6. en el caso 3D sea $\{e_i\}$ define un sistema de coordenadas dextrogiro no necesariamente ortogonal

Demuestre que

(a)

$$e^i = \frac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} , \quad i, j, k = 1,2,3 \text{ y las permutaciones ciclicas } \begin{matrix} 2,3,1 \\ 3,1,2 \end{matrix}$$

intentamos generar de la base dual la base covariante

sea entonces

$$e^1 = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} , \quad e^2 = \frac{e_3 \times e_1}{e_1 \cdot (e_2 \times e_3)} \quad e^3 = \frac{e_1 \times e_2}{e_1 \cdot (e_2 \times e_3)}$$

esto sera cierto si $e^i \cdot e_i = 1 \wedge e^i \cdot e_j = 0$

$$e^1 \cdot e_1 = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot e_1 \Rightarrow \frac{\cancel{e_1 (e_2 \times e_3)}^{\to 1}}{\cancel{e_1 (e_2 \times e_3)}} = 1$$

$$e^2 \cdot e_2 = \frac{e_3 \times e_1}{e_1 \cdot (e_2 \times e_3)} \cdot e_2 \Rightarrow \frac{e_2 \cdot (e_3 \times e_1)}{e_1 \cdot (e_2 \times e_3)}$$ tomando la propiedad del producto tripel las permutaciones ciclicas tenemos $$\frac{\cancel{e_2 \cdot (e_3 \times e_1)}^{1}}{\cancel{e_2 \cdot (e_3 \times e_1)}}$$

$$e^3 \cdot e_3 = \frac{e_1 \times e_2}{e_1 \cdot (e_2 \times e_3)} \cdot e_3 \Rightarrow \frac{e_3 (e_1 \times e_2)}{e_1 \cdot (e_2 \times e_3)} = \frac{\cancel{e_3 \cdot (e_1 \times e_2)}^{\to 1}}{\cancel{e_3 \cdot (e_2 \times e_3)}}$$

tenemos que $e^i e_i = 1$ ✓

$$e^1 \cdot e_2 = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot e_2 = \frac{\cancel{e_2 \cdot (e_2 \times e_3)}^{\to 0}}{e_1 \cdot (e_2 \times e_3)}$$

Por propiedades $(e_2 \times e_3) \perp e_2$ y su producto punto sera cero

$$e^1 \cdot e_3 = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot e_3 = \frac{\cancel{e_3 (e_2 \times e_3)}}{e_1 \cdot (e_2 \times e_3)} = 0$$ ... y asi para todos

Siendo entonces $e^i \cdot e_j = 0$ Q.E.D

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(b) si los volumenes ; $V = e_1 \cdot (e_2 \times e_3)$ y $\tilde{V} = e^1 \cdot (e^2 \times e^3)$ entonces $V\tilde{V} = 1$

sea

$$e^1 = \frac{(e_2 \times e_3)}{e_1(e_2 \times e_3)} , e^2 = \frac{(e_3 \times e_1)}{e_1(e_2 \times e_3)} , e^3 = \frac{(e_1 \times e_2)}{e_1(e_2 \times e_3)}$$

$$\tilde{V} = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot \left(\frac{(e_3 \times e_1)}{e_1 (e_2 \times e_3)} \times \frac{(e_1 \times e_2)}{e_1 (e_2 \times e_3)} \right)$$

$$V\tilde{V} = \cancel{e_1 \cdot (e_2 \times e_3)} \left(\frac{e_2 \times e_3}{\cancel{e_1 \cdot (e_2 \times e_3)}} \cdot \left(\frac{e_3 \times e_1}{\cancel{e_1 \cdot (e_2 \times e_3)}} \times \frac{e_1 \times e_2}{\cancel{e_1 (e_2 \times e_3)}} \right) \right)$$

$$V\tilde{V} = e_2 \times e_3 \cdot (e_2 \times e_3)$$

$$V\tilde{V} = e_1 \cdot e_1 = 1 \quad QED$$

(c) ¿ Que vector satisface $a \cdot e^i = 1$, Demuestre que es unico

sea $e^i = \frac{e_j \times e_k}{e_i \cdot (e_j \times e_k)}$ y $a = (a_1, a_2, a_3)$

$$a \cdot (e_j \times e_k) = e_i \cdot (e_j \times e_k)$$

$$a = \frac{e_i \cdot \cancel{(e_j \times e_k)}^1}{\cancel{(e_j \times e_k)}}$$

$a = e_i$ ✓ entonces $e_i \cdot e^i = 1$ ✓

(d) Encuentre el producto vectorial de dos vectores a y b que estan en el sistema

$$W_1 = 4i + 2j + K$$
$$W_2 = 3\hat{i} + 3\hat{j}$$
$$W_3 = 2K$$

Expresados en esta base son tal que

$$a = (a_1, a_2, a_3)$$
$$b = (b_1, b_2, b_3)$$
$$a = a_1 W_1 + a_2 W_2 + a_3 W_3$$
$$b = b_1 W_1 + b_2 W_2 + b_3 W_3$$

$$a \times b = \begin{vmatrix} i & j & K \\ a_1 w_1 & a_2 w_2 & a_3 w \\ b_1 w_1 & b_2 w_2 & b_3 w \end{vmatrix} = \varepsilon_{ijK}\, a_j\, b_k$$

I Las bases reciprocas $\{e^i\}$

$$e^1 = \frac{(W_2 \times W_3)}{W_1 \cdot (W_2 \times W_3)} = \frac{6i - 6j}{-36}$$

$$e^2 = \frac{(W_3 \times W_1)}{W_1 \cdot (W_2 \times W_3)} = \frac{-12\hat{i}}{-36}$$

$$e^3 = \frac{(W_1 \times W_2)}{W_1 \cdot (W_2 \times W_3)} = \frac{-3\hat{i} + 3\hat{j} - 18\hat{k}}{-36}$$

II Las componentes covariantes y contravariantes de $a = \hat{i} + 2\hat{j} + 3\hat{k}$

Covariantes

$$a|w_1\rangle = 15$$
$$a|W_2\rangle = 9$$
$$a|w_3\rangle = 6$$

Contravariantes

$$\langle e^1 | a = \frac{1}{6}$$
$$\langle e^2 | a = \frac{1}{3}$$
$$\langle e^3 | a = \frac{17}{12}$$