Assignment: Estimating Production Functions

Industrial Organization

This assignment is based on the data posted here for the study by Asker et al. (2014). If you have not already seen this paper, and the earlier article by Hsieh and Klenow, I suggest you read both of them *after* you submit the assignment. I think you will get more out of the exercise that way.

1. *Allocation: Theory.* Firms i in an industry each produce output Y_i using a single input X_i according to the production function

$$Y_i = A_i X_i^{\beta}$$

where A_i is firm productivity and $\beta \in (0,1)$ is a parameter. Say that the industry is *efficient* if it produces the maximum possible output given the total amount of input used. Let lowercase letters denote natural logarithms, so that $x_i = \ln X_i$. Assume that there is a fixed number of firms N.

   (a) What is the coefficient in a regression of y_i on a_i in an efficient industry?

   (b) What is the coefficient in a regression of y_i on x_i in an efficient industry?

   (c) Suppose that output and inputs are each traded in a perfectly competitive market with a price of 1. Is the equilibrium efficient?

   (d) In the competitive equilibrium described in (c), is β identified from the data $\{(x_i, y_i)\}_{i=1}^N$? Is $\{A_i\}_{i=1}^N$ identified?

   (e) A central planner assigns inputs to firms to ensure that *output* is identical across firms, i.e. that $Y_i = Y$ for all i. How much less total output is produced than would be the case under an efficient allocation of the same total amount of inputs?

   (f) In the competitive equilibrium described in (c), are $\{\beta_i, A_i\}_{i=1}^N$ identified if we now posit that $Y_i = A_i X_i^{\beta_i}$? Provide some economic intuition.

2. *Allocation: Evidence.* You may use any software you like, including Stata. We will use the dataset `comprehensive.dta` located in the folder `Replication_File_JPE/stata`. Throughout, we will say that Y_i is equal to sales minus the sum of materials cost and energy consumption, and X_i is equal to the capital stock, all measured as of the preceding year. We will focus on textile manufacturers and we will restrict attention to country-years in which there are at least 50 of these firms in the data with nonmissing values for Y_i and X_i.

(a) Estimate a regression of y_i on x_i separately for each country-year and report the results. Comments on whether they are consistent with efficiency in the sense of question 1.

(b) Suppose that each firm has its own coefficient β_i. Use the assumptions from 1(f) to estimate β_i for each firm. Compute, for each country-year, the percent increase in textile output from moving to an efficient allocation of capital.

(c) Now suppose that all textile firms in the world have the same coefficient β. Using your answers to 1(b) and 2(a), identify the country-year c^* that is closest to an efficient allocation of capital. Estimate β under the assumption that the economy in c^* is competitive in the sense of 1(c) and that the rental rate on capital is 2.5 percent of its value.

(d) Using your estimate of β from 2(c), compute, for each country-year, the percent increase in textile output from moving to an efficient allocation of capital.

(e) Which country shows the smallest gain from reallocation? Why?