Open quantum systems - PHYS3136-1 - Projects

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I. QUANTUM TRANSPORT IN A QUANTUM DOT TUNNEL JUNCTION

Consider a quantum dot system described by the Hamiltonian ($\hbar = 1$)

$$H_S = \sum_{s=\downarrow,\uparrow} \omega c_s^\dagger c_s + U c_\uparrow^\dagger c_\uparrow c_\downarrow^\dagger c_\downarrow, \tag{1}$$

where c_s ($c_s^\dagger$) is the annihilation (creation) operator for an electron of spin $s = \uparrow, \downarrow$ and energy ω, and where $U > 0$ is the Coulomb energy between the electrons. The quantum dot can be viewed as an effective four-level system with the four basis states: $|0\rangle$, $|\downarrow\rangle$,$|\uparrow\rangle$, $|\downarrow\uparrow\rangle$, denoting, respectively, no electron, an electron of spin down, an electron of spin up, and both electrons (spin up and down) in the dot.

In order to study the transport properties of the system, we couple the quantum dot to a source (left) and drain (right) reservoirs consisting of two baths of free electrons. Their Hamiltonians read

$$H_\ell = \sum_{k,s} \omega_k b_{\ell k s}^\dagger b_{\ell k s} \tag{2}$$

where $b_{\ell ks}$ ($b_{\ell ks}^\dagger$) is the annihilation (creation) operator for an electron of spin $s = \uparrow, \downarrow$, momentum k and energy $\hbar\omega_{\ell k}$ in the reservoir $\ell = L, R$. The interaction Hamiltonian read

$$H_I = \sum_{\ell ks} \left(g_k b_{\ell ks}^\dagger c_s + g_k^* b_{\ell ks} c_s^\dagger \right) \tag{3}$$

where the g_k are spin-independent coupling constants.

List of tasks

- Derive a Markovian master equation for the quantum dot using the Born-Markov approximation and assuming
  - baths at thermal equilibrium with states $\rho_{B\ell} = e^{-\beta_\ell (H_\ell - \mu_\ell N_\ell)}/\mathrm{Tr}[e^{-\beta_\ell (H_\ell - \mu_\ell N_\ell)}]$ with $\beta = 1/(k_B T_\ell)$ and where μ_ℓ, T_ℓ and $N_\ell = \sum_k b_{\ell ks}^\dagger b_{\ell ks}$ are the chemical potential, temperature and number operator for the bath ℓ;
  - constant spectral densities via setting $|g(\omega)|^2 = \gamma/\pi$ in the continuum limit where γ is a tunneling rate.
- Calculate the steady state ρ_{ss} of the quantum dot. For which parameters the steady state of the system is at equilibrium ? Confirm your results by solving the master equation numerically.
- Show that the steady state particle current in the right reservoir, defined as $\langle I_R \rangle_{ss} = -\sum_s \mathrm{Tr}[\mathrm{c_s^\dagger c_s} \mathcal{D}_\mathrm{R}[\rho_\mathrm{ss}]]$, where $\mathcal{D}_R[\rho_{ss}]$ denotes the dissipative part of the master equation coming from the right reservoir, is given by

$$\langle I_R \rangle_{ss} = 4\gamma \frac{n_R(\omega - \mu_R)[n_L(U + \omega - \mu_L) - 1] - n_L(\omega - \mu_L)[n_R(U + \omega - \mu_R) - 1]}{n_L(\omega - \mu_L) - n_L(U + \omega - \mu_L) + n_R(\omega - \mu_R) - n_R(U + \omega - \mu_R) + 2} \tag{4}$$

  where $n_\ell(E) = 1/(1 + e^{\beta_\ell E})$ is the Fermi-Dirac distribution for the bath ℓ.
- Show that for low enough temperatures and by setting $\mu_L = +V$ and $\mu_R = -V$, the evolution of the current with respect to V (the so-called current-voltage characteristics of the junction ($I - V$ curve)) is quantized, highlighting the effect of the Coulomb interactions (Coulomb blockade).

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II. DISSIPATIVE PHASE TRANSITION IN THE DICKE MODEL

Consider the Markovian master equation

$$\dot{\rho}_{\rm tot} = -i[H, \rho_{\rm tot}] + \kappa \left(2a\rho_{\rm tot}a^\dagger - \{a^\dagger a, \rho_{\rm tot}\}\right) \tag{5}$$

describing the dynamics of an ensemble of two-level systems coupled to a lossy cavity mode, where

$$H = \frac{\omega_0}{2}\sum_{i=1}^{N}\sigma_z^{(i)} + \omega a^\dagger a + \frac{g}{\sqrt{N}}\sum_{i=1}^{N}\left(\sigma_x^{(i)}a + \sigma_x^{(i)}a^\dagger\right) \tag{6}$$

is the Dicke Hamiltonian, ω_0 the transition frequency of the two-level systems, ω the cavity frequency, g a coupling strength, κ the cavity loss rate, and where $\sigma_m^{(i)}$ $(m = x, y, z;\ i = 1, \ldots, N)$ is the Pauli operator for the i^{th} two-level system and a the annihilation operator for the cavity mode.

List of tasks

- Derive a Redfield master equation governing the dynamics of the ensemble of two-level systems by tracing out over the cavity mode degrees of freedom. Consider the cavity mode to be in the vacuum state, i.e., $\rho_B = |0\rangle\langle 0|$ (bath at zero temperature).
- Solve numerically the Redfield master equation for the two-level systems and compare your solution with the solution of the master equation for the full system given by Eq. (5). In which regimes of parameters the two solutions agree with each other ?
- Show that for large N, the model shows the emergence of a dissipative phase transition. We define a dissipative phase transition (DPT) by the existence of a system observable O, independent of the parameter g, whose steady-state expectation value displays a nonanalytic behavior in the thermodynamic limit $N \to +\infty$. Formally, this definition can be written as

$$\lim_{g\to g_i}\left|\frac{\partial^p}{\partial V^p}\lim_{N\to+\infty}\langle O\rangle_{\rm ss}\right| = +\infty, \tag{7}$$

  where $\langle O\rangle_{\rm ss}$ denotes the steady-state expectation value of O and p is the order of the transition. What is the order of the phase transition ? Give examples of phases transitions of this kind.
- What happens to the dissipative phase transition if individual spontaneous emission and dephasing occurs, i.e., if additional terms of the form

$$\sum_{i=1}^{N}\gamma_-\left(2\sigma_-^{(i)}\rho_{\rm tot}\sigma_+^{(i)} - \{\sigma_+^{(i)}\sigma_-^{(i)}, \rho_{\rm tot}\}\right) \tag{8}$$

$$\sum_{i=1}^{N}\gamma_z\left(2\sigma_z^{(i)}\rho_{\rm tot}\sigma_z^{(i)} - \{\sigma_z^{(i)}\sigma_z^{(i)}, \rho_{\rm tot}\}\right) \tag{9}$$

  appear in the master equation (5) ?

Hint: *For large system sizes, you might want to unravel the master equation and use quantum trajectories.*

References

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III. QUANTUM HEAT ENGINE, HEAT PUMP AND REFRIGERATORS

Consider a three-level system whose the Hamiltonian reads ($\hbar = 1$)

$$H_S(t) = \sum_{i=0}^{2} \omega_i |i\rangle\langle i| \tag{10}$$

where $0 < \omega_0 < \omega_1 < \omega_2$.

The system is coupled to three (hot, cold and work) reservoirs and we assume its dynamics is governed by the following Markovian master equation

$$\dot{\rho}_S = -i[H_S, \rho_S] + \mathcal{D}_h[\rho_S] + \mathcal{D}_c[\rho_S] + \mathcal{D}_w[\rho_S] \tag{11}$$

with

$$\mathcal{D}_h[\rho_S] = \gamma_h[n_h(\omega_2 - \omega_0) + 1]\left(2\sigma_{02}\rho_S\sigma_{02}^\dagger - \left\{\sigma_{02}^\dagger\sigma_{02}, \rho_S\right\}\right) + \gamma_h n_h(\omega_2 - \omega_0)\left(2\sigma_{20}\rho_S\sigma_{20}^\dagger - \left\{\sigma_{20}^\dagger\sigma_{20}, \rho_S\right\}\right) \tag{12}$$

$$\mathcal{D}_c[\rho_S] = \gamma_c[n_c(\omega_1 - \omega_0) + 1]\left(2\sigma_{01}\rho_S\sigma_{01}^\dagger - \left\{\sigma_{01}^\dagger\sigma_{01}, \rho_S\right\}\right) + \gamma_h n_h(\omega_1 - \omega_0)\left(2\sigma_{10}\rho_S\sigma_{10}^\dagger - \left\{\sigma_{10}^\dagger\sigma_{10}, \rho_S\right\}\right) \tag{13}$$

$$\mathcal{D}_w[\rho_S] = \gamma_w\left(2\sigma_{12}\rho_S\sigma_{12}^\dagger - \left\{\sigma_{12}^\dagger\sigma_{12}, \rho_S\right\}\right) + \gamma_h\left(2\sigma_{21}\rho_S\sigma_{21}^\dagger - \left\{\sigma_{21}^\dagger\sigma_{21}, \rho_S\right\}\right) \tag{14}$$

with $\sigma_{ij} = |i\rangle\langle j|$ and $n_\alpha(\omega)$ ($\alpha = h, c$) the Bose-Einstein distribution for the hot and cold reservoirs, i.e.,

$$n_\alpha(\omega) = \frac{1}{e^{\beta_\alpha\omega} - 1} \tag{15}$$

which depends on their temperatures T_α ($T_c < T_h$) through $\beta_\alpha = 1/k_B T_\alpha$.

List of tasks

- Explain why the 'work' reservoir can be seen as a reservoir at infinite temperature.
- Calculate analytically the steady state solution ρ_{ss} of the master equation and the corresponding *extracted power* P and *heat currents* J_α ($\alpha = h, c$) defined as

  $$P = -\mathrm{Tr}[H_S\mathcal{D}_w[\rho_{ss}]] \tag{16}$$
  $$J_\alpha = \mathrm{Tr}[H_S\mathcal{D}_\alpha[\rho_{ss}]]. \tag{17}$$

  Verify your solutions by time-evolving the master equation numerically.
- In which regimes of parameters the system behaves as a heat engine (i.e., $P > 0$, $J_h > 0$ and $J_c < 0$) and as a heat pump (i.e. $P < 0$, $J_h < 0$ and $J_c > 0$) ?
- What is the optimal value of the efficiency η in the two cases ($\eta = P/J_h$ for a heat engine and $\eta = J_c/|P|$ for a heat pump) ?
- Check whether the second law of thermodynamics is satisfied in this model, using the Von Neumann entropy as the definition of entropy. In particular, what is the entropy flow associated with the energy flow from the system to the work bath ?
- What is the effect of unwanted dephasing modeled via terms like

  $$\sum_{i=0}^{2} \gamma_z^{(i)}\left(2\sigma_{ii}^{(i)}\rho_{\mathrm{tot}}\sigma_{ii}^{(i)} - \{\sigma_{ii}^{(i)}\sigma_{ii}^{(i)}, \rho_{\mathrm{tot}}\}\right) \tag{18}$$

  on the performance of the heat engine/heat pump ? Among the three dephasing rates $\gamma_z^{(i)}$ ($i = 0, 1, 2$), which one (ones) is (are) the most detrimental ?

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IV. FLUORESCENCE OF A TWO-LEVEL ATOM

Overview

A two-level atom driven by a coherent laser field and coupled to the quantized electromagnetic field (leading to spontaneous emission) is a paradigmatic open quantum system showing phenomena such as a fluorescence spectrum exhibiting a *Mollow triplet* and non-trivial steady-state behaviour.

In this project, you will explore the dynamics and steady state of this system based on a Lindblad master equation, analyze the spectrum of the Liouvillian superoperator, analytically/numerically solve for the steady state, and analyze the fluorescence spectrum.

System dynamics

Consider a two-level atom with ground state $|g\rangle$ and excited state $|e\rangle$, separated by an energy $\hbar\omega_0$, driven by a coherent laser field of frequency ω_L and Rabi frequency Ω. The atom undergoes spontaneous emission at a rate Γ_e. In a frame rotating at frequency ω_L, the Hamiltonian of the driven atom is given by

$$H = -\frac{\hbar\delta}{2}\sigma_z + \frac{\hbar\Omega}{2}(\sigma_+ + \sigma_-) \tag{19}$$

where $\delta = \omega_L - \omega_0$ is the detuning and $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$. With Markovian dissipation accounting for spontaneous emission, the dynamics of the system is governed by the Lindblad master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \Gamma_e\Big(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\Big). \tag{20}$$

At non-zero temperature, one should replace the dissipative term in Eq. (20) by

$$\Gamma_e(\bar{n}+1)\Big(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\Big) + \Gamma_e\,\bar{n}\Big(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\rho\}\Big) \tag{21}$$

where $\bar{n} = (e^{\hbar\omega_0/k_BT} - 1)^{-1}$ is the mean number of thermal photons of energy $\hbar\omega_0$ at temperature T.

When the atom also undergoes pure dephasing at a rate Γ_d, the following term must be added to the master equation

$$\Gamma_d\Big(\sigma_z\rho\sigma_z - \frac{1}{2}\{\sigma_z\sigma_z,\rho\}\Big) \tag{22}$$

List of tasks

The following is a list of tasks that can be performed initially at zero temperature and in the absence of dephasing. The effect of temperature and dephasing can then be studied separately.

- Solve for the *steady-state* density matrix ρ_{ss} analytically (if possible) or numerically and express it in the Bloch ball representation. Compute the number of fluorescence cycles per second. Discuss specific limiting cases when $\Omega = 0$ (no driving) and $\Omega \gg \Gamma_e$ (strong-driving regime).
- Calculate the fluorescence spectrum $S(\omega)$ via the two-time correlation function $\langle\sigma_+(\tau)\sigma_-(0)\rangle_{\mathrm{ss}}$ by using the *quantum regression theorem*, see e.g. [1-3].
- Plot $S(\omega)$ for various values of Ω/Γ_e and δ/Γ_e, and observe the *Mollow triplet*. Give a physical interpretation of the origin of the Mollow triplet.
- Construct the Liouvillian superoperator $\mathcal{L}$ for the Lindblad master equation. Discuss the physical significance of the Liouvillian spectrum, including relaxation rates and the Liouvillian gap. Compute and plot the characteristic ralexation times as a function of the system parameters.
- Analyze the transient dynamics starting from specific initial states.

References

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[2] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom-Photon Interactions (New York: Wiley-Interscience 1992), complement AV.

[3] M. O. Scully and S. Zubairy, Quantum optics (Cambridge University Press, Cambridge, England, 1997).

V. SUPERRADIANCE AND SUBRADIANCE

Overview

In this project, you will explore the collective spontaneous emission of a set of identical two-level atoms. You will study the dynamics and steady state of this system based on a Lindblad master equation and analyze the spectrum of the Liouvillian superoperator.

System dynamics

The state of a set of N two-level atoms is governed, in the interaction picture with respect to the atomic Hamiltonian

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\sum_{i=1}^{N}\hat{\sigma}_z^{(i)} \quad \text{with} \quad \hat{\sigma}_z^{(i)} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|, \tag{23}$$

by the Lindblad master equation

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}\left[\hat{H}_{\rm dd}, \hat{\rho}(t)\right] + \mathcal{D}\left(\hat{\rho}(t)\right) \tag{24}$$

where $\hat{H}_{\rm dd}$ is the Hamiltonian describing the dipolar interactions between atoms and $\mathcal{D}\left(\hat{\rho}(t)\right)$ is the term describing collective dissipation. These terms have the expression

$$\hat{H}_{\rm dd} = \sum_{i\neq j}^{N}\hbar f_{ij}\,\hat{\sigma}_+^{(i)}\hat{\sigma}_-^{(j)} \tag{25}$$

and

$$\mathcal{D}\left(\hat{\rho}\right) = \sum_{i,j=1}^{N}\gamma_{ij}\left(\hat{\sigma}_-^{(j)}\,\hat{\rho}\,\hat{\sigma}_+^{(i)} - \frac{1}{2}\left\{\hat{\sigma}_+^{(i)}\,\hat{\sigma}_-^{(j)}, \hat{\rho}\right\}\right), \tag{26}$$

where $\hat{\sigma}_+^{(i)} = |e_i\rangle\langle g_i|$ and $\hat{\sigma}_-^{(i)} = |g_i\rangle\langle e_i|$ are the jump operators for the atom i. The coefficients f_{ij} and γ_{ij} appearing in $\hat{H}_{\rm dd}$ and $\mathcal{D}\left(\hat{\rho}\right)$ are given by

$$f_{ij} = \frac{3\gamma_0}{4}\left[(1-3\cos^2\alpha_{ij})\left(\frac{\sin\xi_{ij}}{\xi_{ij}^2} + \frac{\cos\xi_{ij}}{\xi_{ij}^3}\right) - (1-\cos^2\alpha_{ij})\frac{\cos\xi_{ij}}{\xi_{ij}}\right] \tag{27}$$

and, for $i \neq j$,

$$\gamma_{ij} = \frac{3\gamma_0}{2}\left[(1-3\cos^2\alpha_{ij})\left(\frac{\cos\xi_{ij}}{\xi_{ij}^2} - \frac{\sin\xi_{ij}}{\xi_{ij}^3}\right) + (1-\cos^2\alpha_{ij})\frac{\sin\xi_{ij}}{\xi_{ij}}\right] \tag{28}$$

and $\gamma_{ii} = \gamma_0$, with γ_0 the individual spontaneous emission rate, $\xi_{ij} = k_0 r_{ij}$ and α_{ij} the angle between the relative position $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ of atoms i and j and the atomic dipole moment $\mathbf{d}_{eg}$ (see figure).

When the atoms also undergo individual pure dephasing at a rate Γ_d, the following term must be added to the master equation

$$\mathcal{D}_d\left(\hat{\rho}\right) = \Gamma_d\sum_{i=1}^{N}\left(\hat{\sigma}_z^{(i)}\,\hat{\rho}\,\hat{\sigma}_z^{(i)} - \frac{1}{2}\left\{\hat{\sigma}_z^{(i)}\,\hat{\sigma}_z^{(i)}, \hat{\rho}\right\}\right). \tag{29}$$

List of tasks

The following is a list of tasks that can be performed initially at zero temperature and in the absence of dephasing. The effect of temperature and dephasing can then be studied separately.

- In the case of $N = 2$ atoms, calculate the probabilities that the system is in the doubly excited state $|e_1, e_2\rangle$ and the ground state $|g_1, g_2\rangle$ over time for different values of $\xi_{12} = 0.1, 1, 2$ and $\alpha_{12} = 0, \pi/2$. Consider that the system is initially in the symmetric state $|e_1, e_2\rangle$. Compare your results with the case of two separate atoms each decaying with a spontaneous emission rate γ_0 (independent atoms emitting individually).
- Same for the probability of finding the system in the ground state $|g_1, g_2\rangle$ when it is initially in the antisymmetric state $(|e_1, g_2\rangle - |g_1, e_2\rangle)/\sqrt{2}$ or the symmetric state $(|e_1, g_2\rangle + |g_1, e_2\rangle)/\sqrt{2}$. Explain why these states are called subradiant and superradiant respectively.
- Calculate and plot as a function of time the radiated intensity

$$I(t) = -\frac{d}{dt}\langle \hat{H}_0\rangle(t) = -\mathrm{Tr}\left[\hat{H}_0 \frac{d\hat{\rho}(t)}{dt}\right]. \tag{30}$$

- Solve for the *steady-state* density matrix $\rho_{\rm ss}$ analytically for 2 atoms. Is it unique ? Explain.
- Construct the Liouvillian superoperator $\mathcal{L}$ for the master equation under consideration. Discuss the physical significance of the Liouvillian spectrum, including relaxation rates and the Liouvillian gap. Calculate this spectrum and the gap for different numbers of atoms and different geometries (a *1d* chain of atoms, a ring of atoms, a *2d* lattice of atoms, ... see e.g. [4]) and discuss this from a physical point of view.

Hint: *To develop a physical intuition, we recommend plotting the coefficients f_{ij} and ξ_{ij}, characterising dipolar interactions and cooperative effects, as a function of interatomic distance.*

References

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VI. ELECTROMAGNETICALLY INDUCED TRANSPARENCY IN A THREE-LEVEL SYSTEM

Overview

Electromagnetically Induced Transparency (EIT) is a quantum interference phenomenon that renders an otherwise opaque medium transparent to a weak probe field in the presence of a strong control field. This effect arises from the creation of a "dark state" that decouples from the light fields, enabling applications such as slow light, quantum memory, and precision metrology.

In this project, you will explore the theoretical foundations of EIT in a three-level atomic system based on a Lindblad master equation. You will analyze the steady-state solutions, study the transparency window, and investigate the modification of absorption and dispersion properties under EIT conditions.

System Dynamics

Consider a three-level atomic system in the Λ-configuration (see Fig. 1), with states $|1\rangle$ (ground state), $|2\rangle$ (metastable state), and $|3\rangle$ (excited state). The system is driven by a weak probe field with Rabi frequency Ω_p, coupling $|1\rangle \leftrightarrow |3\rangle$, and a strong control field with Rabi frequency Ω_c, coupling $|2\rangle \leftrightarrow |3\rangle$. In an appropriate rotating

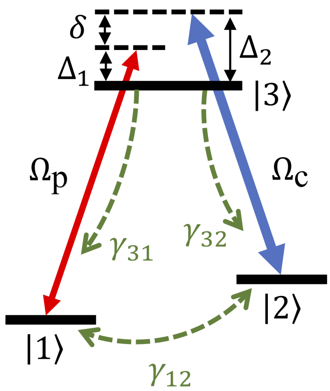

FIG. 1: Three-level system in Λ configuration for EIT. The probe field (red) couples $|1\rangle \leftrightarrow |3\rangle$ and the control field (blue) couples $|2\rangle \leftrightarrow |3\rangle$. In this work, we consider $\gamma_{12} = 0$ for simplicity, unless otherwise stated.

frame with the two laser frequencies, the Hamiltonian is given by

$$H = -\hbar \begin{pmatrix} \Delta_1 & 0 & \Omega_p \\ 0 & \Delta_2 & \Omega_c \\ \Omega_p & \Omega_c & 0 \end{pmatrix} \tag{31}$$

where $\Delta_1 = \nu_1 - \omega_{13}$ and $\Delta_2 = \nu_2 - \omega_{23}$ are the one-photon detunings, and $\delta = \Delta_1 - \Delta_2$ is the two-photon detuning. The open system dynamics is described by the Lindblad master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{D}[\rho] \tag{32}$$

where the dissipator accounts for

- spontaneous emission from $|3\rangle$ to $|1\rangle$ and $|2\rangle$ at rates γ_{31} and γ_{32}
- dephasing processes affecting the states $|2\rangle$ and $|3\rangle$ with rates $\gamma_{2,\mathrm{deph}}$ and $\gamma_{3,\mathrm{deph}}$

and is explicitly given by (show it)

$$\mathcal{D}[\rho] = \sum_{i=1,2} \gamma_{3i} \left(\sigma_{i3}\rho\sigma_{3i} - \frac{1}{2}\{\sigma_{33}, \rho\} \right) + \sum_{j=2,3} \gamma_{j,\mathrm{deph}} \left(\sigma_{jj}\rho\sigma_{jj} - \frac{1}{2}\{\sigma_{jj}, \rho\} \right) \tag{33}$$

where $\sigma_{ij} = |i\rangle \langle j|$.

List of Tasks

- Derive and solve the equations for the steady-state in the weak probe limit ($\Omega_p \ll \Omega_c$) to obtain the probe susceptibility $\chi_p(\Delta_1, \delta)$
- Plot the probe absorption ($\mathrm{Im}[\chi_p]$) and dispersion ($\mathrm{Re}[\chi_p]$) for the resonant case ($\Delta_1 = \Delta_2 = 0$) and the detuned case ($\Delta_1, \Delta_2 \neq 0$)
- Analyze the EIT window width $\gamma_{\mathrm{EIT}} = |\Omega_c|^2/\gamma_{13}$ and its dependence on control field intensity
- Derive the dark state $|D\rangle$ and bright state $|B\rangle$ and show how the dark state becomes decoupled from the excited state
- Investigate the effect of two-photon detuning δ on the dark state evolution
- Numerically simulate the time evolution of the system starting from different initial states
- *(Optional)* Calculate the group velocity reduction and simulate pulse propagation

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