

Exercise 1

$$a) \quad \sigma(z) = \frac{1}{1 + \exp(-z)}$$

$$u(z) = 1 + \exp(-z) \quad u'(z) = -e^{-z}$$

$$\sigma(z) = u(z)^{-1}$$

$$\begin{aligned} \sigma'(z) &= -u(z)^{-2} \cdot u'(z) = -(1 + e^{-z})^2 \cdot (-e^{-z}) \\ &= \frac{e^{-z}}{(1 + e^{-z})^2} \end{aligned}$$

b)

$$\begin{aligned} \sigma(z) \cdot (1 - \sigma(z)) &= \frac{1}{1 + e^{-z}} \cdot \left(1 - \frac{1}{1 + e^{-z}} \right) \\ &= \frac{1}{1 + e^{-z}} \cdot \left(\frac{1 + e^{-z} - 1}{1 + e^{-z}} \right) = \frac{e^{-z}}{(1 + e^{-z})^2} \end{aligned}$$