

Neural-Network-Switched Kalman Filters as Novel Trackers for Multipath Channels

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Abstract—High mobility leads to fast-varying, non-stationary channels in some modern applications, such as wireless communications for High-Speed Railways (HSR). This results in non-linear transitions, namely the potential birth of a new tap in a multipath channel or an active tap's death. A pressing question then is how to make use of unexploited correlations, such as time correlation in each tap of a multipath channel, when the Wide-Sense Stationary Uncorrelated Scattering (WSSUS) condition can no longer be assumed. Whereas Kalman filtering (KF) has been proposed to exploit such time correlation in each tap under WSSUS scenarios, a capital disadvantage of KF is its weak performance when non-linear transitions are considered. This work reviews previous proposals to tackle this birth-death non-linearity problem, as well as their drawbacks, and derives from them a new neural-network switching concept, called Neural-Network-switched Kalman Filter (NNKF). This novel tracker is computationally inexpensive and its simulations hereby show that it outperforms all previously known multipath channel tracking systems. The proposed tracker achieves the performance of the ideal birth/death detection case, thus approximately halving squared error w.r.t. Least-Squares (LS) estimation in Orthogonal Frequency Division Multiplexing (OFDM) systems.

Index Terms—channel estimation, Kalman filter, OFDM, vehicular communications, neural network, high-speed railways

I. INTRODUCTION

Orthogonal Frequency-Division Multiplexing (OFDM) has become the base for many communication technologies, including Long-Term Evolution (LTE) systems, and it has also been proposed as a good candidate for the provision of broadband data services in high-mobility applications [1] such as high-speed railways (HSR) [2]. Moreover, OFDM has been considered in the first release of 5G New Radio (5G-NR) [3], which is currently in its initial roll-out phase in several countries.

Some OFDM environments, particularly vehicular applications, require the tracking of time-variant channels, a task usually proposed to be accomplished through Kalman filtering (KF) and its many adaptations and extensions [4]. KF's advantage lies on the fact that it is the optimal estimator for linear problems, in the sense that it minimizes the mean-squared estimation error for a linear stochastic system using noisy linear sensors; thus, if channel tracking can be approximated as a linear problem, the KF is the optimal solution to the linear problem. This approximation gets harder to make as mobile communications spread into more dynamic channels, such as

those of HSR [5] in rugged terrain [6] or Unmanned Aerial Vehicles (UAVs) [7] in suburban/hilly terrain environments [8], since random intermittent multipath components appear. The evolution of each tap ceases to be linear: phenomena such as path birth and path death [9] are catastrophic for KF-based estimation techniques. The expected continuation of this trend will demand more powerful tap tracking techniques.

So far, tap tracking under birth-death conditions has been analyzed within the framework of Random Set Theory (RST) models. In [10], [11], three possible RST-based estimators were compared; the so-called GMAP-III (see Section IV for its formal definition), which was proven to be the best performer by far, required an impracticable computational cost for any practical applications. In [12], a simpler algorithm was derived: the Simplified Maximum a Posteriori (SMAP) death/birth detector, based on three dynamically-thresholded heuristics. In this work, a different approach, based on Neural Networks (NN), is proposed for the birth/death detector. By switching a KF filter on and off depending on tap activity detection, this low-complexity, NN-switched KF estimation achieves the optimal performance of an ideal death/birth detector.

This paper is organized as follows. In Section II, the problem is formally stated. Then, a quick review of approaches to channel tracking and how to measure their quality is made in Section III. The proposed NN-switched KF estimator is introduced in Section IV, and its simulated results are shown in Section V. Finally, conclusions are extracted in Section VI.

II. SYSTEM MODEL

The OFDM system is modeled with N orthogonal subcarriers. It transmits OFDM symbols with a time duration T_{symb} and a sampling interval T_{samp} . It is assumed there is no out-of-band interference. Each subcarrier has a dual use: data transmission (through data blocks of fixed length M_{inf}) and pilot symbols.

These pilot symbols enable channel estimation. Thus, the first K sent symbols are pilot symbols used to estimate the channel (by averaging over K). The channel estimation so acquired is assumed to be correct and, accordingly, it is used for all M_{inf} subsequent data symbols. Channel changes are both assumed to occur and be tracked only once every estimation period $T_{\text{est}} = (K + M_{\text{inf}}) \cdot T_{\text{symb}}$, which may be

a reasonable approximation when taking its coherence time into account. A cyclic prefix, T_{CP} is reserved between OFDM symbol transmissions, so that $T_{symp} = NT_{samp} + T_{CP}$. It is assumed that the multipath channel duration is smaller than T_{CP} .

Let $k = \{1, \dots, K\}$ be the pilot symbol index in the p th estimation period. The received signal, which is the input for channel estimation, can be given in vector form as:

$$\mathbf{y}_{p,k} = \mathbf{D}_{p,k} \mathbf{F}_p \mathbf{h}_p + \mathbf{z}_{p,k} \quad (1)$$

where $\mathbf{y}_{p,k} = [y_{p,1,k}, \dots, y_{p,N,k}]^T$, with $y_{p,n,k}$ (for $n = \{1, \dots, N\}$) the n th subcarrier observation sample at time $t_{p,k} = (p-1) \cdot T_{est} + (k-1) \cdot T_{symp}$; $\mathbf{D}_{p,k} = \text{diag}(d_{p,1,k}, \dots, d_{p,N,k})$, with $d_{p,n,k}$ the training data on the n th pilot subcarrier at time $t_{p,k}$; $\mathbf{z}_{p,k} = [z_{p,1,k}, \dots, z_{p,N,k}]$, with $z_{p,n,k}$ denoting the zero-mean complex Gaussian additive noise with variance σ_z^2 ; and

$$\{\mathbf{F}_p\}_{n,l} = e^{-j2\pi n \cdot \frac{\tau_l(pT_{est})}{NT_{samp}}} \quad (2)$$

where $\tau_l(pT_{est})$ is the delay of the l th path during the p th interval. If $l = \{1, \dots, L(pT_{est})\}$ and $L(pT_{est})$ is the number of active paths during the p th estimation period, then $\mathbf{h}_p = [h_1(pT_{est}), \dots, h_{L(pT_{est})}(pT_{est})]^T$, where $h_l(pT_{est})$ is the complex gain of the l th path at time-lapse p . As mentioned above, that gain is assumed constant for the duration of the estimation period.

Since each data block is preceded by K pilot symbols, the received signal including reception of all K pilot symbols can be represented by the matrix $\mathbf{Y}_p = [\mathbf{y}_{p,1}, \dots, \mathbf{y}_{p,K}]$.

Additionally, the active path gains follow a Linear Gauss-Markov (LGM) model. If $a_p^{(l)}$ denotes the l th path gain at time p provided that the l th path is active, then the probability density function (pdf) of $a_p^{(l)}$ would be given by the following equations:

$$f(a_1^{(l)}) = \mathcal{N}(a_1^{(l)}; 0, \sigma_{h_l}^2) \quad (3)$$

$$f(a_p^{(l)} | a_{p-1}^{(l)}) = \mathcal{N}(a_p^{(l)}; \lambda a_{p-1}^{(l)}, (1 - \lambda^2) \sigma_{h_l}^2) \quad (4)$$

where $f(\cdot)$ represents the pdf, $\mathcal{N}(\cdot)$ denotes the normal distribution, $\sigma_{h_l}^2$ is the average power of the l th path and λ is the time self-correlation of each active path gain. Nevertheless, if the l th path is dead (inactive), then $h_l(pT_{est})=0$. Each active path has a probability P_{death} of dying (becoming zero-gain); each inactive path has a probability P_{birth} of being born (becoming non-zero-gain). On the other hand, the average signal-to-noise ratio (SNR) is assumed fixed and known.

The problem under consideration can now be formulated as follows: given the observations (1), determine a causal estimator $\hat{\mathbf{h}}_p$ for \mathbf{h}_p , relying upon $\{\mathbf{Y}_{1:p}\}$. The desired solution would minimize the Channel Tracking Mean Squared Error (CTMSE), defined as:

$$CTMSE \triangleq \left\| \sum_p \hat{\mathbf{h}}_p - \mathbf{h}_p \right\|_2^2 \quad (5)$$

III. CHANNEL TRACKING

A. Linear Gauss-Markov (LGM) models

To exploit temporal self-correlation, sometimes it is convenient to consider channels where tap gains can somehow be predicted most of the time by looking at previous estimates for such tap gains. The Linear Gauss-Markov model is adequate for this task. When paths are active in such a model, they follow Eq. (3) and (4). LGM models have been previously used in the literature (e.g. in [10], [11]). Their main advantage: they make it possible to derive a computationally inexpensive, optimal estimation through KF.

Unfortunately, they do not allow for abrupt changes, such as those generated by tap birth/death phenomena; thus, their behaviour may or may not approximate specific real channels.

B. KF-based approaches

Under a LGM model, the optimal estimation is given by KF, an algorithm weighting optimally two information sources: a theoretical one and another one based on noisy measurements.

In our system model, the theoretical source would be the LGM channel model, while the measurements would be provided by the Least-Squares (LS) channel estimation. Notice that LS estimation, $\hat{\mathbf{a}}_p^{LS}$, can be interpreted as a noisy measurement of the true channel $\mathbf{a}_p = [a_p^{(1)}, \dots, a_p^{(L)}]^T$:

$$\mathbf{u}_p \triangleq \hat{\mathbf{a}}_p^{LS} = \mathbf{a}_p + \mathbf{v}_p \quad (6)$$

where $\mathbf{v}_p = [v_p^{(1)}, \dots, v_p^{(L)}]^T$ is the “measurement noise” (in the previous sense) and $\mathbf{u}_p = [u_p^{(1)}, \dots, u_p^{(L)}]^T$ is just another, sometimes more convenient, notation. Since path independence is being assumed, this translates into L scalar equations for real-valued taps, or $2L$ scalar equations for complex-valued taps (as is our case). This is due to the fact that the real and the imaginary parts of each tap gain may be efficiently tracked by independent scalar Kalman filters.

Thus, a bank of independent Kalman filters could be used to track the channel, whereby each Kalman filter would receive either the real or the imaginary part of the LS-estimated tap gain and compute the corresponding part of the KF tap estimation. It would also compute the expected (real/imaginary) part of the gain at time $p+1$, the so-called Kalman prediction, $\hat{u}_p^{(l)}(-)$. This KF approach is optimal in the absence of abrupt changes. However, the death and birth of taps (i.e., the real channel “leaping” out of the idealized LGM) produces a severe distortion and the KF performance degrades catastrophically [12].

C. RST-based approaches

Random sets may be used in the context of this problem to represent not only a number of specific elements, but an unknown number of them. When dealing with channel taps, it is often the case that the number of active taps is unknown [11]. Thus, RST appears as a suitable theoretical framework for such problems.

However, death-birth detection under the RST framework can be difficult to handle and computationally prohibitive,

e.g., in [10], channel tracking using this death-birth detection framework is done through a 10,000-particle filter followed by a KF bank. Much simpler estimators are thus required for practical applications, even if optimality under ideal constraints is compromised. Such a suboptimal estimator, the SMAP, is described in Subsection III.E. Before that, however, a measurement of death-birth detector quality is introduced in the following Subsection.

D. Ideal Switching Systems

What is the theoretically maximum possible reduction in CTMSE when using perfect birth-death information? Since our objective is to reduce the CTMSE defined in (5), that maximum reduction would be our theoretical limit. A possible answer would be simulating perfect death-birth detectors. Such detectors, which would always provide the perfect birth/death detection, were first introduced in [12] with the name of Ideal Switching Systems (ISS).

Such ISS detectors detect 100% of births and 100% of deaths, with no false birth/death detection. It was argued in [12] that it is also possible to easily define an $x\%$ -degraded Ideal Switching System (IIS- $x\%$), which is a switching system detecting $x\%$ of births and $x\%$ of deaths, with no false birth/death detection. The different “iso-detection” (ISS/IIS- $x\%$) lines would thus establish a natural measure for an estimator’s quality, typically in terms of CTMSE. For example, some estimator may outperform ISS-97% while underperforming ISS-98%, meaning that its performance (e.g. CTMSE) would be slightly better than a degraded ISS where detectors detected 97% of births and 97% of deaths, but not as good as a system detecting 98% of both. This performance measure will be used in Section V.

E. Simplified Maximum A Posteriori (SMAP) estimator

In [12], a new death/birth detector was defined: the so-called Simplified Maximum A Posteriori (SMAP) birth/death detector. This detector estimated changes in tap activity/inactivity with the three following heuristics:

1) SMAP detects tap inactivity if the path gain leaps into a narrow, zero-centered range.

$$\ln\left(\frac{P_{death}}{1 - P_{death}}\right) > \frac{\hat{u}_p^{(l)}(-) - 2u_p^{(l)}\hat{u}_p^{(l)}(-)}{2\sigma_v^2(l)} \quad (7)$$

2) SMAP detects tap inactivity if the path gain has slowly converged into zero. To do this, the system needs some memory to store a $(s+1)$ -long sequence $\{|u_p^{(l)}|, \dots, |u_{p+s}^{(l)}|\} < q_B \cdot \sigma_v$, where s is a parameter to be chosen according to the context.

3) SMAP detects new tap activity (“birth”) if the path gain is far from zero. How far? To optimize the threshold (in terms of CTMSE), it should be taken into account that we may have falsely detected zero in a previous time. In that case, we would be right to correct it now. The probability of needing to correct it would be:

$$P_{corr} = P_e \cdot (1 - P_{death}) \cdot 2Q(q_C) \quad (8)$$

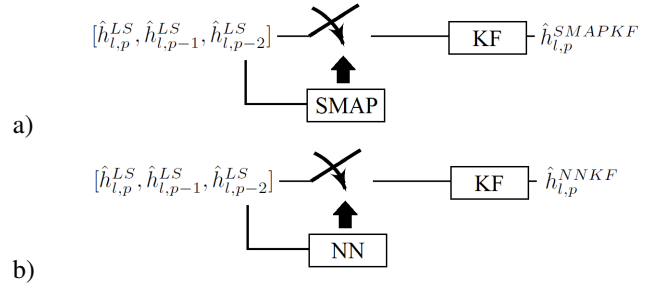


Fig. 1. Tap birth/death detector schemes: a) SMAP; b) NN.

where P_e is the false detection probability when using the heuristic explained in the point above (memory death detector). It may also be the case that the tap gain was really zero and the tap is reborn now. That would happen with probability:

$$P_{birth\ det} = (1 - P_e) \cdot P_{birth} \quad (9)$$

To account for both possibilities, SMAP detects birth when:

$$P_{corr} + P_{birth\ det} > P_{false\ det} \quad (10)$$

where the probability of doing the wrong thing by detecting tap activity (false birth detection) would be:

$$P_{false\ det} = P_e \cdot P_{death} \cdot 2Q(q_C) + (1 - P_e) \cdot (1 - P_{birth}) \cdot 2Q(q_C) \quad (11)$$

where $Q(\cdot)$ is the Q-function. It should be noticed that the SMAP method requires the frontloaded computation of parameters q_A , q_B and q_C , which can be done via machine-learning methods (massive simulations).

The SMAP detector is followed by a KF bank. The complete scheme is shown in Fig. 1a).

IV. NEW PROPOSAL: NEURAL-NETWORK-SWITCHED KALMAN FILTERS (NNKF)

RST-based approaches have proven that, under birth-death conditions, a very good estimator is the so-called GMAP-III, which essentially adds a death-birth detector before the KF step. Thus, it first detects which paths are active, and then it estimates active paths’ gains. The GMAP-III estimator, as first applied to this context in [10], is given by:

$$\text{GMAP-III} : \begin{cases} \widehat{\pi(\mathcal{H}_p)} = \arg \max_{\pi(\mathcal{H}_p)} f_{\pi(\mathcal{H}_p)|\mathcal{Y}_{1:p}}(\pi(\mathcal{H}_p)|\mathcal{Y}_{1:p}) \\ \hat{\mathbf{h}}_p = \int_{\mathbb{R}^{2|\widehat{\pi(\mathcal{H}_p)}|}} \mathbf{h}_p f_{\mathbf{h}_p|\mathcal{Y}_{1:p}}(\mathbf{h}_p|\mathcal{Y}_{1:p}) d\mathbf{h}_p \end{cases} \quad (12)$$

where

$$f_{\pi(\mathcal{H}_p)|\mathcal{Y}_{1:p}}(\pi(\mathcal{H}_p)|\mathcal{Y}_{1:p}) = \int_{\pi'(\mathcal{H}_p)} f(\mathcal{H}_p|\mathcal{Y}_{1:p}) \delta\mathcal{H}_p \quad (13)$$

(For details on notation, please see [10]).

Since previous solutions, like SMAP and GMAP-III, suggested that the tracking problem is separable in nature, it is

hereby proposed to use a NN as birth/death detector for each tap. Such a NN would perform the computation associated to the first line in Eq. (12).

Thus, the proposed NN would switch on the Kalman filter when the tap is active and off in case of death. After the NN estimates which taps are active at a given time, a KF bank would estimate the value for those taps deemed active in a second step. Thus, the KF bank would perform the computation associated to the second line in Eq. (12).

The whole tracking scheme is shown in Fig. 1b), while the detailed 6x4x2 NN is shown in Fig. 2, where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of the complex channel gains, respectively. The specific dimensions of the NN are not fixed; they must be decided according to the specific problem (this is somehow similar to the parameter s in SMAP, which was not fixed either). For most applications, however, the dimension of the input layer should be expected to remain very low, even while providing optimal or close-to-optimal performance.

The NN inputs are the LS estimates for the l th tap gain at times $\{p, p-1, p-2\}$; these LS estimates are separated into their real and imaginary parts. This information is used to decide whether there is a dead tap or an active tap in a way that optimizes CTMSE. To do so, the NN birth/death detector needs to be trained with a reasonable amount of labeled noisy tap gain samples.

A. Covering the KF's eyes before death

For this scheme to work properly, it is important to decide what the KF should do at times of death and at times of life. This proposal advocates a strategy which could be described as “covering the KF's eyes before death”, in a similar manner to covering a young child's eyes so they do not get traumatised by a shocking view. In fact, if the KF were to process dead tap gains following the standard algorithm, it would get “traumatised” and its estimates would get distorted by non-linear transitions. Therefore, it is proposed that the standard algorithm should be put on hold during death: the KF's prediction variance should not be modified until the tap is activated again. Similarly, once the tap is reborn, the KF should modify its previous predictions to match the new tap gain and avoid distortions.

B. Complexity discussion

The proposed tap birth/death detector is low-complexity. It requires a very low number of neurons (e.g. 12 in Fig. 2) for each complex tap. Thus, its complexity grows linearly with the number of taps. In this regards, it is similar to SMAP and different from particle-filter-based methods, where birth/death detection is made for all taps in a single step.

Nevertheless, unlike SMAP, there are several trade-offs you could make when working with the proposed NN switch. For example, you could use either fewer inputs or more of them depending on the required success rate. This could take the form of using only the real part of the gain (and ignoring

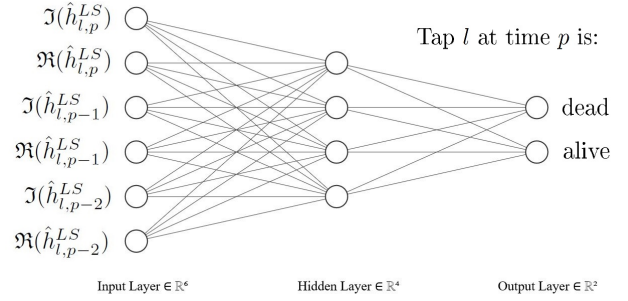


Fig. 2. Proposed NN tap birth/death detector.

the imaginary part) or using only the LS estimate at times $\{p, p-1\}$ instead of a longer sequence.

We hasten to add that the proposed system shown in Fig. 2, while low-complexity, shows a perfect detection record when measured in terms of CTMSE for the simulations in Section V. This suggests the proposed scheme may provide an excellent trade-off between complexity and detection precision for real applications.

V. SIMULATION RESULTS

A system with $N_p = 10$ pilot subcarriers is considered, with $K = 8$ pilot symbols per subcarrier. The pilot symbols have equal average energy, σ_s^2 , and a BPSK modulation scheme is used. The channel follows a LGM model and is assumed to have a uniform multipath delay profile, multipath spread smaller than the guard time, and no path gain correlation across different paths. The overall channel energy is normalized to one. $2.4 \cdot 10^5$ OFDM symbols are transmitted through a channel with $L_{max} = 10$, $P_{birth} = 0.05$ and $P_{death} = 0.05$. This amounts to 5 active taps on average. Individual paths are assumed to have the same average energy σ_h^2 over long periods and $\lambda = 0.999$. This choice of parameters makes it possible to compare the performance of the NNKF tracker (shown in Fig. 2, trained with samples over 70,000 estimation periods) to the computationally heavier methods outlined in [10], the KF system advocated in [14] and the SMAP system in [12].

Fig. 3 shows the precision (in terms of active/inactive tap detection mistakes) of the proposed NN in comparison to a ISS and a degraded ISS. Please notice that not all detection mistakes have the same effect on CTMSE. The ISS-99% fails to detect birth and death in 1% of the cases, but sometimes that means a small MSE error (because the tap gain wrongly detected as dead is close to zero, even though not zero), but a greater MSE error under different circumstances (e. g., when tap gain is wrongly detected, while being far from zero).

Fig. 4 shows the performance (CTMSE, as given in (5)) of our proposed NN birth/death detector + KF estimation vs. LS estimation and scenarios with perfect information (ISS), almost perfect information (ISS-99%) and a conventional stand-alone KF system with no birth/death detector.

It is evident that the NN+KF estimator provides much better performance when compared to ISS-99% and practically

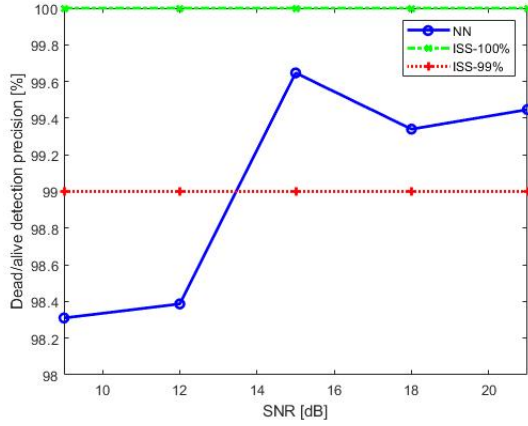


Fig. 3. Tap activity detection precision [%] for NN and two degraded ISS.

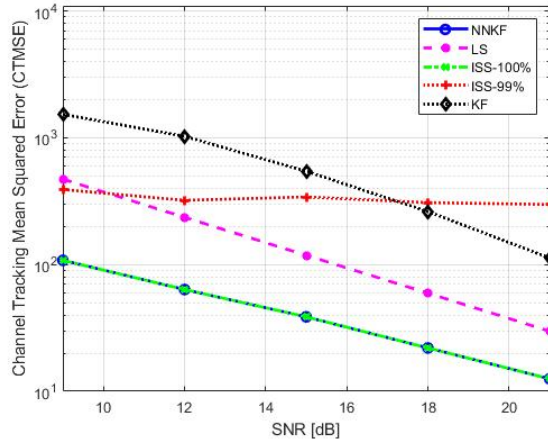


Fig. 4. Tracking precision (CTMSE) for 3 schemes (NNKF, LS, KF) and 2 ISS.

identical to ISS-100% (as shown in Table 1). In other words: unlike ISS-99%, the proposed NN's detection errors contribute almost zero to CTMSE. The NN's detection errors are minor in terms of CTMSE.

This is especially remarkable in view of the fact that the weights for the NN are not directly optimized for the MSE reduction, but for the birth/death detection.

This means that the NN+KF estimator achieves the tracking error reduction attainable by perfect death/alive tap state information. Please notice that, due to its very low computational cost (as discussed in IV.B), an NN detection bank would be suitable for real applications over the entire SNR range of interest.

VI. CONCLUSIONS

The framework and solutions for the birth-death nonlinearity problem, including RST and SMAP, have been reviewed. A low-complexity NN-switched KF tracker has been proposed. Simulations have shown this novel estimator achieves an identical channel-tracking performance to that of the ideal case

SNR [dB]	CTMSE for NNKF	CTMSE for ISS100
9	107.9868	107.9868
12	63.5261	63.5261
15	38.6655	38.6655
18	21.9729	21.9729
21	12.5579	12.5579

TABLE I

CTMSE FOR NNKF AND ISS100 IN DIFFERENT SNR ENVIRONMENTS.

(ISS) where perfect knowledge of tap activations is available. Thus, it outperforms all previously known multipath channel tracking systems for OFDM communications.

VII. ACKNOWLEDGEMENT

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