7 Sea el espacio vectorial de matrices hermíticas con producto interno tal que $\langle a|b\rangle = Tr(A^T B)$, encuentre la base dual asociada a las matrices de pauli y encuentre un vector genérico en este espacio y encuentre su 1-forma asociada

dado $\langle a|b\rangle = \langle b|a\rangle = b_i a^j \langle \hat{e}^i|\hat{e}_j\rangle = b_i a^j \delta^i_j = a^i b_i$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\langle \sigma^1|\sigma_1\rangle = 1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} b_1 & a_1 \\ d_1 & c_1 \end{pmatrix} = \underbrace{b_1 + c_1}_{b_1 + b_1 = 1 \rightarrow \boxed{b_1 = \frac{1}{2}}} = 1$$

$$\langle \sigma^1|\sigma_2\rangle = 0 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} b_1 i & a_1 i \\ d_1 i & -c_1 i \end{pmatrix} = \underbrace{b_1 i - c_1 i}_{\boxed{b_1 = c_1}} = 0$$

$$\langle \sigma^1|\sigma_3\rangle = 0 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a_1 & -b_1 \\ c_1 & -d_1 \end{pmatrix} = \underbrace{a_1 - d_1}_{\boxed{a_1 = d_1}} = 0$$

$$\langle \sigma^1|\sigma_0\rangle = 0 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \underbrace{a_1 + d_1}_{a_1 + a_1 = 0} = 0$$

$$\boxed{a_1 = 0}$$

$$\langle \sigma^1| = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

$$\langle \sigma^2|\sigma_1\rangle = 0 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} b_2 & a_2 \\ d_2 & c_2 \end{pmatrix} = \underbrace{b_2 + c_2}_{b_2 = -c_2} = 0$$

$$\langle \sigma^2|\sigma_2\rangle = 1 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} b_2 i & -a_2 i \\ d_2 i & -c_2 i \end{pmatrix} = \underbrace{b_2 i - c_2 i = 1}_{-c_2 i - c_2 i = 1} = -\frac{1}{2i}$$

$$\langle \sigma^2|\sigma_3\rangle = 0 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a_2 & -b_2 \\ c_2 & -d_2 \end{pmatrix} = a_2 - d_2 = 0 \rightarrow a_2 = d_2$$

$$\langle \sigma^2|\sigma_0\rangle = 0 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = a_2 + d_2 = 0$$

$$2a_2 = 0$$

$$a_2 = 0$$

$$\langle \sigma^2| = \begin{pmatrix} 0 & \frac{1}{2i} \\ -\frac{1}{2i} & 0 \end{pmatrix}$$

Una vez reconocida el patrón dado por el producto interno podemos decir

$$\langle \sigma^3 | \sigma_1 \rangle = 0 = b_3 + c_3 = 0$$
$$\langle \sigma^3 | \sigma_2 \rangle = 0 = b_3 i - c_3 i = 0$$
$$\langle \sigma^3 | \sigma_3 \rangle = 1 = a_3 - d_3 = 1$$
$$\langle \sigma^3 | \sigma_0 \rangle = 0 = a_3 + d_3 = 0$$

$$\begin{cases} b_3 = c_3 \\ c_3 = 0 \\ a_3 = -d_3 \\ d_3 = \frac{1}{-2} \end{cases} \qquad \langle \sigma^3 | = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$\langle \sigma^0 | \sigma_1 \rangle = 0 = b_4 + c_4 = 0$$
$$\langle \sigma^0 | \sigma_2 \rangle = 0 = b_4 i - c_4 i = 0$$
$$\langle \sigma^0 | \sigma_3 \rangle = 0 = a_4 - d_4 = 0$$
$$\langle \sigma^0 | \sigma_0 \rangle = 1 = a_4 + d_4 = 1$$

$$\begin{cases} b_4 = c_4 \\ c_4 = 0 \\ a_4 = d_4 \\ d_4 = \frac{1}{2} \end{cases} \qquad \langle \sigma^0 | = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\langle \sigma^0 | = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}, \quad \langle \sigma^1 | = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}, \quad \langle \sigma^2 | = \begin{pmatrix} 0 & 1/2i \\ -1/2i & 0 \end{pmatrix}, \quad \langle \sigma^3 | = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$