

Machine Learning - Module 01 Univariate Linear Regression

Summary: Today you will implement a method to improve your model's performance: gradient descent. Then you will discover the notion of normalization.

Notions and ressources

Notions of the module

Gradient descent, linear regression, normalization.

Useful Ressources

You are strongly advise to use the following resource: Machine Learning MOOC - Stanford Here are the sections of the MOOC that are relevant for today's exercises:

Week 1

Linear Regression with One Variable

- Gradient Descent (Video + Reading)
- Gradient Descent Intuition (Video + Reading)
- Gradient Descent For Linear Regression (Video + Reading)
- Review (Reading + Quiz)

Week 2

Multivariate Linear Regression

• Gradient Descent in Practice 1 - Feature Scaling (Video + Reading)

Common Instructions

- The version of Python recommended to use is 3.7, you can check the version of Python with the following command: python -V
- The norm: during this piscine, it is recommended to follow the PEP 8 standards, though it is not mandatory. You can install pycodestyle which is a tool to check your Python code.
- The function eval is never allowed.
- The exercises are ordered from the easiest to the hardest.
- Your exercises are going to be evaluated by someone else, so make sure that your variable names and function names are appropriate and civil.
- Your manual is the internet.
- You can also ask questions in the #bootcamps channel in the 42AI or 42born2code.
- If you find any issue or mistakes in the subject please create an issue on 42AI repository on Github.
- We encourage you to create test programs for your project even though this work won't have to be submitted and won't be graded. It will give you a chance to easily test your work and your peers' work. You will find those tests especially useful during your defence. Indeed, during defence, you are free to use your tests and/or the tests of the peer you are evaluating.
- Submit your work to your assigned git repository. Only the work in the git repository will be graded. If Deepthought is assigned to grade your work, it will be run after your peer-evaluations. If an error happens in any section of your work during Deepthought's grading, the evaluation will stop.

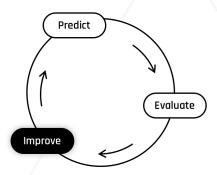
Contents

1	Exercise 00	4
II /	Exercise 01	g
Ш	Exercise 02	13
IV	Exercise 03	16
V	Exercise 04	19
VI	Exercise 05	22
VII	Exercise 06	25
VIII	Conclusion - What you have learned	27

Chapter I

Exercise 00

Improve



In the previous module, you discovered the first two steps of the learning process: starting with a model that makes naive predictions and evaluating it. Now we are going to tackle the third part: improving it!

Lets take a new dataset:

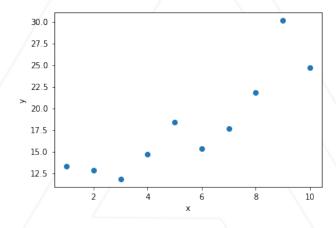


Figure I.1: Scatter plot of a given dataset

Predict

Given our measure of performance, improvement entails **reducing the loss (or cost)** measured by the loss function. If we plot the loss of a model's predictions as a function of its θ_1 parameter (with a fixed value for θ_0), we obtain a curve like this one:

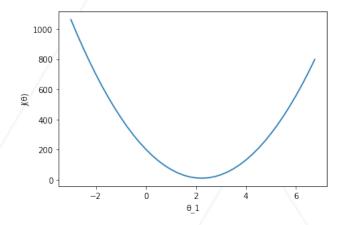


Figure I.2: Loss function given θ_1

On the graphs below, you can see that extreme θ_1 values (which modifies the slope of the hypothesis curve - in orange) correspond to a very high loss. On the other hand, as we get closer to the bottom of the curve, the loss is reduced.

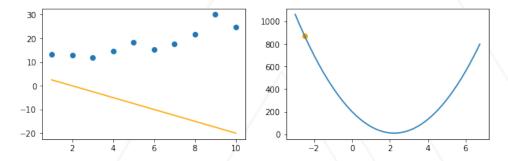


Figure I.3: A quite bad model

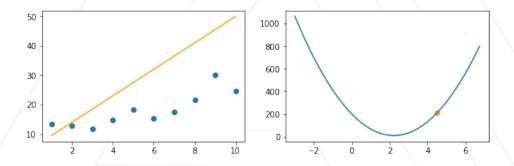


Figure I.4: A better (but still bad) model

The loss function's minimum corresponds to the bottom of the curve. We want θ_1 to get to this sweet spot. It means that wherever θ_1 starts at, as the training goes on, it needs to get closer to the value that matches $J(\theta)$'s minimum.

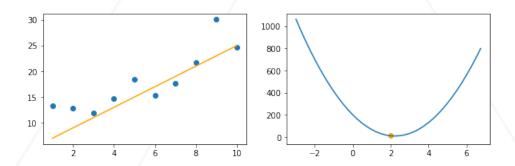


Figure I.5: A good model

But how to get closer to the minimum?

Excellent question dear reader. We're glad you asked! First, the algorithm needs to figure out in which direction θ_1 should be moved (i.e. increased or decreased). It does so by calculating the **slope** of the $J(\theta)$ curve at the current position of θ_1 . If the slope is positive, θ_1 must be decreased. If the slope is negative, it must be increased. If you have studied calculus, you probably sense that all of this involves calculating the derivative of the loss function.

The story gets a little more complicated, however, because we have two parameters to adjust: θ_0 and θ_1 . Not just θ_1 (as we showed in our example to simplify). This means the $J(\theta)$ function doesn't have only one derivative, but two **partial derivatives**. One that computes the slope of J with respect to θ_0 , and a second one for the slope of J with respect to θ_1 . Finally, we package those partial derivatives in a vector of dimension m, which is called **gradient** (noted ∇).

Don't worry if you don't master multivariate calculus yet, we have calculated the partial derivatives for you, all you will need to do is write them in Python.

AZ MITECUL BILLIERSEI	Exercise: 00	
Linear Gradient - Iterative Version		
Turn-in directory: $ex00/$		
Files to turn in : gradient.py		
Forbidden functions: None		

Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. You must write a function that computes the *gradient* of the loss function. It must compute a partial derivative with respect to each theta parameter separately, and return the vector gradient. The partial derivatives can be calculated with the following formulas:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\nabla(J)_1 = \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) x^{(i)}$$

Where:

- $\nabla(J)$ is the gradient vector of size 2×1 , (this strange symbol: ∇ is called nabla)
- x is a vector of dimension m,
- y is a vector of dimension m,
- $x^{(i)}$ is the ith component of vector x,
- $y^{(i)}$ is the ith component of vector y,
- $\nabla(J)_j$ is the jth component of $\nabla(J)$,
- $h_{\theta}(x^{(i)})$ corresponds to the model's prediction of $y^{(i)}$.

Hypothesis Notation

 $h_{\theta}(x^{(i)})$ is the same as what we previously noted $\hat{y}^{(i)}$. The two notations are equivalent. They represent the model's prediction (or estimation) of the $y^{(i)}$ value. If you follow Andrew Ng's course material on Coursera, you will see him using the former notation.

As a reminder: $h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$

Instructions

In the gradient.py file create the following function as per the instructions given below:

```
def simple_gradient(x, y, theta):
    """Computes a gradient vector from three non-empty numpy.array, without any for-loop.
    The three arrays must have compatible shapes.
Args:
    x: has to be an numpy.array, a vector of shape m * 1.
    y: has to be an numpy.array, a vector of shape m * 1.
    theta: has to be an numpy.array, a 2 * 1 vector.
Return:
    The gradient as a numpy.array, a vector of shape 2 * 1.
    None if x, y, or theta are empty numpy.array.
    None if x, y and theta do not have compatible shapes.
    None if x, y or theta is not of the expected type.
Raises:
    This function should not raise any Exception.
"""
... Your code ...
```

```
import numpy as np
x = np.array([12.4956442, 21.5007972, 31.5527382, 48.9145838, 57.5088733]).reshape((-1, 1))
y = np.array([37.4013816, 36.1473236, 45.7655287, 46.6793434, 59.5585554]).reshape((-1, 1))

# Example 0:
theta1 = np.array([2, 0.7]).reshape((-1, 1))
simple_gradient(x, y, theta1)
# Output:
array([[-19.0342574], [-586.66875564]])

# Example 1:
theta2 = np.array([1, -0.4]).reshape((-1, 1))
simple_gradient(x, y, theta2)
# Output:
array([[-57.86823748], [-2230.12297889]])
```

Chapter II

Exercise 01

Linear Algebra Tricks II

If you tried to run your code on a very large dataset, you would find that it takes a long time to execute! That's because it doesn't use the power of Python libraries that are optimized for matrix operations.

Remember the linear algebra trick of in the previous module? Let's use it again! If you concatenate a column of 1's to the left of the x vector, you get what we called matrix X'.

$$X' = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix}$$

This transformation is very convenient because we can rewrite each 1 as $x_0^{(i)}$, and each $x^{(i)}$ as $x_1^{(i)}$. So now the X' matrix looks like this:

$$X' = \begin{bmatrix} x_0^{(1)} & x_1^{(1)} \\ \vdots & \vdots \\ x_0^{(m)} & x_1^{(m)} \end{bmatrix}$$

Notice that each $x^{(i)}$ example becomes a vector made of $(x_0^{(i)}, x_1^{(i)})$. The 0 and 1 indices on the x features correspond to the indices of the θ parameters with which they will be multiplied.

Why does this matter? Well, if we take the equation from the previous exercise:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

We can multiply it by 1 without changing its value:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot 1$$

And rewrite 1 as $x_0^{(i)}$:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

This means that the equation for $\nabla(J)_0$ is now similar to the equation we had for $\nabla(J)_1$, so they can both be captured by ONE **generic equation**:

$$\nabla(J)_j = \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$
 for $j = 0, 1$

And as you probably suspected, a generic equation opens the door to vectorization...

Vectorizing the Gradient Calculation

Now it's time to learn how to calculate the entire gradient in one short, pretty, linear algebra equation!

• First, we'll use the X' matrix and our vectorized hypothesis equation: $h_{\theta}(x) = X'\theta$

$$\nabla(J)_j = \frac{1}{m}(X'\theta - y)X'_j \quad \text{for } j = 0, 1$$

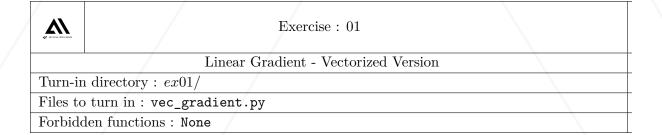
• Second, we need to tweak the equation a bit so that it directly returns a $\nabla(J)$ vector containing both $\nabla(J)_0$ and $\nabla(J)_1$.

$$\nabla(J) = \frac{1}{m} X'^{T} (X'\theta - y)$$

If the equation does not seems obvious, play a bit with your vectors, on paper and in your code, until you get it.

Notation Remark

 X^{T} : You might wonder what the T is for. It means the X^{T} matrix must be **transposed**. Transposing a matrix flips it on its diagonal so that its rows become its columns and vice versa. Here we need to do it so that matrix dimensions are appropriate multiplication and to multiply the right elements together.



Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. You must implement the following formula as a function:

$$\nabla(J) = \frac{1}{m} X'^{T} (X'\theta - y)$$

Where:

- $\nabla(J)$ is a vector of dimension 2×1 .
- X' is a **matrix** of dimensions $(m \times 2)$,
- X^{T} is the transpose of X'. Its dimensions are $(2 \times m)$,
- y is a vector of dimension m,
- θ is a vector of dimension 2×1 .

Be careful:

- the x you will get as an input is an m vector,
- θ is a 2 × 1 vector. You have to transform x to fit the dimension of θ !

Instructions

In the vec_gradient.py file create the following function as per the instructions given below:

```
def gradient(x, y, theta):
    """Computes a gradient vector from three non-empty numpy.array, without any for loop.
    The three arrays must have compatible shapes.
Args:
    x: has to be a numpy.array, a matrix of shape m * 1.
    y: has to be a numpy.array, a vector of shape m * 1.
    theta: has to be a numpy.array, a 2 * 1 vector.
Return:
    The gradient as a numpy.ndarray, a vector of dimension 2 * 1.
    None if x, y, or theta is an empty numpy.ndarray.
    None if x, y and theta do not have compatible dimensions.
Raises:
    This function should not raise any Exception.
"""
... Your code ...
```

```
import numpy as np
x = np.array([12.4956442, 21.5007972, 31.5527382, 48.9145838, 57.5088733]).reshape((-1, 1))
y = np.array([37.4013816, 36.1473236, 45.7655287, 46.6793434, 59.5585554]).reshape((-1, 1))

# Example 0:
theta1 = np.array([2, 0.7]).reshape((-1, 1))
gradient(x, y, theta1)
# Output:
array([[-19.0342574], [-586.66875564]])

# Example 1:
theta2 = np.array([1, -0.4]).reshape((-1, 1))
gradient(x, y, theta2)
# Output:
array([[-57.86823748], [-2230.12297889]])
```

Chapter III

Exercise 02

Interlude - Gradient Descent

So far we've calculated the *gradient*, which indicates whether and by how much we should increase or decrease θ_0 and θ_1 in order to reduce the loss. What we have to do next is update the theta parameters accordingly, step by step, until we reach the minimum. This iterative process, called **Gradient Descent**, will progressively improve the performance of your regression model on the training data.

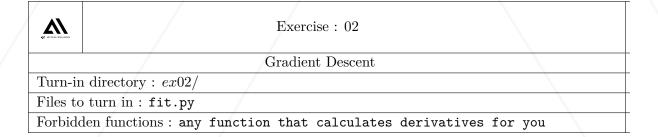
The gradient descent **algorithm** can be summarized like this: for a certain number of cycles, at each step, both θ parameters are slightly moved in the opposite directions than what the gradient indicates.

The algorithm can be expressed in pseudocode as the following:

```
repeat until convergence: {
\operatorname{compute} \nabla(J)
\theta_0 := \theta_0 - \alpha \nabla(J)_0
\theta_1 := \theta_1 - \alpha \nabla(J)_1
}
```

Few remarks on this algorithm:

- If you directly subtracted the gradient from θ , your steps would be too big and you would quickly overshoot past the minimum. That's why we use α (alpha), called the *learning rate*. It's a small float number (usually between 0 and 1) that decreases the magnitude of each update.
- The pseudocode says "repeat until convergence", but in your implementation, you will not actually check for convergence at each iteration. You will instead set a number of cycles that is sufficient for your gradient descent to converge.
- When training a linear regression model on a new dataset, you will have to choose appropriate alpha and the number of cycles through trial and error.



Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. Be able to explain what it means to *fit* a Machine Learning model to a dataset. Implement a function that performs **Linear Gradient Descent** (LGD).

Instructions

In this exercise, you will implement linear gradient descent to fit your model to the dataset

The pseudocode for the algorithm is the following:

```
repeat until convergence: {  \text{compute } \nabla(J) \\ \theta_0 := \theta_0 - \alpha \nabla(J)_0 \\ \theta_1 := \theta_1 - \alpha \nabla(J)_1  }
```

Where:

- α (alpha) is the *learning rate*. It's a small float number (usually between 0 and 1),
- For now, "reapeat until convergence" will mean to simply repeat for max_iter (a number that you will choose wisely).

You are expected to write a function named fit as per the instructions below:

Hopefully, you have already written a function to calculate the linear gradient.



- ullet You can create more training data by generating an x array with random values and computing the corresponding y vector as a linear expression of x. You can then fit a model on this artificial data and find out if it comes out with the same θ coefficients that first you used.
- ullet It is possible that $heta_0$ and $heta_1$ become "nan". In that case, it means you probably used a learning rate that is too large.

Chapter IV

Exercise 03

S WALCH WITHOUGH	Exercise: 03	
Linear Regression with Class		
Turn-in directory: $ex03/$		
Files to turn in: my_linear_regression.py		
Forbidden functions: any functions from sklearn		

Objective

Write a class that contains all methods necessary to perform linear regression.

Instructions

In this exercise, you will not learn anything new but don't worry, it's for your own good. You are expected to write your own MyLinearRegression class which looks similar to the class available in Scikit-learn: sklearn.linear_model.LinearRegression

You will add the following methods:

- fit_(self, x, y),
- predict_(self, x),
- cost_elem_(self, y, y_hat),
- cost_(self, y, y_hat).

You have already implemented these functions, you just need a few adjustments so that they all work well within your MyLinearRegression class.

```
from mylinearregression import MyLinearRegression as MyLR
x = np.array([[12.4956442], [21.5007972], [31.5527382], [48.9145838], [57.5088733]])
y = np.array([[37.4013816], [36.1473236], [45.7655287], [46.6793434], [59.5585554]])
lr1 = MyLR([2, 0.7])
lr1.predict_(x)
array([[10.74695094],
MyLR.cost_elem_(y, lr1.predict(x))
array([[710.45867381],
[364.68645485],
                   [108.97553412],
MyLR.cost_(y, lr1.predict(x))
1r2 = MyLR([1, 1], 5e-8, 1500000)
lr2.fit_(x, y)
lr2.thetas
# Output:
array([[1.40709365],
[1.1150909]])
lr2.predict_(x)
array([[15.3408728],
[25.38243697],
MyLR.cost_elem_(y, lr2.predict(x))
array([[486.66604863],
MyLR.cost_(y, lr2.predict(x))
# Output:
```

Chapter V

Exercise 04

CZ, MITTERN BILLIANS	Exercise: 04	
Practicing Linear Regression		
Turn-in directory: $ex04/$		
Files to turn in : linear_model.py, are_blue_pills_magics.csv		
Forbidden functions: sklearn		

Objective

Evaluate a linear regression model on a very small dataset, with a given hypothesis function h. Manipulate the loss function J, plot it, and briefly analyze the plot.

Instructions

You can find in the resources folder a tiny dataset called are_blue_pills_magics.csv which gives you the driving performance of space pilots as a function of the quantity of the "blue pills" they took before the test. You have a description of the data in the file named are_blue_pills_magics.txt. As your hypothesis function h, you will choose:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Where x is the variable, and θ_0 and θ_1 are the coefficients of the hypothesis. The hypothesis is a function of x.

You are strongly encouraged to use the class you have implement in the previous exercise.

Your program must:

- Read the dataset from the csv file,
- perform a linear regression,

Then you will model the data and plot 2 different graphs:

• A graph with the data and the hypothesis you get for the spacecraft piloting score versus the quantity of "blue pills" (see figure V.1)

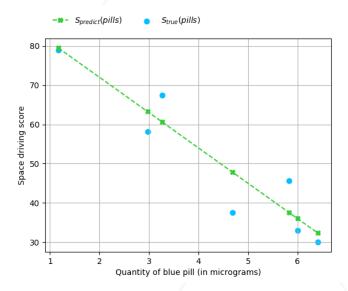


Figure V.1: Space driving score as a function of the quantity of blue pill (in micrograms). In blue the real values and in green the predicted values.

• The cost function $J(\theta)$ in function of the θ values (see figure V.2),

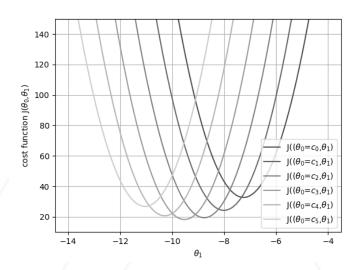


Figure V.2: Evolution of the loss function J as a function of θ_1 for different values of θ_0 .

• You will calculate the MSE of the hypothesis you chose (you know how to do it already).

```
import pandas as pd
import numpy as np
from sklearn.metrics import mean_squared_error
from mylinearregression import MyLinearRegression as MyLR

data = pd.read_csv("are_blue_pills_magic.csv")
    Xpill = np.array(data[Micrograms]).reshape(-1,1)
    Yscore = np.array(data[Score]).reshape(-1,1)

linear_model1 = MyLR(np.array([[89.0], [-8]]))
    linear_model2 = MyLR(np.array([[89.0], [-6]]))
    Y_model1 = linear_model1.predict_(Xpill)
    Y_model2 = linear_model2.predict_(Xpill)

print(MyLR.mse_(Yscore, Y_model1))
# 57.6030/285714282

print(mean_squared_error(Yscore, Y_model1))
# 57.6030/2857142825

print(MyLR.mse_(Yscore, Y_model2))
# 232.16344285714285

print(mean_squared_error(Yscore, Y_model2))
# 232.16344285714285
```

Here, the use of scikit learn is to ensure that our code is performing as expected. The use of scikit learn is forbidden in the code you will turn-in.



There is no method named .mse_ in sklearn's LinearRegression class, but there is also a method named .score. The .score method corresponds to the R^2 score. The metric MSE is available in the sklearn.metrics module.

Chapter VI

Exercise 05

Interlude - Normalization

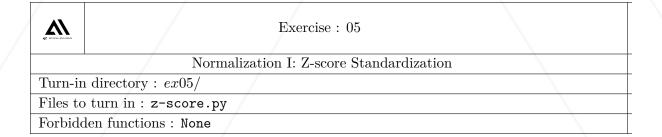
The values inside the x vector can vary quite a lot in magnitude, depending on the type of data you are working with. For example, if your dataset contains distances between planets in km, the numbers will be huge. On the other hand, if you are working with planet masses expressed as a fraction of the solar system's total mass, the numbers will be very small (between 0 and 1). Both cases may slow down convergence in Gradient Descent (or even sometimes prevent convergence at all). To avoid that kind of situation, normalization is a very effective way to proceed.

The idea behind this technique is straightforward: scaling the data.

With normalization, you can transform your x vector into a new x' vector whose values range between [-1,1] more or less. Doing this allows you to see much more easily how a training example compares to the other ones:

- If an x' value is close to 1, you know it's among the largest in the dataset
- If an x' value is close to 0, you know it's close to the median
- If an x' value is close to -1, you know it's among the smallest

So with the upcoming normalization techniques, you'll be able to map your data to two different value ranges: [0,1] or [-1,1]. Your algorithm will like it and thank you for it.



Objective

Introduction to standardization/normalization methods. You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - \frac{1}{m} \sum_{i=1}^{m} x^{(i)}}{\sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (x^{(i)} - \frac{1}{m} \sum_{i=1}^{m} x^{(i)})^2}} \quad \text{for } i \text{ in } 1, ..., m$$

Where:

- x is a vector of dimension m,
- $x^{(i)}$ is the ith component of the x vector,
- x' is the normalized version of the x vector.

The equation is much easier to understand in the following form:

$$x'^{(i)} = \frac{x^{(i)} - \mu}{\sigma}$$
 for i in $1, ..., m$

This should remind you something from TinyStatistician...

Nope?

Ok let's do a quick recap:

- μ is the mean of x,
- σ is the standard deviation of x.

Instructions

In the zscore.py file, write the zscore function as per the instructions given below:

Chapter VII

Exercise 06

A WAREN BHITME	Exercise: 06	
Normalization II: Min-max Standardization		
Turn-in directory: $ex06/$		
Files to turn in: minmax.py		
Forbidden functions: None		

Objective

Introduction to standardization/normalization methods. Implement another normalization method.

You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - min(x)}{max(x) - min(x)}$$
 for $i = 1, ..., m$

Where:

- x is a vector of dimension m,
- $x^{(i)}$ is the ith component of vector x,
- min(x) is the minimum value found among the components of vector x,
- max(x) is the maximum value found among the components of vector x.

You will notice that this min-max standardization doesn't scale the values to the [-1,1] range. What do you think the final range will be?

Instructions

In the minmax.py file, create the minmax function as per the instructions given below:

Chapter VIII

Conclusion - What you have learned

The excercises serie is finished, well done! Based on all the knowledges tackled today, you should be able to discuss and answer the following questions:

- 1. What is a hypothesis and what is its goal?
- 2. What is the loss function and what does it represent?
- 3. What is Linear Gradient Descent and what does it do? (hint: you have to talk about J, its gradient and the theta parameters...)
- 4. What happens if you choose a learning rate that is too large?
- 5. What happens if you choose a very small learning rate, but still a sufficient number of cycles?
- 6. Can you explain MSE and what it measures?

Contact

You can contact 42AI association by email: contact@42ai.fr You can join the association on 42AI slack and/or posutale to one of the association teams.

Acknowledgements

The modules Python & ML is the result of a collective work, we would like to thanks:

- Maxime Choulika (cmaxime),
- Pierre Peigné (ppeigne),
- Matthieu David (mdavid).

who supervised the creation, the enhancement and this present transcription.

- Amric Trudel (amric@42ai.fr)
- Benjamin Carlier (bcarlier@student.42.fr)
- Pablo Clement (pclement@student.42.fr)

for your investment for the creation and development of these modules.

- Richard Blanc (riblanc@student.42.fr)
- Solveig Gaydon Ohl (sgaydon-@student.42.fr)
- Quentin Feuillade Montixi (qfeuilla@student.42.fr)

who betatest the first version of the modules of Machine Learning.

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 4.0 International" license.

