QWORLD

UNIVERSITY OF LATVIA
FACULTY OF COMPUTING

# QCLASS23/24

Introductory & Intermediate Level Quantum Courses

# Lecture 1

# Complex number

Real part

Imaginary part

$$z = a + ib$$

set of complex numbers

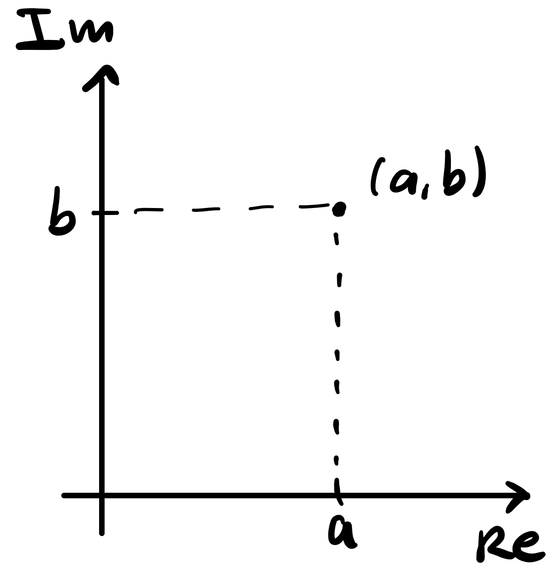
$$z \in \mathbb{C}$$

$$i^2 = -1 \longrightarrow i = \sqrt{-1}$$

$$a, b \in \mathbb{R}$$

belongs to

set of real numbers



$$\bar{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ a_n \end{pmatrix} \qquad \bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{pmatrix} \qquad \bar{a} + \bar{b} = \begin{pmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{pmatrix}$$

1 column
$n$ rows

column "vector"

matrix

$$\begin{array}{c} \longrightarrow n \text{ columns} \\ M = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \downarrow m \text{ rows} \end{array}$$

$M\bar{a}$

$(m \times n)(n \times p)$

$\bar{a}^T \bar{b}$

$(n \times 1)(1 \times n) = n \times n$

$\bar{a} \quad \bar{b}$

$(1 \times n)(1 \times n)$

$\bar{a}^T = (a_1 \quad a_2 \cdots a_n)$

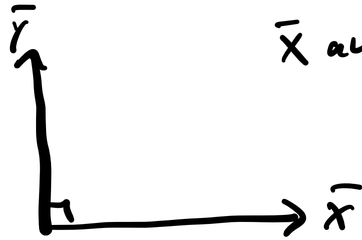
$n \times 1$

$\bar{b}\,\bar{a}^T \Rightarrow 1 \times 1$

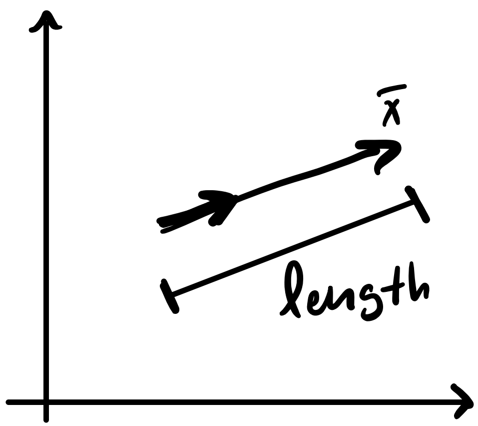
dot product

$$\bar{x} \cdot \bar{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

$\bar{x} \cdot \bar{y} = 0$




$\bar{x}$ and $\bar{y}$ are orthogonal to each other




norm

$$\|\bar{x}\| = \sqrt{\bar{x} \cdot \bar{x}}$$

normalize

$$\bar{x}_{normalized} = \frac{\bar{x}}{\|\bar{x}\|}$$

$\bar{a} + \bar{b}$

$5\bar{a}$

Vector space

$$A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix} \qquad \bar{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$A\bar{u} = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+6 \\ 5+2 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$$

$$A\bar{u} = 7\,\bar{u}$$

eigenvalue

eigenvector

eigenstates (in QC)

in general:

$$A\bar{v} = \lambda\bar{v}$$

Linear operators

$$O = \begin{pmatrix} 2 & -3 & 4 \\ 1 & 7 & -5 \end{pmatrix} \qquad \bar{b} = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix}$$

$$(2 \times 3) \quad (3 \times 1)$$

$$O\bar{b} = \begin{pmatrix} 23 \\ -11 \end{pmatrix}$$

$$O : U \longrightarrow V$$

3-dim  2-dim

# Unitary Operators

congugate transpose

$$U^{-1}U = U^{*}U = UU^{*} = I$$

identity matrix $\rightarrow$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 & \\ & & & \ddots \end{pmatrix}$

$$U = U^{-1}$$

$$U^{*} = (U^{T})^{*}$$

$$z = a + ib$$

$$z^{*} = a - ib$$

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \rightarrow A^{T} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

Tensor product

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

Dirac notation

$$\bar{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = |a\rangle$$

ket

$a_i \in \mathbb{C}$

$|u\rangle$

$|b\rangle$

$\downarrow$ transpose conjugate

$\langle a|$ ← bra

$\langle a|b\rangle$

bra-ket

$\langle a|a\rangle = c$ (one number, a scalar)

$(1\times n)(n\times 1) \to 1\times 1$

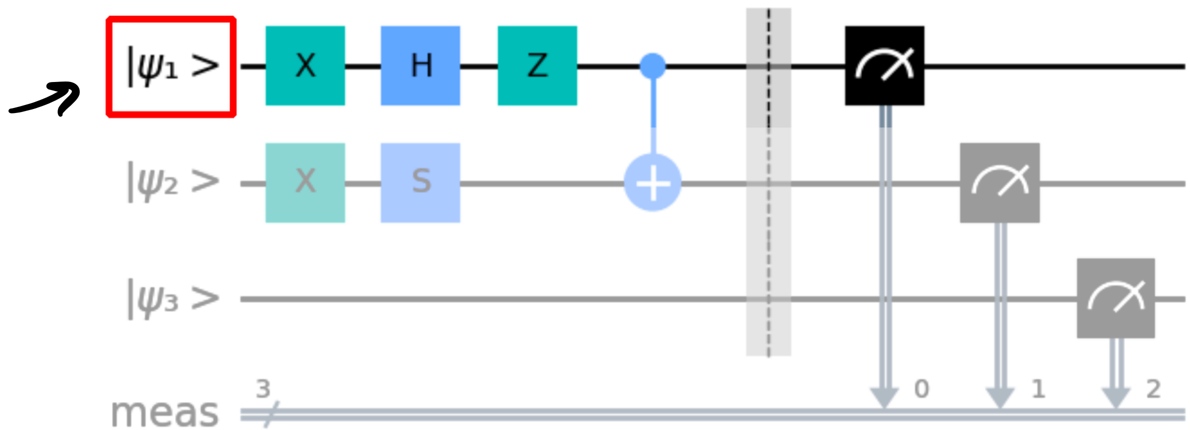
$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Hilbert space

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

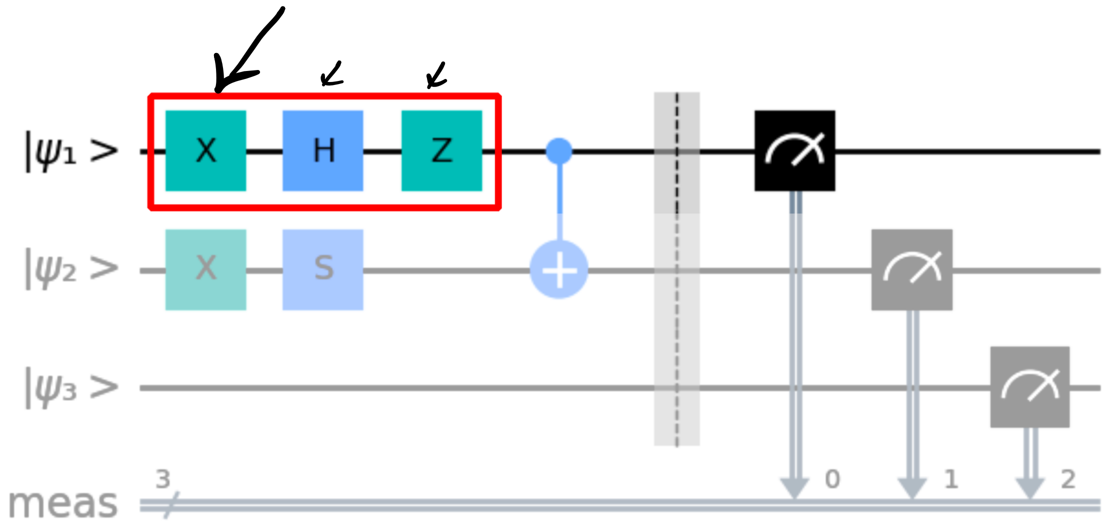
## P1. How to describe states of quantum systems

$$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\psi_2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

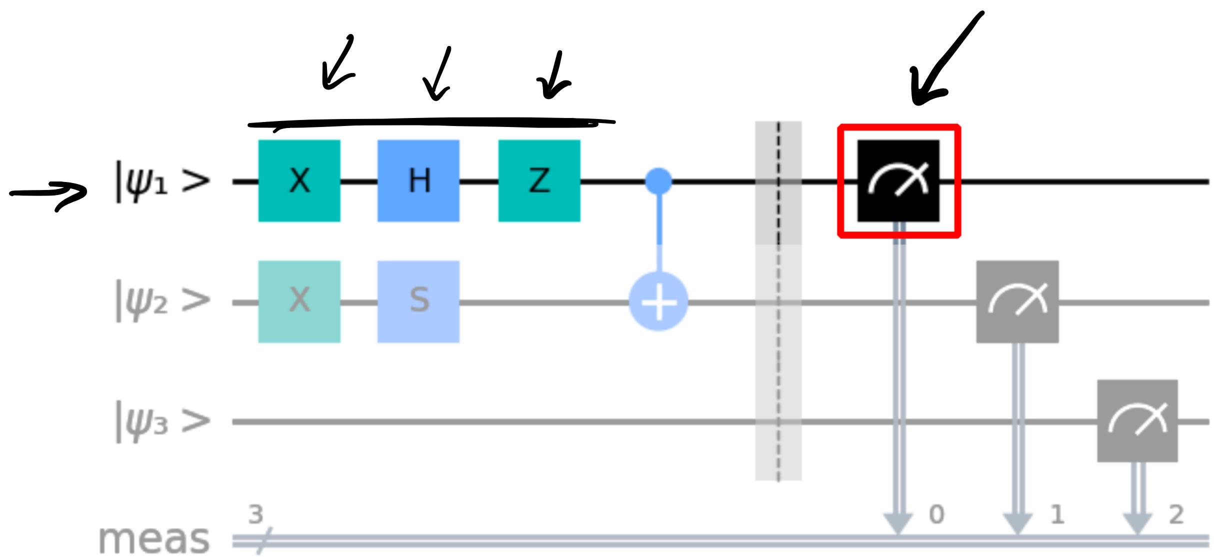


P2. How quantum states evolve

$$X\,|\psi_1\rangle = |\psi_1'\rangle$$



# P3. How to extract information from quantum systems



# P4. How to combine quantum systems

$$\underbrace{|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle}$$