DOC 221 Dinámica orbital y control de actitud

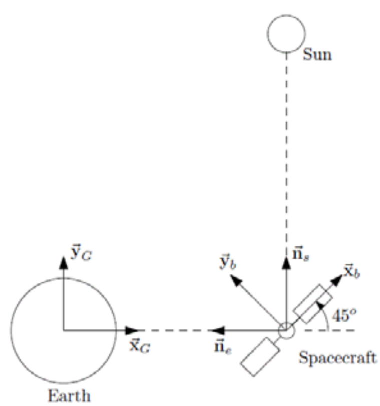
Problems Lecture ADCS - VII

Problem 1:

A spacecraft is orbiting the Earth as shown in figure below. As shown in the figure, at this particular location in the orbit, the Earth-pointing and Sun-pointing vectors are given in the ECI frame as

$$\vec{\mathbf{n}}_e = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad , \quad \vec{\mathbf{n}}_s = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Also, as shown in figure below, the spacecraft attitude is obtained by rotating of 45° about vector $\mathbf{z}_G$. The ECI coordinate have sub-indices G and the body coordinates sub-indices b.



(a) Determine rotation matrix $\mathbf{C}_{bG}$ from body coordinates to ECI coordinates.

(b) Determine the coordinates of Earth and Sun vectors $\mathbf{n}_e$ and $\mathbf{n}_s$ respectively, in the spacecraft body frame.

(c) As we learned in the TRIAD method, construct the body frame triad with vectors $\mathbf{t}_{1b}$, $\mathbf{t}_{2b}$ and $\mathbf{t}_{3b}$ with the spacecraft coordinates of the unit vectors. Construct the reference frame triad with vectors $\mathbf{t}_{1i}$, $\mathbf{t}_{2i}$ and $\mathbf{t}_{3i}$ with the ECI coordinates of the unit vectors.

(d) Obtain the rotation matrices $[\mathbf{t}_{1b}, \mathbf{t}_{2b}, \mathbf{t}_{3b}]$ and $[\mathbf{t}_{1i}, \mathbf{t}_{2i}, \mathbf{t}_{3i}]$.

(e) Using your solution to part (c) above, compute rotation matric $\mathbf{C}_{bG}$ using TRIAD method. Compare this with the result in part (a).

(f) Using the measured vectors obtained in part (b), compute $\mathbf{C}_{bG}$ using the q-method and QUEST method. Verify that you obtain the same result as in part (d). Note that it should be exactly the same, since no measurement noise has been added.

Problem 2:

Suppose a spacecraft has two attitude sensors that provide the following measurements of the two normalized vectors $\mathbf{v}_1$ and $\mathbf{v}_2$

$$\mathbf{v}_{1b} = \begin{bmatrix} 0.8273 & 0.5541 & -0.0920 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{v}_{2b} = \begin{bmatrix} -0.8285 & 0.5522 & -0.0955 \end{bmatrix}^{\mathrm{T}}$$

These vectors have known inertial frame components of:

$$\mathbf{v}_{1i} = \begin{bmatrix} -0.1517 & -0.9669 & 0.2050 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{v}_{2i} = \begin{bmatrix} -0.8393 & 0.4494 & -0.3044 \end{bmatrix}^{\mathrm{T}}$$

Apply the TRIAD algorithm and find the rotation matrix $\mathbf{C}_{bi}$.

Problem 3:

This problem is related to Lecture ADCS-VI. The initial yaw, pitch, and roll angles (of a 3-2-1 Euler sequence) of a vehicle are $(\Theta_1, \Theta_2, \Theta_3) = (80^\circ, 30^\circ, 40^\circ)$ at time t_0. Assume that the angular velocity vector of the vehicle is given in body frame components as

$$\boldsymbol{\omega} = \begin{bmatrix} \sin(0.1t) \\ 0.01 \\ \cos(0.1t) \end{bmatrix} 5^o / s \,,$$

where the time t is assumed to be given in units of seconds. Write a program to numerically integrate the yaw, pitch, and roll angles over a simulation time of 1 minute. Note that you must integrate these equations using radians as the angle units. However, plot the yaw, pitch and roll angles time histories and show the results in terms of degrees.

Problem 4:

This problem is related to Lecture ADCS-VI. The initial yaw, pitch, and roll angle (of a 3-2-1 Euler sequence) of a vehicle are $(\Theta_1, \Theta_2, \Theta_3) = (80^o, 30^o, 40^o)$. Assume that the angular velocity vector of the vehicle is given in body frame components as

$$\boldsymbol{\omega} = \begin{bmatrix} \sin(0.1t) \\ 0.01 \\ \cos(0.1t) \end{bmatrix} 50^o / s$$

where the time t is assumed to be given in units of seconds.

- **(a)** Translate this initial attitude description into the corresponding quaternion (or Euler parameters).
- **(b)** Write a program to numerically integrate the quaternion (or Euler parameters) over a simulation time of 1 minute. Note that you must integrate these equations using radians as the angle units. Plot the quaternion (or four Euler parameters) time histories.
- **(c)** Given the result of the numerical integration, plot the quaternion (or Euler parameters) constraint $|\mathbf{q}|=1$ and comment on this constraint.