Robotic Perception

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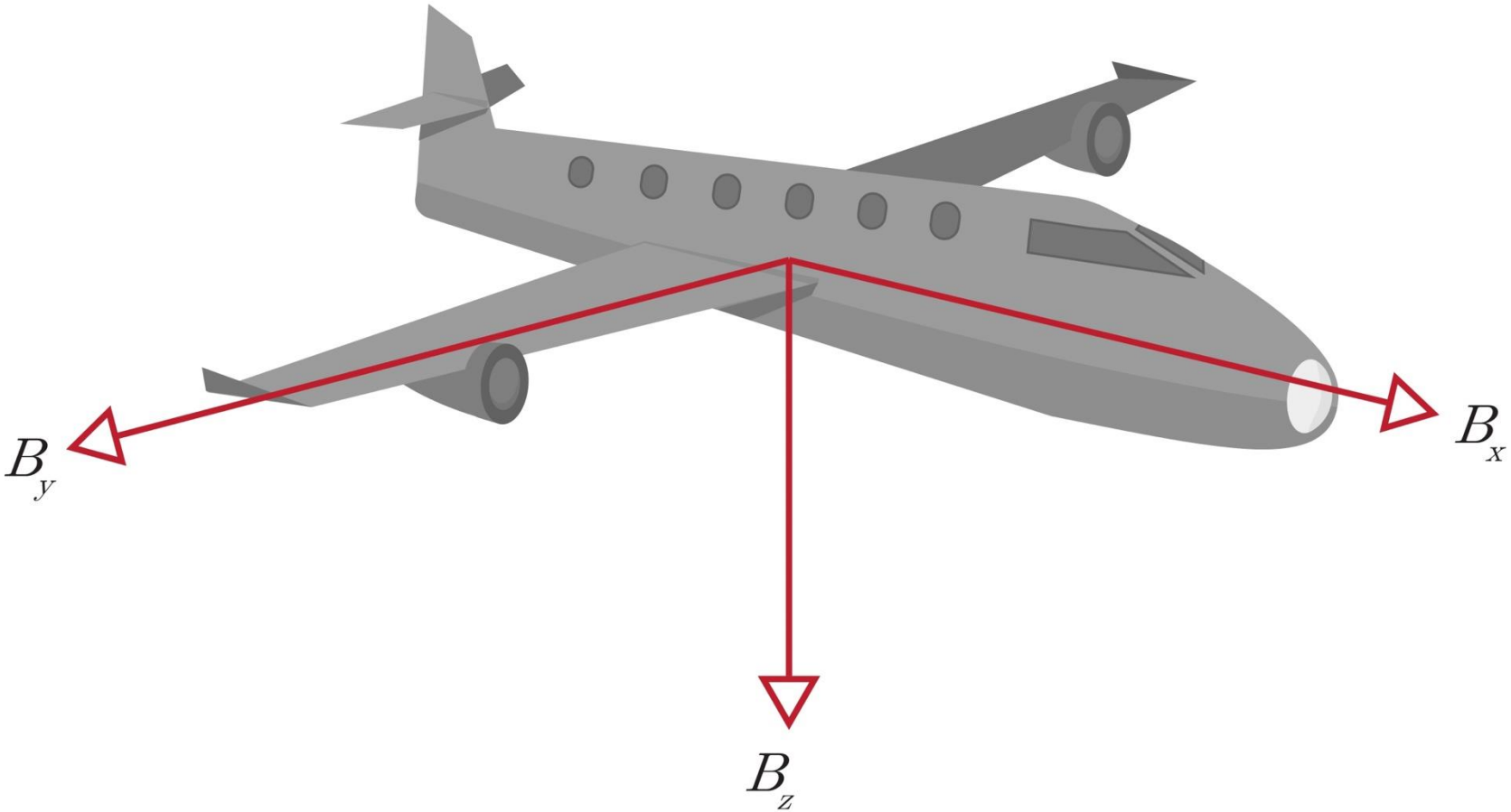
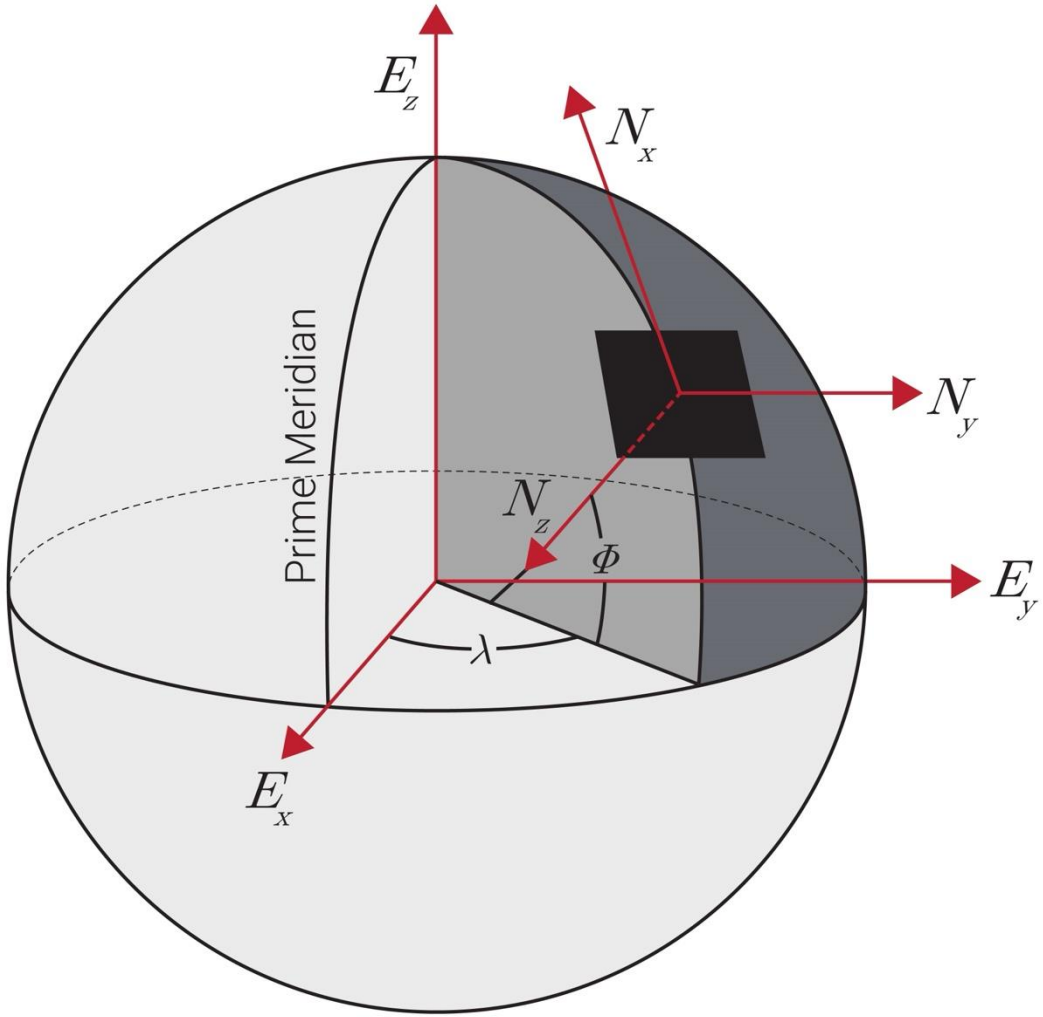


ECEF and NED



Postion, Orientation

Position

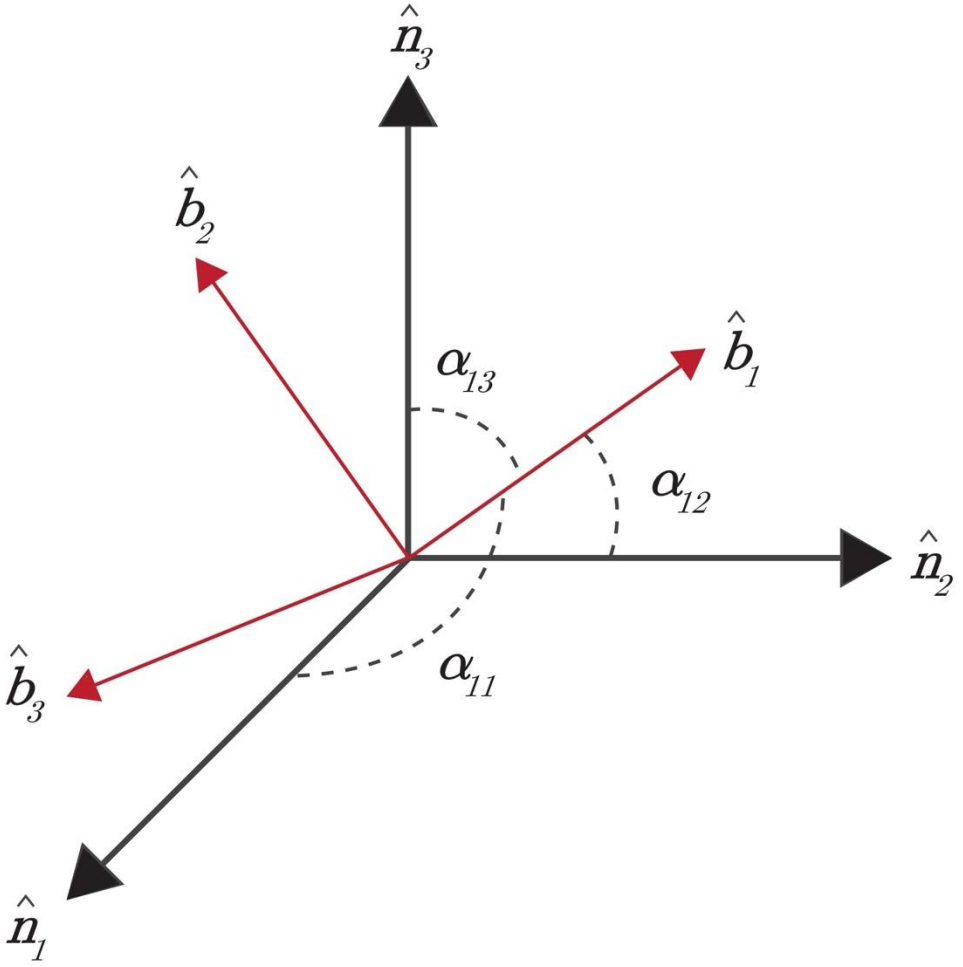
x, y, z

Velocity

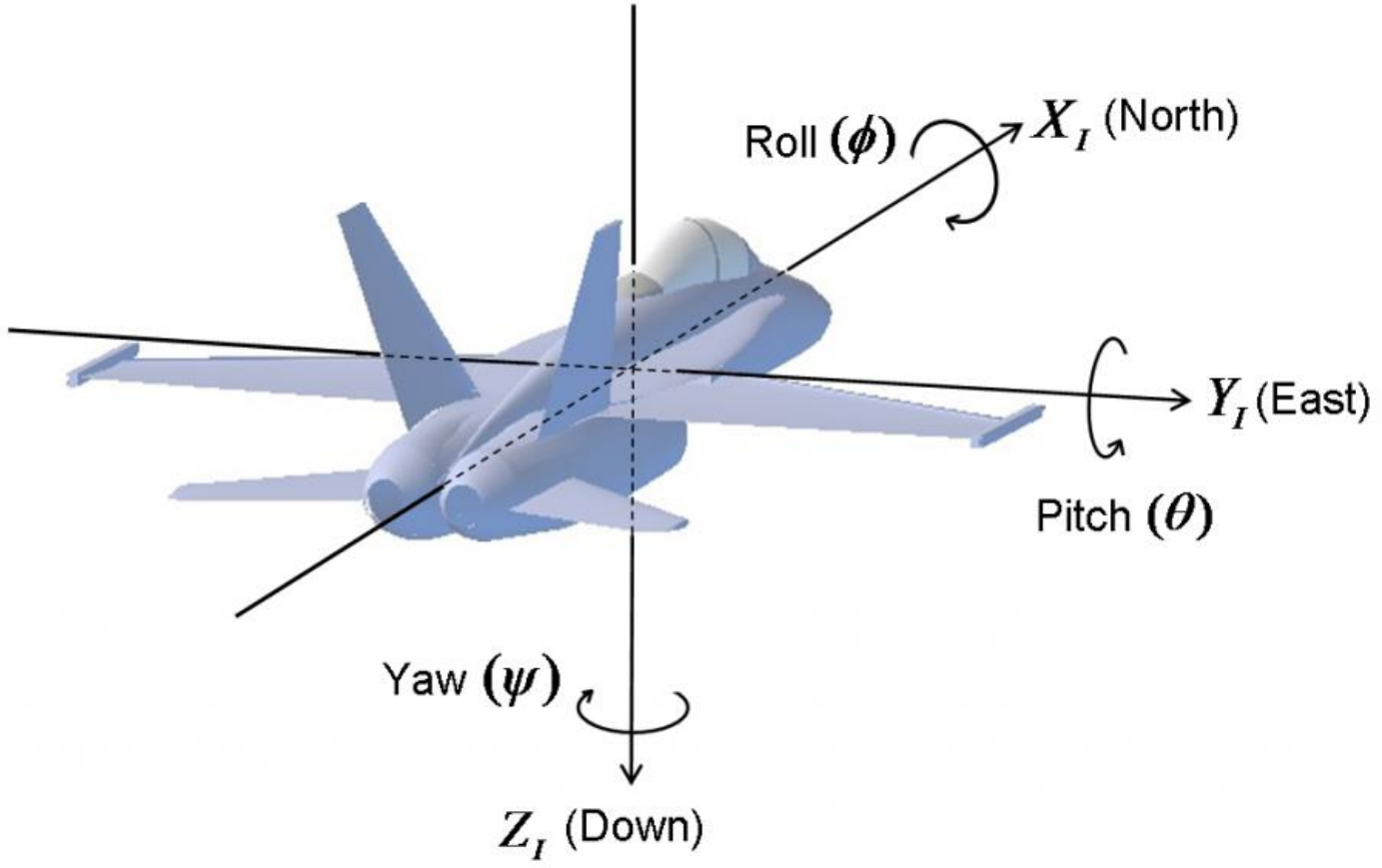
vx, vy, vz

Orientations?

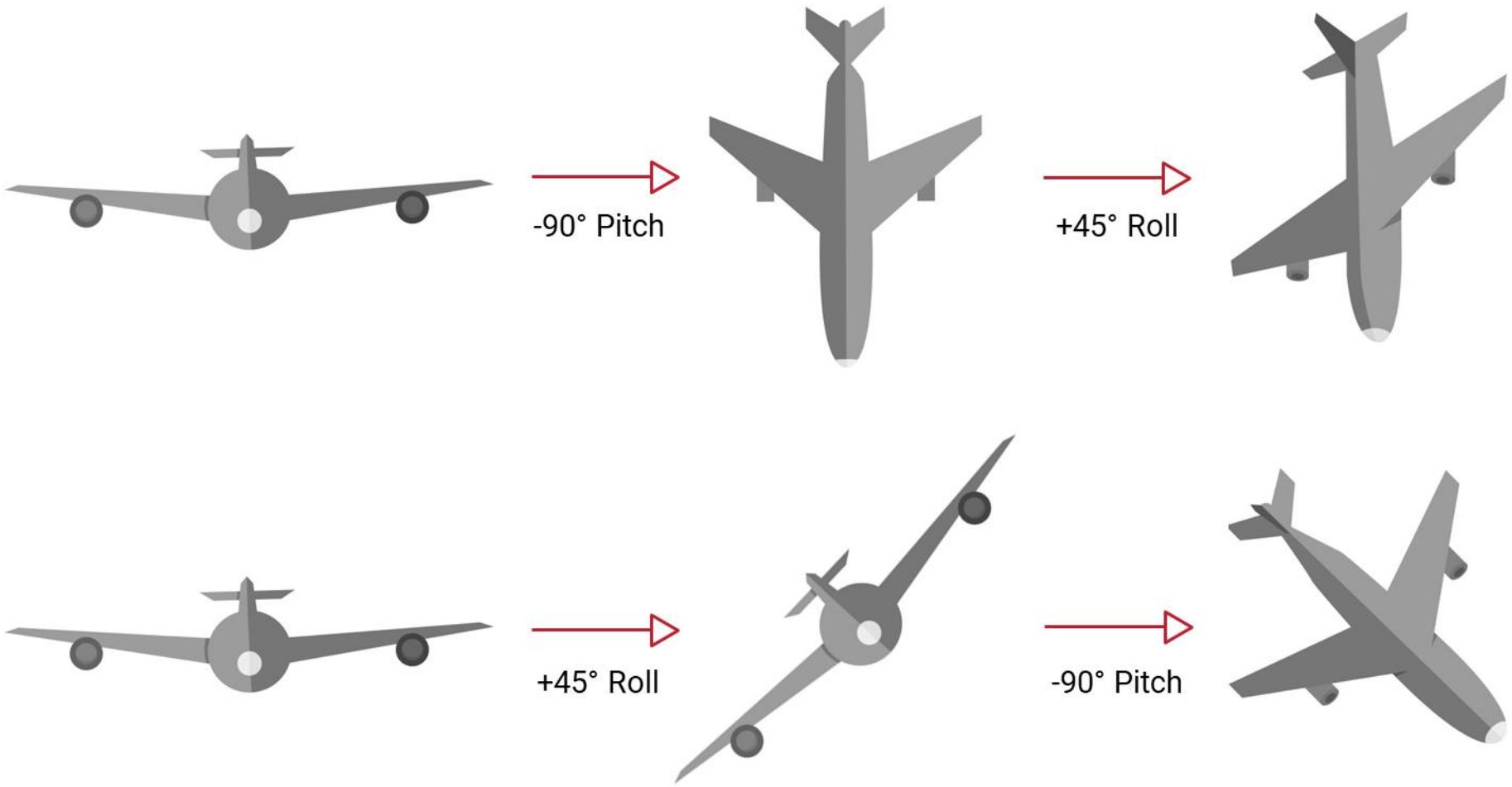
DCM


$\hat{n}_3$
$\hat{b}_2$
$\hat{b}_1$
α_{13}
α_{12}
$\hat{n}_2$
$\hat{b}_3$
α_{11}
$\hat{n}_1$

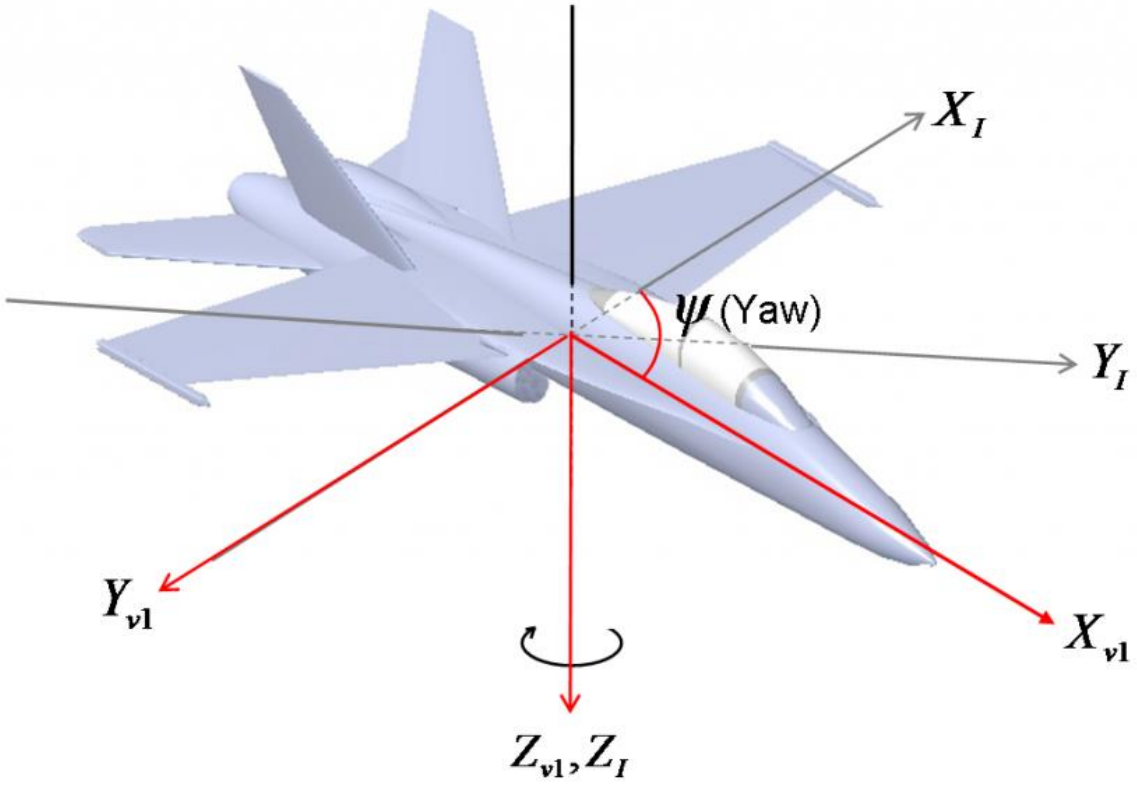
Euler Angles (Yaw, Pitch, Roll)



Order of Angles



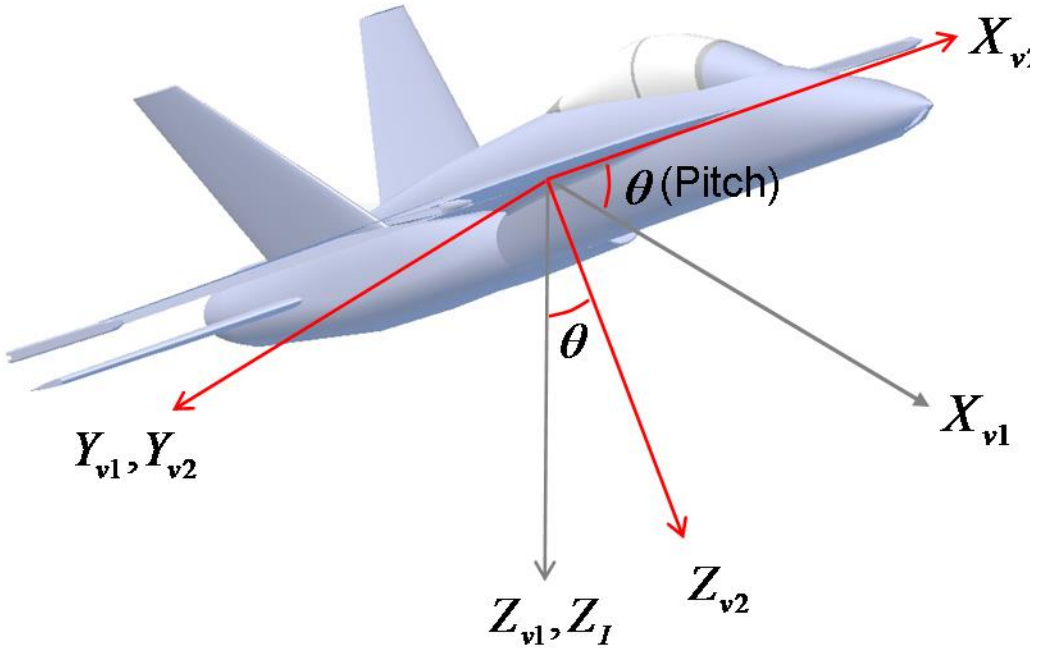
Yaw



$$R_I^{v1}(\psi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

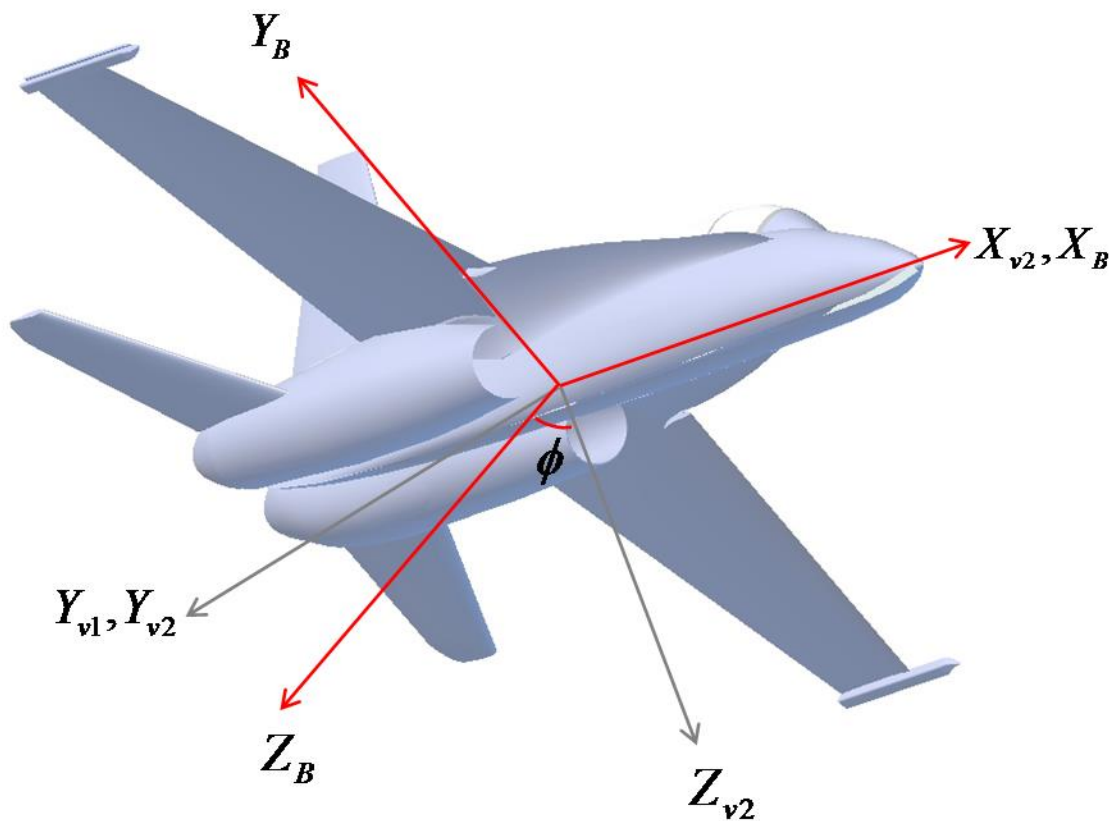
Pitch

$$R_{V1}^{V2}(\theta) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}.$$



$$R_I^{V2}(\theta, \psi) = R_{V1}^{V2}(\theta) R_I^{V1}(\psi).$$

Body Frame



$$R_{V2}^{B}(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}.$$

$$R_{I}^{B}(\phi,\theta,\psi) = R_{V2}^{B}(\phi) R_{V1}^{V2}(\theta) R_{I}^{V1}(\psi).$$

$$R_{I}^{B}(\phi,\theta,\psi) = \begin{pmatrix} c(\psi)c(\theta) & c(\theta)s(\psi) & -s(\theta) \\ c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi) & c(\phi)c(\psi) + s(\phi)s(\psi)s(\theta) & c(\theta)s(\phi) \\ s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) & c(\phi)c(\theta) \end{pmatrix}.$$

Body to Inertial

$$R_B^I(\phi,\theta,\psi) = R_I^{V1}(-\psi)R_{V1}^{V2}(-\theta)R_{V2}^{B}(-\phi).$$

$$R_I^B(\phi,\theta,\psi) = \begin{pmatrix} c(\psi)c(\theta) & c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi) & s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta) \\ c(\theta)s(\psi) & c(\phi)c(\psi) + s(\phi)s(\psi)s(\theta) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) \\ -s(\theta) & c(\theta)s(\phi) & c(\phi)c(\theta) \end{pmatrix}.$$

Quaternions

$$\mathbf{q}_1\mathbf{q}_2 = \begin{pmatrix} a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 \\ a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2 \\ a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2 \\ a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2 \end{pmatrix}.$$

$$\mathbf{q}_i^b = (a \quad b \quad c \quad d)^T.$$

b.c.d are the vector about which rotations are performed and “a” specified the amount of rotation

If \theta is the rotation about (vx,vy,vz)

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} \cos(0.5\theta) \\ v_x \sin(0.5\theta) \\ v_y \sin(0.5\theta) \\ v_z \sin(0.5\theta) \end{pmatrix}.$$

$$\mathbf{v}_B = \mathbf{q}_i^b \begin{pmatrix} 0 \\ \mathbf{v}_I \end{pmatrix} (\mathbf{q}_i^b)^{-1}.$$

Body to Inertial

$$R_i^b(\mathbf{q}_i^b) = \begin{pmatrix} a^2+b^2-c^2-d^2 & 2bc-2ad & 2bd+2ac \\ 2bc+2ad & a^2-b^2+c^2-d^2 & 2cd-2ab \\ 2bd-2ac & 2cd+2ab & a^2-b^2-c^2+d^2 \end{pmatrix}.$$

$$\mathbf{v}_B = R_i^b(\mathbf{q}_i^b)\mathbf{v}_I.$$

$$\phi = \arctan\left(\frac{2(ab+cd)}{a^2-b^2-c^2+d^2}\right),$$

$$\theta = -\arcsin(2(bd-ac)), \text{ and}$$

$$\psi = \arctan\left(\frac{2(ad+bc)}{a^2+b^2-c^2-d^2}\right).$$

Gimbal Lock – Body Frame to Inertial

Pitch 90 degrees

$$\mathbf{v}_I = R_B^I(\phi, \theta, \psi)\mathbf{v}_B.$$

$$D(\phi, \theta, \psi) = \begin{pmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{pmatrix}.$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\ q\cos(\phi) - r\sin(\phi) \\ q\sin(\phi)/\cos(\theta) + r\cos(\phi)/\cos(\theta) \end{pmatrix}.$$