PageRank

- An Hyperlink that point to me works like a *recommendation* from other page.
- With more recommendation, more **important** is my page.
- The more important are my recommenders, the more **important** is my page.

Rank

A **numeric value** to represent the importance of a page.

We aren't inspecting the content. We are looking for recommendations

PageRank

- We can view the importance of a web page i as the probability that a random surfer on the Internet opens a browser to any page and starts following **hyperlinks** visits i.
- Weights on edges in a transition matrix could be assigned in a *probabilistic way*.
- We can model the process as a **random walk** on graphs. Each page has equal probability $\frac{1}{n}$ to be chosen as a starting point.
- Then, probability that page i is visited after one step is equal to Ax and so on.
- The probability that page i will be visited after k steps is equal to $A^k x$.
- That sequence converges to a **unique probabilistic vector** v^* called the *stationary distribution*.
- **This will be our PageRank**.

PageRank

- $\Pi = (r_i)$ The vector of Rank number, the rank r_i for page p_i
- **Each page that point to me, add a fraction of its own rank for my total rank**
- That's means it is a **linear combination**: $r_i = \sum_k T_{ik} r_{ik}$
- $T = (t_{ik})$ The transition matrix; and it is stochastic: $\sum_k t_{ik} = 1$ (the total rank is conserved).
- **What can we do with this?**

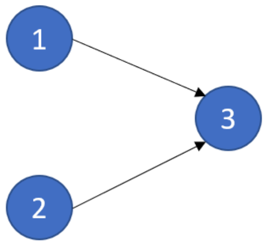
PageRank

Perron-Frobenius Theorem

- Is about the **eigenvectors** and **eigenvalues** of *non-negative and irreducible matrices*.
- A matrix is called *non-negative* if all of its entries are ≥ 0.
- A matrix is called *irreducible* if for any of its entries (i, j) there is k such that the (i, j) entry of A^k is **positive**.
- It says that for A a non-negative and irreducible matrix, it is an *eigenvalue* $\lambda_{\max}$ and for all other eigenvalues λ we have $|\lambda| \leq \lambda_{\max}$.
- Moreover, if the sum of all entries of each column of A is 1, then $\lambda_{\max} = 1$.

But, there is a little problem: **the matrices we obtain from the Web are not always irreducible**

There could be: **nodes with no outgoing edges**, also called **dangling nodes**.



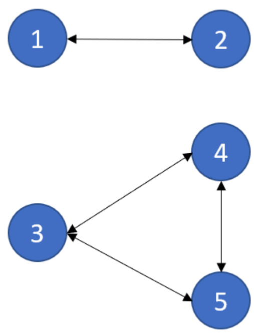
$$v_0 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

PageRank

There could be: **disconnected components**.



$$A = \left[\begin{array}{cc|ccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right]$$

$$v = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Both eigenvectors with eigenvalue = 1

PageRank

- They propose a fix positive constant d between 0 and 1 called the **dampling factor** (with a typical value $d = 0.85$).
- This models the random surfer behaviour!
- Now, the *transition matrix* used to compute PageRank will be a new matrix M such that:

$$M = dT + \frac{(1-d)}{n} E$$

  with E as a matrix of ones.
- M is called the **Google matrix**.

PageRank

The assumption is the **importance of a page** is given for the **importance of the pages that pointed it**.

$$r_p^{(k+1)} = (1-d)\frac{1}{n} r_p^{(k)} + d \sum_{\forall q, p \in P \mid q \to p} \frac{1}{N_q} r_q^{(k)}$$

$$r_p^{(k+1)} = (1-d)\frac{1}{n}r_p^{(k)} + d \sum_{\forall q, p \in P \mid q \to p} \frac{1}{N_q} r_q^{(k)}$$

Where:
- r_p: Importance of page p
- n: Number of pages in the web graph
- d: Probability that the surfer follows some out-links of q when visit that page
- N_q: Number of out-links from page q
- r_q: Importance of page q
- $\frac{1}{N_q}$: Conditional probability of going to another page

The PageRank is the fixed point value of this recurrence!

PageRank

In matrix form:

$$\Pi^{(k+1)} = \left(dT + \left(\frac{1-d}{n}\right)E\right)\Pi^{(k)}$$

- d: The probability of going out the n node
- T: An transition matrix (stochastic) that is interpreted as the transition probability. But in the Google way they consider **equiprobability** $\frac{1}{N_q}$
- E: Is the *1 matrix*. A matrix filled with 1.

Basically, replace the usual transition matrix with the Google matrix and compute the eigenvector with eigenvalue equal to 1

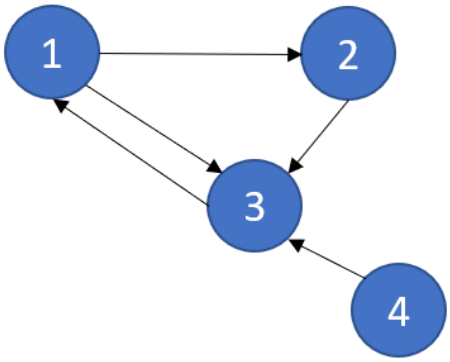
PageRank

Intuitive Justification

- Thought as a **model of user behavior**.
- **Random surfer** that keep clicking on links **starting from one random page**, and **some time get bored** and **continue** from other random page.
- **PageRank** value correspond to the **probability that the random surfer visit the page**.
  - The **stationary probability.**
- Then the **d parameter** correspond to the **probability to start again from another page**.

PageRank

1. Initialize: $\Pi^{(0)} = \left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$
2. Set d, usually $d = 0.85$
3. Calculate: $\Pi^{(k+1)} = \left(dT + \frac{(1-d)}{n}E\right)\Pi^{(k)}$
4. If $\left\|\Pi^{(k+1)} - \Pi^{(k)}\right\| < \xi$ stop and return. Else, $\Pi^{(k)} = \Pi^{(k+1)}$ and go to the point 3.



Considering the graph above, get its PageRank

PageRank

Disadvantages

- If a page is pointed by another one, it means that the page receives a vote for the PageRank calculus.
- If a page is pointed by a lot of pages, it means that the page is important.
- **Only the good pages are pointed by others one**, but:
  - **Reciprocal link**: If the page A links page B, then page B will link page A.
  - **Link Requirements**: Some web pages give electronic gifts, like programs, documents etc., if another page points it.
  - **Near persons community**: For instance, friends and relatives that from their pages point another friend or relative only because of the human relationship between them.

PageRank

- **Not the real Google algorithm.** It is a very carefully hidden secret.
- The original PageRank doesn't consider text content (keywords) on the page. But the real one **does**. The real algorithm also considers **n-grams**
- The real algorithm also considers user behavior. They capture it with:
  - Click on links
  - Google toolbar
  - Google web-accelerator (a Google proxy)
  - Gmail and Youtube

PageRank

- Could a group of people artificially reference pages between them in order to increase the rank?
- A **parallel algorithm search for spammer** and lowers its rank.
- The sensibility from a spamming update is:

$$\left\|\Pi - \tilde{\Pi}\right\|_1 < \frac{2d}{1-d} \sum_{i \in U} r_i$$

  - U: A community of spammers
  - Π: Change on each $i \in U$, r_i are the original rank

HITS

HITS Algorithm

- Proposed by John Kleinberg in 1998.
- Stands for Hypertext Induced Topic Search.
- Expand the list of relevant pages returned by a search engine.
- Produce two rankings: Authority ranking and Hub ranking.

Assumptions

- A credible page will point to credible pages.
- Credible pages are pointed by others.

The page ranking depends on the user query and the hyperlink structure that follows from paths of the most credible pages

HITS Algorithm

Definitions

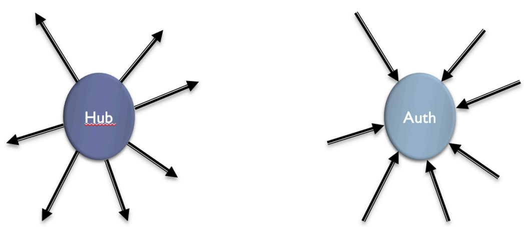
- Authority: A page with many in-links.
- Hub: A page with many out-links.

The idea is that:

- A page may have authoritative content on some topic and thus many people trust it and thus link to it.
- When a user comes to a hub page, he/she will find many useful links wich take him/her to good content pages on the topic.

HITS Algorithm

- If q it is good hub, then q point to many good authority pages p.
- If p it is a good authority, then p is pointed by many good hub pages q.
- Authorities and Hubs have a **mutual reinforcement** relationship.
- We can give a measure of the quality of *goodness* for authority and hubs. We call it: Authority and Hub weight (a_p, h_q)



HITS Algorithm

Assumptions

- The authority level (or rank) came from in-edges.

authorities.

A **simple method** to differentiate the page's relevance is:

- First **assigning non-negative weights**, depending if the page is hub or authoritative. Well, finally the page have both of them.
- Next, the weights are **adjusted by an iterative process** and the **relative page's importance** in the community is calculated.

HITS Algorithm

Given a query q, HITS collects a set of pages as follows:

1. It send the query q to a search engine system and collects t highest ranked pages, which assume to be highly relevant to the search query. This is called the **root** set W.
2. It then grows W by including any page pointed to by a page in W and any page that points to a page in W. It restrict its size by allowing each page in W to bring at most k pages. This set is called the **base set** S.

HITS Algorithm

Assuming we have a set S of pages connected where:

- V is the set of pages (or nodes).
- E is the set of directed edges (or links).
- Then $G = (V, E)$
- We use L to denote the adjacency matrix of the graph, where:

$$L = \begin{cases} 1 & \text{if} \quad (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$

HITS Algorithm

- Let the authority score of the page i be $a(i)$, and the hub score of the page i be $h(i)$.
- The mutual reinforcing relationship of the two scores is represented as:

$$a(i) = \sum_{(j,i) \in E} h(j)$$

The authority score of the page i correspond to the sum of all the hubs that are point to it.

$$h(i) = \sum_{(i,j) \in E} a(j)$$

The hub score of the page i correspond to the sum of all the authorities it points to.

HITS Algorithm

- Then, we use $\boldsymbol{a}$ to denote the column vector with all the authority scores and $\boldsymbol{h}$ to denote the column vector with all the hub scores.
- This is equivalent to say:

$$a = L^T h$$

$$h = La$$

are:

$$a_k = L^T L a_{k-1} \iff a_{k+1} = L^T L a_k$$

$$h_k = L L^T h_{k-a} \iff h_{k+1} = L L^T h_k$$

HITS Algorithm

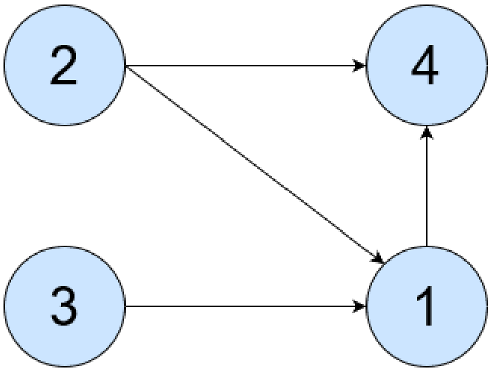
1. Initialize $a = (1, ..., 1)$ and $h = (1, ..., 1)$
2. Calculate:

$$a_{k+1} = L^T L a_k$$

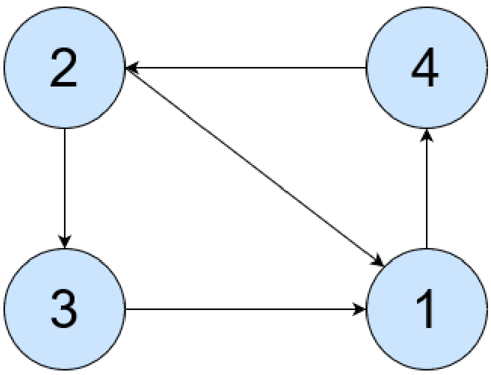
$$h_{k+1} = L L^T h_k$$

3. Normalize a_k and h_k
4. Repeat until: $\left\|a_{k+1} - a_k\right\|_1 < \varepsilon_a \wedge \left\|h_{k+1} - h_k\right\|_1 < \varepsilon_h$
5. Get the most auth pages and the most hub pages.

HITS Algorithm



HITS Algorithm



HITS Algorithm

By construction of HITS:

ignored in the page's rank task.
- Being the algorithm purely **hyperlink-based computation**.

CLEVER Project (Chakrabarti S.)

- Addresses the problem by considering query's terms in the calculus of the above equations.
- A non-negative weight, whose initial value is basis on the text that surround the hyperlink expression (a tag in HTML)

HITS Algorithm

Advantages

- Double ranking (by Authority and by Hub).
- Rank pages according to a query topic.

Disadvantages

- Doesn't have the anti-spam capability of PageRank.
- Topic drift.
- Query-dependence.
- Query time evaluation.

SALSA

SALSA

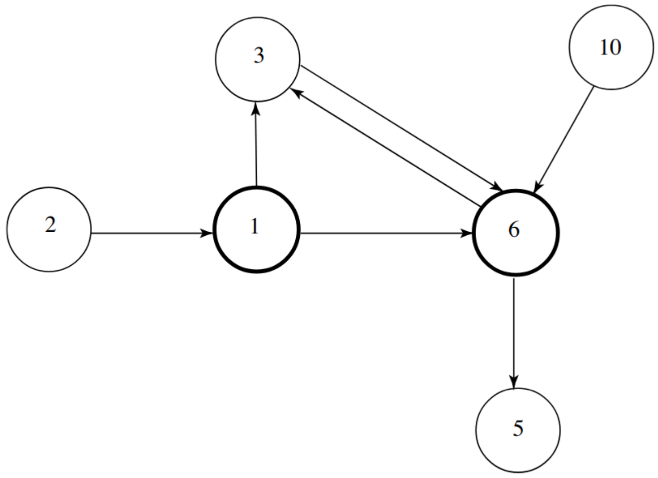
Source

"Google's PageRank and Beyond: The Science of Search Engine Rankings". Amy N. Langville and Carl D. Meyer

- Proposed by R. Lempel and S. Moran in 2001.
- SALSA stands for Stochastic Approach to Link Structure Analysis.
- Like HITS, SALSA create both Authority and Hub scores for webpages.
- SALSA creates a neighborhood graph showing the closeness between Authority pages and Hub pages.

Other Algorithms

- Rather than forming and adjacency matrix L for the neighborhood graph N, a bipartite unidirected graph, denoted G, is built.
- G is defined by the sets:
  - V_h: Set of Hub nodes (all nodes in N with outdegree > 0).
  - V_a: Set of Authority nodes (all nodes in N with indegree > 0).
  - E: Set of directed edges in N.
- Note that a node in N may be in both V_h and V_a.

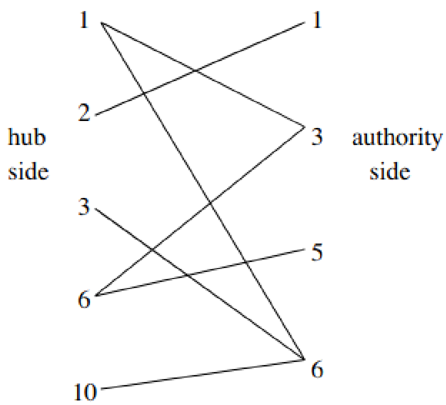


Other Algorithms

- From the previous graph, we have that:

$$V_h = \{1, 2, 3, 6, 10\} \qquad V_a = \{1, 3, 5, 6\}$$

- Then, we can create the bipartite undirected graph G, as shown.
- Every directed edge in E is represented by an unidirectedted edge in G.



Other Algorithms

- Matrices H and A can be derived from the adjacency matrix L used in the HITS and PageRank methods.
- HITS used unweighted matrix L.
- PageRank uses a row weighted version of matrix L.
- SALSA uses both row and column weighting.

Other Algorithms

To get A and H matrices, we have to calculate normalize versions of L:

- Let L_r be L with each nonzero row divided by its row sum.
- Let L_c be L with each nonzero column divided by its column sum.

In the previous example, it will be:

$$L = \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 10 \end{array} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad L_r = \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 10 \end{array} \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad L_c = \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 10 \end{array} \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Other Algorithms

- H: SALSA's hub matrix, consists of the nonzero rows and columns of $L_r L_c^T$
- A: SALSA's authority matrix, consists of the nonzero rows and columns of $L_c^T L_r$

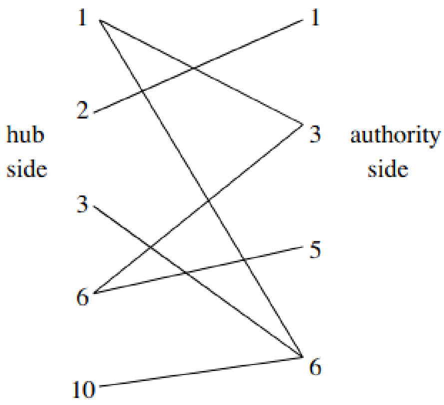
$$L_r L_c^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 5 & 6 & 10 \end{array} \\ \begin{pmatrix} \frac{5}{12} & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \end{array} \qquad L_c^T L_r = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 5 & 6 & 10 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Other Algorithms

$$H = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 6 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 6 & 10 \end{array} \\ \begin{pmatrix} \frac{5}{12} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{2}{12} \\ 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \end{array} \qquad A = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} \end{pmatrix} \end{array}$$

Other Algorithms

- If the bipartite graph G is *connected*, then A and H are both irreductible Markov chains and π_h^T, the stationary vector of H, gives the hub scores for the query with neighborhood graph N, and π_a^T gives the authority scores.
- If G is not *connected*, then A and H contain multiple irreducible components. In this case, the global authority and hub scores must be pasted together from the stationary vectors for each individual irreducible component.



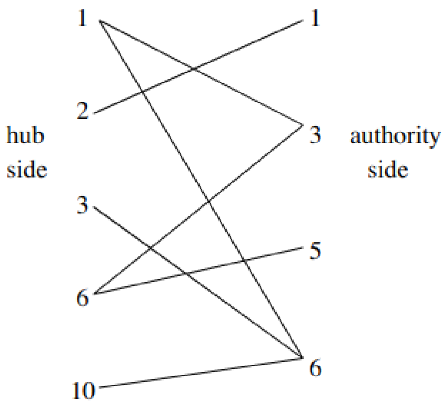
- Because G is **not connected**, A and H contain multiple connected components.
- H contains two connected components:

$$C = \{2\} \qquad D = \{1, 3, 6, 10\}$$

- A contains two connected components:

$$E = \{1\} \qquad F = \{3, 5, 6\}$$

Other Algorithms



- The stationary vectors for the two irreducible components of H are:

$$\pi_h^{T(C)} = (1) \qquad \pi_h^{T(D)} = \left(\tfrac{1}{3}\ \tfrac{1}{6}\ \tfrac{1}{3}\ \tfrac{1}{6}\right)$$

- The stationary vectors for the two irreducible components of H are:

$$\pi_a^{T(E)} = (1) \qquad \pi_a^{T(F)} = \left(\tfrac{1}{3}\ \tfrac{1}{6}\ \tfrac{1}{2}\right)$$

Other Algorithms

- Now, we can join the two components together for each matrix.
- We must multiply each entry in the vector by its appropiate weight.

Other Algorithms

- Thus the global Hub vector is:

$$\pi_h^T = \left(\tfrac{4}{5} \times \tfrac{1}{3}\ \ \tfrac{1}{5} \times 1\ \ \tfrac{4}{5} \times \tfrac{1}{6}\ \ \tfrac{4}{5} \times \tfrac{1}{3}\ \ \tfrac{4}{5} \times \tfrac{1}{6}\right)$$

$$\pi_h^T = (0.2667\ \ 0.2\ \ 0.1333\ \ 0.2667\ \ 0.1333)$$

Other Algorithms

- And the global Authority vector is:

$$\pi_a^T = \left(\tfrac{1}{4} \times 1\ \ \tfrac{3}{4} \times \tfrac{1}{3}\ \ \tfrac{3}{4} \times \tfrac{1}{6}\ \ \tfrac{3}{4} \times \tfrac{1}{2}\right)$$

$$\pi_a^T = (0.25\ \ 0.25\ \ 0.125\ \ 0.375)$$

Advantages and Disadvantages of SALSA

Advantages

- Not affected as much my topic drift like HITS.
- Less affected susceptible to spamming.
- Dual rank (Authority and Hubs).

Disadvantages

- Query-dependence.
- Query time evaluation.