1

1.1 Sea $X_i \sim Exp(1)$ tq $f_{X_i}(x) = \lambda e^{-\lambda x}$ para $x \geq 0$ entonces:

$$f_{X_{k:n}}(x) = \begin{cases} \frac{n!}{(k-1)!\,(n-k)!} (F(x))^{k-1} (1-F(x))^{n-k} f(x) & \forall x \in \text{Soporte}(f) \\ 0 & \text{si no.} \end{cases}$$

Con $F_{X_i}(x) = 1 - e^{-x}$ nos queda:

$$f_{X_{k:n}}(x) = \frac{n!}{(k-1)!\,(n-k)!} (1-e^{-x})^{k-1} (e^{-x})^{n-k+1} \quad \text{para } x \geq 0, \quad 0 \text{ si no}$$

1.2 Sea $(X_i) \sim Exp(\alpha)$, $\alpha > 0$

Sea M_n el máximo, considere $M_n - \frac{\ln(n)}{\alpha}$. Entonces:

$$\begin{aligned} \mathbb{P}\left(M_n - \frac{\ln(n)}{\alpha} \leq x\right) &= \mathbb{P}\left(M_n \leq x + \frac{\ln(n)}{\alpha}\right) \\ &= \left(1 - \tilde{\mathbb{F}}\left(x + \frac{\ln(n)}{\alpha}\right)\right)^n \\ &= \left(1 - \exp\left(-\alpha\left(x + \frac{\ln(n)}{\alpha}\right)\right)\right)^n \\ &= \left(1 - \exp\left(-\left(\alpha \cdot x + \frac{\ln(n)}{\alpha}\right)\right)\right)^n = \left(1 - \frac{e^{-\alpha x}}{n}\right)^n \\ &\xrightarrow[n\to\infty]{} e^{-e^{-\alpha x}} \end{aligned}$$

2

2.1 Sean $(X_n)_n$ iid distribuidas Pareto (α), $\alpha>0$.

Entonces, $f_{X_i}(x) = \dfrac{\alpha}{x^{\alpha+1}}$ Para $x>1$, 0 si no

$F_{X_i}(x) = 1 - \dfrac{1}{x^{\alpha}}$ para $x>1$, 0 si no.

Entonces:

$$f_{X_{k:n}}(x) = \frac{n!}{(k-1)!\,(n-k)!}\left(1-\frac{1}{x^{\alpha}}\right)^{k-1}\cdot\left(\frac{1}{x^{\alpha}}\right)^{n-k}\cdot\frac{\alpha}{x^{\alpha+1}} \quad \text{para } x>1$$

2.2 Sea $\bar{F}(x) = \begin{cases} 1/x^{\alpha} & x>1 \\ 1 & x\leq 1 \end{cases}$

Considere $M_n/n^{1/\alpha}$ entonces:

$$\mathbb{P}\left(\frac{M_n}{n^{1/\alpha}}\leq x\right) = \mathbb{P}(M_n\leq n^{1/\alpha}\cdot x) = \left(1-\frac{1}{x^{\alpha}n}\right)^{n} \to e^{-1/x^{\alpha}}$$

3 $(X_i)_n$ iid con $\bar{F}(x) = (1-x)^\alpha$, $x \in [0,1]$

3.1 $F(x) = 1 - (1-x)^\alpha$ $f(x) = \alpha(1-x)^{\alpha-1}$

a) $$f_{X_{k:n}}(x) = \begin{cases} \frac{n!}{(k-1)!(n-k)!} (1-(1-x)^\alpha)^{k-1} ((1-x)^\alpha)^{n-k} \cdot \alpha (1-x)^{\alpha-1} \end{cases}$$

3.2 Sean $(X_n)_n$ tq $\bar{F}(x) = \begin{cases} 1 & x \leq 0 \\ (1-x)^\alpha & x \in [0,1] \\ 0 & x > 1. \end{cases}$

Considere $n^{1/\alpha}(M_n - 1)$, entonces:

$$\mathbb{P}((n^{1/\alpha}(M_n - 1)) \leq x) = \mathbb{P}\left(M_n \leq \frac{x}{n^{1/\alpha}} + 1\right)$$

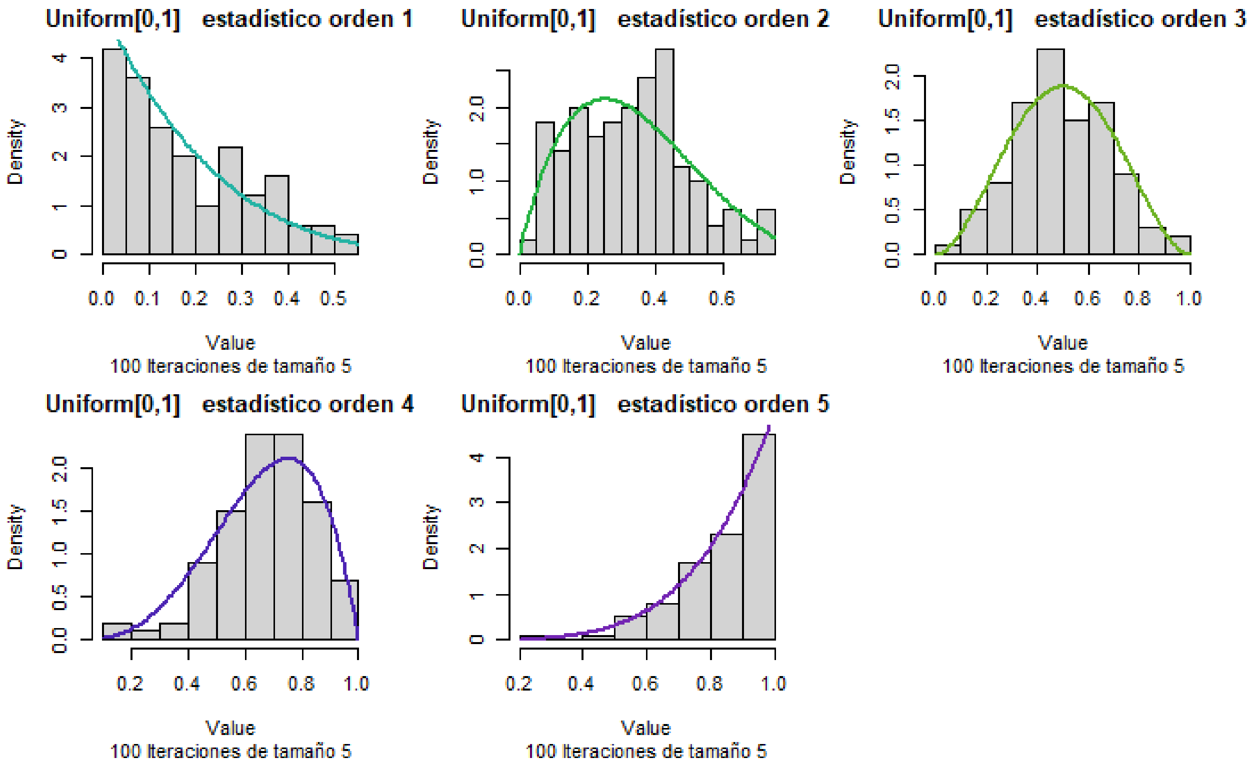
$$= \left(1 - \left(1 - \left(\frac{x}{n^{1/\alpha}} + 1\right)\right)^\alpha\right)^n$$

$$= \left(1 - \frac{(-x)^\alpha}{n}\right)^n \xrightarrow[n \to \infty]{} e^{-(-x)^\alpha}$$

4.

El Notebook con el código en R va como archivo adjunto en el correo.

Densidades obtenidas:



CDF Teórico vs empírico: