
```
x = [0.5, 1.5]
plt.bar(x, [AIC_backward, BIC_backward], width=0.5)
plt.xticks(x, labels=["AIC", "BIC"])
plt.ylabel("Score")
plt.title("AIC and BIC for backward selection OLS model")
plt.show()
plt.close()
```



[33]:
```
x = [0.5, 1.5]
plt.bar(x, [MSE_backward_train, MSE_backward_test], width=0.5)
plt.xticks(x, labels=["Training MSE", "Test MSE"])
plt.ylabel("Error")
plt.title("MSE for backward selection OLS model")
plt.show()
plt.close()
```



[34]:
```
x = [0.5, 1.5]
plt.bar(x, [R2_backward_train, R2_backward_test], width=0.5)
plt.xticks(x, labels=["Training", "Test"])
plt.ylabel("$R^2$")
plt.title("$R^2$ for backward selection OLS model")
plt.show()
plt.close()
```



Compare the training data errors and testing data errors, which model works better? Provide a summary of your results.

[35]:
```
print("MSE forward test error:", MSE_forward_test)
print("MSE forward train error:", MSE_forward_train)
print("R-squared forward test:", R2_forward_test)
print("R-squared forward train:", R2_forward_train)
print("AIC forward:", AIC_forward)
print("BIC forward:", BIC_forward)
```

```
MSE forward test error: 6.26450254546285
MSE forward train error: 0.004685830924388568
R-squared forward test: 0.9494722582022483
R-squared forward train: 0.9878671570446053
AIC forward: -6126.5453027080075
BIC forward: -5958.245686056394
```

[36]:
```
print("MSE backward test error:", MSE_backward_test)
print("MSE backward train error:", MSE_backward_train)
print("R-squared backward test:", R2_backward_test)
print("R-squared backward train:", R2_backward_train)
print("AIC backward:", AIC_backward)
print("BIC backward:", BIC_backward)
```

```
MSE backward test error: 5.6939683521808355
MSE backward train error: 0.004781754675615563
R-squared backward test: 0.954074028924781
R-squared backward train: 0.9876187853410356
AIC backward: -6100.918013230734
BIC backward: -6002.259617262547
```

Answer:

We can see from the results that the model from forward selection seems to be overfitting more that the model from backward selection. This is evident from the fact that the training error is smaller for the forward selection model, but the testing error is smaller for the backward selection model.

The R^2 values show that the forward selection model explains a slightly higher amount of the variance from the test set, but this comes at the cost of a much more complex model (more degrees of freedom) that overfits the data. In fact, we can see that for the test data, R^2 for the backward selection model is closer to 1.

We used different metrics (AIC and BIC) to select optimal models in each case. In this case, it seems that the model from backward selection with BIC performs better.

More details of each model are shown below.

[37]:
```
best_model_forward.summary()
```

[37]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.988
Model:                            OLS   Adj. R-squared:                  0.988
Method:                 Least Squares   F-statistic:                     7037.
Date:                Mon, 22 Feb 2021   Prob (F-statistic):               0.00
Time:                        05:16:36   Log-Likelihood:                 3092.3
No. Observations:                2449   AIC:                            -6127.
Df Residuals:                    2420   BIC:                            -5958.
Df Model:                          28
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0186      0.006      3.216      0.001       0.007       0.030
x1             0.9438      0.034     28.135      0.000       0.878       1.010
x2             0.0177      0.005      3.678      0.000       0.008       0.027
x3             0.0169      0.005      3.463      0.001       0.007       0.027
x4             0.0228      0.003      7.170      0.000       0.017       0.029
x5            -0.0948      0.017     -5.461      0.000      -0.129      -0.061
x6             0.0461      0.012      3.775      0.000       0.022       0.070
x7            -0.0389      0.037     -1.061      0.289      -0.111       0.033
```

```
x8             0.0069      0.002      3.493      0.000       0.003       0.011
x9             0.8064      0.078     10.307      0.000       0.653       0.960
x10          -20.9790     24.480     -0.857      0.392     -68.983      27.025
x11           -0.0562      0.011     -5.098      0.000      -0.078      -0.035
x12            0.0641      0.014      4.741      0.000       0.038       0.091
x13           -0.0154      0.018     -0.843      0.399      -0.051       0.020
x14            0.0452      0.012      3.681      0.000       0.021       0.069
x15            0.0501      0.015      3.408      0.001       0.021       0.079
x16           -0.1026      0.032     -3.193      0.001      -0.166      -0.040
x17            0.0133      0.004      2.981      0.003       0.005       0.022
x18           -0.0296      0.014     -2.125      0.034      -0.057      -0.002
x19            0.0238      0.010      2.421      0.016       0.005       0.043
x20           -0.0054      0.003     -1.868      0.062      -0.011       0.000
x21           19.5360     24.541      0.796      0.426     -28.588      67.660
x22           -0.0581      0.023     -2.559      0.011      -0.103      -0.014
x23            0.0091      0.005      1.993      0.046       0.000       0.018
x24           -0.0357      0.017     -2.105      0.035      -0.069      -0.002
x25           -0.0041      0.003     -1.520      0.129      -0.009       0.001
x26           -0.0035      0.002     -1.799      0.072      -0.007       0.000
x27           -0.0458      0.022     -2.043      0.041      -0.090      -0.002
x28            0.0282      0.019      1.478      0.139      -0.009       0.066
x29            0.8050      0.078     10.307      0.000       0.652       0.958
==============================================================================
Omnibus:                      851.724   Durbin-Watson:                   0.482
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           143502.491
Skew:                          -0.478   Prob(JB):                         0.00
Kurtosis:                      40.489   Cond. No.                     2.25e+15
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 1.1e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

[38]:
```python
best_model_backward.summary()
```

[38]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.988
Model:                            OLS   Adj. R-squared:                  0.988
Method:                 Least Squares   F-statistic:                 1.212e+04
Date:                Mon, 22 Feb 2021   Prob (F-statistic):               0.00
Time:                        05:16:36   Log-Likelihood:                 3067.5
```

```
No. Observations:                2449   AIC:                            -6101.
Df Residuals:                    2432   BIC:                            -6002.
Df Model:                          16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0147      0.003      4.427      0.000       0.008       0.021
x1            -0.0949      0.031     -3.020      0.003      -0.157      -0.033
x2             0.0606      0.013      4.693      0.000       0.035       0.086
x3             0.6804      0.069      9.879      0.000       0.545       0.815
x4            -1.2378      0.127     -9.757      0.000      -1.487      -0.989
x5            -0.0308      0.013     -2.288      0.022      -0.057      -0.004
x6             0.0213      0.004      4.796      0.000       0.013       0.030
x7             0.9459      0.032     29.713      0.000       0.883       1.008
x8            -0.0469      0.009     -5.485      0.000      -0.064      -0.030
x9             0.0069      0.002      3.475      0.001       0.003       0.011
x10            0.0621      0.011      5.880      0.000       0.041       0.083
x11            0.0490      0.010      4.822      0.000       0.029       0.069
x12           -0.0380      0.012     -3.288      0.001      -0.061      -0.015
x13           -0.0835      0.016     -5.300      0.000      -0.114      -0.053
x14            0.0210      0.003      7.052      0.000       0.015       0.027
x15           -0.0429      0.015     -2.933      0.003      -0.072      -0.014
x16            0.6780      0.069      9.878      0.000       0.543       0.813
x17            0.0207      0.005      4.373      0.000       0.011       0.030
==============================================================================
Omnibus:                      802.717   Durbin-Watson:                   0.490
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           145573.692
Skew:                          -0.293   Prob(JB):                         0.00
Kurtosis:                      40.766   Cond. No.                     1.85e+16
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 9.95e-29. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

1.3.4 [15 Points] Best Subset Selection

Fit the best subset selection to the dataset and report the best model of each model size (up to 7 variables, excluding the intercept) and their prediction errors. Make sure that you simplify your output so that it only presents the essential information. If the algorithm cannot handle this many variables, then consider using just day 1 and 2 information.

Note: since there are too many combinations and the code would take too long, I will apply PCA to reduce the number of variables to a manageable number.

[39]:
```
from sklearn.decomposition import PCA
pca_transformer = PCA(n_components=20).fit(train_set[:, :-1])
X = pca_transformer.transform(train_set[:, :-1])
X_test = pca_transformer.transform(test_set[:, :-1])
```

[40]:
```
X.shape
```

[40]:
```
(2449, 20)
```

[41]:
```
X_test.shape
```

[41]:
```
(410, 20)
```

[42]:
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
```

[43]:
```
def fit_and_score(X, y):
    n_samples = X.shape[0]
    model = LinearRegression().fit(X, y)
    RSS = mean_squared_error(y, model.predict(X)) * n_samples
    return model, RSS
```

[44]:
```
from itertools import combinations
```

[45]:
```
y = train_set[:, -1]
max_params = 7
total_params = X.shape[1]
best_models = []
best_params = []

for k in range(1, max_params + 1):

    print("Step", k, "of 7.")
    rss_prev = np.inf

    for comb in combinations(np.arange(total_params), k): #All combinations of␣
 ↪k parameters
        X_partial = X[:, np.asarray(comb)]
        temp_model, rss = fit_and_score(X_partial, y)
        if (rss < rss_prev):
            best_model = temp_model
            rss_prev = rss
            best_p = np.asarray(comb)

    best_models.append(best_model)
```

```
    best_params.append(best_p)
```

```
Step 1 of 7.
Step 2 of 7.
Step 3 of 7.
Step 4 of 7.
Step 5 of 7.
Step 6 of 7.
Step 7 of 7.
```

[46]:

```
for k in range(max_params):
    print("Coefficient(s) of best model with", k+1, "parameter(s):",␣
 ↪best_models[k].coef_)
    print("Parameters used (indices):", best_params[k])
    print("Intercept:", best_models[k].intercept_)
    print('--------------------------------------------------------------')
```

```
Coefficient(s) of best model with 1 parameter(s): [0.07972952]
Parameters used (indices): [3]
Intercept: 0.2651805639256272
--------------------------------------------------------------
Coefficient(s) of best model with 2 parameter(s): [0.07972952 0.10281607]
Parameters used (indices): [3 4]
Intercept: 0.2651805639256272
--------------------------------------------------------------
Coefficient(s) of best model with 3 parameter(s): [ 0.07972952  0.10281607
-0.03936328]
Parameters used (indices): [3 4 7]
Intercept: 0.2651805639256273
--------------------------------------------------------------
Coefficient(s) of best model with 4 parameter(s): [-9.03270133e-05
7.97295227e-02  1.02816068e-01 -3.93632770e-02]
Parameters used (indices): [0 3 4 7]
Intercept: 0.2651805639256272
--------------------------------------------------------------
Coefficient(s) of best model with 5 parameter(s): [-9.03270133e-05
7.97295227e-02  1.02816068e-01 -3.93632770e-02
  4.46400126e-02]
Parameters used (indices): [ 0  3  4  7 18]
Intercept: 0.26518056392562733
--------------------------------------------------------------
Coefficient(s) of best model with 6 parameter(s): [-9.03270133e-05
7.97295227e-02  1.02816068e-01  8.49840108e-03
 -3.93632770e-02  4.46400126e-02]
Parameters used (indices): [ 0  3  4  5  7 18]
Intercept: 0.26518056392562733
--------------------------------------------------------------
```

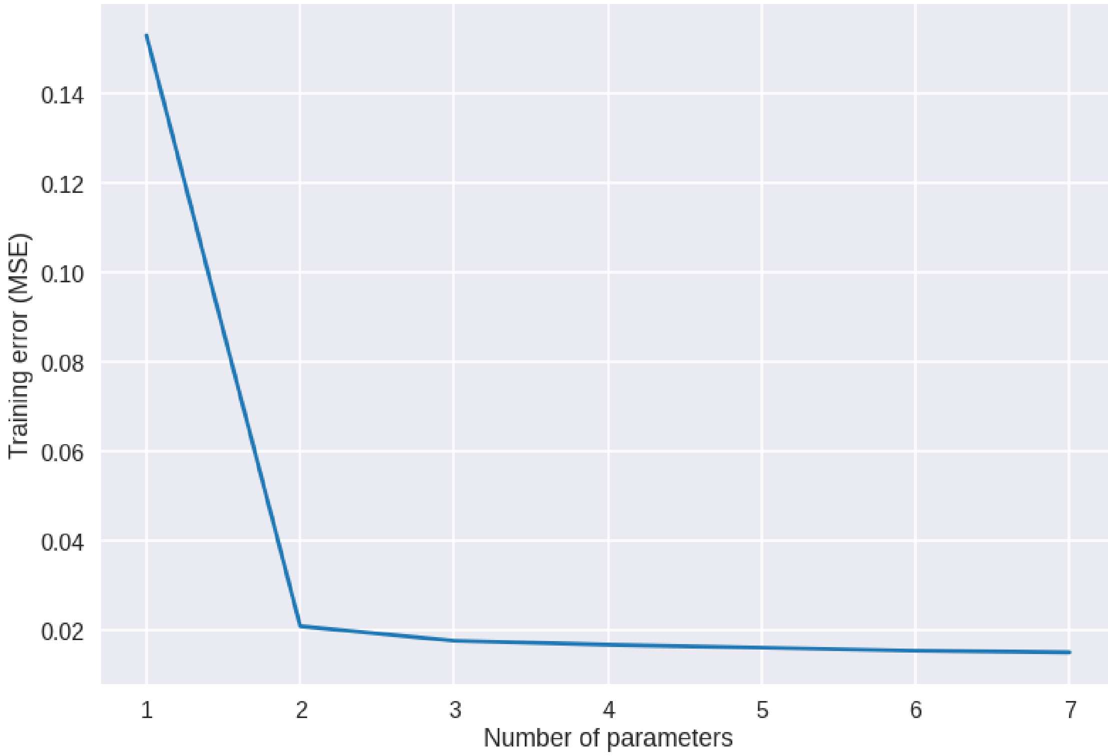
```
Coefficient(s) of best model with 7 parameter(s): [-9.03270133e-05
7.97295227e-02  1.02816068e-01  8.49840108e-03
 -3.93632770e-02  4.46400129e-02 -4.01345559e-02]
Parameters used (indices): [ 0  3  4  5  7 18 19]
Intercept: 0.26518056392562733
--------------------------------------------------------------
```

[47]:
```python
y_train = train_set[:, -1]
y_test = test_set[:, -1]
train_errors = []
test_errors = []

for k in range(max_params):
    train_error = mean_squared_error(y_train, best_models[k].predict(X[:,␣
↪best_params[k]]))
    test_error = mean_squared_error(y_test, best_models[k].predict(X_test[:,␣
↪best_params[k]]))

    train_errors.append(train_error)
    test_errors.append(test_error)
```

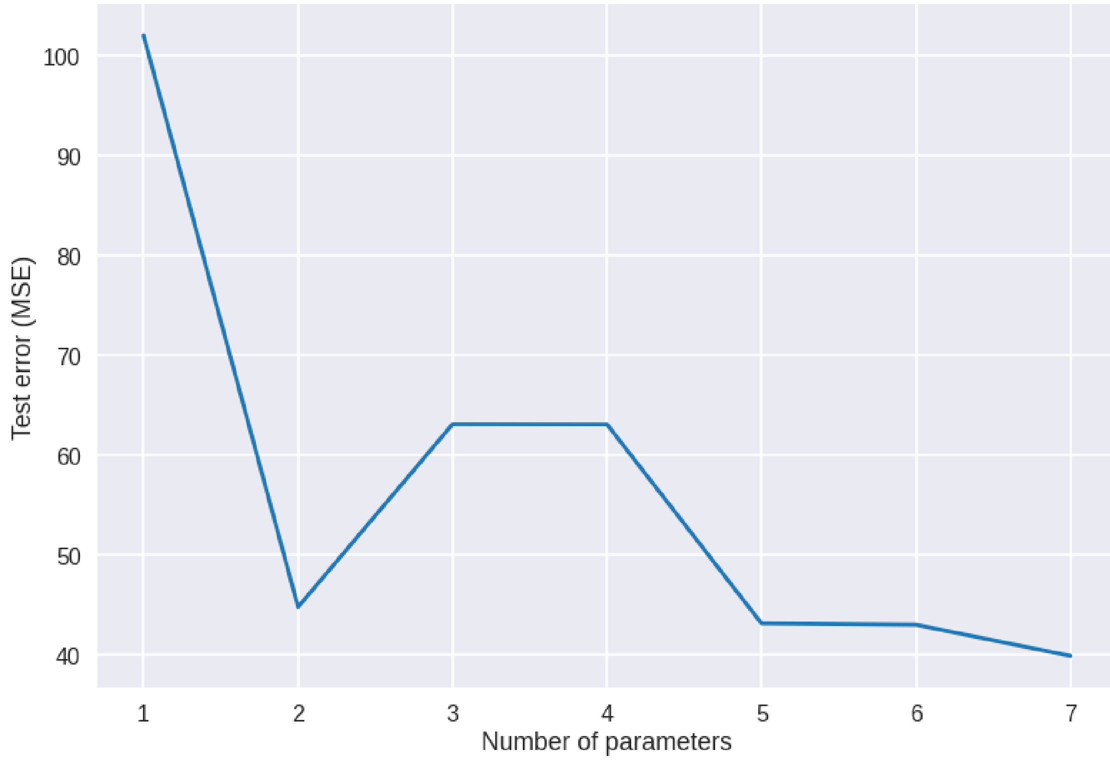
[56]:
```python
x = np.arange(1, max_params+1)
plt.plot(x, train_errors)
plt.xlabel("Number of parameters")
plt.ylabel("Training error (MSE)")
plt.show()
plt.close()
```



[49]:
```
x[np.asarray(train_errors).argmin()]
```

[49]: 7

[55]:
```
x = np.arange(1, max_params+1)
plt.plot(x, test_errors)
plt.xlabel("Number of parameters")
plt.ylabel("Test error (MSE)")
plt.show()
plt.close()
```



[51]:
```
x[np.asarray(test_errors).argmin()]
```

[51]: 7

Conclusion: the best model is the one with 7 parameters.

1.3.5 [15 Points] KNN

Use KNN to perform this prediction task. Do you expect KNN to perform better or worse than the linear model, and why? Does the analysis result match your intuition? Report your model fitting results.

[52]:
```
from sklearn.neighbors import KNeighborsRegressor
knn_regressor = KNeighborsRegressor(n_neighbors=5).fit(train_set[:, :-1],␣
 ↪train_set[:, -1])
train_error_knn = mean_squared_error(train_set[:, -1], knn_regressor.
 ↪predict(train_set[:, :-1]))
test_error_knn = mean_squared_error(test_set[:, -1], knn_regressor.
 ↪predict(test_set[:, :-1]))
```

[53]:
```
print("Training error:", train_error_knn)
print("Test error:", test_error_knn)
```

```
Training error: 0.004251550755666209
Test error: 227.19038460993252
```

[54]:
```python
print("Model parameters:", knn_regressor.get_params())
```

```
Model parameters: {'algorithm': 'auto', 'leaf_size': 30, 'metric': 'minkowski',
'metric_params': None, 'n_jobs': None, 'n_neighbors': 5, 'p': 2, 'weights':
'uniform'}
```

Do you expect KNN to perform better or worse than the linear model, and why? Does the analysis result match your intuition? Report your model fitting results.

Answer:

I expected KNN to perform worse, because these models tend to have high variance and low bias. It is clear from the test error and training error that the KNN model is overfitting (so the high variance problem proved real). The best linear model (found with backward selection) performed much better: although it had a similar training error, it also showed a much lower test error. For these reasons, I can conclude that the results matched my intuition.