CHAPTER 5

Computing VAR

> The Daily Earnings at Risk (DEaR) estimate for our combined trading activities averaged approximately $15 million.
>
> —*J.P. Morgan 1994 Annual Report*

Value at risk (VAR) is a statistical measure of downside risk based on current positions. Its greatest advantage is that it summarizes risk in a single, easy-to-understand number. No doubt this explains why VAR is fast becoming an essential tool for conveying trading risks to senior management, directors, and shareholders. J.P. Morgan (now J.P. Morgan Chase) was one of the first banks to disclose its VAR. It revealed in its 1994 Annual Report that its trading VAR was an average of $15 million at the 95 percent level over 1 day. Based on this information, shareholders then can assess whether they are comfortable with this level of risk. Before such figures were released, shareholders had only a vague idea of the extent of trading activities assumed by the bank.

This chapter turns to a formal definition of value at risk (VAR). VAR assumes that the current portfolio is "frozen" over the horizon, like all traditional risk measures, and combines current positions with the uncertainty in the risk factors at the end of the horizon.

Section 5.1 shows how to derive VAR as a summary statistic of the entire probability density function of profits and losses. This can be done in two basic ways, either by considering the actual empirical distribution or by using a parametric approximation, such as the normal distribution. In the first case, VAR is derived from the sample quantile; in the second, from the standard deviation.

Section 5.2 then discusses the choice of the quantitative factors, the confidence level and the horizon. Criteria for this choice should be guided by the use of the VAR measures. If VAR is simply a benchmark for risk, the choice is totally arbitrary. In contrast, if VAR is used to set equity capital, the choice is quite delicate. This section also discusses a generalization of VAR to losses during the horizon as opposed to solely on the target date. Criteria for parameter selection are also explained in the context of the Basel Accord rules.

The next section turns to an important and often ignored issue, which is the precision of the reported VAR number. VAR is an *estimator*, or function of the observed data. One can think of the observed data as samples, or realizations, from some underlying distribution for which we are trying to assess VAR. Different samples will lead to different VAR estimates. Thus there is some inherent imprecision in VAR numbers. It would be useful to give users some sense of this imprecision. Section 5.3 provides a framework for measuring sampling variation using confidence bands.

Section 5.4 then introduces *extreme-value theory* (EVT) to measure VAR. EVT is a semiparametric method that can be used to smooth out the tails of the density function. This allows extrapolation of quantiles to higher confidence levels and increases the precision of the VAR measures.

This chapter considers a simple situation with one risk factor only. More generally, large bank portfolios can have millions of positions that must be simplified and aggregated at a higher level. This will be the subject of Chapters 10 and 11.

5.1 COMPUTING VAR

With all the requisite tools in place, we now can formally define the value at risk (VAR) of a portfolio. *VAR is the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger.* This definition involves two quantitative factors, the horizon and the confidence level.

Define c as the confidence level and L as the loss, measured as a positive number. VAR is also reported as a positive number. A general definition of VAR is that it is the smallest loss, in absolute value, such that

$$P(L > \text{VAR}) \leq 1 - c \tag{5.1}$$

Take, for instance, a 99 percent confidence level, or $c = 0.99$. VAR then is the cutoff loss such that the probability of experiencing a greater loss is less than 1 percent.

5.1.1 Steps in Computing VAR

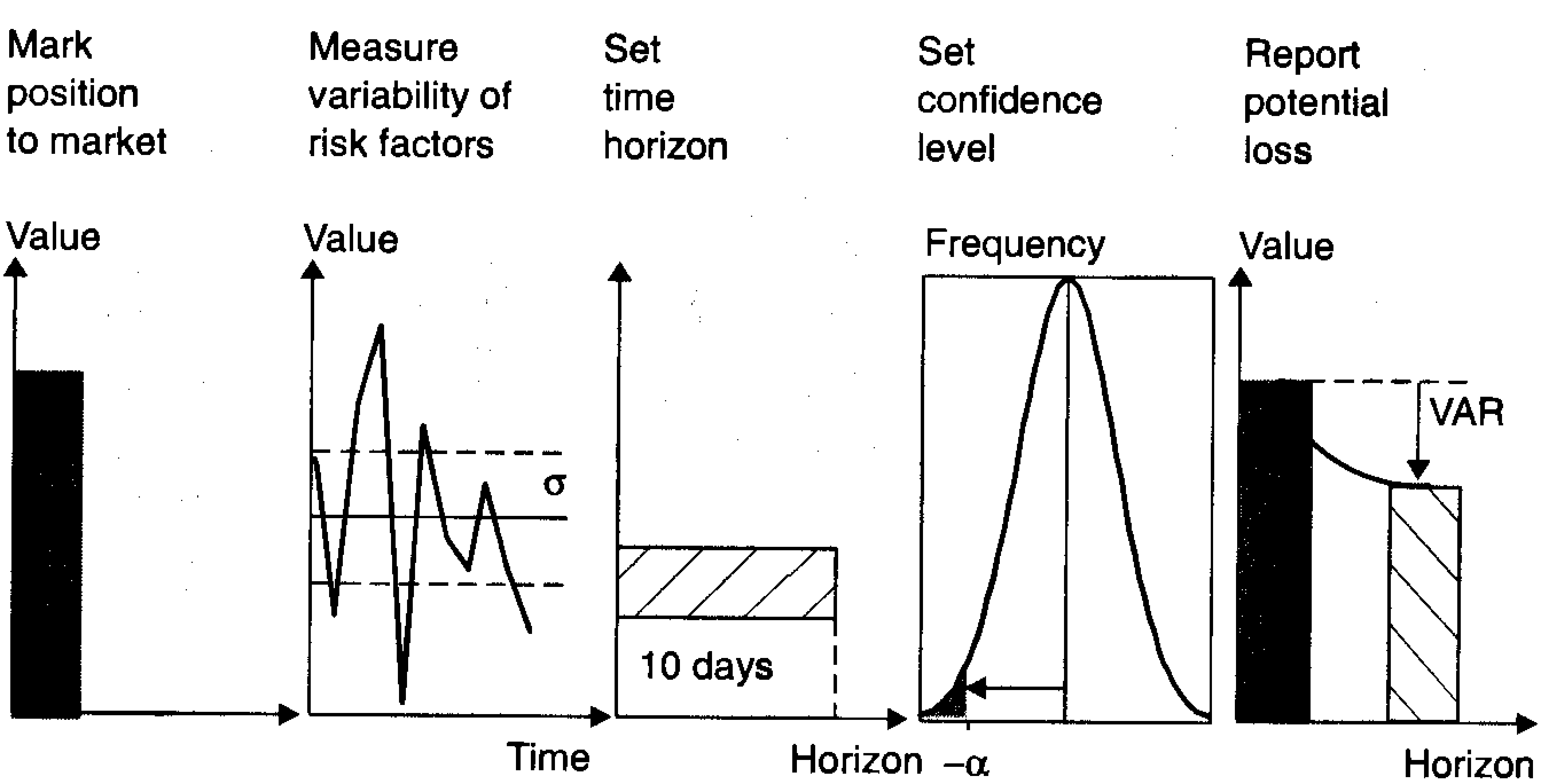
Assume, for instance, that we need to measure the VAR of a $100 million equity portfolio over 10 days at the 99 percent confidence level. The following steps are required to compute VAR:

- *Mark to market* the current portfolio (e.g., $100 million).
- *Measure the variability of the risk factor* (e.g., 15 percent per annum).
- *Set the time horizon*, or the holding period (e.g., adjust to 10 trading days).
- *Set the confidence level* (e.g., 99 percent, which yields a 2.33 factor, assuming a normal distribution).
- *Report the worst potential loss* by processing all the preceding information into a probability distribution of revenues, which is summarized by VAR (e.g., $7 million at the 99 percent confidence level).

These steps are illustrated in Figure 5-1. The detail of the computation is described next.

FIGURE 5-1

Steps in computing VAR.



Sample computation:

$100M	×	15%	×	$\sqrt{(10/252)}$	×	2.33	=	$7M

Before we start, however, we should briefly explain the square-root-of-time adjustment. Chapter 4 explained that with independently and identically distributed (i.i.d.) returns, variances are additive over time, which implies that volatility grows with the square root of time. Time, however, is measured in terms of *trading days* instead of *calendar days*. This is so because, empirically, volatility arises more uniformly over trading days.[1] This explains why the adjustment for time is expressed in terms of the square root of the number of trading days (10 trading days over a 2-week calendar period), divided by 252, which is usually taken as the number of trading days in a year.

5.1.2 Nonparametric VAR

The most general method makes no assumption about the shape of the distribution of returns. Define W_0 as the initial investment and R as its rate of return, which is random. Assuming that the position is fixed, or that there is no trading, the portfolio value at the end of the target horizon is $W = W_0\,(1 + R)$. The expected return and volatility of R are defined as μ and σ. Define now the lowest portfolio value at the given confidence level c as $W^* = W_0\,(1 + R^*)$. VAR measures the worst loss at some confidence level, so it is expressed as a positive number. One issue is, relative to what? The *relative VAR* is defined as the dollar loss relative to the mean on the horizon:

$$\text{VAR(mean)} = E(W) - W^* = -W_0(R^* - \mu) \tag{5.2}$$

Often trading VAR is defined as the *absolute VAR*, that is, the dollar loss relative to zero or without reference to the expected value:

$$\text{VAR(zero)} = W_0 - W^* = -W_0 R^* \tag{5.3}$$

If the horizon is short, the mean return could be small, in which case both methods will give similar results. Otherwise, relative VAR is conceptually more appropriate because it views risk in terms of a deviation from the mean, or "budget," on the target date, appropriately accounting for the time value of money. This approach is also more conservative if the mean value is positive. It is also more consistent with definitions of *unexpected loss*, which have become common for measuring credit risk over long horizons.

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[1] Fama (1965) and French (1980) show that the variance of stock returns over the weekend (Friday to Monday) is basically similar to the variance over trading days (e.g., Monday to Tuesday). The interpretation is that not much new information is generated during the weekend.

In its most general form, VAR can be derived from the probability distribution of the future portfolio value $f(w)$. At a given confidence level c, we wish to find the worst possible realization W^* such that the probability of exceeding this value is c, that is,

$$c = \int_{W^*}^{\infty} f(w)dw \tag{5.4}$$

or such that the probability of a value lower than W^*, $p = P(w \leq W^*)$, is $1 - c$, that is,

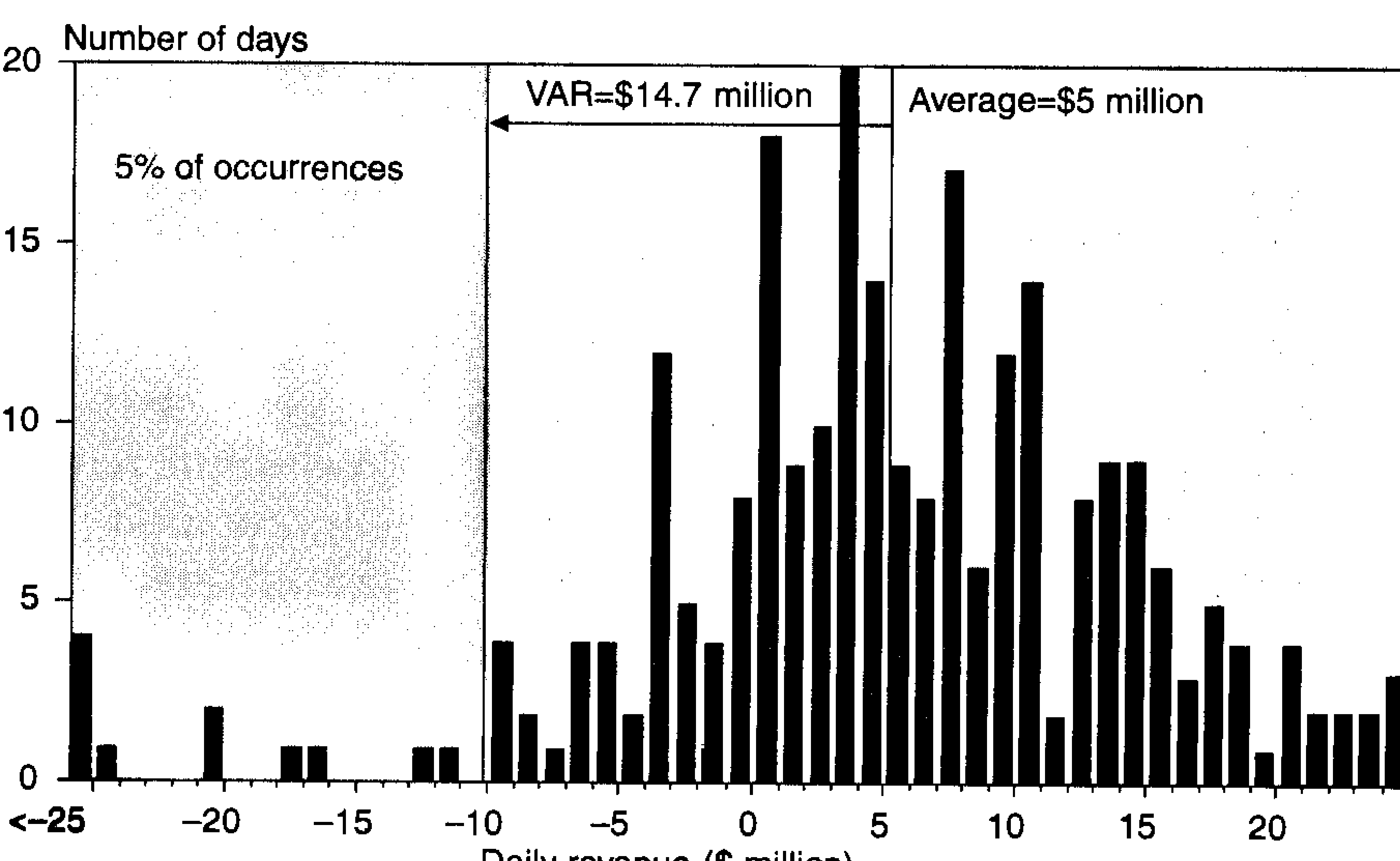
$$1 - c = \int_{-\infty}^{W^*} f(w)dw = P(w \leq W^*) = p \tag{5.5}$$

In other words, the area from $-\infty$ to W^* must sum to $p = 1 - c$. The number W^* is called the *quantile* of the distribution, which is the cutoff value with a fixed probability of being exceeded. Note that we did not use the standard deviation to find the VAR. This specification is valid for any distribution, discrete or continuous, fat- or thin-tailed.

Assume that this can be used to define a forward-looking distribution, making the hypothesis that daily revenues are identically and independently distributed. We can derive the VAR at the 95 percent confidence level from the 5 percent left-side "losing tail" in the histogram. Figure 5-2, for

FIGURE 5-2

Computation of nonparametric VAR.



instance, reports J.P. Morgan's distribution of daily revenues in 1994. The graph shows how to compute nonparametric VAR.

From this graph, the average revenue is about \$5.1 million. There is a total of 254 observations; therefore, we would like to find W^* such that the number of observations to its left is 254×5 percent $= 12.7$. We have 11 observations to the left of $-\$10$ million and 15 to the left of $-\$9$ million. Interpolating, we find $W^* = -\$9.6$ million.

The VAR of daily revenues, measured relative to the mean, is $\text{VAR} = E(W) - W^* = \$5.1 - (-\$9.6) = \$14.7$ million. If one wishes to measure VAR in terms of absolute dollar loss, VAR then is \$9.6 million. Finally, it is useful to describe the average of losses beyond VAR, which is \$20 million here. Adding the mean, we find an expected tail loss (ETL) of \$25 million.

5.1.3 Parametric VAR

The VAR computation can be simplified considerably if the distribution can be assumed to belong to a parametric family, such as the normal distribution. When this is the case, the VAR figure can be derived directly from the portfolio standard deviation using a multiplicative factor that depends on the confidence level. This approach is called *parametric* because it involves estimation of parameters, such as the standard deviation, instead of just reading the quantile off the empirical distribution.

This method is simple and convenient and, as we shall see later, produces more accurate measures of VAR. The issue is whether the distributional assumption is realistic.

Say that we pick a normal distribution to fit the data. First, we need to translate the general distribution $f(w)$ into a standard normal distribution $\Phi(\epsilon)$, where ϵ has mean zero and standard deviation of unity. We associate W^* with the cutoff return R^* such that $W^* = W_0\,(1+R^*)$. Generally, R^* is negative and can be written as $-|R^*|$. Further, we can associate R^* with a standard normal deviate $\alpha > 0$ by setting

$$-\alpha = \frac{-|R^*| - \mu}{\sigma} \tag{5.6}$$

It is equivalent to set

$$1 - c = \int_{\infty}^{W^*} f(w)dw = \int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-\alpha} \Phi(\epsilon)d\epsilon \tag{5.7}$$

Thus the problem of finding VAR is equivalent to finding the deviate α such that the area to the left of it is equal to $1 - c$. For a defined probability p, the deviate α can be found from tables of the *cumulative standard normal distribution function*, that is,

$$p = N(x) = \int_{-\infty}^{x} \Phi(\epsilon) d\epsilon \tag{5.8}$$

This function also plays a key role in the Black-Scholes option pricing model. It increases monotonically from 0 (for $x = -\infty$) to 1 (for $x = +\infty$), going through 0.5 as x passes through 0. From Table 4-2, the deviate that corresponds to a one-tailed level of 95 percent is $\alpha = 1.645$.

We then retrace our steps, back from the α we just found to the cutoff return R^* and VAR. From Equation (5.6), the cutoff return is

$$R^* = -\alpha\sigma + \mu \tag{5.9}$$

For more generality, assume now that the parameters μ and σ are expressed on an annual basis. The time interval considered is Δt, in years. We can use the time aggregation results developed in the preceding chapter, which assume uncorrelated returns.

Replacing in Equation (5.2), we find the VAR relative to the mean as

$$\text{VAR(mean)} = -W_0\,(R^* - \mu) = W_0\,\alpha\sigma\sqrt{\Delta t} \tag{5.10}$$

In other words, the VAR figure is simply a multiple of the standard deviation of the distribution times an adjustment factor that is related directly to the confidence level and horizon.

When VAR is defined as an absolute dollar loss, we have

$$\text{VAR(zero)} = -\,W_0\,R^* = W_0\,(\alpha\,\sigma\sqrt{\Delta t} - \mu\Delta t) \tag{5.11}$$

Figure 5-3 show how to compute this parametric VAR. The standard deviation of the distribution is \$9.2 million. According to Equation (5.10), the normal-distribution VAR is $\alpha \times (\sigma W_0) = 1.645 \times \$9.2 = \$15.2$ million. Note that this number is very close to the VAR obtained from the general distribution, which was \$14.7 million.

Thus, the two approaches give similar results in this case. For confidence levels that are not too high, typically below 99 percent, the normal distribution adequately represents many empirical distributions, especially for large, well-diversified portfolios. Indeed, Figure 5-4 presents the cumulative distribution functions obtained from the histogram in Figure 5-2 and from its normal approximation. The two lines are generally very close, suggesting that the normal approximation provides a good fit to the actual data.

FIGURE 5-3

Computation of parametric VAR.

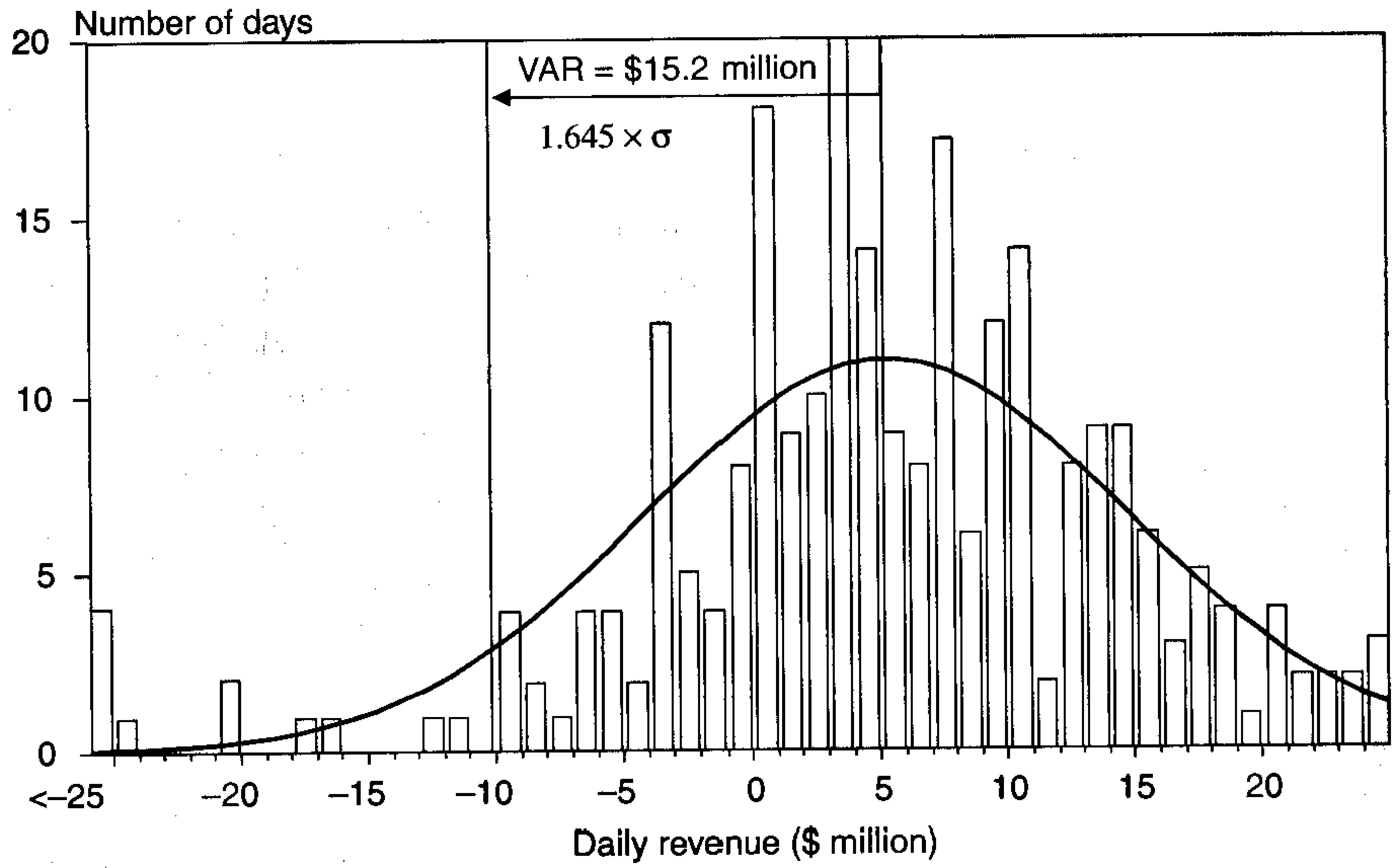
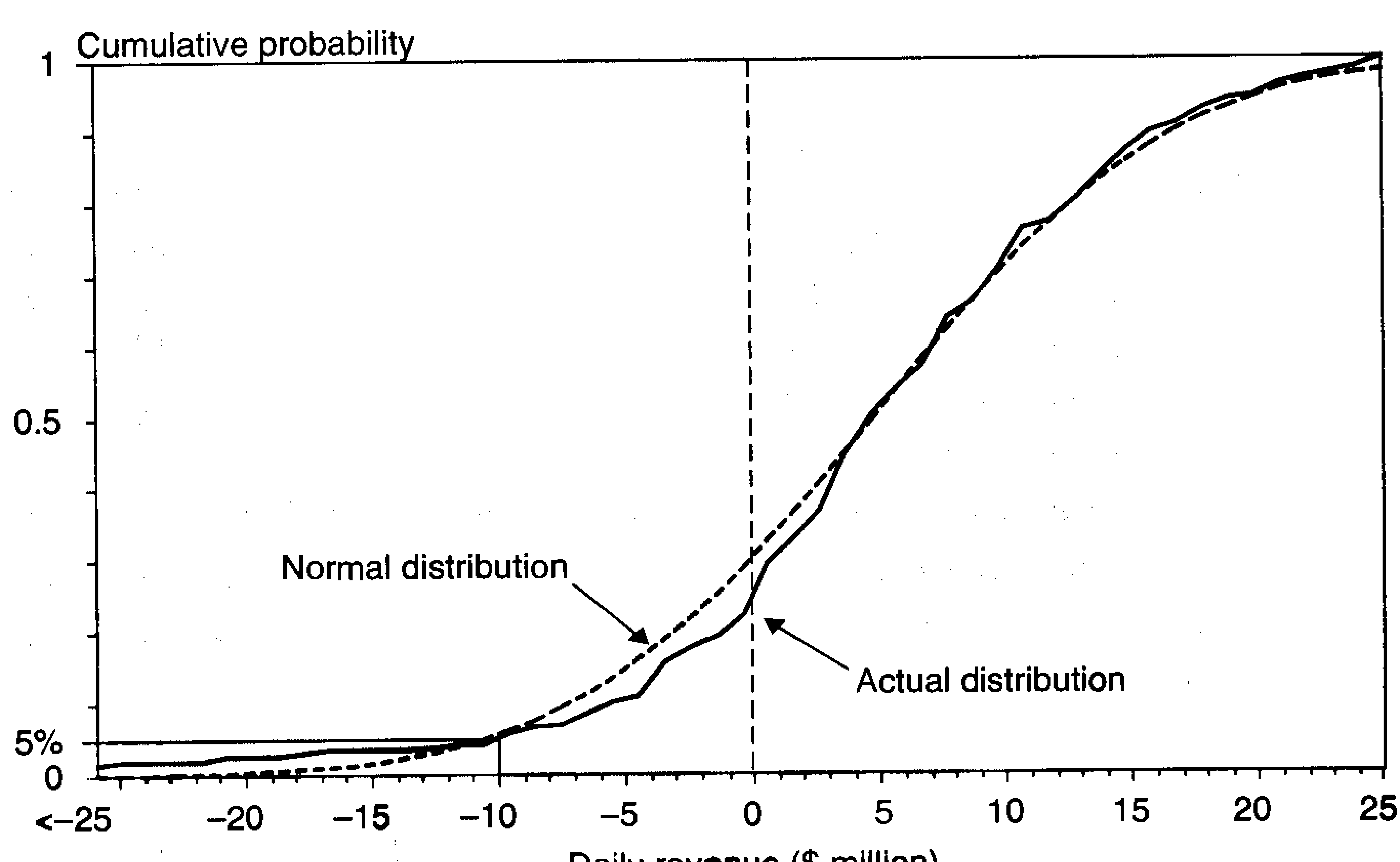


FIGURE 5-4

Comparison of cumulative distributions.



This method generalizes to other distributions as long as all the uncertainty is contained in σ. Other distributions entail different values of α. Instead of a normal distribution, we could select a student t with 6 degrees of freedom, for example. Such distribution has fatter tails than the normal. From Table 4-3, the multiplier α is 2.57 at the 99 percent level of confidence. This gives a parametric VAR of $2.57 \times \$9.2 = \$24$ million using the student distribution instead of $21 million using the normal distribution.

5.1.4 Why VAR as a Risk Measure?

VAR's heritage can be traced to Markowitz's (1952) seminal work on portfolio choice. He noted that, "You should be interested in risk as well as return" and advocated the use of the standard deviation as an intuitive measure of dispersion.

Much of Markowitz's work was devoted to studying the tradeoff between expected return and risk in the mean-variance framework, which is appropriate when either returns are normally distributed or investors have quadratic utility functions. Perhaps the first mention of confidence-based risk measures can be traced to Roy (1952), who presented a "safety first" criterion for portfolio selection. He advocated choosing portfolios that minimize the probability of a loss greater than a disaster level. Baumol (1963) also proposed a risk measurement criterion based on a lower confidence limit at some probability level $L = \alpha\,\sigma - \mu$, which is an early description of Equation (5.11).

Other measures of risk also have been proposed, including semi-deviation, which counts only deviations below a target value, and lower partial moments, which apply to a wider range of utility functions.

More generally, VAR is a statistical measure of risk that summarizes the distribution of returns into a single number $\rho(W)$. The question is, Why should it be preferred over other measures?[2]

Artzner et al. (1999) provide an interesting approach to the choice of risk measures by postulating four desirable properties for capital adequacy purposes:

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[2] Kaplanski and Krol (2002) discuss conditions under which risk measures are appropriate. VAR, or equivalently, the standard deviation, can be used if distributions are normal. Otherwise, the use of VAR is only compatible with utility functions that display a strange behavior, linear with a spike around the VAR loss. On the other hand, ETL is compatible with better-behaved utility functions.

- *Monotonicity.* If $W_1 \leq W_2$, then $\rho(W_1) \geq \rho(W_2)$. In other words, if portfolio 1 has systematically lower returns than portfolio 2 for all states of the world, its risk must be greater.
- *Translation invariance.* $\rho(W+k) = \rho(W) - k$. Adding cash in the amount k to a portfolio should reduce its risk by k.
- *Homogeneity.* $\rho(bW) = b\rho(W)$. Increasing the size of a portfolio by b should simply scale its risk by the same factor (this rules out liquidity effects for large portfolios, however).
- *Subadditivity.* $\rho(W_1+W_2) \leq \rho(W_1) + \rho(W_2)$. Merging portfolios cannot increase risk.

A risk measure that satisfies these properties is said to be *coherent.* Artzner et al. (1999) show that the quantile-based VAR measure fails to satisfy the last property. Indeed, one can come up with pathologic examples of short option positions that can create large losses with a low probability and hence have low VAR yet combine to create portfolios with larger VAR. In contrast, the expected tail loss (ETL) measure $E(-X \mid X \leq -\text{VAR})$ satisfies these desirable "coherence" properties. Thus, in theory, ETL has better properties than VAR.[3]

When returns are normally distributed, however, the standard deviation–based VAR satisfies the last property, $\sigma(W_1+W_2) \leq \sigma(W_1) + \sigma(W_2)$. Indeed, as Markowitz had shown, the volatility of a portfolio is less than the sum of volatilities.[4]

It is true that VAR fails to describe the shape of losses beyond VAR. Some portfolios may have losses close to VAR. Others may have potential losses several times the size of VAR. In this situation, reporting ETL is a useful addition to VAR. This is most likely to be the case with option trading desks, which can create portfolios with some low probability of large losses by selling options, or with undiversified portfolios exposed to credit risk.

At the highest level of a financial institution, however, the portfolio benefits from the central limit theorem, which states that the sum of independent random variables converges to a normal distribution. Indeed, the distributions of aggregate bank portfolios disclosed in annual reports

[3] In addition, Rockafeller and Uryasev (2000) show that when a nonparametric method is used, it is easier to optimize portfolio risk using ETL rather than VAR.

[4] More generally, VAR is also coherent with elliptical distributions, which are symmetric and unimodal. These include the student t distribution and, obviously, the normal pdf.

BOX 5-1

VAR IN PRACTICE AT DEUTSCHE BANK

Here is how Deutsche Bank explains its use of VAR.

> We use the value-at-risk approach to derive quantitative measures for our trading book market risks under normal market conditions. Our value-at-risk figures play a role in both internal and external (regulatory) reporting. For a given portfolio, value at risk measures the potential future loss (in terms of market value) that, under normal market conditions, will not be exceeded with a defined confidence level in a defined period.
>
> The value-at-risk measure enables us to apply a constant and uniform measure across all of our trading businesses and products. It also facilitates comparisons of our market risk estimates both over time and against our daily trading results.
>
> We calculate value at risk for both internal and regulatory reporting using a 99 percent confidence level, in accordance with BIS rules. For internal reporting, we use a holding period of one day.

This demonstrates that VAR is an essential tool to measure the bank's market risk. Its trading VAR, in millions of euros, has evolved as follows.

Year end	1998	1999	2000	2001	2002	2003	2004
VAR	37	45	38	41	33	60	66

Thus the bank has increased its trading risk substantially over this period.

generally look symmetric and close to a normal distribution. In practice, there is not much difference in rankings provided by different risk measures.[5] No doubt this explains why the industry continues to use VAR as the benchmark for measuring financial risk. An illustration is given in Box 5-1.

5.2 CHOICE OF QUANTITATIVE FACTORS

We now turn to the choice of two quantitative factors: the length of the holding horizon and the confidence level. In general, VAR will increase with either a longer horizon or a greater confidence level. Under certain conditions, increasing one or the other factor produces equivalent VAR numbers. This section provides guidance on the choice of c and Δt, which should depend on the use of the VAR number.

[5] Pfingsten et al. (2004) compare risk measures for actual trading portfolios and find that they are highly correlated.

5.2.1 VAR as a Benchmark Measure

The first, most general use of VAR is simply to provide a companywide yardstick to compare risks across different markets. In this situation, the choice of the factors is arbitrary. By now, the commercial banking industry has settled on a 99 percent confidence level and daily horizon to be compatible with the Basel Accord rules.

For this application, the focus is on cross-sectional or time differences in VAR. For instance, the institution wants to know if a trading unit has greater risk than another. Or whether today's VAR is in line with yesterday's. If not, the institution should "drill down" into its risk reports and find whether today's higher VAR is due to increased volatility or bigger bets. For this purpose, the choices of the confidence level and horizon do not matter much as long as *consistency* is maintained.

5.2.2 VAR as a Potential Loss Measure

Another application of VAR is to give a broad idea of the worst loss an institution can incur. If so, the horizon should be determined by the nature of the portfolio.

A first interpretation is that the horizon is defined by the *liquidation period.* Commercial banks currently report their trading VAR over a daily horizon because of the liquidity and rapid turnover in their portfolios. In contrast, investment portfolios such as pension funds generally invest in less liquid assets and adjust their risk exposures only slowly, which is why a 1-month horizon generally is chosen for investment purposes. Since the holding period should correspond to the longest period needed for an orderly portfolio liquidation, the horizon should be related to the liquidity of the securities, defined in terms of the length of time needed for normal transaction volumes. A related interpretation is that the horizon represents the *time required to hedge* the market risks.

An opposite view is that the horizon corresponds to the period over which the portfolio remains relatively constant. Since VAR assumes that the portfolio is frozen over the horizon, this measure gradually loses significance as the horizon extends.

However, perhaps the main reason for banks to choose a daily VAR is that this is consistent with their *daily profit and loss (P&L) measures.* This allows an easy comparison between the daily VAR and the subsequent P&L number.

For this application, the choice of the confidence level is relatively arbitrary. Users should recognize that VAR does not describe the worst-ever loss but is rather a probabilistic measure that should be exceeded with some frequency.

5.2.3 VAR as Equity Capital

On the other hand, the choice of the factors is crucial if the VAR number is used directly to set a capital cushion for the institution. If so, a loss exceeding the VAR would wipe out the equity capital, leading to bankruptcy.

For this purpose, however, we must assume that the VAR measure adequately captures all the risks facing an institution, which may be a stretch. Thus the risk measure should encompass market risk, credit risk, operational risk, and other risks.

The choice of the confidence level should reflect the degree of risk aversion of the company and the cost of a loss exceeding VAR. Higher risk aversion or greater cost implies that a greater amount of capital should cover possible losses, thus leading to a higher confidence level.

At the same time, the choice of the horizon should correspond to the time required for corrective action as losses start to develop. Corrective action can take the form of reducing the risk profile of the institution or raising new capital.

To illustrate, assume that the institution determines its risk profile by targeting a particular credit rating. The expected default rate then can be converted directly into a confidence level. Higher credit ratings should lead to a higher confidence level. Table 5-1, for instance, shows that a Baa investment-grade credit rating corresponds to a default rate of 0.31 percent over the next year. Therefore, an institution that wishes to carry this credit rating should carry enough capital to cover its annual VAR at the 99.69 percent confidence level, or 100.00–0.31.

Longer horizons inevitably lead to higher default frequencies. Institutions with an initial Baa credit rating have a default frequency of 7.63 percent over the next 10 years. The same credit rating can be achieved by extending the horizon or decreasing the confidence level appropriately.

Finally, it should be noted that the traditional VAR analysis only considers the worst loss at the horizon only. It ignores intervening losses, which may be important if the portfolio is marked to market and is subject to margin calls. Figure 5-5 illustrates a situation where the portfolio

TABLE 5-1

Credit Rating and Default Rates

	Default Rate	
Desired Rating	**1 Year**	**10 Years**
Aaa	0.00%	1.01%
Aa	0.06%	2.57%
A	0.08%	3.22%
Baa	0.31%	7.63%
Ba	1.39%	19.00%
B	4.56%	36.51%

Source: Adapted from Moody's default rates over 1920 to 2004.

FIGURE 5-5

Losses at and during the horizon.

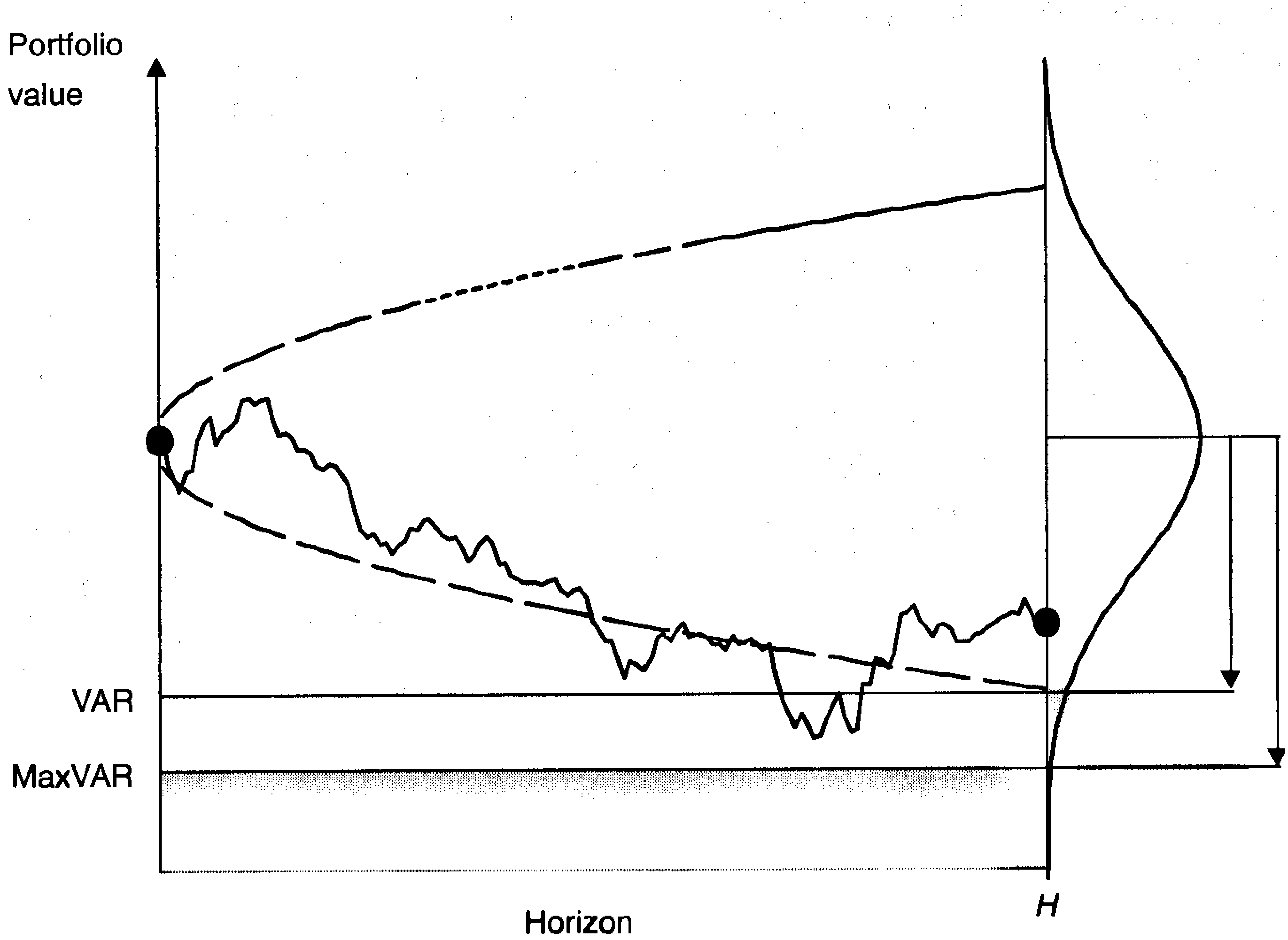


TABLE 5-2

VAR and MaxVAR, Normal Distribution

Confidence	VAR	MaxVAR	Ratio	MaxVAR (N=10)
95	1.645	1.960	1.192	1.802
99	2.326	2.576	1.107	2.420

value breaches VAR during the period but ends up above VAR at the horizon. This is a problem if this *interim* loss could cause liquidation.

This issue is addressed with *maxVAR*, which is defined as the worst loss at the same confidence level but *during* the horizon period H. This must be greater than the usual VAR, as shown in Table 5-2.[6] At the 99 percent confidence level, the maxVAR is 11 percent higher than the traditional VAR.

This assumes, however, that the portfolio value is observed continuously during the interval. In practice, the value is measured at discrete intervals, for example, daily. This will miss some of the drawdowns followed by reversals, however, leading to a lower maxVAR. For example, with $N=10$ observations, the maxVAR is slightly reduced.

5.2.4 Criteria for Backtesting

The choice of the quantitative factors is also important for backtesting considerations. Model backtesting involves systematic comparisons of VAR with the subsequently realized P&L in an attempt to detect biases in the reported VAR figures and will be described in Chapter 6. The goal should be to set up the tests so as to maximize the likelihood of catching biases in VAR forecasts.

Longer horizons reduce the number of independent observations and thus the power of the tests. For instance, using a 2-week VAR horizon means that we have only 26 independent observations per year. A 1-day VAR horizon, in contrast, will have about 252 observations over the same year. Hence a shorter horizon is preferable to increase the power of the tests. This explains why the Basel Committee performs backtesting over

[6] See Boudoukh et al. (2004).

a 1-day horizon, even though the horizon is 10 business days for capital adequacy purposes.

Likewise, the choice of the confidence level should be such that it leads to powerful tests. Too high a confidence level reduces the expected number of observations in the tail and thus the power of the tests. Take, for instance, a 95 percent level. We know that just by chance we expect a loss worse than the VAR figure in 1 day out of 20. If we had chosen a 99 percent confidence level, we would have to wait, on average, 100 days to confirm that the model conforms to reality. Hence, for backtesting purposes, the confidence level should not be set too high. In practice, a 95 percent level performs well for backtesting purposes.

5.2.5 Application: The Basel Parameters

The VAR approach is used in a variety of practices, as shown in Box 5-2. One illustration of the use of VAR as equity capital is the internal-models approach of the Basel Committee, which imposes a 99 percent confidence level over a 10-business-day horizon. The resulting VAR then is multiplied by a safety factor of 3 to provide the minimum capital requirement for regulatory purposes.

Presumably, the Basel Committee chose a 10-day period because it reflects the tradeoff between the costs of frequent monitoring and the benefits of early detection of potential problems. Presumably also, the Basel Committee chose a 99 percent confidence level that reflects the tradeoff between the desire of regulators to ensure a safe and sound financial system and the adverse effect of capital requirements on bank returns.

Even so, a loss worse than the VAR estimate will occur about 1 percent of the time, on average, or once every 4 years. It would be unthinkable for regulators to allow major banks to fail so often. This explains the multiplicative factor $k=3$, which should provide near-absolute insurance against bankruptcy.

At this point, the choice of parameters for the capital charge should appear rather arbitrary. There are many combinations of the confidence level, the horizon, and the multiplicative factor that would yield the same capital charge. This is an overidentified problem, with too many input parameters that can combine to give the same output.

The justification for the value of the multiplicative factor k also looks rather mysterious. As explained before, it effectively increases the confidence level. Presumably, k also accounts for a host of additional risks not

BOX 5-2

VAR FOR MARGIN REQUIREMENTS

Clearing corporations use a VAR approach to decide how much margin they require from investors who take positions in futures and options contracts on organized exchanges. Because the clearing corporation guarantees the performance of all contracts, it needs to protect itself from the possibility of defaults by investors who lose money on their positions. This protection is obtained by requiring traders to post a *margin*. Like VAR, the margin provides a buffer against losses.

The size of the margin is defined by the horizon and confidence level. Higher margins provide more safety to the clearinghouse. With a high confidence level, it is unlikely that the margin will be wiped out by a large loss. On the other hand, if margins are too high, investors may decide not to enter the markets, and some business will be driven away. The horizon is the time required for corrective action. For clearinghouses, this is 1 day. If traders lose money on their positions and do not replenish their margin account, the positions can be liquidated within a day.

As an example, consider the futures contract on the dollar/euro exchange rate (EC) traded on the Chicago Mercantile Exchange (CME). The notional amount is 125,000 euros. Assume that the annual volatility is 12 percent and that the current price is \$1.05 per euro.

Assuming a normal distribution, the margin that provides a sufficient buffer at the 99 percent confidence level over 1 day is

$$\text{VAR} = 2.33 \times (0.12/\sqrt{252}) \times (\text{euro } 125{,}000 \times 1.05\$/\text{euro}) = \$2310$$

This is indeed close to the maintenance margin for an outright futures position, which is \$2300 for this contract. When markets are more volatile, the margin can be increased.

modeled by the usual application of VAR that fall under the category of *model risk*. For example, the bank may be understating its risk owing to simplifications in the modeling process, to unstable correlation, or simply to the fact that it uses a normal approximation to a distribution that really has more observations in the tail, as explained in Appendix 5.A.

In the end, however, the capital charge seems adequate. For example, even during the extreme turbulence of the second half of 1998, the BCBS (1999b) found that no institution lost more than the market-risk charge.

5.2.6 Conversion of VAR Parameters

Using a parametric distribution such as the normal distribution is particularly convenient because it allows conversion to different confidence levels (which define α). Conversion across horizons (expressed as $\sigma\sqrt{\Delta t}$) is also feasible if we assume a constant risk profile, that is, portfolio positions and volatilities. Formally, the portfolio returns need to be (1) independently distributed, (2) normally distributed, and (3) with constant parameters.

As an example, we can convert the RiskMetrics risk measure into the Basel Committee internal-models measure. RiskMetrics provides a 95 percent confidence interval (1.645σ) over 1 day. The Basel Committee rules define a 99 percent confidence interval (2.326σ) over 10 days. The adjustment takes the following form:

$$\mathrm{VAR}_{\mathrm{BC}} = \mathrm{VAR}_{\mathrm{RM}} \frac{2.326}{1.645} \sqrt{10} = 4.45 \times \mathrm{VAR}_{\mathrm{RM}}$$

Therefore, the VAR under the Basel Committee rules is more than four times the VAR from the RiskMetrics system.

More generally, Table 5-3 shows how the Basel Committee parameters translate into combinations of confidence levels and horizons, taking an annual volatility of 12 percent, which is typical of the euro/$ exchange rate.

TABLE 5-3

Equivalence between Horizon and Confidence Level, Normal Distribution, Annual Risk = 12% (Basel Parameters: 99% Confidence over 2 Weeks)

Confidence Level c	Number of SD α	Horizon Δt	Actual SD $\sigma\sqrt{\Delta t}$	Cutoff Value $\alpha\sigma\sqrt{\Delta t}$
Baseline				
99%	−2.326	2 weeks	2.35	−5.47
57.56%	−0.456	1 year	12.00	−5.47
81.89%	−0.911	3 months	6.00	−5.47
86.78%	−1.116	2 months	4.90	−5.47
95%	−1.645	4 weeks	3.32	−5.47
99%	−2.326	2 weeks	2.35	−5.47
99.95%	−3.290	1 week	1.66	−5.47
99.99997%	−7.153	1 day	0.76	−5.47

These combinations are such that they all produce the same value for $\alpha\sigma\sqrt{\Delta t}$. For instance, a 99 percent confidence level over 2 weeks produces the same VAR as a 95 percent confidence level over 4 weeks. Or conversion into a weekly horizon requires a confidence level of 99.95 percent.

5.3 ASSESSING VAR PRECISION

This chapter has shown how to estimate essential parameters for the measurement of VAR, means, standard deviations, and quantiles from actual data. These estimates, however, should not be taken for granted entirely. They are affected by *estimation error*, which is the natural sampling variability owing to limited sample size. Adding a couple of new observations will change the results. The issue is by how much.

Often VAR numbers are reported to the public with many significant digits. This is ridiculous and even harmful because it gives the mistaken impression that the VAR number is estimated precisely, which is not the case. This section shows how to compute *confidence bands* around reported VAR estimates to account for sampling variability.[7]

5.3.1 The Problem of Measurement Errors

From the viewpoint of VAR users, it is useful to assess the degree of precision in the reported VAR. In a previous example, the daily VAR was \$15 million. The question is, How confident is management in this estimate? Could we say, for example, that we are 95 percent sure that the true estimate is within a \$14 million to \$16 million range? Or is it the case that the range is \$5 million to \$25 million? The two confidence bands give a very different picture of VAR. The first is very precise; the second is less informative (although it tells us that it is not in the hundreds of millions of dollars).

VAR, or any statistic θ, is estimated from a fixed window of T days. This yields an estimate $\hat{\theta}(x,T)$ that depends on the sample realizations and on the sample size. The reported statistic $\hat{\theta}$, is only an *estimate* of the true value and is affected by sampling variability. In other words, different choices of the window T or realizations will lead to different VAR figures.

[7] In addition to sampling variability, there are many more sources of approximation errors when constructing large-scale VAR numbers, but these are more difficult to identify. See also Chapter 21 on model risk.

One possible interpretation of the estimates (the view of "frequentist" statisticians) is that they represent samples from an underlying distribution with unknown parameters. With an infinite number of observations $T \to \infty$ and a perfectly stable system, the estimates should converge to the true values. In practice, sample sizes are limited, either because some financial series are relatively recent or because structural changes make it meaningless to go back too far in time. Since some estimation error may remain, the natural dispersion of values can be measured by the *sampling distribution* for the parameter $\hat{\theta}$. This can be used to generate confidence bands for the VAR estimate. Note that a confidence level must be chosen to define the confidence bands, which has nothing to do with the VAR confidence level.

5.3.2 Estimation Errors in Means and Variances

When the underlying distribution is normal, the exact distribution of the sample mean and variance is known. The estimated mean $\hat{\mu}$ is distributed normally around the true mean:

$$\hat{\mu} \sim N(\mu, \sigma^2/T) \tag{5.12}$$

where T is the number of independent observations in the sample. Note that the standard error in the estimated mean converges toward 0 at a speed of $\sqrt{1/T}$ as T increases. This is a typical result.

As for the estimated variance $\hat{\sigma}^2$, the following ratio has a chi-square distribution with $(T-1)$ degrees of freedom:

$$\frac{(T-1)\hat{\sigma}^2}{\sigma^2} \sim \chi^2(T-1) \tag{5.13}$$

In practice, if the sample size T is large enough (e.g., above 20), the chi-square distribution converges rapidly to a normal distribution, which is more convenient:

$$\hat{\sigma}^2 \sim N\left(\sigma^2, \sigma^4 \frac{2}{T-1}\right) \tag{5.14}$$

As for the sample standard deviation, its standard error in large samples is

$$\text{SE}(\hat{\sigma}) = \sigma\sqrt{\frac{1}{2T}} \tag{5.15}$$

Of course, we do not know the true value of σ for this computation, but we could use our estimated value. We can use this result to construct confidence bands for the point estimates. Assuming a normal distribution and a two-tailed confidence level of 95 percent, we have to multiply SE by 1.96.

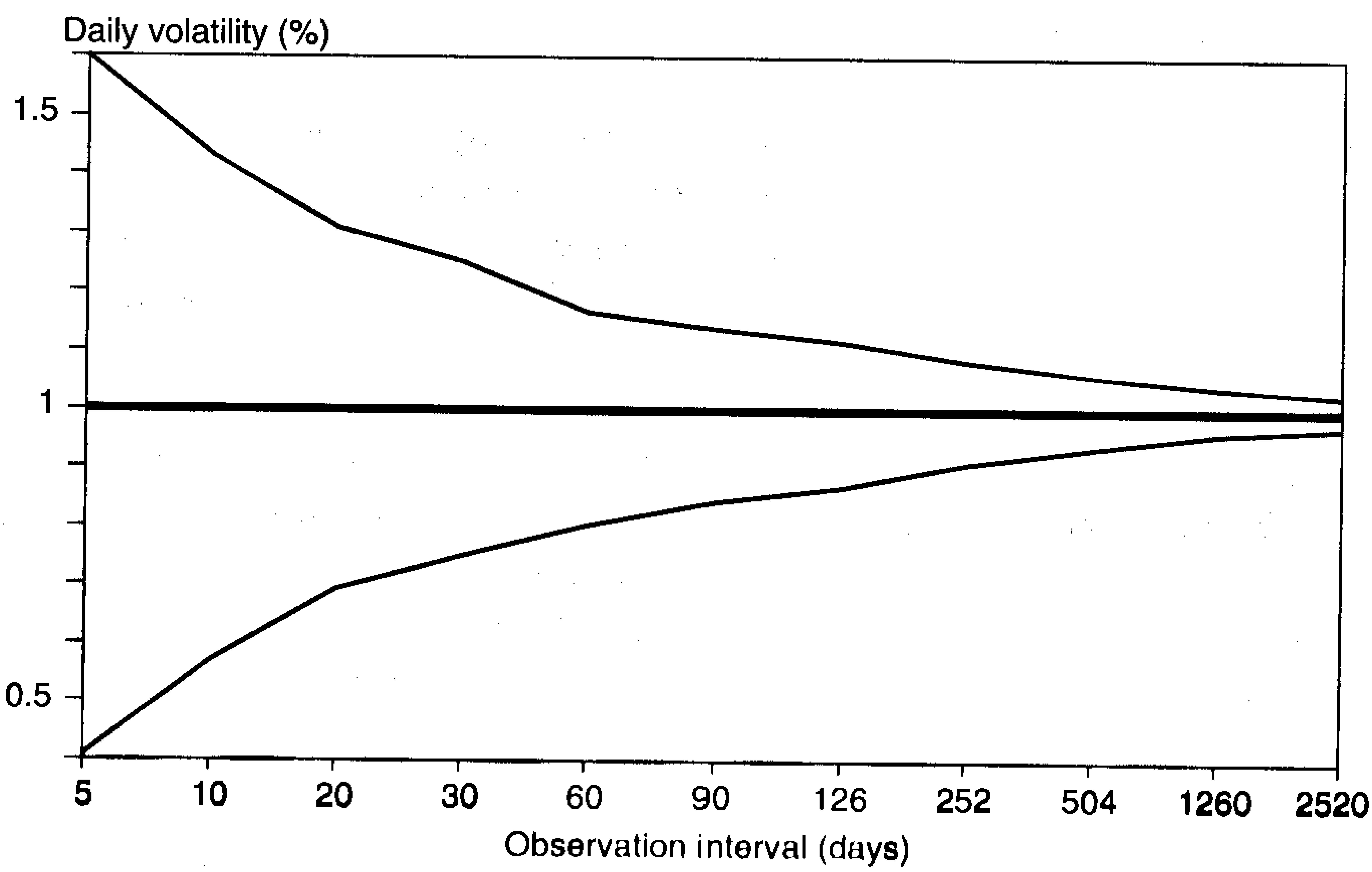
For instance, consider monthly returns on the euro/$ rate from 1973 to 2004. Sample parameters are $\hat{\mu}=-0.15$ percent, $\hat{\sigma}=3.39$ percent, and $T=384$ observations. The standard error of the estimate indicates how confident we are about the sample value; the smaller the error, the more confident we are. One standard error in $\hat{\mu}$ is $SE(\hat{\mu})=\hat{\sigma}\sqrt{1/T} = 3.39\sqrt{1/384} = 0.17$ percent. Therefore, the point estimate of $\hat{\mu}=-0.15$ percent is less than one standard error away from 0. Even with 32 years of data, μ is measured very imprecisely.

In contrast, one standard error for $\hat{\sigma}$ is $SE(\hat{\sigma}) = \hat{\sigma}\sqrt{1/2T} = 3.39\sqrt{1/768} = 0.12$ percent. Since this number is much smaller than the estimate of 3.39 percent, we can conclude that the volatility is estimated with much greater accuracy than the expected return—giving some confidence in the use of VAR systems. Alternatively, a 95 percent confidence interval around the point estimate of $\hat{\sigma}$ can be computed as $(3.39-1.96 \times 0.12, 3.39+1.96 \times 0.12) = [3.15, 3.63]$, which is rather tight.

As the sample size increases, so does the precision of the estimate. To illustrate this point, Figure 5-6 depicts 95 percent confidence bands around the estimate of volatility for various sample sizes, assuming a true daily volatility of 1 percent.

FIGURE 5-6

Confidence bands for sample volatility.



With 20 trading days, or 1 month, the band is rather imprecise, with upper and lower values set at [0.69%, 1.31%]. After 1 year, the band is [0.91%, 1.08%]. As the number of days increases, the confidence bands shrink to the point where, after 10 years, the interval narrows to [0.97%, 1.03%]. Thus, as the observation interval lengthens, the estimate should become arbitrarily close to the true value.

This example can be used to estimate confidence bands for a *sigma-based* quantile, which is

$$\hat{q}_\sigma = \alpha\,\hat{\sigma} \tag{5.16}$$

For instance, with a normal distribution and 95 percent VAR confidence level, $\alpha = 1.645$. Confidence bands for $\hat{q}_\sigma$ then are obtained by multiplying the confidence bands for $\hat{\sigma}$ by 1.645. This also applies to statistics, such as the expected tail loss, that are based on the volatility.

5.3.3 Estimation Error in Sample Quantiles

For arbitrary distributions, the cth quantile can be determined from the empirical distribution as $\hat{q}(c)$, which is a *nonparametric* approach. There is, as before, some sampling error associated with this statistic. Kendall (1994) reports that the asymptotic standard error of $\hat{q}$ is

$$\mathrm{SE}(\hat{q}) = \sqrt{\frac{c(1-c)}{Tf(q)^2}} \tag{5.17}$$

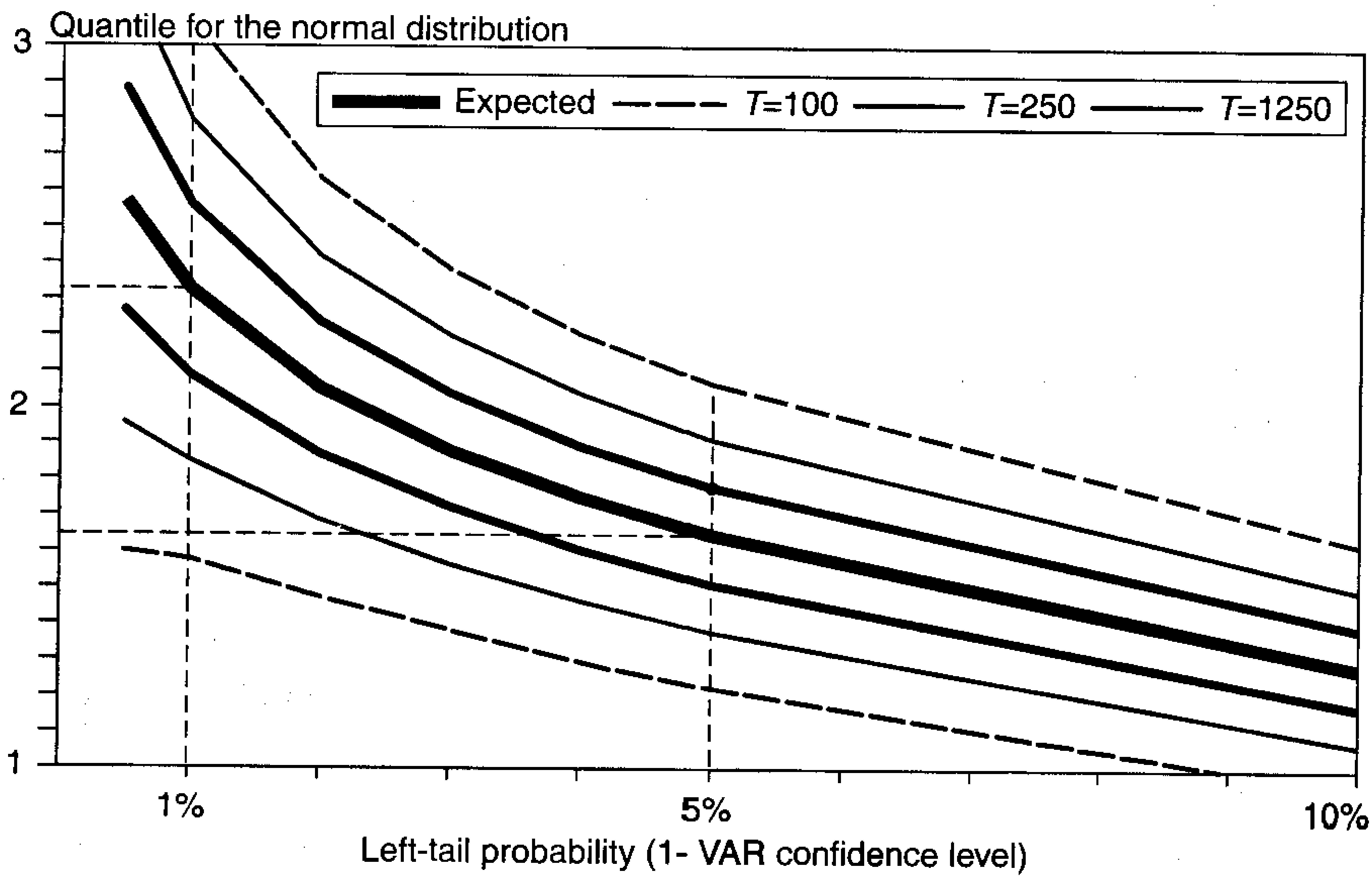
where T is the sample size, and $f(\cdot)$ is the probability distribution function evaluated at the quantile q. The effect of estimation error is illustrated in Figure 5-7, where the expected quantile and 95 percent confidence bands are plotted for quantiles from the normal distribution.

For the normal distribution, the 5 percent left-tailed interval is centered at 1.645. With T=100, the confidence band is [1.24, 2.04], which is quite large. With 250 observations, which correspond to 1 year of trading days, the band is still [1.38, 1.91]. With T=1250, or 5 years of data, the interval shrinks to [1.52, 1.76].

These intervals widen substantially as one moves to more extreme quantiles. The expected value of the 1 percent quantile is 2.33. With 1 year of data, the band is [1.85, 2.80], which is 60 percent around the true value. The interval of uncertainty is about twice that at the 5 percent interval. With 1 year of data, the band is [1.85, 2.80], which is 41 percent of the true value. With 1000 observations, or about 4 years of data, the

FIGURE 5-7

Confidence bands for sample quantiles.



band is [2.09, 2.56], which is 20 percent of the true value.[8] Thus sample quantiles are increasingly unreliable as one goes farther in the left tail. There is more imprecision as one moves to lower left-tail probabilities because fewer observations are involved. This is why VAR measures with very high confidence levels should be interpreted with extreme caution.

In practice, Equation (5.17) has limited usefulness when the underlying distribution $f(\cdot)$ is unknown. The standard error can be measured, however, by *bootstrapping* the data. This involves resampling from the sample, with replacement, T observations and recomputing the quantile. Repeating this operation K times then generates a distribution of sample quantiles that can be used to assess the precision in the original estimate. Christoffersen and Goncalves (2005) illustrate this method, which can be used for the expected tail loss (ETL) as well. They show that the estimation error in ETL is substantially larger than that in VAR. For the normal distribution and a 99 percent confidence level, the standard error is greater

[8] Most institutions use between 1 and 4 years of data for this nonparametric approach.

by 20 percent; this gets worse when the distribution has fat tails. Intuitively, this can be explained by the fact that ETL is an average of a small number of observations that can experience extreme swings in value.

5.3.4 Comparison of Methods

So far we have developed two approaches for measuring a distribution's VAR: (1) by reading the quantile directly from the distribution $\hat{q}$ and (2) by calculating the standard deviation and then scaling by the appropriate factor $\alpha\hat{\sigma}$. The issue is: Is any method superior to the other?

Intuitively, the parametric σ-based approach should be more precise. Indeed, $\hat{\sigma}$ uses information about the whole distribution (in terms of all squared deviations around the mean), whereas a quantile uses only the ranking of observations and the two observations around the estimated value. And in the case of the normal distribution, we know exactly how to transform $\hat{\sigma}$ into an estimated quantile using α. For other distributions, the value of α may be different, but we still should expect a performance improvement because the standard deviation uses all the sample information.

Table 5-4 compares 95 percent confidence bands for the two methods.[9] The σ-based method leads to substantial efficiency gains relative to the sample quantile. For instance, at the 95 percent VAR confidence level, the interval around 1.65 is [1.38, 1.91] for the sample quantile; this is

TABLE 5-4

Confidence Bands for VAR Estimates, Normal Distribution, T=250

	VAR Confidence Level c	
	99%	95%
Exact quantile	2.33	1.65
Confidence band		
Sample $\hat{q}$	[1.86, 2.80]	[1.38, 1.91]
σ-based, $\alpha\hat{\sigma}$	[2.12, 2.53]	[1.50, 1.79]

[9] For extensions to other distributions such as the student, see Jorion (1996).

reduced to [1.50, 1.78] for $\alpha\hat{\sigma}$, which is quite narrower than the previous interval.

A number of important conclusions can be derived from these numbers. First, there is substantial estimation error in the estimated quantiles, especially for high confidence levels, which are associated with rare events and hence difficult to measure. Second, parametric methods are inherently more precise because the sample standard deviation contains far more information than sample quantiles. The difficulty, however, is choice of the proper distribution.

Returning to the \$15.2 million VAR figure at the beginning of this chapter, we can now assess the precision of this number. Using the parametric approach based on a normal distribution, the standard error of this number is $\mathrm{SE}(\hat{q}_\sigma) = \alpha \times \mathrm{SE}(\hat{\sigma}) = 1.65 \times (1 / \sqrt{2 \times 254}) \times \$9.2$ million $= \$0.67$ million. Therefore, a two-standard-error confidence band around the VAR estimate is [\$13.8 million, \$16.6 million]. This narrow interval should provide reassurance that the VAR estimate is indeed meaningful.

5.4 EXTREME-VALUE THEORY

We now introduce a class of parametric models, based on sound theory, that can be used to provide better fits of the distributions tails. Extreme-value theory (EVT) extends the central limit theorem, which deals with the distribution of the *average* of i.i.d. variables drawn from an unknown distribution to the distribution of their *tails*.[10] Note that EVT applies only to the tails. It is inaccurate for the center of the distribution. This is why it is sometimes called a *semiparametric* approach (see Box 5-3).

5.4.1 The EVT Distribution

Gnedenko (1943) proved the celebrated *EVT theorem*, which specifies the shape of the cumulative distribution function (cdf) for the value x beyond a cutoff point u. Under general conditions, the cdf belongs to the following family:

$$\begin{aligned} F(y) &= 1 - (1 + \xi y)^{-1/\xi} \qquad \xi \neq 0 \\ F(y) &= 1 - \exp(-y) \qquad \xi = 0 \end{aligned} \tag{5.18}$$

[10] For a good introduction to EVT in risk management, see McNeil (1999). Embrechts et al. (1997) have written a book that provides a complete and rigorous exposition of the topic.

BOX 5-3

EVT AND NATURAL DISASTERS

EVT has been used widely in applications that deal with the assessment of catastrophic events in fields as diverse as reliability, reinsurance, hydrology, and environmental science. Indeed, the impetus for this field of statistics came from the collapse of sea dikes in the Netherlands in February 1953, which flooded large parts of the country, killing over 1800 people. (Netherlands also means "low countries.")

After this disaster, the Dutch government created a committee that used the tools of EVT to establish the necessary dike heights. As with VAR, the goal was to choose the height of the dike system so as to balance the cost of construction against the expected cost of a catastrophic flood.

Eventually, the dike system was built to withstand a 1250-year storm at a cost of $3 billion. By comparison, flood defenses in the United States are designed to withstand events that would occur every 30 to 100 years. This surely explains why the dike system, called *levees* in the United States, failed miserably for New Orleans in 2005.

where $y = (x - u)/\beta$, with $\beta > 0$ a *scale* parameter. For simplicity, we assume that $y > 0$, which means that we take the absolute value of losses beyond a cutoff point. Here, ξ is the all-important shape parameter that determines the speed at which the tail disappears. We can verify that as ξ tends to zero, the first function will tend to the second, which is exponential. It is also important to note that this function is only valid for x beyond u.

This distribution is defined as the *generalized Pareto distribution* (GPD) because it subsumes other known distributions, including the Pareto and normal distributions as special cases. The normal distribution corresponds to $\xi = 0$, in which case the tails disappear at an exponential speed. For typical financial data, $\xi > 0$ implies *heavy tails* or a tail that disappears more slowly than the normal. Estimates of ξ are typically around 0.2 to 0.4 for stock-market data. The coefficient can be related to the student t, with degrees of freedom approximately $n = 1/\xi$. Note that this implies a range of 3 to 6 for n.

Heavy-tailed distributions do not necessarily have a complete set of moments, unlike the normal distribution. Indeed, $E(X^k)$ is infinite for $k \geq 1/\xi$. For $\xi = 0.5$ in particular, the distribution has infinite variance (such as the student t with $n = 2$).

5.4.2 Quantiles and ETL

In practice, EVT estimators can be derived as follows. Suppose that we need to measure VAR at the 99 percent confidence level. We then choose a cutoff point u such that the left tail contains 2 to 5 percent of the data. The EVT distribution then provides a parametric distribution of the tails above this level. We first need to use the actual data to compute the ratio of observations in the tail beyond u, or N_u/N, which is required to ensure that the tail probability sums to unity. Given the parameters, the *tail* distribution and density function are, respectively,

$$F(x) = 1 - \left(\frac{N_u}{N}\right)\left[1 + \frac{\xi}{\beta}(x-u)\right]^{-1/\xi} \tag{5.19}$$

$$f(x) = \left(\frac{N_u}{N}\right)\left(\frac{1}{\beta}\right)\left[1 + \frac{\xi}{\beta}(x-u)\right]^{-(1/\xi)-1} \tag{5.20}$$

Various approaches are possible to estimate the parameters β and ξ.[11]

The quantile at the cth level of confidence is obtained by setting the cumulative distribution to $F(y) = c$, and solving for x, which yields

$$\widehat{\text{VAR}} = u + \frac{\hat{\beta}}{\hat{\xi}}\left\{[(N/N_u)(1-c)]^{-\hat{\xi}} - 1\right\} \tag{5.21}$$

This provides a *quantile estimator* of VAR based not only on the data but also on our knowledge of the parametric distribution of the tails. Such an estimator has lower estimation error than the ordinary sample quantile, which is a nonparametric method.

Next, the expected tail loss (ETL), or average beyond the VAR, is

$$\widehat{\text{ETL}} = \frac{\widehat{\text{VAR}}}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}u}{1-\hat{\xi}} \tag{5.22}$$

[11] Longin (1996) presents various methods to do so. For instance, the ***maximum likelihood method*** chooses parameters that maximize the likelihood function. Assuming independent observations, the likelihood of the sample is the product of the likelihood of each observation. Or, the log-likelihood is the sum of the log-likelihoods. Given the observed data x, the parameters can be found by numerically maximizing the function $\ln f(\beta, \xi) = \Sigma_{i=1}^{N} \ln f(x|\beta, \xi)$. Another particularly simple method is the *Hill estimator*, which uses an approximation to the pdf. The estimator is $\hat{\xi} = (1/N_u)\,\Sigma_{i=1}^{N_u} \ln(x_i/u)$, for all $x_i > u$. In practice, long samples are required to estimate the parameters with reasonable precision.

As an example, consider the distribution of daily returns on a broad index of U.S. stocks, the S&P 500. This series has a volatility around 1 percent per day but very high kurtosis. Figure 5-8 illustrates the fitting of the lower tails of the distribution.

The empirical distribution simply reflects the historical data. It looks irregular, however, owing to the discrete and sparse nature of data in the tails. As a result, the quantiles are very imprecisely estimated. The fitted normal distribution is smoother but drops much faster than the empirical distribution. It assigns unrealistically low probability to extreme events. Instead, the EVT tails provide a smooth, parametric fit to the data without imposing unnecessary assumptions.

These results are illustrated in Table 5-5, which compares VAR estimates across various confidence levels and across days. The numbers are scaled so that the normal 1-day VAR at the 95 percent level of confidence is 1.0. The table confirms that for 1-day horizons, the EVT VAR is higher than the normal VAR, especially for higher confidence levels. At the 99.9 percent confidence level, the EVT VAR is 2.5, against a normal VAR of 1.9.

FIGURE 5-8

Distribution of S&P 500 lower-tail returns: 1984–2004.

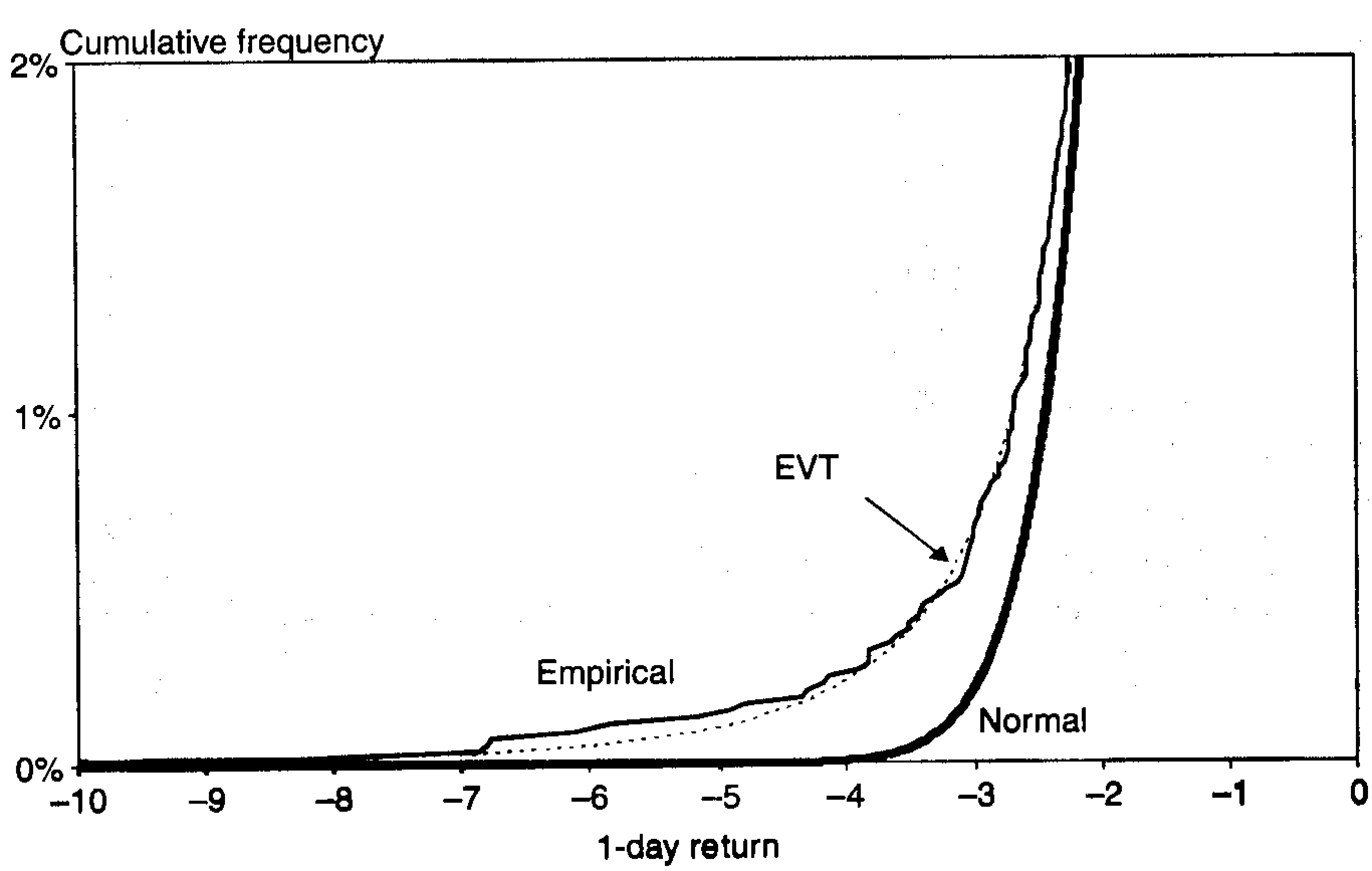


TABLE 5-5

The Effect of Fat Tails and Multiple Periods on VAR

	Confidence				
	95%	99%	99.5%	99.9%	99.95%
Extreme value					
1-day	0.9	1.5	1.7	2.5	3.0
10-day	1.6	2.5	3.0	4.3	5.1
Normal					
1-day	1.0	1.4	1.6	1.9	2.0
10-day	3.2	4.5	4.9	5.9	6.3

Source: Danielsson and de Vries (1997).

5.4.3 Time Aggregation

Another issue is that of *time aggregation.* When the distribution of 1-day returns is normal, we know that the distribution of 10-day returns is likewise, with the scaling parameter adjusted by the square root of time, or $T^{1/2}$, where T is the number of days.

EVT distributions are stable under addition; that is, they retain the same tail parameter for longer-period returns. Danielsson and de Vries (1997), however, have shown that the scaling parameter increases at the approximate rate of T^{ξ}, which is slower than the square-root-of-time adjustment. For instance, with $\xi = 0.22$, we have $10^{\xi} = 1.65$, which is less than $10^{0.5}=3.16$. Intuitively, because extreme value are more rare, they aggregate at a slower rate than the normal distribution as the horizon increases.

The fat-tail effect, therefore is offset by time aggregation. The 10-day EVT VAR is 4.3, which is now less than the normal VAR of 5.9. For longer horizons, therefore, the conclusion is that the usual Basel square-root-of-time scaling factor may provide sufficient protection.

EVT has other limitations. It is *univariate* in nature. As a result, it does not help to characterize the joint distribution of the risk factors. This is an issue because the application of EVT to the total revenue of an institution does not explain the drivers of potential losses.

5.4.4 EVT Evaluation

To summarize, the EVT approach is useful for estimating tail probabilities of extreme events. For routine confidence levels such as 90, 95, and perhaps even 99 percent, conventional methods may be sufficient. At higher confidence levels, however, the normal distribution generally underestimates potential losses. Empirical distributions suffer from a lack of data in the tails, which makes it difficult to estimate VAR reliably. This is where EVT comes to the rescue. EVT helps us to draw smooth curves through the extreme tails of the distribution based on powerful statistical theory.

The EVT approach need not be difficult to implement. For example, the student *t* distribution with 4 to 6 degrees of freedom is a simple distribution that adequately describes the tails of most financial data.

Even so, we should recognize that fitting EVT functions to recent historical data is still fraught with the same pitfalls as VAR. The most powerful statistical techniques cannot make short histories reveal once-in-a-lifetime events. This is why these methods need to be complemented by stress testing, which will be covered in Chapter 14.

5.5 CONCLUSIONS

In this chapter we have seen how to measure VAR using two alternative methodologies. The general approach is based on the empirical distribution and its sample quantile. The parametric approach, in contrast, attempts to fit a parametric distribution such as the normal to the data. VAR then is measured directly from the standard deviation. Systems such as RiskMetrics are based on a parametric approach.

The advantage of such methods is that they are much easier to use and create more precise estimates of VAR. The disadvantage is that they may not approximate well the actual distribution of profits and losses. Users who want to measure VAR from empirical quantiles, however, should be aware of the effect of sampling variation or imprecision in their VAR number.

This chapter also has discussed criteria for selecting the confidence level and horizon. On the one hand, if VAR is used simply as a benchmark risk measure, the choice is arbitrary and needs to be consistent only across markets and across time. On the other hand, if VAR is used to decide on the amount of equity capital to hold, the choice is extremely important and can be guided, for instance, by default frequencies for the targeted credit rating.

Finally, this chapter has discussed alternative measures of risk. Because VAR is just a quantile, it does not describe the extent of average losses that exceed VAR. Another measure, known as *expected tail loss* (ETL), has several advantages relative to VAR, in theory.

In practice, however, no institution reports its ETL at the aggregate level. This is so because the distribution of these portfolios generally is symmetric, in which case various risk measures give similar risk rankings.

In addition, VAR is by now recognized as a measure of loss "under normal market conditions." If users are worried about extreme market conditions, the recent historical data used can be extrapolated to higher confidence levels using extreme-value theory.

Even so, the use of historical data has limitations because this history may not include extreme but plausible scenarios. This explains why institutions complement VAR methods with *stress testing*, which is a more flexible method for dealing with losses under extreme conditions. Because of its importance, Chapter 14 will be devoted to stress testing.