Approximation of Optimum Thrust Nozzle Contour

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FOR A prescribed length and area ratio the nozzle contour producing maximum thrust can be uniquely determined (1,2).[2] The thrust performance of such optimum contoured nozzles, for various area ratios and lengths, was discussed in (3). Even though such nozzle contour is unique, determination of it is by the method of characteristics and requires the use of high speed computing machines (1,2). Besides the nozzle thrust, a design engineer would like to know its weight and cooling requirements to enable him to make the proper choice. For this purpose an approximation to the optimum thrust contour is enough, and a simple geometric method is described here.

The first step in defining a nozzle contour is the description of the throat region. After a certain amount of initial expansion along the prescribed throat wall, the contour is so formed as to turn the expanding gases to a near axial direction. For the family of nozzle contours considered here and in (3), the throat region is assumed given by two circular arcs as shown in Fig. 1. With these initial throat conditions, let us consider a nozzle having a certain area ratio A_e/A_t and length ratio L/R_t. The nozzle contour optimized for thrust will have unique values for θ_M, the wall slope at the inflection point M, and θ_E, the wall slope at the end point E. These values of θ_M and θ_E can be evaluated directly (3) prior to computing the contour itself, and are shown in a parametric form in Fig. 2. For the sake of clarity, even values for θ_M and odd values for θ_E are indicated in the figure. The nozzle end point given by (R_E/R_t, L/R_t) and the respective values of θ_M and θ_E are sufficient to describe a parabola. In Fig. 1 is shown a simple geometric construction for the parabolic contour. Through the nozzle end point E, line EQ is drawn inclined at an angle θ_E to the nozzle axis. Line MQ is drawn inclined at an angle θ_M to the nozzle axis and tangent to the prescribed wall profile in the throat region. The line segments MQ and QE are then divided into an equal number of parts by points a, b, c and e, f, g, respectively. The required parabola is the envelope of lines joining a to e, b to f and c to g. If necessary, further subdivision of the line segments MQ and QE can be made to obtain closer tangent points. The parabola thus constructed yields a close approximation to the contour computed by the method of characteristics. For example, in the case of a nozzle of area ratio 25:1 and length ratio 12:1, the parabolic approximation does not differ from the computed contour (3) by more than 3 per cent in radial dimension. The maximum deviation in this example occurs at one third of the distance from throat to the exit.

The values of θ_M and θ_E shown in Fig. 2 are the results of calculations using $\gamma = 1.23$ and throat conditions such as shown in Fig. 1. A different value of γ does not appear to appreciably change the nozzle contour when the area ratio and length are prescribed (3). Similarly, one can expect that minor changes in the throat region would not cause large changes in the contour. Hence, the values shown in Fig. 2 and the parabolic contours can be used in general to approximately describe the optimum thrust nozzle contours.

Received Jan. 4, 1959

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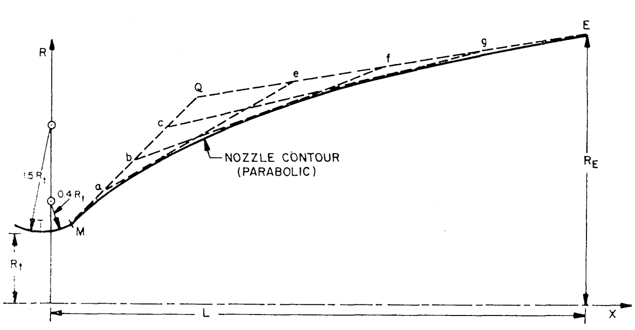

Fig. 1 Sketch of a nozzle showing nomenclature and construction of the parabola

References

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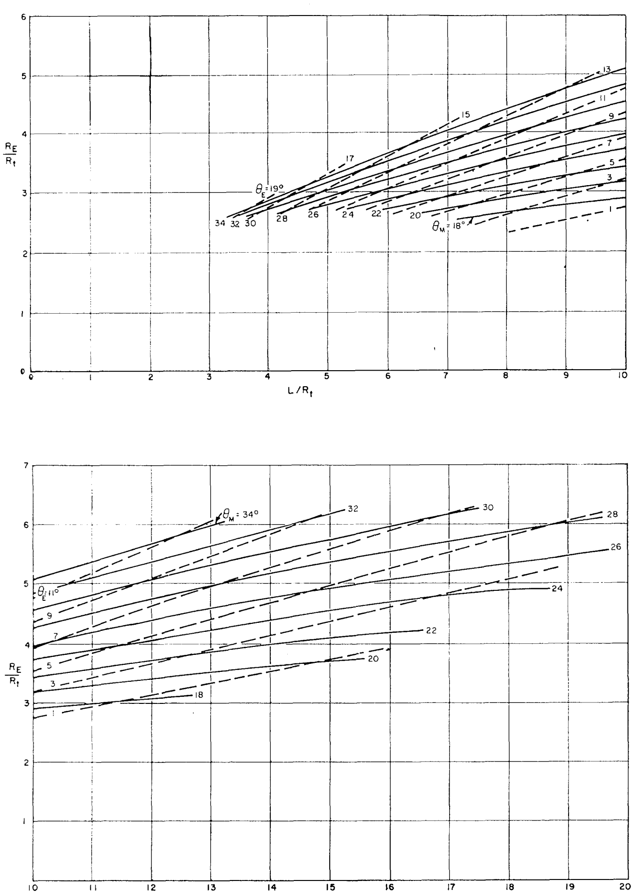

Fig. 2 Parametric values of θ_M and θ_E corresponding to various exit radius ratios and nozzle length ratios