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Bookmark

Bayes' Rule for Random Variables (Also Called Bayes' Theorem for Random Variables)

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Video
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These notes cover roughly the same content as the video:

BAYES' THEOREM FOR RANDOM VARIABLES (COURSE NOTES)

In inference, what we want to reason about is some unknown random variable X , where we get to observe some other random variable Y , and we have some model for how X and Y relate. Specifically, suppose that we have some “prior” distribution p_X for X ; this prior distribution encodes what we believe to be likely or unlikely values that X takes on, *before we actually have any observations*. We also suppose we have a “likelihood” distribution $p_{Y|X}$.

After observing that Y takes on a specific value y , our “belief” of what X given $Y = y$ is now given by what's called the “posterior” distribution $p_{X|Y}(\cdot | y)$. Put another way, we keep track of a probability distribution that tells us how plausible we think different values X can take on are. When we observe data Y that can help us reason about X , we

proceed to either upweight or downweight how plausible we think different values \mathbf{X} can take on are, making sure that we end up with a probability distribution giving us our updated belief of what \mathbf{X} can be.

Thus, once we have observed $\mathbf{Y} = \mathbf{y}$, our belief of what \mathbf{X} is changes from the prior $p_{\mathbf{X}}$ to the posterior $p_{\mathbf{X}|\mathbf{Y}}(\cdot | \mathbf{y})$.

Bayes' theorem (also called Bayes' rule or Bayes' law) for random variables explicitly tells us how to compute the posterior distribution $p_{\mathbf{X}|\mathbf{Y}}(\cdot | \mathbf{y})$, i.e., how to weight each possible value that random variable \mathbf{X} can take on, once we've observed $\mathbf{Y} = \mathbf{y}$. Bayes' theorem is the main workhorse of numerous inference algorithms and will show up many times throughout the course.

Bayes' theorem: Suppose that \mathbf{y} is a value that random variable \mathbf{Y} can take on, and $p_{\mathbf{Y}}(\mathbf{y}) > 0$. Then

$$p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y}) = \frac{p_{\mathbf{X}}(\mathbf{x})p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x})}{\sum_{\mathbf{x}'} p_{\mathbf{X}}(\mathbf{x}')p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x}')}$$

for all values \mathbf{x} that random variable \mathbf{X} can take on.

Important: Remember that $p_{\mathbf{Y}|\mathbf{X}}(\cdot | \mathbf{x})$ could be undefined but this isn't an issue since this happens precisely when $p_{\mathbf{X}}(\mathbf{x}) = 0$, and we know that $p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) = 0$ (for every \mathbf{y}) whenever $p_{\mathbf{X}}(\mathbf{x}) = 0$.

Proof: We have

$$p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y}) \stackrel{(a)}{=} \frac{p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y})}{p_{\mathbf{Y}}(\mathbf{y})} \stackrel{(b)}{=} \frac{p_{\mathbf{X}}(\mathbf{x})p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x})}{p_{\mathbf{Y}}(\mathbf{y})} \stackrel{(c)}{=} \frac{p_{\mathbf{X}}(\mathbf{x})p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x})}{\sum_{\mathbf{x}'} p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}', \mathbf{y})} \stackrel{(d)}{=} \frac{p_{\mathbf{X}}(\mathbf{x})p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x})}{\sum_{\mathbf{x}'} p_{\mathbf{X}}(\mathbf{x}')p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x}')}.$$

where step (a) uses the definition of conditional probability (this step requires $p_{\mathbf{Y}}(\mathbf{y}) > 0$), step (b) uses the product rule (recall that for notational convenience we're not separately writing out the case when $p_{\mathbf{X}}(\mathbf{x}) = 0$), step (c) uses the formula for marginalization, and step (d) uses the product rule (again, for notational convenience, we're not separately writing out the case when $p_{\mathbf{X}}(\mathbf{x}') = 0$). \square

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