2 Considere que:

- $r = x\hat{i} + y\hat{j} + z\hat{k} = x^i \hat{i}_i$
- $a = a(r) = a(x,y,z) = a^i(x,y,z)\hat{i}_i$ y $b = b(r) = b(x,y,z) = b^i(x,y,$
- $\phi = \phi(r) = \phi(x,y,z)$ y $\psi = \psi(r) = \psi(x,y,z)$

2a) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

sea $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ con indices $\nabla_i = \left(\frac{\partial}{\partial x^i}\right)$

ϕ y ψ son ambas funciones que depende de (r) y $r(x,y,z)$

$$\nabla_i \phi\psi = \frac{\partial \phi(x,y,z)\psi(x,y,z)}{\partial x^i}$$

usando las reglas de derivacion sobre multiplicacion de funciones tenemos

$$\frac{\partial\phi(x,y,z)}{\partial x^i} \cdot \psi(x,y,z) + \frac{\partial\psi(x,y,z)}{\partial x^i} \cdot \phi(x,y,z)$$

simplificando

$$\psi\nabla_i\phi + \phi\nabla_i\psi \quad \text{y reordenado} \quad \psi\nabla\phi + \phi\nabla\psi = \checkmark$$

2d) $\nabla\cdot(\nabla\times a)$ ¿Que se puede decir de $\nabla\times(\nabla\cdot a)$?

sea $\varepsilon_{ijk} := \begin{cases} 1, & (i,j,k) = (1,2,3) \text{ o } (2,3,1) \text{ o } (3,1,2) \\ -1, & (i,j,k) = (3,2,1) \text{ o } (2,1,3) \text{ o } (1,3,2) \\ 0, & \text{el resto} \end{cases}$

y sabemos que

$(a\times b)_i = \sum_{j,k=1}^{3} \varepsilon_{ijk} a_j b_k =$ sea entonces a un campo vectorial

$(\nabla\times a)_i = \varepsilon_{ijk}\nabla_j a_k |e_i\rangle$

$$\nabla\cdot(\nabla\times a) = \nabla_i\varepsilon_{ijk}\nabla_j a_k = \varepsilon_{ijk}\nabla_i\nabla_j a_k = \begin{vmatrix} \frac{\partial}{\partial x^1} & \frac{\partial}{\partial x^2} & \frac{\partial}{\partial x^3} \\ \frac{\partial}{\partial x^1} & \frac{\partial}{\partial x^2} & \frac{\partial}{\partial x^3} \\ a_1 & a_2 & a_3 \end{vmatrix} = 0$$

es cero porque se derivan sobre variables diferentes y sobre $\nabla \times (\nabla \cdot a)$

sabemos que $\nabla \cdot a = \nabla_i a_i = \frac{\partial a_i}{\partial x_i} + \frac{\partial a_j}{\partial x_j} + \frac{\partial a_k}{\partial x_k}$ = Campo escalar

$\nabla \times (\nabla \cdot a) =$

$\varepsilon_{ijk} \nabla_j (\nabla \cdot a)_k \Rightarrow \varepsilon_{ijk} \cancel{\nabla_j (\nabla_k a_k)} \to 0 = 0$ lo cual tiene sentido físico porque los campos escalares no tienen rotacional

$\varepsilon_{ijk}\, \partial_j\, \varepsilon_{ijk} \nabla a_{jk}$

2F) $\nabla \times (\nabla \times a) = \nabla(\nabla \cdot a) - \nabla^2 a$

sea

$$\nabla \times (\nabla \times a) = \varepsilon_{ijk} \nabla_j (\nabla \times a)_k$$
$$= \varepsilon_{ijk} \nabla_j (\varepsilon_{klm} \nabla_l a_m)$$
$$= \varepsilon_{ijk} \nabla_j \varepsilon_{klm} \nabla_l a_m$$

y sabemos que

$$\varepsilon_{kij} \varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$= \varepsilon_{kij} \varepsilon_{klm} \nabla_j \nabla_l a_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) \nabla_j \nabla_l a_m$$

$\delta_{il}\delta_{jm} \nabla_j \nabla_l a_m - \delta_{im}\delta_{jl} \nabla_j \nabla_l a_m$ aplicando los deltas

$$\nabla_m \nabla_i a_m - \nabla_j \nabla_j a_i = \nabla_i (\nabla_m a_m) - \nabla^2 a_i$$
$$= \nabla(\nabla \cdot a) - \nabla^2 a$$