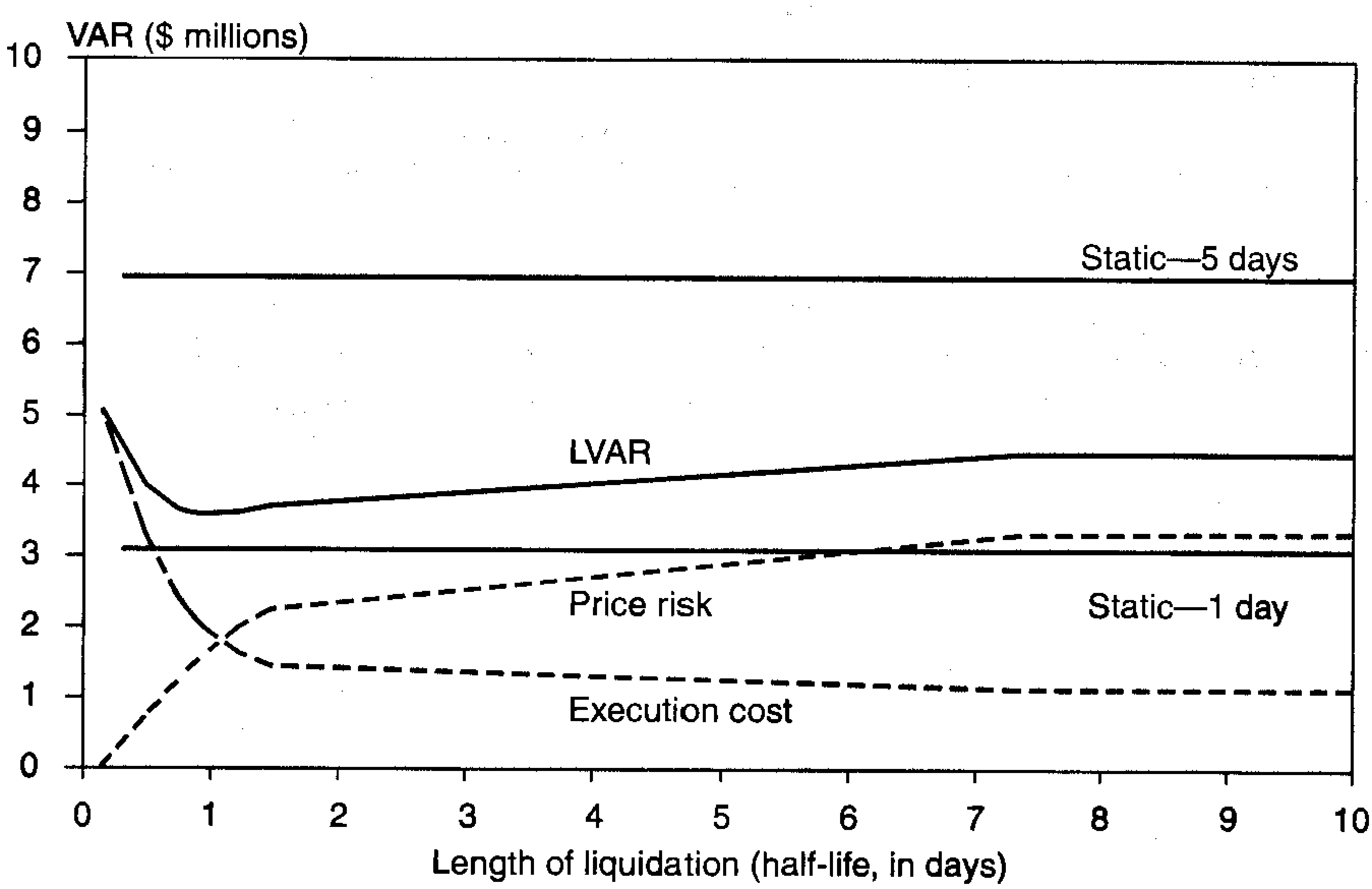
closed-form solutions for efficient execution strategies. Their paper is an important contribution that helped lay the groundwork for *algorithmic trading* on Wall Street.[4]

An optimal trajectory is described in Figure 13-2. This is defined by a set of daily positions $x_0, x_1, x_2, \ldots, x_n$. On the first day, the optimal position drops by more than the uniform sale: $x_0 - x_1 > 1/n$. Intuitively, this is so because it helps lowering price risk over the total horizon. Note that the strategy can be described by its *half-life*, which is the time required to liquidate half the portfolio. In this case, this takes 1 day.

Figure 13-3 compares various VAR measures for different speeds of execution. The "static" 1- and 5-day VARs correspond to the usual mark-to-market VAR measures with 30 percent annual volatility at the

FIGURE 13-3

Liquidity-adjusted VAR.



[4] *Algorithmic trading* is commonly defined as the automatic slicing of trading orders according to a predefined strategy to meet a specific benchmark. Estimates suggest that 60 percent of U.S. buy-side firms now use algorithmic trading. The increase in algorithmic trading, combined with the decimalization of bid-ask spreads, explains why trade sizes are getting smaller on U.S. exchanges. Algorithmic trading slices trades into small pieces, preserving anonymity and decreasing price impact. These methods, however, are most effective when trading liquid stocks, for which the price-impact function can be measured reasonably accurately.

95 percent level of confidence. Under these conditions, the daily volatility is 1.9 percent, and the 1-day VAR is $1.645 \times 0.019 \times \$100 = \$3.1$ million for this \$100 million portfolio, assuming a normal distribution. Under liquidation, however, we have to account for market impact.

The LVAR measure incorporates the total execution-cost and price-risk components in a consistent fashion. As we extend the length of liquidation, the execution-cost component decreases, but the price-risk component increases. Here, the total LVAR is minimized at a half-life of 1 day. In this case, a 5-day static VAR would provide a conservative measure of liquidation VAR.

The real benefit of this approach is that it draws attention to market-impact effects in portfolio liquidation. It also illustrates that execution strategies should pay close attention to execution costs and price volatility.

Other strategies can be used for liquidation. In the case of stock portfolios, for instance, the portfolio manager could cut the price risk by immediately putting in place a hedge with stock-index futures. In this case, the remaining price risk is "specific" to the security. Orders to sell then could be transmitted so as to minimize their price impact.

13.2.5 Example

In practice, the computational requirements to adjust the conventional VAR numbers are formidable. The method requires a price-quantity function for all securities in the portfolio. Combined with the portfolio position, this yields an estimate of the price impact of a liquidation.

Table 13-3 provides an example of such an analysis, as provided by Morgan Stanley for a four-country \$50 million equity portfolio. The data for Switzerland are expanded at the individual-stock level. To estimate the total impact cost, we need information about the historical bid-ask spreads, the median trading volume, and recent volatility. The portfolio relative size then is defined as the number of shares held as a percentage of median trading volume. The total impact cost then is computed as a function of half the bid-ask spread, the price-impact function, and the size of the position.

Here, the total cost of immediate (1-day) liquidation is estimated to be 21.5 basis points. This can be compared with the daily mark-to-market volatility of this portfolio, which is 110 basis points. Using Equation (13.9), if the portfolio were to be liquidated at the end of the next day, the worst LVAR loss at the 95 confidence percent level would be about $\$50 \times (1.645 \times 0.011 + 0.0022) = \$0.9$ million + \$0.1 million. This adds up to \$1.0

TABLE 13-3

Market Impact-Cost Report

	Portfolio			Cost Analysis			
Asset	**Value (US$)**	**Shares Held**	**Price**	**Spread (bp)**	**Median Volume**	**Shares/ Volume**	**Impact Cost (bp)**
France	19,300,182	184,063	104.9	19.9		1.3%	18.2
Germany	19,492,570	322,550	60.4	26.1		2.5%	29.3
U.K.	5,860,371	424,373	13.8	20.2		0.6%	17.6
Switzerland	5,351,851	9,355	572.1	12.5		1.1%	9.5
Novartis	2,369,367	1,630	1,453.6	11.7	123,554	1.3%	8.8
Swatch	64,678	400	161.7	32.9	42,559	0.9%	15.5
Nestle	1,752,009	935	1,873.8	6.4	76,004	1.2%	7.3
CS Group	1,165,797	6,390	182.4	22.2	978,168	0.7%	14.1
Total	50,004,974	940,341	53.2	21.6		1.7%	21.5

Source: Morgan Stanley (1999).

million, most of which is price risk. The relative importance of liquidity no doubt would be much greater for a larger portfolio.

13.3 ASSESSING FUNDING LIQUIDITY RISK

Assessing funding liquidity risk involves examining the asset-liability structure of the institution and potential demands on cash and other sources of liquidity. Some lessons are available from the Counterparty Risk Management Policy Group (1999), which was established in the wake of the LTCM near failure to strengthen practices related to the management of market, counterparty credit, and liquidity risk.[5]

The CRMPG proposes to evaluate funding risk by comparing the amount of cash an institution has at hand with to what it could need to meet payment obligations. It defines *cash liquidity* as the ratio of cash equivalent over the potential decline in the value of positions that may create cash-flow needs.

[5] The CRMPG consists of senior-level practitioners from the financial industry, including many banks that provided funding to LTCM.

TABLE 13-4

Computing Funding Liquidity Ratio

	Case 1	Case 2
Assets		
Cash	$5	$5
Liabilities		
Equity	$5	$5
Derivatives		
Long 10-year swap	$100, two-way mark to market	$100, unsecured
Short 10-year swap	$100, two-way mark to market	$100, two-way mark to market
Cash equivalent	$5	$5
Funding VAR	$1.1 (1-day)	$3.5 (10-day)
Ratio	4.5	1.4

Suppose that an institution has two swap positions that identically offset each other with two different counterparties. Thus there is no market risk, and the usual VAR is zero. The swaps are structured with different credit terms, however. Table 13-4 summarizes the positions.

In Case 1, each position is a *two-way mark-to-market* swap, also called *bilateral mark-to-market.* Because the two swaps are both marked to market, any cash payment in one swap must be offset by a receipt on the other leg. The only risk is that of a delay in the receipt, say, over 1 day. Assume that the worst move on a $100 million swap at the 99 percent level over 1 day is $1.1 million. Since this is the worst cash need, the funding ratio is $5/$1.1 = 4.5, which indicates sufficient cash coverage.

In Case 2, one of the positions is an unsecured *one-way mark-to-market* swap. Under this arrangement, the institution is required to make payments if the position loses money; it will not, however, receive intermediate payments if the position gains. Because of this asymmetry, the institution is subject to mismatches in the timing of collateral payments if the first swap loses money. We now need to consider a longer horizon, say, 10 days. This gives a VAR of $3.5 million and a funding ratio of 1.4. This seems barely enough to provide protection against funding risk. Thus some of the elements of traditional VAR can be used to compute funding risk, which can be quite different from market risk when the institution is highly leveraged. Box 13-2 illustrates how credit-rating agencies evaluate liquidity risk.

BOX 13-2

HOW RATING AGENCIES ASSESS LIQUIDITY RISK

Liquidity risk is an important component of the risk of a trading operation. Credit-rating agencies do take this risk into account when assessing the credit risk of an institution with a large trading desk.

Standard & Poor's defines *liquidity risk* as the risk that a trading operation's need for cash collateral may exceed its total liquidity resources. Exposure to collateral calls is evaluated under a stress scenario where the institution is downgraded to a speculative rating. Standard & Poor's then determines whether the institution has sufficient dedicated liquidity resources to cover these collateral calls.

The size of the worst collateral calls is estimated by the sum of all positions that have negative market values. This is so because positions with positive values are not subject to margin calls. For instance, if an institution owes $1 million to each of counterparties A and B but is owed $5 million each by counterparties C and D, it may have to post $2 million in the worst-case scenario. This is so because collateral is not transferable. In other words, even if the institution held $10 million from C and D, these funds could not be used to honor margin calls from A and B. When setting its credit rating, Standard & Poor's estimates the probability that the institution would not be able to post $2 million in the worst-case scenario.

13.4 LESSONS FROM LTCM

The story of Long-Term Capital Management (LTCM) provides a number of lessons in liquidity risk. LTCM was founded by John W. Meriwether in 1994, who left Salomon Brothers after the 1991 bond scandal. Meriwether took with him a group of traders and academics and set up a hedge fund that tried to take advantage of "relative value," or "convergence arbitrage" trades, betting on differences in prices, or spreads, among closely related securities.

13.4.1 LTCM's Leverage

Since such strategies tend to generate tiny profits, leverage has to be used to create attractive returns. By December 1997, the total equity in the fund was $5 billion. LTCM's balance sheet was about $125 billion.

This represented an astonishing leverage ratio of 25:1. Even more astonishing was the off-balance-sheet position, including swaps, options, and other derivatives, that added up to a notional amount of $1.25 trillion. This represents the total of *gross positions*, measured as the sum of the absolute value of the trade's notional principal amounts.

To give an idea of the magnitude of these positions, the Bank for International Settlements reported a total swap market of $29 trillion in 1998. Hence LTCM's swap positions accounted for 2.4 percent of the global swap market. Many of these trades, however, were offsetting each other, so this notional amount is practically meaningless. What mattered was the net risk of the fund. LTCM, however, failed to appreciate that these gross positions were so large that attempts to liquidate them would provoke large market moves.

13.4.2 LTCM's "Bulletproofing"

LTCM was able to leverage its balance sheet through sale-repurchase agreements (repos) with commercial and investment banks. Under *repo* agreements, the fund sold some of its assets in exchange for cash and a promise to repurchase them back at a fixed price at some future date. Normally, the value of the assets or collateral exceeds the cash loaned, by an amount known as a *haircut*, which creates a limit to the leverage. LTCM, however, was able to obtain unusually good financing conditions, with next-to-zero haircuts, because it was widely viewed as "safe" by its lenders. In addition, the swaps were subject to two-way marking to market.

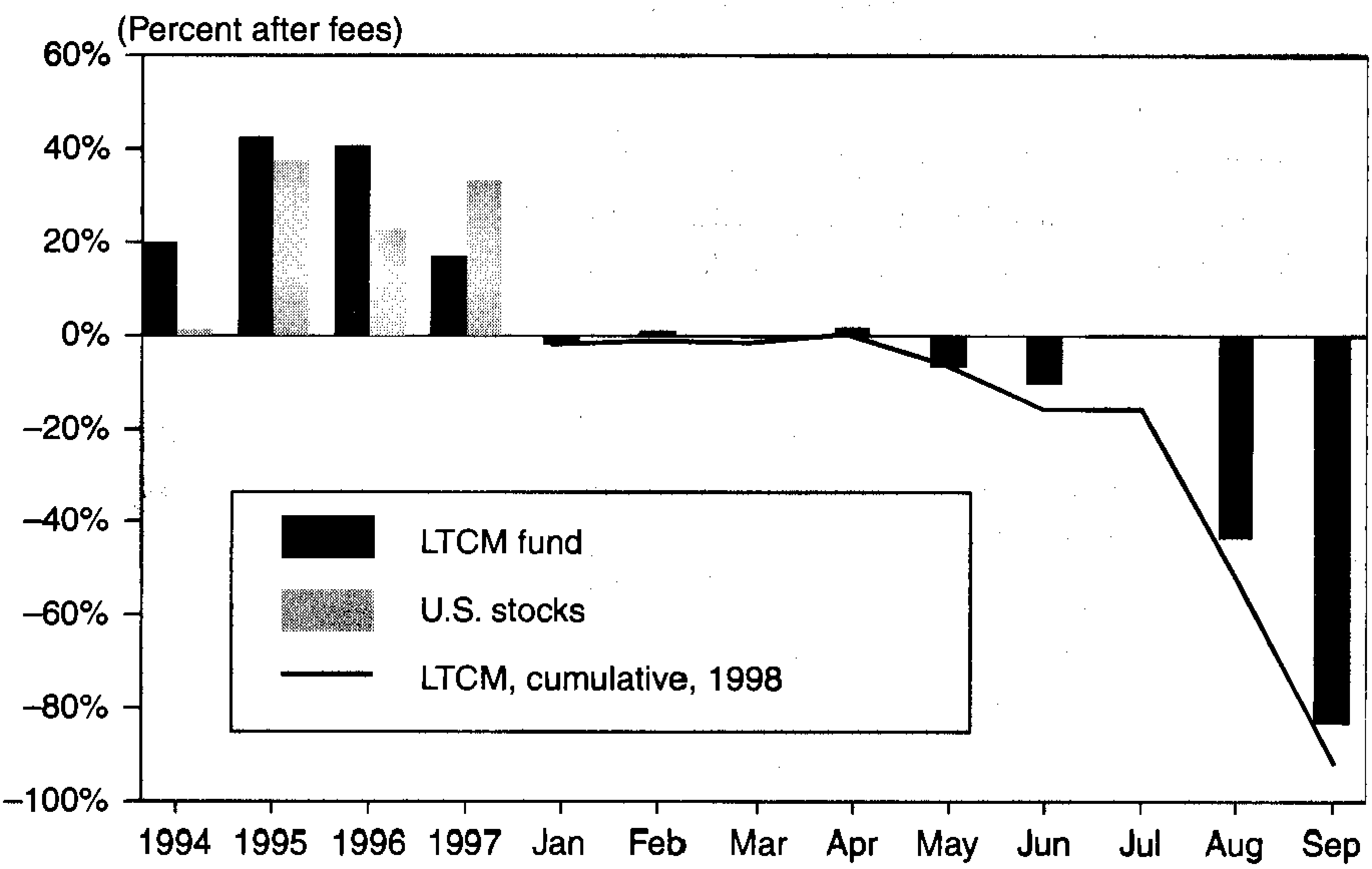
On the supply side, LTCM had "bulletproofed" itself against a liquidity squeeze. LTCM initially had required investors to keep their money in the fund for a minimum of 3 years. The purpose of this so-called lockup clause was to avoid forced sales in case of poor performance. LTCM also secured a $900 million credit line from Chase Manhattan and other banks. Even though LTCM had some protection against funding liquidity risk, it was still exposed to market risk and asset-liquidity risk.

13.4.3 LTCM's Downfall

LTCM's strategy profited handsomely from the narrowing of credit spreads during the early years, leading to after-fees returns above 40 percent, as shown in Figure 13-4. Troubles began in May and June of 1998. A

FIGURE 13-4

LTCM's returns.



downturn in the mortgage-backed securities market led to a 16 percent loss in LTCM's capital. Then came August 17. Russia announced that it was "restructuring" its bond payments—de facto defaulting on its debt. This bombshell led to a reassessment of credit and sovereign risks across all financial markets. Credit spreads, risk premiums, and liquidity spreads jumped up sharply. Stock markets dived. LTCM lost $550 million on August 21 alone.

By August, the fund had lost 52 percent of its December 31 value. With assets still at $126 billion, the leverage ratio had increased from 28:1 to 55:1. LTCM badly needed new capital. It desperately tried to find new investors, without success.

In September, the portfolio's losses accelerated. Bear Stearns, LTCM's prime broker, faced a large margin call from a losing LTCM T-bond futures position. It then required increased collateral, which depleted the fund's liquid resources.

LTCM now was caught in a squeeze between *funding risk*, as its reserves dwindled, and *asset risk*, as the size of its positions made it impractical to liquidate assets.

A liquidation of the fund would have forced the brokers to sell off tens of billions of dollars of securities and to cover their numerous derivatives trades with LTCM. Because lenders had required next-to-zero haircuts, there was a potential for losses to accrue while the collateral was being liquidated. In credit risk terms, lenders had low current exposure but significant *potential exposure*.

The potential disruption in financial markets was such that the New York Federal Reserve felt compelled to act. On September 23, it organized a bailout of LTCM, encouraging 14 banks to invest \$3.6 billion in return for a 90 percent stake in the firm. These fresh funds came just in time to avoid meltdown. By September 28, the fund's value had dropped to \$400 million only. LTCM investors had lost a whopping 92 percent of their year-to-date investment.

13.4.4 LTCM's Liquidity

LTCM failed because of its inability to manage its risk. This was due in no small part to the fact that LTCM's trades were rather undiversified. LTCM was reported to have lost about \$1.5 billion from interest-rate swap positions and a similar amount from short positions on equity volatility. As we will show in Chapter 21, this was a result of an ill-fated attempt to manage risk through portfolio optimization.

Table 13-5 describes the exposure of various reported trades to fundamental risk factors. All the trades were exposed to increased market volatility. Most were exposed to increased liquidity risk (which is itself

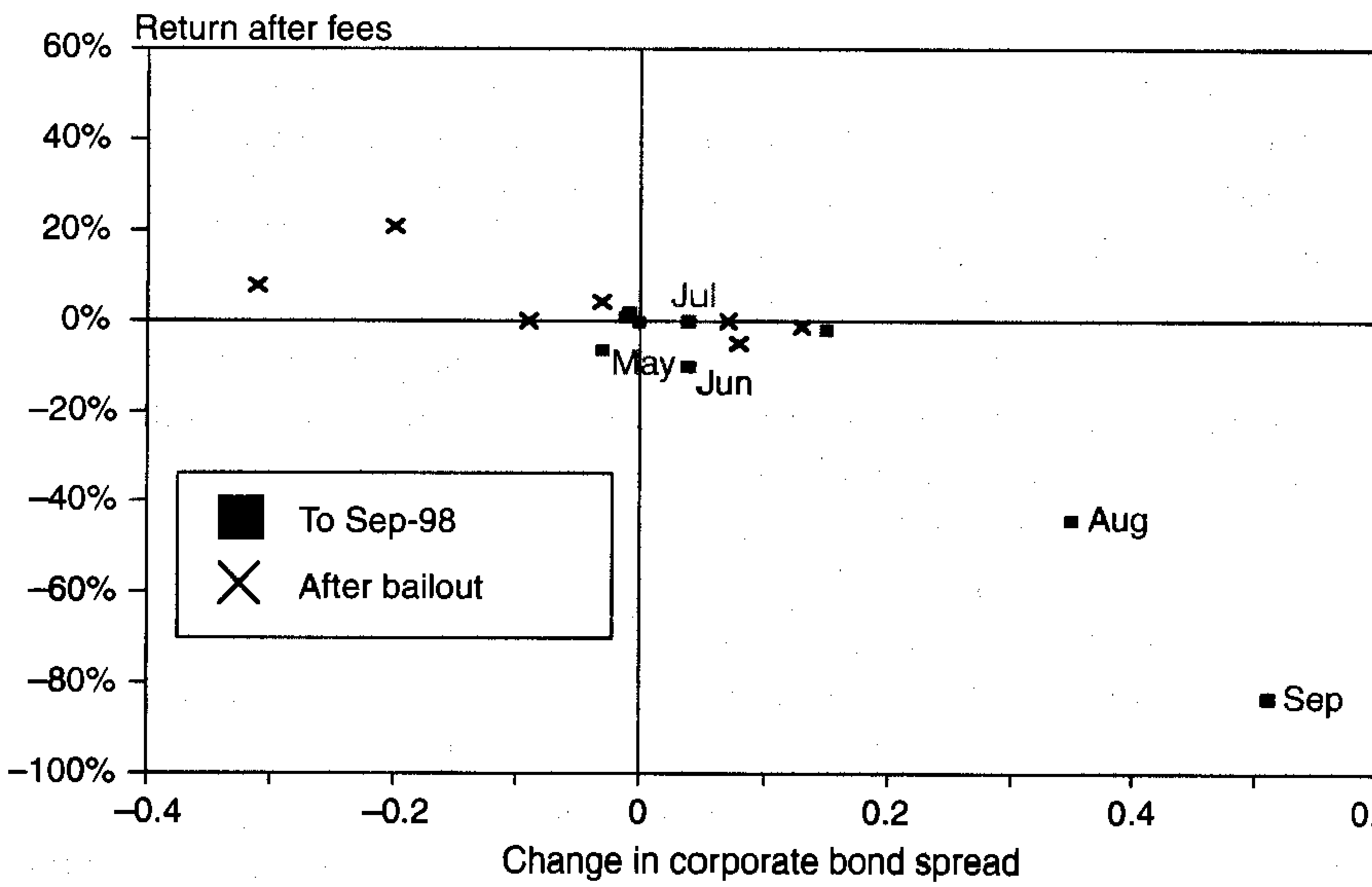
TABLE 13-5

Exposure of LTCM's Portfolio to Risk Factors

	Loss if Risk Factor Increases		
Trade	**Volatility**	**Default**	**Illiquidity**
Long interest-rate swap	Yes	Yes	Yes
Short equity options	Yes		
Long off-the-run/short on-the-run Treasuries			Yes
Long mortgage-backed securities (hedged)	Yes		Yes
Long sovereign debt	Yes	Yes	Yes

FIGURE 13-5

Explaining LTCM's returns.



positively correlated with volatility). Many were exposed to increased default risk.

To illustrate the driving factor behind LTCM's risks, Figure 13-5 plots the monthly returns against monthly changes in credit spreads. The fit is remarkably good, indicating that a single risk factor would explain 90 percent of the variation up to the September bailout. Thus there was little diversification across risk factors.

In addition, LTCM was a victim of both asset and funding liquidity risk. Although it had taken some precautions against withdrawal of funds, it did not foresee that it would be unable to raise new funds as its performance dived. The very size of the fund made it very difficult to organize an orderly portfolio liquidation.

The episode also raised questions about the soundness of the brokers' risk management systems. The brokers lulled themselves into thinking that they were protected because their loans were "fully collateralized." Even so, their loans carried no haircuts and were exposed to the risk that LTCM could default at the same time as the collateral lost value. One of the lessons of this near disaster was to accelerate the integration of credit and market risk management.

13.5 CONCLUSIONS

This chapter has shown how to account for liquidity risk. Traditional VAR models measure the worst change in mark-to-market value over the horizon but do not account for the actual cost of liquidation. These costs depend on the price-impact function, as well as the size of the positions. This leads to a "hybrid" liquidity-adjusted VAR measure that combines price volatility with liquidation costs.

In general, bid-ask spread effects are less important than traditional VAR measures. What matters more are the large price drops owing to liquidating large positions. In normal markets, liquidity effects are fairly predictable. Whether these LVAR measures apply to stressed markets, however, is more doubtful.

An alternative approach is to value positions at the conservative bid-ask quote and even to take a reserve to account for illiquidity. In such cases, there is no need to take liquidity risk into account in VAR because it is already factored into the valuation of positions.

Funding liquidity risk, in contrast, arises when financing for the portfolio cannot be maintained. Here again, VAR can be altered to estimate the risk that a portfolio could run out of cash.

Thus liquidity risk involves the two sides of the balance sheet, assets and liabilities. The greater the liquidation horizon for a portfolio, the greater is the need for extended financing of the portfolio.

The CRMPG recently reviewed the progress made since the original 1999 study in an update dubbed CRMPG II (2005). Many banks have responded that the CRMPG recommendations provided "a useful framework." CRMPG II reports that institutions now have "a greater focus on liquidity-based adjustments to closeout values and on the interaction of asset liquidity and funding liquidity." Still, the CRMPG warns that crises will "inevitably occur" and that "investments in risk management systems should continue to be a high priority."

While LVAR may be somewhat difficult to measure, some rules of thumb are useful. We do know that bid-ask spreads are positively correlated with volatility. A position in illiquid assets will incur greater execution costs as volatility increases. Thus liquidity risk can be mitigated by taking offsetting positions in assets, or businesses, that benefit from increased volatility or have positive vega. Examples are long positions in options and customer trading, which typically benefit when trading volatility and volume spike up.

As with other applications of VAR, the main benefit of this analysis is not so much to come up with one summary risk number but rather to provide a systematic framework for thinking about the interactions among market risk, asset liquidity risk, and funding liquidity risk.

QUESTIONS

1. Define asset and funding liquidity risk.
2. What is a potential problem for the marking-to-market assumption underlying the measurement of VAR if VAR is to measure the worst loss over a liquidation period?
3. Explain how the analysis of market microstructure, or demand and supply curves, is useful to assess liquidity risk.
4. What is the common characteristic of *deep* markets in terms of liquidity risk?
5. Define *normal market sizes*.
6. How is asset liquidity risk controlled?
7. A hedge fund has a position in 1 million shares of a stock whose midprice is \$100. The bid-ask spread is \$0.40, up to a volume of 100,000. Beyond that, prices fall by \$0.50 per share for every 100,000 shares transacted in one day. Compute the loss from the midprice if the entire position is liquidated over 1 day. This should be computed in dollars and in fraction of the initial position value.
8. Repeat with two other scenarios: (a) The sale is spread uniformly over 10 days. (b) The sale is spread over 5 days. Assume that prices are not expected to move.
9. Assuming a daily stock volatility of 1 percent and uncorrelated returns, compute the volatility of holding the original position over 10 days. Then compare the volatility of the three strategies in the previous questions. Ignore intraday risk.
10. What is the tradeoff between liquidating quickly or slowly.
11. Can you explain why hedge funds do not accept new investors after they have reached some size? Would you expect a large or small size for strategies investing in government bonds or high-yield bonds?
12. Some hedge funds have *lockup periods* for their investors, which prevent them from pulling their money within some period. Which type of strategies are more likely to use such clauses: leveraged funds investing in government bonds or high-yield bonds?

13. Explain how funding liquidity risk can arise for leveraged institutions.
14. Do pension funds, which are not leveraged, face funding liquidity risk?
15. Explain what a haircut is (in the context of liquidity risk).
16. Why do companies issue debt with credit triggers? Do you think these are useful features?
17. Among U.S. stocks, bonds, and Treasury bills, which class of assets has the lowest bid-ask spread?
18. What are sources of bid-ask spreads in market microstructure theory?
19. How is the liquidity-adjusted VAR, LVAR, different from the traditional VAR?
20. Is the relative importance of the liquidity term in LVAR greater or smaller as the number of assets increases in a portfolio?
21. What is cash liquidity, as defined by the Counterparty Risk Management Policy Group?
22. What instruments did LTCM use to leverage its balance sheet? Explain.
23. What were the major risks involved in the LTCM debacle?
24. Reviewing the types of trades done by LTCM, do you think this was a well-diversified fund?

CHAPTER 14

Stress Testing

This is one of those cases in which the imagination is baffled by the facts.

—*Winston Churchill*

The main purpose of value-at-risk (VAR) measures is to quantify potential losses under "normal" market conditions, where *normal* is defined by the confidence level, typically 99 percent. In principle, increasing the confidence level could uncover progressively larger but less likely losses. In practice, VAR measures based on recent historical data can fail to identify extreme unusual situations that could cause severe losses. This is why VAR methods should be supplemented by a regular program of stress testing. Stress testing is a *nonstatistical* risk measure because it is not associated with a probability statement like VAR.

Stress testing is indeed required by the Basel Committee as one of seven conditions to be satisfied to use internal models. It is also endorsed by the Derivatives Policy Group and by the Group of Thirty. *Stress testing* can be described as a process to identify and manage situations that could cause extraordinary losses. This can be made with a set of tools, including (1) scenario analysis; (2) stressing models, volatilities, and correlations; and (3) policy responses.

Scenario analysis consists of evaluating the portfolio under various extreme but probable states of the world. Typically, these involve large movements in key variables, which requires the application of full-valuation methods. The earliest application of stress tests consisted of sequentially moving key variables by a large amount. This is also called *sensitivity testing*.

This approach, however, ignores correlations, which are crucial to large-scale risk measurement. More generally, scenarios provide a description of the joint movements in financial variables. Scenarios can be *historical*, that is, drawn from historical events, or *prospective*, that is, drawn from plausible economic and political developments. Prospective scenarios are also called *hypothetical*. More recently, the industry has realized that the identification of scenarios should be driven by the particular portfolio at hand. Scenarios that matter are those that could generate extreme losses.

Stress tests are used primarily for understanding the risk profile of a firm. Increasingly, however, they are also used, in conjunction with VAR, for *capital allocation*. Whenever the stress tests reveal some weakness, management must take steps to manage the identified risks. One solution could be to set aside enough capital to absorb potential large losses. Too often, however, this amount will be cripplingly large, reducing the return on capital. Alternatively, positions can be altered to reduce the exposure. The goal is to ensure that the institution can ride out the turmoil.

Section 14.1 discusses why stress testing is required at all. In theory, extreme losses could be identified by increasing the confidence level of VAR measures. Section 14.2 shows how to use scenarios to generate portfolio losses. Sections 14.3 and 14.4 then examine scenario analysis in great detail. This is no easy matter owing to the large number of risk factors that global financial institutions are exposed to. Next, Section 14.5 turns to stress testing of models and parameters. Section 14.6 then discusses management actions that can be taken in response to stress-test results.

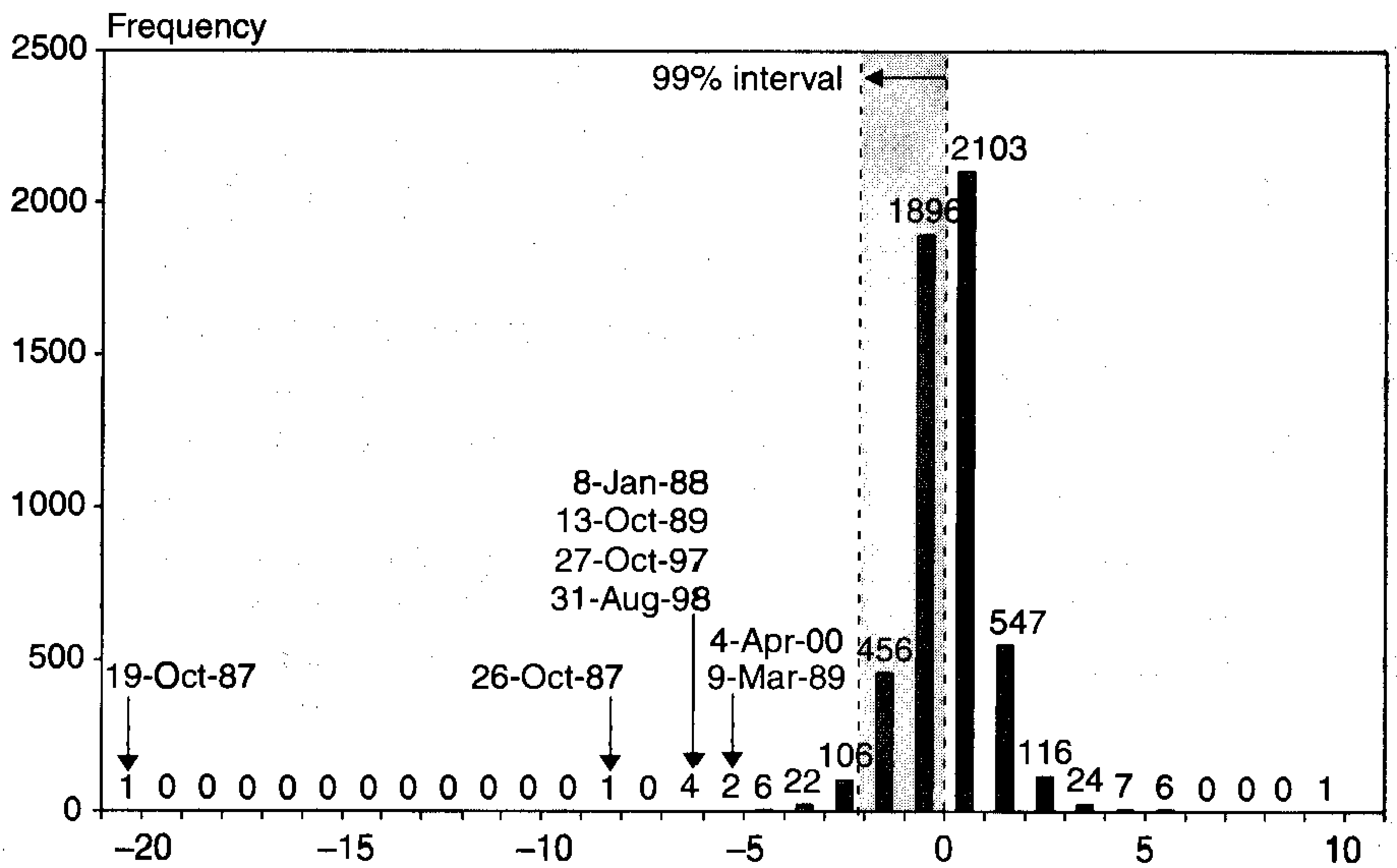
14.1 WHY STRESS TESTING?

Compared with VAR methods, stress testing appears refreshingly simple and intuitive. The first step is scenario analysis, which examines the effect of simulated large movements in key financial variables on the portfolio. Such scenarios have the advantage of linking the loss to a specific event, which is more intuitive to many managers than a draw from a statistical distribution. Owing to its simplicity, this approach actually predates VAR methods.

To understand the need for scenario analysis, consider, for instance, the stock market crash of October 19, 1987. Figure 14-1 displays the distribution of U.S. daily stock returns using data from 1984 to 2004. Over this period, the average volatility was about 1 percent per day. On Monday, October 19, the Standard & Poor's (S&P) Index lost 20 percent of its value.

FIGURE 14-1

Distribution of daily U.S. stock returns, 1984–2004.



Even if there was some time variation in volatility, this 20 standard deviation event was so far away in the tail that it never have should happened under a normal distribution. The figure also shows that a standard 99 percent VAR interval would have totally missed the magnitude of the actual loss.

More generally, Bookstaber (1997) says that there is

> . . . a general rule of thumb that every financial market experiences one or more daily price moves of 4 standard deviations or more each year. And in any year, there is usually at least one market that has a daily move that is greater than 10 standard deviations.

These observations, however, are an indictment of the distributional assumption rather than VAR itself. In theory, one could fit a better distribution to the data and vary the confidence level so as to cover more and more of the left-tail events. This can be accomplished with historical simulations or, if a smoother distribution is required, through the use of extreme-value theory (EVT). In other words, the generation of a scenario is akin to a particular point in the distribution drawn from historical data. So what is special about stress testing?

The goal of stress testing is to identify unusual scenarios that would not occur under standard VAR models. Berkowitz (2000) classifies these scenarios into the following categories:

1. Simulating shocks that have never occurred or are more likely to occur than historical observation suggests.
2. Simulating shocks that reflect permanent structural breaks or temporarily changed statistical patterns.

Thus one reason to stress test is that VAR measures typically use recent historical data. Stress testing, in contrast, considers situations that are absent from historical data or not well represented but nonetheless likely. Alternatively, stress tests are useful to identify states of the world where historical relationships break down, either temporarily or permanently.

A direct example of the need for stress testing is Niederhoffer's belief, described in Box 14-1, that the market would not drop by more

BOX 14-1

VICTOR NIEDERHOFFER: THE EDUCATION OF A SPECULATOR

Victor Niederhoffer outlined his investment philosphy in his book, *Education of a Speculator*, which quickly became a best-seller. An eccentric and brilliant investor, he was a legend of the hedge-fund business. Indeed, he had compiled an outstanding track record—a 32 percent compound annual return since 1982.

Niederhoffer's mission was to "apply science" to the market. Although he had a Ph.D. in business from the University of Chicago, he did not believe in efficient markets and traded on statistical anomalies. He believed, for instance, that the market would never drop by more than 5 percent in a single day. Putting this theory into practice, Niederhoffer sold naked out-of-the-money puts on stock index futures. When the stock market plummeted by 7 percent on October 27, 1997, he was unable to meet margin calls for some $50 million. His brokers liquidated the positions, wiping out his funds.

Apparently, his views were narrowly based on recent history. It is true that the worst loss had been 3 percent in the previous 5-year period. Larger losses do occur once in a while, however. Most notably, the market lost 20 percent on October 19, 1987.

than 5 percent in a day. Indeed, this never happened from 1990 to October 1997. This does not mean that a loss of this magnitude can never happen.

Another illustration is a breakup of a fixed exchange-rate system. In the summer of 1992, it would have been useful to assess potential vulnerabilities in the European monetary system. Indeed, in September 1992, the Italian lira and the British pound abandoned their fixed exchange rates, which led to a disastrous fall in their value. Historical volatilities based on the previous 2 years would have completely missed the possibility of a devaluation. Thus scenario analysis forces risk managers to consider events they otherwise might ignore.

14.2 PRINCIPLES OF SCENARIO ANALYSIS

We now consider the implementation of scenario analysis. Define s as a selected scenario. This is constructed as a set of changes in risk factors $\Delta f_{k,s}$ for various k. Based on the new hypothetical risk-factor values, $f_{k,0} + \Delta f_{k,s}$, all the securities in the portfolio are revalued, preferably using a full-valuation method if the portfolio has nonlinear components. The portfolio return then is derived from changes in the portfolio value V, which depends on positions and risk factors, that is,

$$R_{p,s} = V_s - V_0 = V(f_{1,0}+\Delta f_{1,s} \ldots, f_{K,0}+\Delta f_{K,s}) - V(f_{1,0} \ldots, f_{K,0}) \qquad (14.1)$$

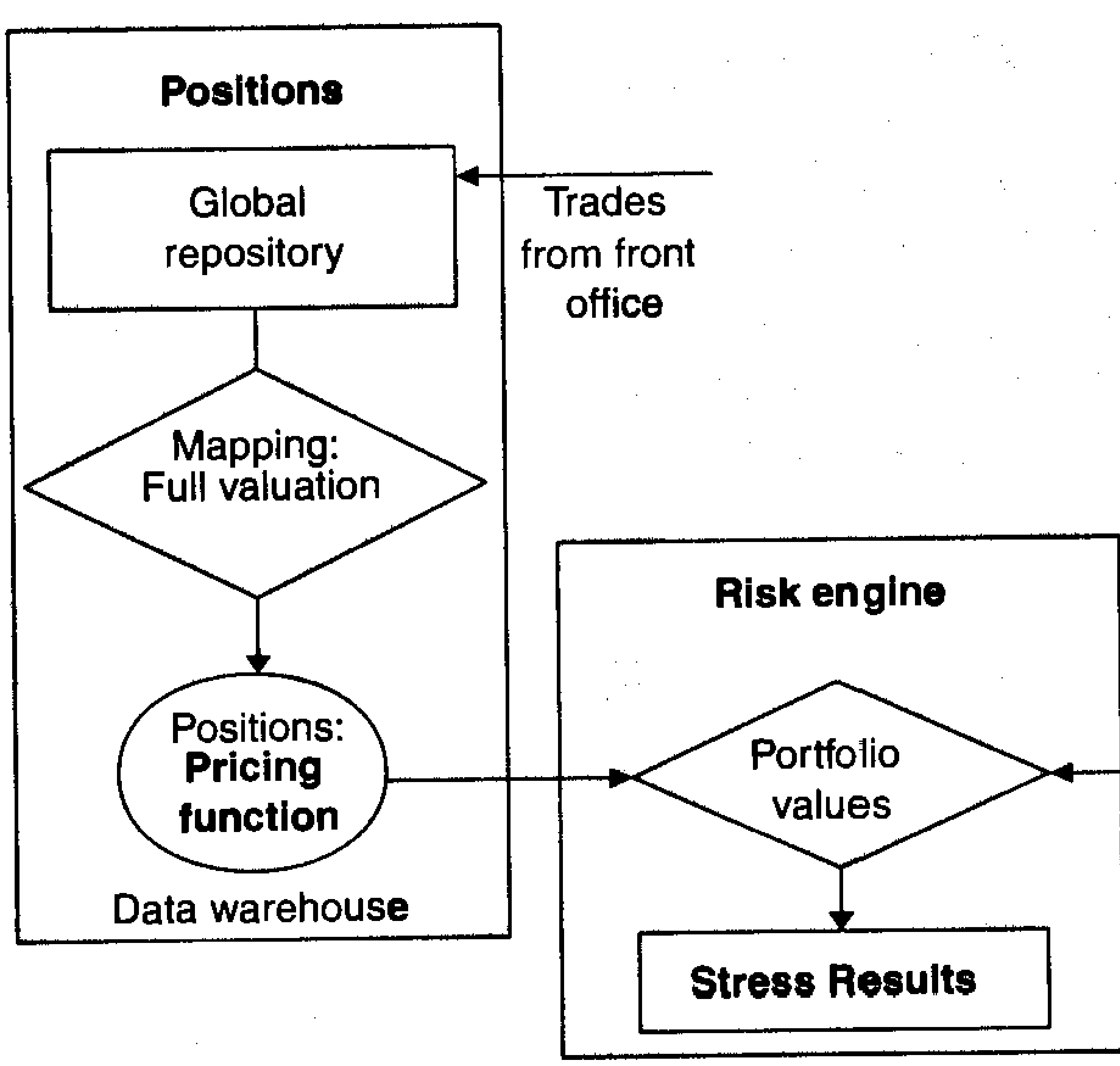
Note that this is a special case of the historical simulation method. Thus scenario analysis can be implemented easily once a VAR system is in place. Figure 14-2 details the steps involved in this approach. The question is how to generate realistic scenarios.

14.2.1 Portfolio- versus Event-Driven

The generation of scenarios can be either *event-driven* or *portfolio-driven*. In the first case, the scenario is formulated from plausible events that generate movements in the risk factors. In the second case, risk vulnerabilities in the current portfolio are identified first that translate into adverse movements in risk factors. These lead to the generation of scenarios. For instance, institutions that invest in long-term bonds funded by short-term debt are vulnerable to upward movements in the yield curve. It is therefore essential to consider scenarios that reflect such changes.

FIGURE 14-2

Scenario-analysis approach.



14.3 GENERATING UNIDIMENSIONAL SCENARIOS

14.3.1 Sensitivity Tests

The traditional approach to scenario analysis focuses on one variable at a time. For instance, the Derivatives Policy Group (DPG) provides specific guidelines for scenarios. It recommends focusing on a set of specific movements:

1. Parallel yield-curve shifting by ±100 basis points
2. Yield-curve twisting by ±25 basis points
3. Each of the four combinations or yield-curve shifts and twist
4. Implied volatilities changing by ±20 percent of current values
5. Equity index values changing by ±10 percent
6. Currencies moving by ±6 percent for major currencies and ±20 percent for others
7. Swap spreads changing by ±20 basis points

While these movements are quite large for a daily horizon, the DPG's goal was to provide comparable results across institutions in order to assess zones of vulnerabilities. By specifying consistent guidelines, it tried to ensure that all the models used by brokers "possess broadly similar performance."

These scenarios shock risk factors generally one at a time. The loss in value, scaled by the size of the factor movement, is a *sensitivity* measure. These tests can be run relatively quickly and are intuitive.

This approach is appropriate in situations where the portfolio depends primarily on one source of risk. The Office of Thrift Supervision (OTS), for instance, uses scenario analysis to assess the market risk of savings and loans associations (S&Ls).[1] The OTS requires institutions to estimate what would happen to their economic value under parallel shifts in the yield curve varying from −400 to +400 basis points. The OTS recently has imposed a risk-based capital requirement linked directly to the interest-rate exposure of supervised institutions.

14.3.2 An Example: The SPAN System

The standard portfolio analysis of risk (SPAN) system is a good example of a scenario-based method for measuring portfolio risk. SPAN was introduced in 1988 by the Chicago Mercantile Exchange (CME) to calculate collateral requirements on the basis of overall *portfolio risk* as opposed to position by position. Since its inception, SPAN had become widely used by futures and options exchanges as a mechanism to set margin requirements.

The objective of the SPAN system is to identify movements in portfolio values under a series of scenarios. SPAN then searches for the largest loss that a portfolio may suffer and sets the margin at that level. The SPAN system only aggregates futures and options on the same underlying instrument. It uses full-valuation methods, which is important given that portfolios usually include options.

Consider a portfolio of futures and options on futures involving the dollar/euro exchange rate. SPAN scans the portfolio value over a range of prices and volatilities. These ranges are selected so that they cover a fixed percentage of losses, for example, 99 percent. Contracts have a notional of 125,000 euros. Assume a current price of \$1.05/euro and a

[1] The OTS is a U.S. agency created in 1989 to supervise S&Ls.

12 percent annual volatility. The value range for the contract is set at the daily VAR, that is,

Price range = 2.33 × 12 percent $\sqrt{252}$) × (euro125,000 × 1.05\$/euro) = \$2310

This is indeed close to the daily margin for an outright futures position, which is around \$2500 for this contract. This corresponds to a price range of \$0.0176 around the current price of \$1.05 per euro. Next, the volatility range is set at 1 percent.

Table 14-1 presents an example of scenario generation. We select scenarios starting from the initial rate plus and minus three equal steps that cover the price range, as well as an up-and-down move for the volatility range. In addition, to provide protection for short positions in deep

TABLE 14-1

Example of SPAN Scenario System

	Scenario			Gain/Loss	
Number	Fraction Considered for P&L	Price Scan Expressed in Range	Volatility Scan Expressed in Range	Long Call	Long Futures
1	100%	0	1	\$198	\$0
2	100%	0	−1	−\$188	\$0
3	100%	+1/3	1	\$395	\$767
4	100%	+1/3	−1	−\$21	\$767
5	100%	−1/3	1	\$23	−\$767
6	100%	−1/3	−1	−\$332	−\$767
7	100%	+2/3	1	\$615	\$1,533
8	100%	+2/3	−1	\$170	\$1,533
9	100%	−2/3	1	−\$132	−\$1,533
10	100%	−2/3	−1	−\$455	−\$1,533
11	100%	+1	1	\$858	\$2,300
12	100%	+1	−1	\$388	\$2,300
13	100%	−1	1	−\$268	**−\$2,300**
14	100%	−1	−1	**−\$559**	**−\$2,300**
15	35%	+2	0	\$517	\$1,610
16	35%	−2	0	−\$240	−\$1,610
Ranges:		\$0.0176	1%		

Note: Euro-FX futures and option on futures with notional of 125,000 euros, spot of 1.05\$/euro, strike of \$1.10, 12 percent annual volatility, 90 days to maturity, and interest rate of 5 percent.

out-of-the-money options, two scenarios are added with extreme price movements, defined as double the maximum range. Because such price changes are rare, the margin required is 35 percent of the resulting loss.

Next, the value of each option and futures position is calculated under each scenario, using full valuation. The table presents calculations for two positions only, long one call and long one futures, under each of the 16 scenarios. The result of the computation for each risk scenario is called a *risk-array value*. The set of risk array values for the position is called the *risk array*.

The long-call position would suffer the most under scenario 14, with a large downward move in the futures accompanied by a drop in the volatility. Similarly, the worst loss for a long-futures position also occurs under a large downward move. This analysis is repeated for all options and futures in the portfolio and aggregated across all positions. Finally, the margin is set to the worst portfolio loss under all scenarios.

The SPAN system is a scenario-based approach with full valuation. Its systematic scanning approach is feasible because it considers only two risk factors. The number of combinations, however, soon would become unmanageable for a greater number of factors. This is perhaps the greatest hurdle to systematic scenario analysis.

Another drawback is that the approach essentially places the same probability on most of the scenarios, which ignores correlations between risk factors. And as we have seen, correlations are an essential component of portfolio risk.

14.4 MULTIDIMENSIONAL SCENARIO ANALYSIS

Unidimensional scenarios provide an intuitive understanding of the effect of movements in key variables. The problem is that these scenarios do not account for correlations. This is where multidimensional scenarios are so valuable. The approach consists of (1) positing a state of the world and (2) inferring movements in market variables.

14.4.1 Prospective Scenarios

Prospective scenarios represent *hypothetical* one-off surprises that are analyzed in terms of their repercussions on financial markets. One might want to examine, for instance, the effect of an earthquake in Tokyo, of

Korean reunification, of a war in an oil-producing region, or of a major sovereign default. The definition of scenarios should be done with input from top managers, who are most familiar with the firm's business and extreme events that may affect it.

Let us go back to the example of a scenario analysis for a potential breakup in the exchange-rate mechanism (ERM), evaluated as of summer 1992. The risk manager could hypothesize a 20 percent fall in the value of the Italian lira against the German mark. One could surmise further that if the Italian central bank let the lira float, short-term rates likewise could drop, and the stock market would rally. Beyond the effect on Italian interest rates and equity prices, however, it may not be obvious to come up with plausible movements for other financial variables. The problem is that the portfolio may have large exposures to these other risk factors that remain hidden. Thus this type of subjective scenario analysis is not well suited to large, complex portfolios.

Factor Push Method

Some implementations of stress testing try to account for multidimensionality using a rough two-step procedure. First, push up and down all risk-factor variables individually by, say, $\alpha = 2.33$ standard deviations, and then compute the changes to the portfolio. Second, evaluate a worst-case scenario, where all variables are pushed in the direction that creates the worst loss. For instance, variable 1 could be pushed up by $\alpha\sigma_1$, whereas variable 2 could be pushed down by $\alpha\sigma_2$, and so on.

This approach is very conservative but completely ignores correlations. If variables 1 and 2 are positively correlated, it makes little sense to consider moves in opposite directions. Further, looking at extreme movements may not be appropriate. Some positions such as combinations of long positions in options will lose the most money if the underlying variables do not move at all.

Conditional Scenario Method

There is a systematic method, however, to incorporate correlations across all variables consistently. Let us represent the "key" variables that are subject to some extreme movements as R^*. The other variables are simply represented by R. The usual approach to stress testing focuses solely on R^*, setting the other values to zero. Simplifying Equation (14.1) to a linear movement, what we call the *narrow stress loss* (NSL), is $\Sigma_i w_i^* R_i^*$.

To account for multidimensionality, we first regress the R variables on the controlled R^* variables, obtaining the conditional forecast from

$$R_j = \alpha_j + \sum_i \beta_{j,i} R_i^* + \epsilon_j = E(R_j \mid R^*) + \epsilon_j \tag{14.2}$$

This allows us to predict other variables conditional on movements in key variables using information in the covariance matrix. We construct a *predicted stress loss* (PSL) as $\Sigma_i w_i^* R_i^* + \Sigma_j w_j E[R_j | R^*]$. This can be compared with the realized *actual stress loss* (ASL), which is $\Sigma_i w_i^* R_i^* + \Sigma_j w_j R_j$.

Kupiec (1998) illustrates this method with episodes of large moves from 1993 to 1998 using a \$1 million portfolio invested in global equity, bond, and currency markets. Table 14-2 presents typical results. For the Philippine peso, for instance, the event was a devaluation, which was a 5.50 standard deviation move. The notional value of the position on this risk factor was \$40,700, which led to a narrow stress loss (NSL) of \$3070. This number, however, fails to account for other markets, such as Philippine equities, that moved in the opposite direction. Taking this correlation into account, the predicted stress loss (PSL) is much smaller than the NSL, even close to zero. In some other cases, PSL is much worse than NSL.

Interestingly, the table shows that, in all cases, the PSL produces results that are much closer to the ASL than in the simple, narrow model that zeroes out nonkey variables. The conclusion is that the covariance matrix, which is at the core of conditional normal VAR modeling, does provide useful information for stress-testing analysis.

TABLE 14-2

Comparison of Forecast Losses on a \$1 Million Portfolio

				Stress Loss		
Key Variable	**Period**	**Event Size (σ)**	**Position on Key Variable**	**Narrow**	**Predicted**	**Actual**
Philippine peso	11 Jul 1997	−5.50	\$40,700	−\$3,070	\$43	\$190
Japanese equities	23 Jan 1995	−5.23	\$72,120	−\$2,700	−\$7,730	−\$11,700
U.S.equities	27 Oct 1997	−4.93	\$136,480	−\$6,650	−\$5,330	−\$5,420
U.K.bonds	29 Dec 1994	−4.84	\$122,910	−\$2,640	−\$3,550	−\$3,030
U.S.bonds	20 Feb 1996	4.86	\$122,970	−\$1,210	−\$7,070	−\$10,380

The main drawback of this conditional scenario method is that it relies on correlations derived from the entire sample period. This includes normal periods and *hectic* periods. Should correlations change systematically across these periods, however, the results will differ. For portfolios with long positions only, increases in correlations will increase the worst loss. Banks often reexamine the stress-test results using correlations estimated over hectic periods.[2]

As an example, the correlation between stocks and bonds typically is positive in normal times. In times of stress, however, this correlation often turns negative. When equity markets drop sharply, the demand for Treasury bonds typically increases, reflecting a flight to quality. At the short end of the yield curve, this effect usually is reinforced when the central bank injects liquidity into the financial system, pushing down short-term rates. Thus government bonds are good diversifiers for stocks in times of stress.

A related, useful approach relies on the output from a VAR Monte Carlo analysis or historical simulation. The risk manager could examine the worst losses from the simulation, which specifically accounts for correlations. Such analysis provides valuable insight into the vulnerabilities of a particular portfolio and could guide the construction of scenarios.

14.4.2 Historical Scenarios

Alternatively, scenario analysis can examine historical data to provide examples of joint movements in financial variables. The role of the risk manager is to identify scenarios, such as those listed in Table 14-3, that may be outside the VAR window. Each of these scenarios then will yield a set of joint movements in financial variables that automatically takes correlations into account.

Table 14-3 displays a list of scenarios, both historical and prospective, used by a large group of banks. The largest category of stress tests focuses on interest rates. Historical scenarios include the 1994 bond market crash, the 1997 Asian currency crisis, the LTCM and Russia crises, and the terrorist attack on the World Trade Center, all of which led to global interest-rate shocks. Also common are stress tests involving equities, currencies, commodities, and credit.

[2] See, for instance, Kim and Finger (2000). Using volatility measures to sort the sample, however, creates potential biases in correlation estimates, as explained in Boyer et al. (1999). Another issue is that because hectic periods cover, by definition, fewer observations than the entire sample, correlations are estimated less precisely.