



Bookmarks



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▼ 1. Probability and Inference

Introduction to Probability (Week 1)

Exercises due Sep 21, 2016 at 21:00 UTC

Probability Spaces and Events (Week 1)

Exercises due Sep 21, 2016 at 21:00 UTC

Random Variables (Week 1)

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Jointly Distributed Random Variables (Week 2)

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Conditioning on Events (Week 2)

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Homework 1 (Week 2)

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Inference with Bayes' Theorem for Random Variables (Week 3)

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Independence Structure (Week 3)

Exercises due Oct 05, 2016 at 21:00 UTC

Homework 2 (Week 3)

1. Probability and Inference > Inference with Bayes' Theorem for Random Variables (Week 3) > Bayes' Theorem for Random Variables: A Computational View

BAYES' THEOREM FOR RANDOM VARIABLES: A COMPUTATIONAL VIEW

Computationally, Bayes' theorem can be thought of as a two-step procedure. Once we have observed $Y = y$:

1. For each value x that random variable X can take on, initially we believed that $X = x$ with a score of $p_X(x)$, which could be thought of as how plausible we thought ahead of time that $X = x$. However now that we have observed $Y = y$, we weight the score $p_X(x)$ by a factor $p_{Y|X}(y | x)$, so

$$\text{new belief for how plausible } X = x \text{ is: } \alpha(x | y) \triangleq p_X(x)p_{Y|X}(y | x), \quad \left(\begin{array}{l} 2 \\ \cdot \\ 4 \end{array} \right)$$

where we have defined a new table $\alpha(\cdot | y)$ which is *not* a probability table, since when we put in the weights, the new beliefs are no longer guaranteed to sum to 1 (i.e., $\sum_x \alpha(x | y)$ might not equal 1)! $\alpha(\cdot | y)$ is an *unnormalized* posterior distribution!

Also, if $p_X(x)$ is already 0, then as we already mentioned a few times, $p_{Y|X}(y | x)$ is undefined, but this case isn't a problem: no weighting is needed since an impossible outcome stays impossible.

To make things concrete, here is an example from the medical diagnosis problem where we observe $Y = \text{positive}$:

Homework due Oct 05,
2016 at 21:00 UTC

Notation Summary (Up Through Week 3)

Mini-project 1:

Movie

Recommendations (Week 3)

Mini-projects due Oct
12, 2016 at 21:00 UTC

p_X		$p_{Y X}$		X	
healthy infected				healthy infected	
0.999	0.001	positive	0.01	0.99	
		negative	0.99	0.01	

entry-wise multiply to get
unnormalized posterior

healthy infected	
0.00999	0.00099

2. We fix the fact that the unnormalized posterior table $\alpha(\cdot | y)$ isn't guaranteed to sum to 1 by renormalizing:

$$p_{X|Y}(x | y) = \frac{\alpha(x | y)}{\sum_{x'} \alpha(x' | y)} = \frac{p_X(x)p_{Y|X}(y | x)}{\sum_{x'} p_X(x')p_{Y|X}(y | x')}.$$

An important note: Some times we won't actually care about doing this second renormalization step because we will only be interested in what value that X takes on is more plausible relative to others; while we could always do the renormalization, if we just want to see which value of x yields the highest entry in the unnormalized table $\alpha(\cdot | y)$, we could find this value of x without renormalizing!

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