Quantum Mechanics I HW1

Diego Ramírez Milano (201214691)

February 18, 2016

Given the the wave function

$$\Psi(x,t) = e^{-iEt/\hbar} N e^{-x^2/2\lambda^2} \tag{1}$$

$$\Psi^*(x,t) = e^{iEt/\hbar} N e^{-x^2/2\lambda^2} \tag{2}$$

Where N is the normalization factor and $\lambda = \sqrt{\frac{\hbar}{m\omega}}$.
The Hamiltonian operator for the harmonic oscillator reads

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}kx^2 = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}m\omega^2 x^2 \tag{3}$$

Help

$$\int_0^\infty e^{-a^2x^2} = \frac{\sqrt{\pi}}{2a},\ a > 0 \tag{4}$$

$$\int_0^\infty x^2 e^{-a^2x^2} = \frac{\sqrt{\pi}}{4a^3},\ a > 0 \tag{5}$$

$$\int_{\mathbb{R}} \Psi\Psi^* \mathrm{d}x = 1 \tag{6}$$

1. Find the normalization factor N.

2. Calculate the probability for the particle to be in the interval $[0, \infty]$.

3. Calculate the expected values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle \hat{P}_x \rangle$ and $\langle \hat{P}_x^2 \rangle$.

4. Find the eigenvalue E in $\hat{H}\Psi = E\Psi$.

5. Calculate the probability current j_x.

$$j_x = \frac{i\hbar}{2m}\left(\Psi \frac{\mathrm{d}}{\mathrm{d}x}\Psi^* - \Psi^* \frac{\mathrm{d}}{\mathrm{d}x}\Psi\right) \tag{7}$$