



École Polytechnique Fédérale de Lausanne  
Laboratoire de Physique des Hautes Energies  
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# Simulation of the impact of LHCb kaon detection asymmetry on the Charm Mixing Parameter $y_{CP}$

*Author*

Diego Mendoza

*Supervisor*

Dr. Frédéric BLANC

*Course: Computer Simulation on Physical systems*

*Professor*

Prof. Alfredo Pasquarello

## **Abstract**

The Charm Mixing Parameter  $y_{CP}$  measured at the LHCb experiment could be affected by its detection asymmetry for kaons. This study demonstrates that the possible change on the  $y_{CP}$  parameter is of the order of  $10^{-6}$ , insignificant compared to the statistical uncertainty of the  $y_{CP}$  measurement.

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# 1 Introduction: theory and motivation

In this Project, the field which we are particularly interested in is "Particle Physics" or "High Energy Physics". Its objective is the understanding of the elementary particles, which constitute matter and the fundamental interactions between themselves through the actual theory called the Standard Model (SM). One could think that this theory has no more consequences than in the subatomic world. Nevertheless, it has an enormous impact on macro-scale theories, such as Cosmology. Indeed, one of the most important opened question at present related to the evolution of the Universe is the large difference of matter over anti-matter that we observe today. In order to explain this, a new concept was introduced in the past: the CP Violation (CPV), one of many symmetries that the SM possesses and which under certain conditions is not conserved. The first evidence of the CPV was measured at Brookhaven Laboratory in the US in 1964 in neutral  $K$  meson [1] and more recently in May 2019 in the LHCb experiment at CERN the first evidence of CPV in  $D^0$  meson [2].

Until today, the SM predicts a small CPV but not enough to explain the huge difference between matter and antimatter in our Universe. This could mean that new Physics beyond the SM need to be discovered. One of the interest fields is the charm sector, where there are some evidences that CPV could be measured experimentally. Although that is an important study at the moment, the reliability of these experiments is not clear yet because of the effects that the detector efficiency could have on it. This is the principal aim to present in this study, that has made use of a simulation to analyse the impact of these effects.

In order to achieve that, this project will be divided into several parts. Firstly, the general concepts needed to understand the theory will be presented: the Standard Model, symmetries and CP violation. Secondly, the LHCb detector and its main sub-detectors will be explained and the model developed to simulate its behaviour in terms of efficiency and event selection. Next, a whole section will be dedicated to the Simulating methods and Benchmarking checks, going deep into some important computational concepts. Finally, a discussion of results will close this Project.

## 1.1 The Standard Model, Symmetries and CPV

The Standard Model of Particle Physics is a theory that describes three of the most fundamental interactions of Nature: the electromagnetic force and the weak and strong nuclear forces, excluding the gravitational force (described in Relativity). The particles predicted by the SM are twelve particles with half-odd-integer spin (called fermions), divided in two groups: quarks and leptons; and six integer-spin particles (called bosons). Fermions constitute the fundamental blocks of matter, while bosons intermediate the three interactions between them. In image (1) these particles are presented<sup>1</sup>

The SM is constituted by a Lagrangian which introduces some continuous and discrete symmetries.<sup>2</sup> The continuous symmetries in the SM are divided into those corresponding to the strong force theory and to the electroweak<sup>3</sup>. However, other types of symmetries can be generated by discrete transformations and they become essential in this theory. For instance, one transformation is the charge conjugation (C), which consists on changing the sign of all quantum quantities<sup>4</sup>, so each

<sup>1</sup>Image taken from [5]

<sup>2</sup>A symmetry is defined as a type of invariability. Under a set of transformations or operations, a mathematical object remains unchanged

<sup>3</sup>The electroweak interaction is the unified description of the electromagnetic and the weak force

<sup>4</sup>It does not only involve the electric sign, but also the charges of other forces

	fermions			bosons
quarks	$u$	$c$	$t$	$g$
	$d$	$s$	$b$	$\gamma$
leptons	$e^-$	$\mu^-$	$\tau^-$	$Z$
	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$W^\pm$
				$H^0$

Figure 1: Elementary particles in SM

particle is transformed into its anti-particle; another one is the parity (P) which involves reversing the spatial coordinates: ( $\vec{r} \rightarrow -\vec{r}$ ) and finally the time reversal (T) in order to reverse the temporal coordinate  $t \rightarrow -t$ . Any application of these transformation does not need to be invariant, but the combination of all three is called CPT, and it is an exact symmetry of Nature<sup>5</sup>

The case which we are interested in is the CP symmetry, that supposes that the physic laws should not change when a particle is turned into its anti-particle and the spatial coordinates are inverted. Nevertheless, our Universe is made of matter in its majority and if CP symmetry was not violated, the Big Bang should have generated the same amount of matter and antimatter. Consequently, a complete cancellation of both would have occurred. The SM is able to predict CPV through three different sources.

The first one is related to the quark sector through the Calibbo-Kobayashi-Maskawa matrix (CKM) which was introduced in 1973 by the Japanese physicists Makoto Kobayashi and and Toshihide Maskawa. In order to introduce this mechanism, they needed to predict the existence of a third generation of quarks, the Top and the Bottom. The Top quark was discovered in 1995 and the Bottom in 1977, both at Fermilab [3]. This CPV has been proved experimentally although the observed value is a small portion of the CPV that the actual matter-antimatter asymmetry in the Universe requires. The second source corresponds to the strong force, but until today it has not been confirmed to exist in a enough amount to explain the previous discrepancy. Finally, the third possible source is found in the lepton sector and until today there is no experimental evidence which shows that it is the reason of the matter-antimatter asymmetry.

In case of the CPV predicted in the lepton sector were insufficient (as the case of CPV in the quark sector) to explain the matter-antimatter asymmetry, it would mean that new physics beyond the SM must be developed.

## 1.2 CPV in the Charm Sector and the $y_{CP}$ parameter

Our interest remains in one type of particles, called neutral mesons:  $K^0, D^0, \dots$ , which are formed by a pair of quark and anti-quark both up-type (up, charm, top) or down-type (down, strange, beauty). For them, it exists a phenomenon called flavour oscillation or mixing in the neutral mesons, which permits that these particles can turn into their corresponding anti-meson via weak interaction. In particular, we will focus on the  $D^0$  meson, which belongs to the Charm Sector.

<sup>5</sup>This is known as the CPT Theorem, which claims that CPT symmetry is conserved for all field theory in Minkowski spacetime

Mathematically, this is due to the fact that the solution of the weak eigenstate equation can be written as the superposition of both states  $|D^0\rangle$  and  $|\bar{D}^0\rangle$ .

If the CP symmetry is conserved, both states are present in the same proportion for a eigenstate:

$$|D\rangle = a|D^0\rangle \pm b|\bar{D}^0\rangle$$

with  $a = b$ <sup>6</sup>.

If CPV is present, the ratio  $|a/b|$  is different from 1, so each solution of the eigenstate equation has different mixing rates of  $|D^0\rangle$  and  $|\bar{D}^0\rangle$ . In this case, it is called "Indirect CPV". The SM is able to predict a small level of CPV, although more important effects can be obtained outside the outlook of the SM. This would open a source of CPV that could explain the previous asymmetry mentioned. In order to study this phenomenon, the mixing parameter is defined

$$y_{CP} \equiv \frac{\Gamma_{CP+}}{\Gamma} - 1 = \frac{\Gamma(D^0 \rightarrow h^+h^-) + \Gamma(\bar{D}^0 \rightarrow h^+h^-)}{2\Gamma} - 1 \quad (1)$$

that estimates the difference between the decay width  $\Gamma$  for the  $D^0$  and its anti-particle in decays of the type  $K \rightarrow h^+h^-$  ( $h = K, \pi$ ). This value is equal to the mixing parameter  $y$  if there is no CPV, so any deviation from that value shows that this symmetry is not conserved in  $D^0 - \bar{D}^0$  mixing. Using the relation between  $\Gamma$  and decay time  $\tau$

$$\Gamma = \frac{1}{\tau} \quad (2)$$

the equation (3) can be transformed into the ratio of decay times

$$y_{CP} = \frac{D^0 \rightarrow K^+K^-}{D^0 \rightarrow \pi^+\pi^-} \quad (3)$$

This is an indirect way of measuring the mixing parameter and which is used experimentally. The current average value of  $y_{CP}$  estimated is  $(0.719 \pm 0.113) \%$  and the last measurement of the  $y_{CP}$  at the LHCb is  $[0.57 \pm 0.13 \pm 0.09] \%$  corresponding to proton-proton collision an integrated luminosity of  $8.4 \text{ pb}^{-1}$  and  $\sqrt{s} = 7, 8, 13 \text{ TeV}$  from study [8].

### 1.3 Detector efficiency and possible consequences in the $y_{CP}$ parameter

To calculate the value of this parameter, it is needed to reconstruct all events from the particles measured by the detector. Usually, these particles are the results of interactions and decays, and they mix with the background: particles that are directly created from the collisions, they are not the result of any secondary process. In order to solve this problem, a minimum distance from the collision center (from now called Primary Vertex, PV) is introduced. In other words, all particles measured that come from the proximity of the PV will be considered as background. In this situation, two important facts must be taken into account.

One the one side, the low-lifetime of the  $D^0$  meson. This means that some of this particles that are created at the PV would decay in its proximity, so when analysing the result particles they will be considered as background and these events will be lost.

On the other side, it has been demonstrated in previous studies [5] that the LHCb detector shows

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<sup>6</sup>The normalization condition  $|a|^2 + |b|^2 = 1$  is always satisfied

an asymmetry in some particle detection, it is more efficient to detect some particles than their antiparticles. This is more important in the case of detecting kaons than pions, as they interact differently with matter. As a consequence, decay time distributions for  $D^0 \rightarrow K^+ K^- (\pi^+ \pi^-)$  and  $D^0 \rightarrow K^+ \pi^- (\pi^+ K^-)$  are affected in a different way by these inefficiencies and it could force a variation on the  $y_{CP}$  obtained. To estimate how much the parameter would change because of the effect of inefficiencies, one can use the ratio of the mixing parameter with and without efficiencies, which is the same as the ratio of decay times

$$R = \frac{D^0 \rightarrow K^+ K^-}{D^0 \rightarrow K^+ K^-} \quad (4)$$

The result on equation (4) would directly be the change of the mixing parameter, the main aim to achieve on this study. In order to estimate it, a computational study has been developed making use of Monte Carlo techniques to generate events that follows some probabilities distributions we observe experimentally and subsequent methods which apply the asymmetry efficiency of the detector on the result data.

## 2 Model

In this section, the real experiment and LHCb detector will be presented and its main characteristics, accompanied by the model developed to simulate it, the potential events and the particles behaviour.

### 2.1 The LHCb experiment

The speciality of the "Large Hadron Collider beauty" is the study of the "beauty quark" and its relation with the matter-antimatter asymmetry in our Universe. The detector is quite different from others as ATLAS or CMS, it only detects forward particles emitted by the collision in one direction, with small angles to respect to the beam axis. The detector is made of several sub-detectors<sup>7</sup>. The most interesting ones in our study are:

- VELO (Vertex LOcator). The closest to the PV where the proton-proton collision occurs. It can measure where the particle decays (with very low-lifetime as  $D^0$ )
- RICH. It identifies particles as pions and kaons from neutral meson decay through the Cherenkov radiation that they emit. There are two at the detector, before and after the magnet.
- Tracker. There are four in total (TT, T1, T2 and T3) and their function is to reconstruct the particles trajectories.
- Magnet. It measures the particle momentum by bending their trajectories.
- Muon detectors (M1 to M5). They are situated at the end of the LHCb detector.

All these parts can be visualized in the figure (2)

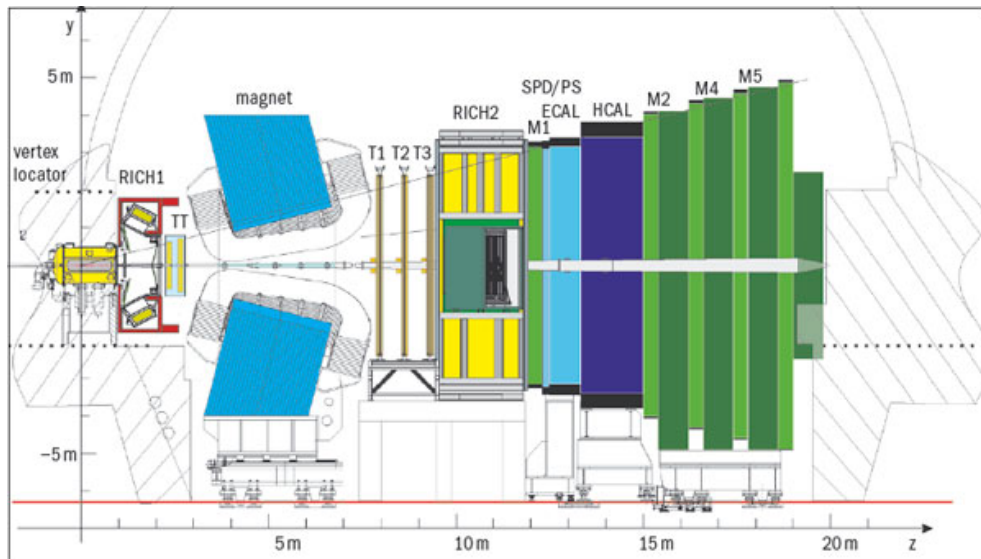


Figure 2: Drawing of the LHCb detector

<sup>7</sup>A complete description of the detector is in [collaboration2008lhcb]

In the case of the  $D^0$  meson, the particle cannot reach the first sub-detector because of its low lifetime, so it decays nearby the PV (typical decay length  $\lambda \approx 3mm$ ).

## 2.2 Computational Model

Some hypothesis has been assumed in order to set the behaviour of particles and detector as close as possible to the real data at LHCb as follow. In this part, these assumptions will be presented from a physical point of view, but the computational methods used are explained in the next section.

### 2.2.1 Producing the $D^0$ meson

It is the only initial particle generated at the PV as we are interested just in this type of meson. Each meson will be completely determined by its decay time and momentum (vector).

- Lifetime. Its probability density<sup>8</sup> is an normalized exponential function, determined by the mean lifetime<sup>9</sup>.

$$P(t)dt = \frac{1}{\tau} e^{-\frac{t}{\tau}} \quad (5)$$

Different lifetimes will be generated following this distribution, see method (3.2.1)

- Momentum (modulus). The quantity of movement of the neutral mesons are generated following real data obtained at LHCb experiment, see method (3.2.1)
- Momentum (direction). Using spherical coordinates<sup>10</sup>, the  $\tan(\theta)$  is generated using the following probability density<sup>11</sup>

$$P(\tan(\theta))d\tan(\theta) = 25e^{-25\tan(\theta)} \quad (6)$$

so the  $\theta$  angle is directly determined, see method (3.2.1). The  $\phi$  angle is generated with equal probability in the range  $[0, 2\pi]$

$$P(\phi)d\phi = \frac{1}{2\pi} \quad (7)$$

using method (3.2.1)

From these quantities, all information can be obtained for the neutral meson. The distance that it travels from the PV is

$$\vec{d} = \vec{\beta}\gamma ct \quad (8)$$

being  $t$  the  $D^0$  decay time.

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<sup>8</sup>The probability density of a physical quantity, for example lifetime, defines the probability of a particle to live from the initial to a time  $t$  integrating this function in this interval

<sup>9</sup>The mean lifetime is a quantity used in decay that characterized one particle. It is the time that takes in a population of  $N$  particles to decay and remain  $\frac{N}{2}$  particles

<sup>10</sup>In spherical coordinates, the  $\theta$  variable defines the angle between the vector and the Z axis, while the  $\phi$  variable defines the angle between the vector projection on the XY plane and the X axis, see image 3

<sup>11</sup>This function is obtained from a fit of real data at LHCb and proposed by the Supervisor



### 2.2.2 Producing the $h^+$ and $h^-$ decay particles

The  $D^0$  meson travels a distance and decays at the point  $\vec{d}$ , producing two particles  $h^+$  and  $h^-$ . If the decay is analysed from the  $D^0$  rest frame, where the meson has zero velocity, the total zero momentum must be conserved after the decay. The result particles will be both pions or kaons. They are completely determined by only the spherical angles in the rest-frame axis, that will be generated in the following way.

- The probability density for the  $\cos \theta$  is uniform between the range  $[-1, 1]$

$$P(\cos \theta) d \cos \theta = \frac{1}{2} \quad (9)$$

so it is generated using method (3.2.1) and  $\theta$  angle is completely determined for one particle just applying the inverse function to this value generated.

- The  $\phi$  angle is generated using (7) with method (3.2.1)

By default, the particle created with these angles will be the positive one,  $h^+$ . In order to preserve the total zero momentum, the way of  $h^-$  must be just the opposite.

Using the tools presented in sections A.2 and A.3, the two 4-D vectors can be built for both particles at the rest frame and the lab frame using the Lorentz boost.

### 2.2.3 Determining the Impact Parameter

As it has been said before, the detector also identifies particles created in the p-p collision at the PV. In order to distinguish them from the particles created at the  $D^0$  decay, a minimum IP<sup>12</sup> must be considered. For a particle having a linear momentum  $\vec{p}$  and which is at a point  $\vec{d}$  (where the  $D^0$  meson decays), the IP can be calculated with the following relation.

$$IP = \frac{|\vec{d} \times \vec{p}|}{|\vec{p}|} \quad (10)$$

In those decays in which the result particles have an IP lower than the minimum IP, the model would not take this data as useful. For instance, in image (3), the two cases can be visualised in 2 dimension. Although the real case is in 3 dimension, the process is still the same.

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<sup>12</sup>The Impact Parameter is the closest distance from the direction of a particle to the PV

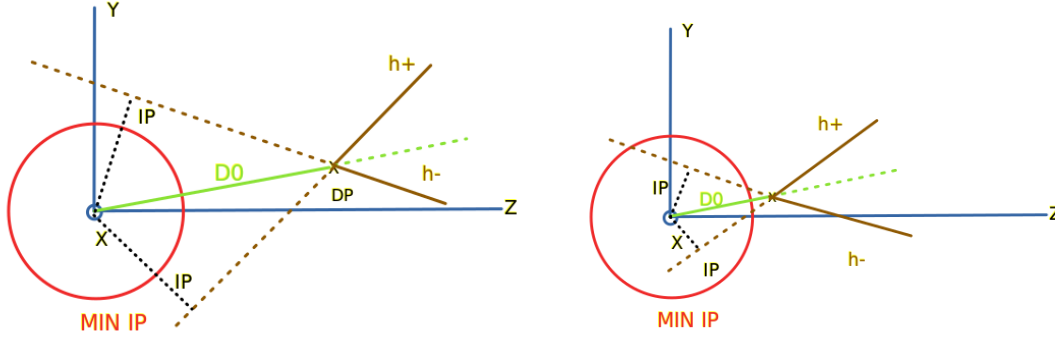


Figure 3: (LEFT) A decay measured by the detector. (RIGHT) A decay missed from data

The value of the IP for  $h^+$  and  $h^-$  will depend on the random angles generated for the result particles and the  $D^0$ , but the most important factor is the flight distance of the meson. This will introduce an important dependence on its lifetime, even though momentum is also influential.

#### 2.2.4 Effect of detector efficiency

The detector efficiency asymmetry in the measurement of one type of particle and its antiparticle can be the responsible of any deviation on the obtained value of the  $y_{CP}$  parameter. These inefficiencies are more noticeable in the case of kaons than pions, so in this study only the first ones will be considered. By definition, the efficiency detecting  $K^+$  will be equal to one while for the  $K^-$  the detector efficiencies will be the values on the table (1), data taken from the study [5].

$p(K)[GeV/c]$	$\epsilon_{detector}$	$\sigma_\epsilon$
[0,6]	0.9756	0.0067
[6,12]	0.9793	0.0046
[12,18]	0.9828	0.0045
[18,24]	0.9848	0.0042
[24,30]	0.9856	0.0042
[30,36]	0.9843	0.0042
[36,42]	0.9869	0.0042
[42,48]	0.9855	0.0043
[48,54]	0.9885	0.0045
[54,60]	0.9891	0.0045
[60,66]	0.9896	0.0045
[66,72]	0.9882	0.0046
[72,78]	0.9912	0.0052
[78,84]	0.9921	0.0053
[84,90]	0.9893	0.0058

Table 1: Detector efficiency for  $K^-$

### 3 Computational methods and benchmarking checks

#### 3.1 ROOT

ROOT is a software framework used in data analysis for its advanced graphical interface and effectiveness as interpreter of C++ objects. It was specially created for research at high-energy physics at CERN in order to perform simulations and process data. It provides a binding to integrate other languages as Python, called PyROOT. This work has made use of it to benefit from all useful tools on ROOT and such a highly readable coding language as Python.

The most important functional parts of ROOT used in this program are histograms, functions, physical vectors and random number and variable generators. This part will present the classes needed to work with these objects.

- **TRandom3**: It is a random number generator class based on a 623-dimensional equidistributed uniform pseudorandom number generator. It has a large period of  $2^{19937} - 1$  and even though a large internal state of 624 integers, it is the most recommended class. The function members used in this class are `Rndm()`, which can generate uniform pseudorandom numbers on the range  $[0,1]$  or on other different range with the extension `Rndm.Uniform()`; and `SetSeed()` that permits to choose the seed for the random number generator. In the present program, the machine clock has been used as seed.
- **TF1**: Class that provides 1-Dim functions defined in ranges and associated to parameters. The constructor is `TF1()` and these functions are used in the program to fit histograms of type `TH1F`.
- **TH1**: Base class for 1-dimensional histograms which enable operations as filling, binning, sum, division and drawing. The constructor taken in this work is `TH1F()`, which uses a float number per channel. Other methods provided by this class are: `Fill()`, `Fit()`, method that uses TF1 objects; `GetMean()`, `Divide()`, which gives an histogram result from the quotient of two with the same characteristics; `Scale()` which normalizes the histogram; `Sumw2()`, to store sum of squares of weights; or `Draw()`.
- **GenVector**: General Vector class to work with vectors and transformations. It provides other template classes as `DisplacementVector3D`, to generate 3-dimensional vectors in cartesian coordinates; `LorentzVector` class, which works with 4-dimensional vectors in the Minkowski space; and `Boost` class to build Lorentz transformations represented by 4x4 matrix `A`.

#### 3.2 Methods

This program has made use of some specific techniques which need to be explained apart from the whole code because of their complexity. There are four main computational methods used to generate variables and simulate the detector behaviour, they are the most interesting components from a computational point of view.

### 3.2.1 Random variable generation

To reproduce events on the detector, it is necessary to simulate some of the characteristics of the particles used in this work: momentum, angles and decay time.

- Uniform distributed variables. Some variables have a probability density function<sup>13</sup> which is uniform in a specific range. For example in this study, in the emission of the  $D^0$  meson, the spherical angle  $\phi$  has the probability density function in equation (7); or for the generation of decay particles at the rest frame, the spherical angle  $\theta$  is generated from the random variable  $\cos\theta$  from the equation (9) and  $\phi$  is generated in the same way as the previous case of the neutral meson. To generate all these variables, the class TRandom3 and the object Rndm.Uniform() have been used.
- Nonuniform distributed variables. Some more interesting variables have a probability density function more complex. In this case variables cannot be generated directly from the TRandom3 class, so a method must be developed to generate them from the random number generator. In our case, the variables are the  $D^0$  decay time and spherical angle emission  $\theta$ , whose probability density functions are equations (5) and (6) respectively. Both distributions are exponential, so the method will be the same. Using the conservation of probability theorem, the probability of finding two related variables: X uniformly distributed in  $[0,1]$  and Y distributed by  $Q(Y)$  an exponential between  $[0,+\infty]$  must be constant:

$$|P(X)dX| = |Q(Y)dY| = |dX| = \left| \frac{e^{-Y}}{K} \right| \quad (11)$$

This enable to express Y from X, shown in equation (12), form equation (11) just doing a change of variable

$$Y = -K \ln(1 - X) \quad (12)$$

being K a constant that comes from the normalization of the probability density function. In the case of the decay time  $K = \tau$  while for the  $\theta$  angle,  $K = \frac{1}{25}$ . This method is enough to generate random variables from the object Rndm() on TRandom3 class.

- Acceptance-rejection method of von Neumann. This method is used when proceeding through the previous one is impractical (sometimes because the function is too difficult). This method will be explained applied to the interest case: generating momentum values of  $D^0$  from a real histogram obtained from [8]. First step is finding a function  $h(x)$  such that the histogram is always enclosed. The corresponding probability density is given by equation (14).

$$h^N(x) = \frac{h(x)}{\int_{-\infty}^{+\infty} h(x) dx} \quad (13)$$

In our case, an horizontal slope has been chosen as function<sup>14</sup>, whose high is the maximum value of the real histogram. The range or work is  $[0, 3 * 10^5]$ , so the method calculate the normalized function as

$$h^N(x) = \frac{1}{3 * 10^5 * hist_{max}} \quad (14)$$

<sup>13</sup>The probability density function is a mathematical functions which expresses the probability of finding a variable in a range  $[a,b]$  when this function is integrated in this range. They always have to be normalized

<sup>14</sup>It was not worthy to improve the method with other more suitable function because the time that this method takes is very small

Now, the method generates a first random number, which would be the variable generated, in the large range and takes the value of the histogram on it. Also, a random number in  $[0,1]$  is generated and kept. Next, it uses an if condition: the first random number is stored if the histogram value on it is greater than the second random number times the function used (slope, so constant value). Else, the method starts again. It uses a while condition and a auxiliary number to count the variables generated until it is one million.

### 3.2.2 IP cut method

This method is thought to erase those events from data which mix with background. It is a simple method that uses a for-loop and the logic operator OR to compare in each event ( $D^0$  meson generated) the IP of decay particles  $h^+$  and  $h^-$  with the minimum IP set. If any of them is lower than the value, the event is lost from data.

### 3.2.3 Efficiency method

This method is created in order to simulate the detector asymmetry efficiency detecting  $K^-$  over  $K^+$ . It also uses a for-loop in which for each event, it calls a function that returns the value on table (1) depending on the  $K^-$  momentum. Once obtained this value, a random number between  $[0,1]$  is generated using `Rndm()` from `TRandom3` and both are compared using an if condition: if the random number is greater than the efficiency, that event will be erased from data. Else, the event remains and the method studies the next event.

## 3.3 Program and benchmarking checks

The program starts setting the initial parameters for the simulation:  $D^0$  lifetime, decay particles masses and number of events to generate, that in the present study has been  $10^6$ . This means that it will generate  $10^6$  events for each decay  $D^0 \rightarrow \pi^+\pi^-$  and  $D^0 \rightarrow K^+K^-$ . Next, the program creates the arrays which will be filled with the future values obtained: floats, 4-D and 3-D vectors, boosts...<sup>15</sup> and it gives them zero initial values. All arrays are defined inside a list of dimension 2 because in that way one can change their lengths independently, what is impossible if a matrix were used, and can simulate both decays only using a for-loop<sup>16</sup>

The program starts now with the decay  $D^0 \rightarrow \pi^+\pi^-$ , the neutral mesons are created generating the decay time from method (3.2.1), linear momentum from (3.2.1) and direction from (3.2.1) to  $\theta$  and (3.2.1) to  $\phi$  and the arrays are filled with these values. A way to check that these variables are being created correctly is filling a histogram and fit it with a suitable function in each case and compare the result parameters. For example, it has been done for decay time, equation (5), obtaining a lifetime of  $409.5 \pm 0.4$  fs or in the case of the angle, equation (6), a value  $24.87 \pm 0.19$ . To check  $D^0$  momentum method (3.2.1), one can compare the mean of a result histogram filled with the variables and the real histogram used. In this case, the values are 55440 and 55430 GeV/c, so all methods are working correctly.

<sup>15</sup>ROOT enables working in Python with arrays of any ROOT object: vectors, histograms, functions...

<sup>16</sup>First index of all lists are the arrays to keep variables in one decay type and the second to store the other decay.

Next step is generating the decay particles at the rest frame: direction for  $h^+$  is determined from the angles, variables obtained from the methods (3.2.1) for  $\phi$  and  $\theta$  (see section 2.2.2). For  $h^-$  it is the same direction but opposite way. Energy of these particles is taken from the  $D^0$  energy on the rest frame (its mass) using equation in A and therefore their linear momentum  $\vec{p}$  are completely determined. The program creates now the 4-D vectors from LorentzVector class for each particle and it uses the TBoost class to make a Lorentz transformation to the lab frame, so the direction of these particles in the detector are now accessible. Repeating the previous steps with the for-loop, events for decay  $D^0 \rightarrow K^+ K^-$  are generated in the same ways using now their corresponding mass.

Once all events are created, the IP method (3.2.2) is applied with the minimum IP desired. It erases positions in arrays with the function `np.delete()` from Numpy library. The method starts again for the other decay type. One way of checking if this method is working is using a for-loop to check if any IP is greater than the minimum IP. Immediately, results are copied in order not to lose them and the efficiency method (3.2.3) is applied to data for the nominal inefficiency and twice it, but this time it only affects to events from kaons. Both results are kept separately to compare them.

- For 50  $\mu m$  IP, there are 444478 events from pions and 409985 from kaons. Applying inefficiencies, there are left 403540 and 203973 for nominal and twice inefficiencies.
- For 100  $\mu m$  IP, there are 228445 events from pions and 196484 from kaons. Applying inefficiencies, there are left 193426 and 97779 for nominal and twice inefficiencies.
- For 200  $\mu m$  IP, there are 70065 events from pions and 52561 from kaons. Applying inefficiencies, there are left 51774 and 25998 for nominal and twice inefficiencies.

Efficiency is near 99% for nominal inefficiency and 49% for twice it. One way of checking the method is compare if the number of events that remain after applying it correspond to these percentages. In our study the results seems correct.

## 4 Results and Discussion

The effect of the detector efficiency on the  $y_{CP}$  value is also affected by the minimum IP chosen to clean the measurements from the background. In other words, kinematics (efficiency) and lifetime (IP cut) are related to the possible repercussion on the interest parameter. In order to study this relation, different minimum IP have been considered at the same time that the detector inefficiencies are varied among complete efficiency (there is no asymmetry in the detector measurement of particles and anti-particles), nominal inefficiency (values at table 1) and twice the nominal inefficiency (to predict how uncertainties can affect the result). The three minimum IP studied are 50, 100 and 200  $\mu m$ .

In Figure (4) we can see the result for the  $y_{CP}$  from equation (3) for different minimum IP and nominal inefficiency and in table (2) the values obtained for all cases.

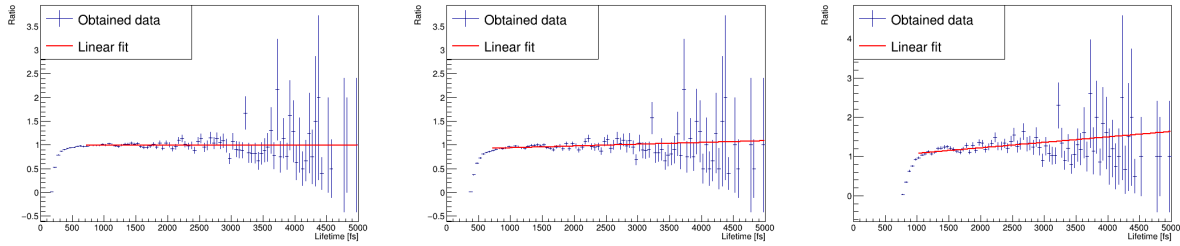


Figure 4: Results for min IP 50 (left), 100 (center) and 200 (right)  $\mu m$  for nominal inefficiency

min IP [ $\mu m$ ]	Ratio $\frac{K^+K_{\epsilon=1}^-}{\pi^+\pi^-}$	Ratio $\frac{K^+K_{\epsilon}^-}{\pi^+\pi^-}$	Ratio $\frac{K^+K_{2\epsilon}^-}{\pi^+\pi^-}$
50	$(-1.68 \pm 8.21)10^{-6}$	$(-2.02 \pm 2.87)10^{-6}$	$(-5.65 \pm 9.81)10^{-6}$
100	$(41.61 \pm 3.28)10^{-6}$	$(41.35 \pm 3.29)10^{-6}$	$(39.04 \pm 4.03)10^{-6}$
200	$(148.9 \pm 5.6)10^{-6}$	$(146.8 \pm 5.8)10^{-6}$	$(123.03 \pm 6.8)10^{-6}$

Table 2: Slope values of the linear fit

For the lower IP, the slope is compatible with zero in all 3 cases of inefficiencies, so at this point no effect is observable. However, the ratio is always lower than the unit. This makes sense with the theory because kaons are more probable to be produced with a smaller angle respect to the direction of the  $D^0$  meson than pions, as the mass of the first ones is greater. In this way, the IP cut will affect more to kaons.

In the case of 100  $\mu m$  the slope is no longer compatible with zero, even though the effect is very small. However, the case of 200  $\mu m$  confirms this assumption as the difference between the value of the slope and the error is two order of magnitude. From the evolution of the result we can conclude that the IP cut affects more to kaons than pions in the range of low lifetime. This is due to the difference in kinematics between the two decays  $D^0 \rightarrow \pi^+\pi^-$  and  $D^0 \rightarrow +^-$ , from equations at Appendix (A) it is clear that kaons will have momentum values smaller than pions as their mass is greater, so there will be much more relation between low lifetime and low linear momentum in kaons than in the case of pions.

From this results we can conclude that the  $y_{CP}$  is significantly affected only when the inefficiencies are large (twice the nominal value in table (1)) or a tight cut is made on the IP ( $\geq 200 \mu m$ ). This means that for the nominal cut that is made on the real detector ( $\geq 50 \mu m$ ) and nominal efficiency, the value of  $y_{CP}$  is not affected by the kaon inefficiencies.

Additionally, the results can be improved to avoid the huge difference in low decay time seen before and compare  $y_{CP}$  parameters obtained for nominal inefficiency and no inefficiency, or for twice nominal inefficiency and no inefficiency using the equation (4). Results are shown on table (3) and at images (5), (6) and (7) for three different minimum IP.

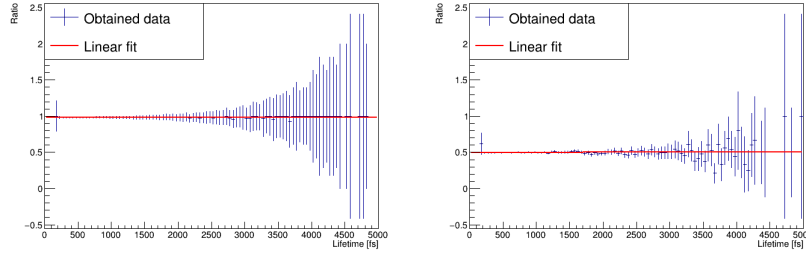


Figure 5: Results for min IP  $50 \mu m$  for nominal inefficiency (left) and twice the nominal inefficiency (right)

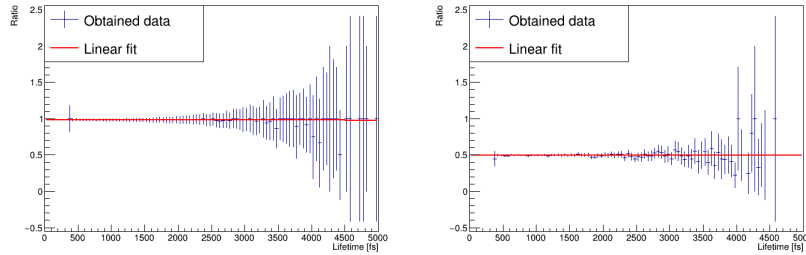


Figure 6: Results for min IP  $100 \mu m$  for nominal inefficiency (left) and twice the nominal inefficiency (right)

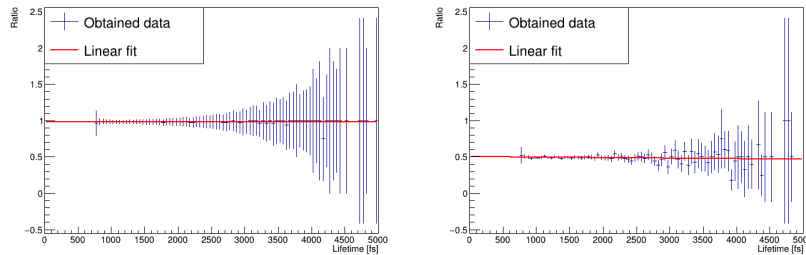


Figure 7: Results for min IP  $200 \mu m$  for nominal inefficiency (left) and twice the nominal inefficiency (right)



min IP [ $\mu m$ ]	Ratio $\frac{K^+K_{\epsilon}^-}{K^+K_{\epsilon=1}^-}$	Ratio $\frac{K^+K_{2\epsilon}^-}{K^+K_{\epsilon=1}^-}$
50	$(-0.7 \pm 2.6)10^{-6}$	$(-5.05 \pm 3.22)10^{-6}$
100	$(-1.62 \pm 2.97)10^{-6}$	$(-0.42 \pm 3.64)10^{-6}$
200	$(-0.44 \pm 4.16)10^{-6}$	$(-14.21 \pm 5.0)10^{-6}$

Table 3: Slope values of the linear fit

All the results of R are compatible with zero, except for the slopes with twice the inefficiencies and 50  $\mu m$  min IP, which is order  $10^{-6}$ ; or 200  $\mu m$  min IP, which is order  $10^{-5}$ . This means that the effect of kaon efficiency of the detector would have no impact in real LHCb measure of  $y_{CP}$  with minimum IP 50  $\mu m$  and assuming the nominal efficiencies. Moreover, the minimum precision that the detector can obtain is order  $10^{-4}$  due to statistical fluctuations of data that we have not mentioned here. This is a very important and result and it means that even in the worst-case scenario, the effects of the kaon detector efficiency are impossible to be reflected on real measurements.

## 5 Conclusion

In this study, the possible effects of kaon detector inefficiency on the charm mixing  $y_{CP}$  parameter has been simulated. The values of  $y_{CP}$  were extracted from a linear fit from the ratio between  $D^0 \rightarrow \pi^+\pi^-$  and  $D^0 \rightarrow \pi^+\pi^-$  as a function of  $D^0$  decay time for different minimum IP considered. Also, the two values of  $y_{CP}$  applying nominal and twice inefficiencies are compared to the  $y_{CP}$  value without inefficiencies for each minimum IP in order to predict how the value could change.

In the first part of results, there is only significant change of the mixing parameter when a large IP cut of 100 and 200  $\mu m$  and twice the nominal inefficiency values are applied. It differs from the real case on LHCb detector, where the minimum IP is 50  $\mu m$  and nominal efficiency is assumed. However, due to the differences on kinematics between pions and kaons, no reliable results are obtained at low  $D^0$  decay times, so this range is not taken into account in this first analysis.

The second part of the results solves this problem at low decay times comparing only kaon decays. From the values obtained when the  $y_{CP}$  parameter is compared using nominal and twice the value of inefficiencies respect to no kaon inefficiencies, the effect on the interest case of minimum IP 50  $\mu m$  and nominal efficiency is not significant. Only when the minimum IP is large enough (200  $\mu m$ ) and the inefficiencies are twice the nominal values, the impact is significant and of order  $10^{-5}$ . However, the expected uncertainty on LHCb measurements of the  $y_{CP}$  parameter is expected to be order  $10^{-4}$ .

In conclusion, this study has demonstrated that the kaon efficiency effect of the LHCb detector has no significant impact on the measurement of  $y_{CP}$ .

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# Appendices

## A Physical constants and mathematical formalism

### A.1 Physical constant

In this Project, the physical constants<sup>17</sup> have been obtained from ref

- $D^0$  mass:  $1864.84 MeV/c^2$
- $D^0$  mean lifetime:  $(4.101 \pm 0.015) * 10^{-13} s$
- $K^{+-}$  mass:  $493.667 MeV/c^2$
- $\pi^{+-}$  mass:  $139.57 MeV/c^2$

### A.2 Special Relativity and Lorentz Transformations

Every calculus done in this Project has been inside the framework of Special Relativity, so the aim of this section is to summarize some concepts and tools for the non-expert reader. Some important quantities are:

- Total energy of a particle:  $E = \sqrt{p^2 c^2 + m^2 c^4}$
- Relative velocity respect to light speed:  $\vec{\beta} = \frac{\vec{p}c}{E}$
- Lorentz Factor:  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

In Special Relativity, instead of using 3-dimensional vectors to describe position, velocity..., 4-dimensional ones are used because we are working using other different metric<sup>18</sup> from the Euclidean metric. The 4-D vectors that have been used in this Project have the following form:

$$A = (E_0, \vec{p}c) = (E_0, p_x c, p_y c, p_z c)$$

It is possible to transform these vectors in different ways. A Lorentz boost can transform a vector from one coordinate frame in space-time to another one which moves at constant speed. For instance, if a particle moves with an arbitrary velocity in any direction  $\vec{v}$ , the boost matrix that represents the Lorentz transformation is:

$$B(\mathbf{v}) = \begin{bmatrix} \gamma & -\gamma v_x/c & -\gamma v_y/c & -\gamma v_z/c \\ -\gamma v_x/c & 1 + (\gamma - 1) \frac{v_x^2}{v^2} & (\gamma - 1) \frac{v_x v_y}{v^2} & (\gamma - 1) \frac{v_x v_z}{v^2} \\ -\gamma v_y/c & (\gamma - 1) \frac{v_y v_x}{v^2} & 1 + (\gamma - 1) \frac{v_y^2}{v^2} & (\gamma - 1) \frac{v_y v_z}{v^2} \\ -\gamma v_z/c & (\gamma - 1) \frac{v_z v_x}{v^2} & (\gamma - 1) \frac{v_z v_y}{v^2} & 1 + (\gamma - 1) \frac{v_z^2}{v^2} \end{bmatrix},$$

<sup>17</sup>Mass is always equal for both particle and antiparticle

<sup>18</sup>A metric is the mathematical way settled to measure distances between vectors in a dimensional space

and the 4-D vectors transform in the following way:

$$A' = B(\mathbf{v})A$$

### A.3 Two-body decay

In this type of decay, the momentum and energy of the result particles are completely determined. In the decay studied in this Project, both result particles have the same mass, so their energy and momentum would be the same (also the direction).

- Energy disponible at the rest-frame:  $\sqrt{s} = E^* = mc^2$
- Result particle energy:  $E_1^* = E_2^* = \frac{E^*}{2}$