ADCS - V
Space Mission

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Summary of last lecture

Objective and description of attitude

Hardware

- Sensors (determination)

  Sun-, Earth- and Star-Sensor, Magnetometers, Gyroscope
- Actuators (control)

  Thruster, Reaction wheel, Momentum wheel,
  Control momentum gyroscope, Magnetic torque

Attitude control methods

- Passive and active

Introduction to attitude concepts

- Conservation of angular momentum, Inertia matrix
- Attitude kinematics and dynamics → Euler equation

Environmental torques

- Aerodynamic, Gravity-gradient, Magnetic, Solar pressure

Outline

Mission analysis for attitude dynamics and control

Preliminary design of attitude determination and control system (ADCS) for given satellite

Steps of ADCS design process

Bibliography

- James R. Wertz and Wiley Larson (1999)
  "*Space mission analysis and design*"
  Chapter 11: Attitude determination and control

- F. N. Medina (~2006)
  Proyecto fin de carrera
  "Satélite de recursos naturales Zahori"

Introduction

Function of ADCS

Hold orientation of satellite in desired direction during mission despite possible external torques act on it

Reorientation (slew maneuvers) according to requirements of mission (payload, solar panels or antennas)

Introduction

Active spacecraft attitude control system consists of:

- Attitude sensors
- Attitude actuators
- Program on processor

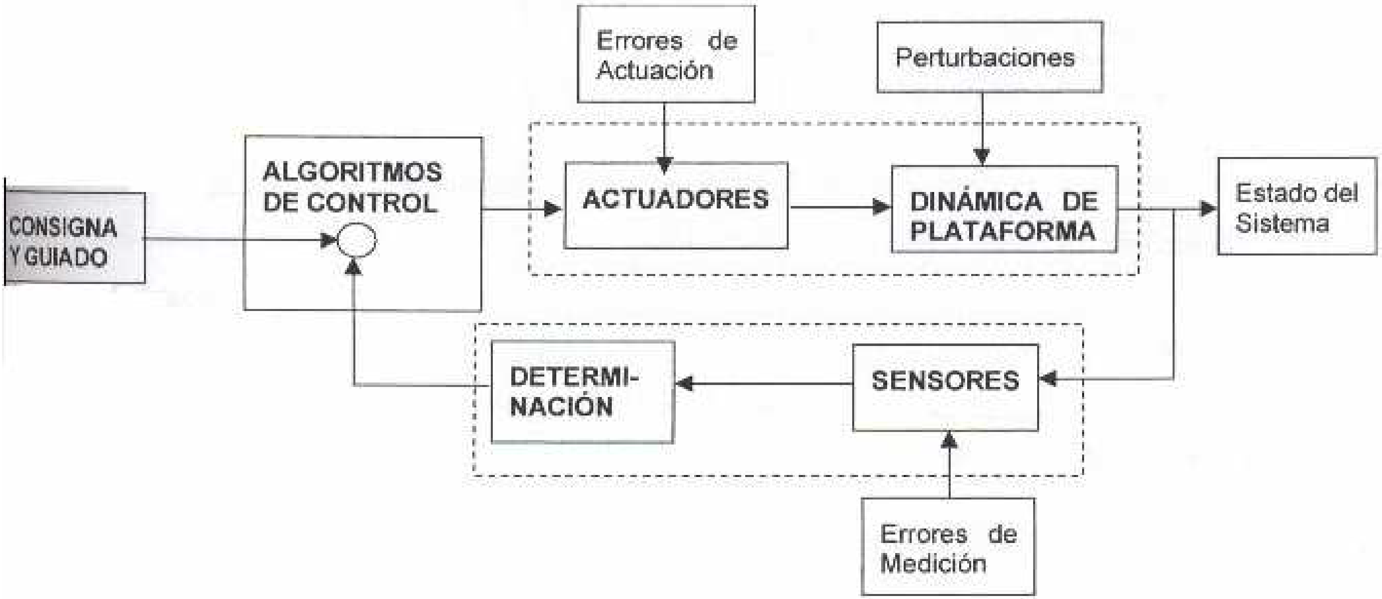
Attitude sensors take measurements which are used to compute current spacecraft attitude and/or angular velocity

Attitude actuators then supply torques to correct difference between measured and desired attitude

Program on processor has implemented mathematical relationships between measured attitude and corrective torques (so called **control law**)

More on three axis stabilization in lecture ADCS - X

Introduction



Sensors make measurements (with error) for determining orientation of vehicle

Sensor measured current state values together with supposed values from control algorithm generates orders for actuators

Actuators produce necessary torques (with errors) on vehicle to move in desired direction

Torques caused by actuators as well as by disturbances produce vehicle dynamics

New state of vehicle is determined by sensors

Steps of ADCS design process

1. Define control modes and requirements of ADCS
2. Select type of attitude control
3. Quantify external and internal disturbance environment
4. Select and size ADCS hardware
5. Define ADCS algorithm

Iterate and document process

LEO satellite

Characteristics of satellite

Mass (kg)	95
Dimensionss (m x m x m)	1 x 0,4 x 0,7
Power (W)	163
Orbit inclination (º)	35
Height above Earth (km)	395
Pointing precision (º)	0,034

Remark:
Scientific goal of Zahori satellite is to detect water under Earth surface

Steps of ADCS design process

1 Define control modes and requirements of ADCS

2 Select type of attitude control

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Iterate and document process

LEO satellite

- Payload must point according to nadir of trajectory
- Attitude determination should be autonomous
- Capacity to reorient vehicle: ±20º, 0.3 º/s
- Reorientation: Once each month for 10 orbits (2% of lifespan)
- Accuracy for pointing: 0.034º
- Mass of ADCS subsystem 3 kg (6% dry mass)
- Mass of fuel 14 kg (1 kg for attitude control)
- Use orbital maintenance system
- Lifespan: 2 years

Typical attitude control modes

1

Orbit insertion

Initial acquisition of nominal attitude

Normal mode of operation

Reorientation/Slew

Safe mode

Special conditions

Typical attitude control modes

1

Mode	Comment
~~Orbit insertion~~	Assume launcher inserts satellite in final orbit without need of ADCS
Initial acquisition of nominal attitude	Design operation for these modes
Normal mode of operation	
Reorientation/Slew	
Safe mode	Consider only in more advanced design stage
~~Special conditions~~	Do not take into account any special operation conditions

Attitude determination and control performance requirements

1

		Required actions
Determination	Accuracy	0.034º
	Range	±20º of nadir
Control	Accuracy	0.034º
	Range	±50º of nadir
	Angular velocity max.	0.3 º/s

Steps of ADCS design process

1. Define control modes and requirements of ADCS
2. Select type of attitude control
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Iterate and document process

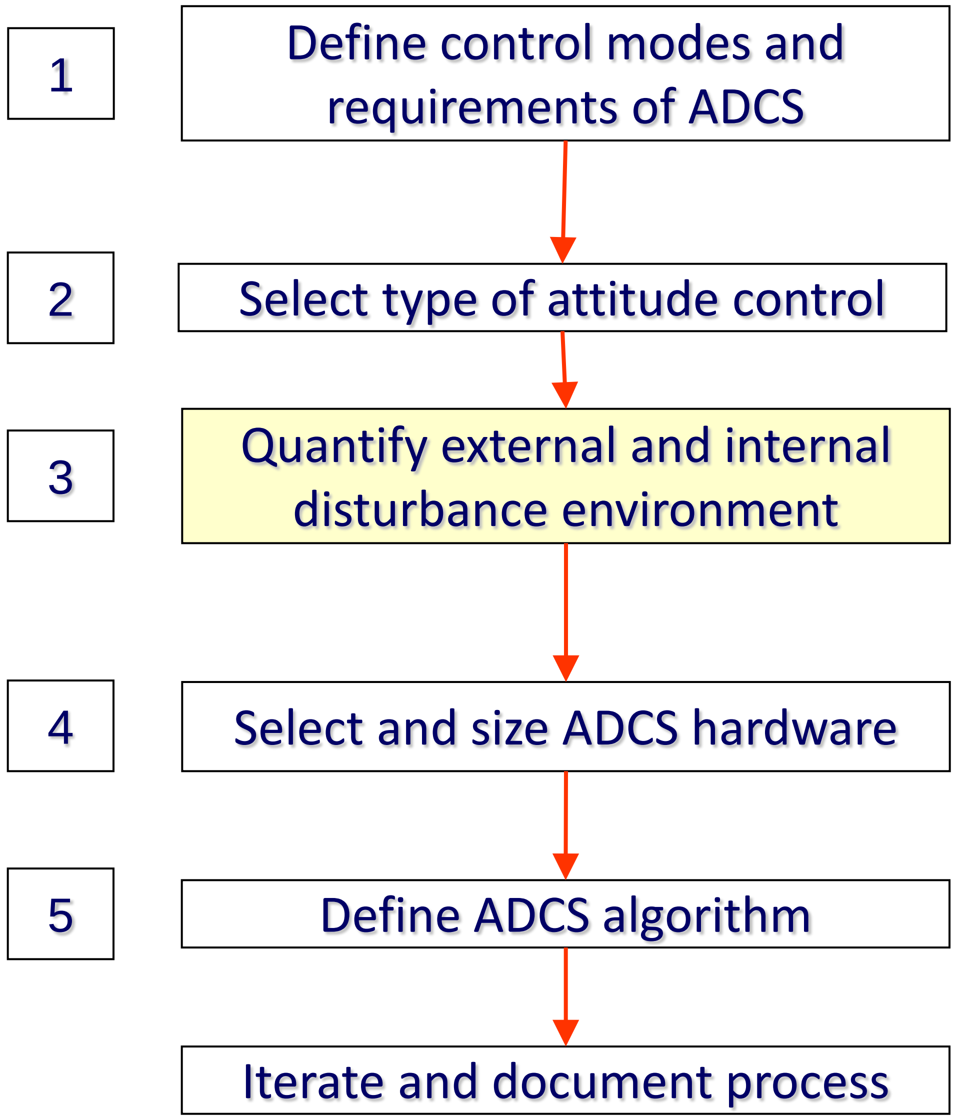
Selection of spacecraft control type

2

Type of control	Typical accuracy
• Gravity-gradient	±5º (two axes)
• Gravity-gradient and momentum wheel	±5º (three axes)
• Passive Magnetic	±5º (two axes)
• Pure spin stabilization	±0.1º - 1º (two axes)
• Dual-spin stabilization	±0.1º - 1º (two axes)
• Momentum wheel (1 axis)	±0.1º - 1º
• Thruster (3 axes)	±0.1º - 5º
• Reaction wheels (3 axes) *	±0.001º - 1º
• Control momentum gyroscopes (3 axes)	±0.001º - 1º

* Requires momentum dumping (thruster or magnetic), when wheels reach maximum speed

Steps of ADCS design process



Quantify environment disturbances

3

Sources of torques for LEO orbit:

- Gravity-gradient
- Magnetic field torque
- Solar radiation pressure
- Aerodynamic torque

Disturbances affected by:

- Orientation
- Mass properties
- Design symmetry

Quantify for worst case scenario

Environmental disturbances

3

Gravity-gradient torque

Influenced primarily by:

- Spacecraft inertia
- Orbit altitude

$$T_g = \frac{3\mu}{2R^3} \left| I_z - I_y \right| \sin\left(2\theta\right)$$

With:

- T_g: Maximal gravity torque
- μ: Earth´s gravity constant [3.986×10^{14} m^3/s^2]
- R: Orbit radius [in m]
- Θ: Maximum deviation of z-axis from local vertical
- I_z, I_y: Moments of inertia about z and y axes (or I_x , is smaller) [in kg m^2]

Environmental disturbances

Gravity-gradient torque

$$T_g = \frac{3\mu}{2R^3}\left|I_z - I_y\right|\sin\left(2\theta\right)$$

Application to example:

- μ = 3.986 x 10^{14} m^3/s^2
- R = 6.77 x 10^6 m
- I_z = 32.8 kg m^2
- I_y = 18.9 kg m^2
- (I_x=24.7 kg m^2)

- θ = 1º → $\boxed{T_g = 6.2\times10^{-7}\ \mathrm{N\,m}}$
- θ = 20º → $\boxed{T_g = 1.2\times10^{-5}\ \mathrm{N\,m}}$

Environmental disturbances

3

Solar radiation pressure

Influenced primarily by:

- Spacecraft geometry
- Spacecraft center of gravity and center of solar pressure location
- Spacecraft surface reflectivity

Transparent
Absorbent
Reflector (diffuse o specular)

Environmental disturbances

3

Solar radiation pressure

Worst case solar radiation torque is

$$T_{sp} = \frac{F_s}{c} A_s (1+q) \cos i \left(c_{ps} - c_g\right)$$

With:

- F_s: Solar constant [1362 W/m^2]
- c: Speed of light [3 x 10^8 m/s]
- A_s: Surface area opposing Sun
- q: Reflectance factor (ranging from 0 to 1)
- i: Angle of incidence of Sun
- c_{ps}: Location of center of solar pressure
- c_g: Location of center of gravity

Environmental disturbances

3

Solar radiation pressure

Worst case solar radiation torque is

$$T_{sp} = \frac{F_s}{c} A_s (1+q) \cos i \left| c_{ps} - c_g \right|$$

Application to example:

- F_s = 1362 W/m^2
- c = 3 x 10^8 m/s
- A_s = 5.96 m^2
- q = 0.5 (assume some mean values)
- i = 0^o

Assume maximal possible distance:

- c_{ps} - c_g = 6 x 10^{-3} m

$$\boxed{T_{sp} = 2.4 \times 10^{-7} \text{ Nm}}$$

Environmental disturbances

3

Magnetic field torque

Influence primary by:

- Orbital altitude
- Orbital inclination
- Residual spacecraft magnetic dipole

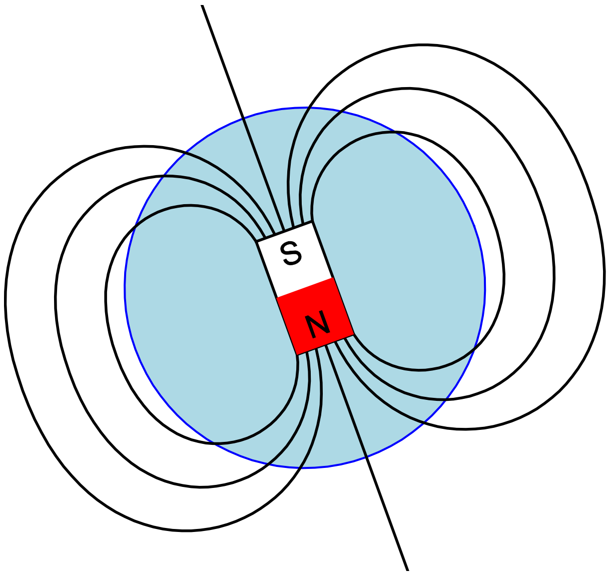
$$T_m = D \cdot B$$

With:

- T_m: Magnetic torque on spacecraft
- D: Residual dipole of spacecraft [A m^2]
- B: Earth´s magnetic field [T]

Environmental disturbances

Magnetic field



In first order, dipole Earth magnetic field is given by

$$|B| = \frac{M}{R^3}\left(1 + 3\sin^2\theta_m\right)^{0.5}$$

With

- B: Magnitude of dipole magnetic field
- M: Magnetic moment of Earth ($7.96 \cdot 10^{15}$ T m^3)
- R: Orbit radius
- Θ_m: Magnetic latitude (angle Θ_m measured northwards from equator)

Worst case:

$$\theta_m = i - 11.5^{o} = 23.5^{o} \Longrightarrow B = 1.2\frac{M}{R^3}$$

Environmental disturbances

Magnetic field torque

Residual dipole of vehicle

- Size of vehicle
- System of compensation
- Typical range between 0.1 and 20 A m^2

Typical value for small vehicle D = 1 A m^2

Finally…

$$T_m = \frac{D\,M}{R^3}\left[1+3\sin^2\left(i-11.5^{\circ}\right)\right]^{0.5}$$

$$\boxed{T_m = 3.1\times 10^{-5}\ \mathrm{Nm}}$$

Environmental disturbances

3

Aerodynamic torque

- Orbital altitude
- Spacecraft geometry
- Relative position of center of gravity and center of atmospheric pressure

$$T_a = \frac{1}{2}\rho v^2 A c_d \left(c_{pa} - c_g\right)$$

With:

- ρ: Atmospheric density (ρ = 10^{-11} kg/m^3 for 400 km)
- v: Spacecraft velocity (7.67 x 10^3 m/s)
- A: Surface area normal to velocity vector (A = 0.42 m^2)
- c_d: Drag coefficient (between 2 and 2.5)
- c_{pa} - c_g: relative location between center of pressure and center of gravity (assume 6 x 10^{-3} m)

$$\boxed{T_a = 1.7\times 10^{-6}\ \mathrm{Nm}}$$

Environmental disturbances

3

Summary...

Gravity-gradient	→	$\boxed{T_g = 1.2\times10^{-5}\ \mathrm{N\,m}}$	Constant
Magnetic field	→	$\boxed{T_m = 3.1\times10^{-5}\ \mathrm{Nm}}$	Cyclic
Solar radiation pressure	→	$\boxed{T_{sp} = 2.4\times10^{-7}\ \mathrm{Nm}}$	Cyclic
Aerodynamic torque	→	$\boxed{T_a = 1.7\times10^{-6}\ \mathrm{Nm}}$	Constant

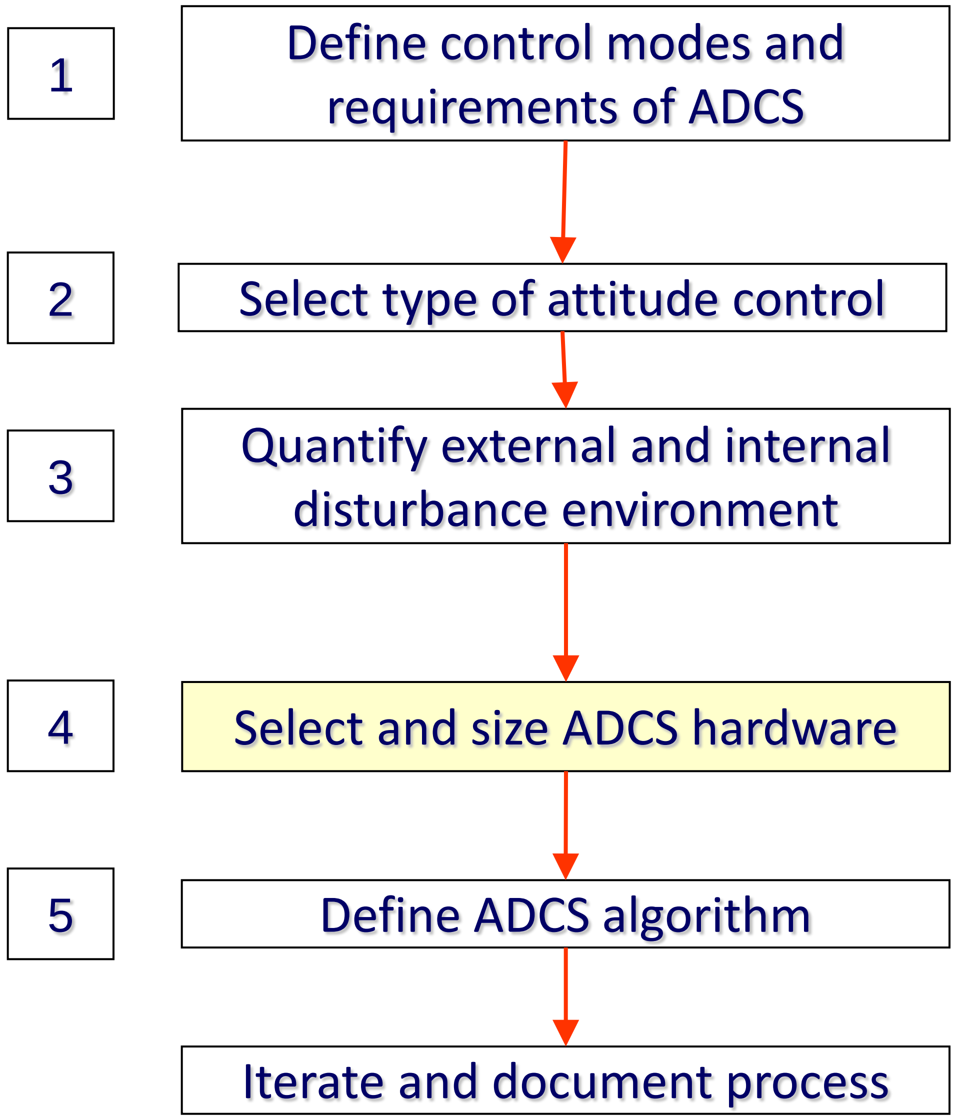
Internal disturbances

- In general have control of internal disturbance torques
- Check if **internal disturbance torque** are smaller than environmental one

Examples:

- Uncertainty of center of gravity (1 - 3 cm)
- Thruster misalignment (0.1º – 0.5º)
- Mismatch of thruster outputs (±5%)
- Rotating machinery (e.g.: pumps, tape recorder)
- Liquid sloshing (movements of liquid)
- Dynamics of flexible bodies (e.g.: solar panels or antennas)
- Thermal shocks on flexible appendages (e.g.: attitude disturbance when entering/leaving eclipse)

Steps of ADCS design process



Select and size hardware

4

Reaction wheels
- Disturbance
  - Cyclic
  - Secular
- Performance
  - Angular momentum storage capacity
  - Torque control capability

- Secular torque and total storage capacity define how frequent angular momentum must be dumped
- Torque capability determined by slew requirements or need for control above peak disturbance torque in order to maintain pointing accuracy

Select and size hardware

4

- Torque from reaction wheel must compensate disturbance

Reaction wheel torque (T_{RW}) must equal worst-case anticipated disturbance torque (T_D) plus some margin (C_{MF}):

$$T_D = T_m = 3.1 \times 10^{-5}\ \text{Nm} \qquad C_{MF} = 1.1 - 1.5$$

$$T_{RW} \geq T_D\, C_{MF} = 3.7 \times 10^{-5}\ \text{Nm}$$

- Slew torque for repointing of satellite

$$\frac{\theta}{2} = \frac{1}{2}\frac{T}{I_X}\left(\frac{t}{2}\right)^2 \Rightarrow T = \frac{4\theta I_X}{t^2} \Leftarrow \begin{aligned} \theta &= 20^{\circ} = 0.35\,\text{rad} \\ t &= 4\,\text{min} = 240\,\text{s} \\ I_X &= 24.7\,\text{kg}\,\text{m}^2 \end{aligned}$$

(half of time accelerated and other half decelerated)

$$\Downarrow$$

$$T_{RW} = 6 \times 10^{-4}\ \text{Nm}$$

Select and size hardware

- Ability of momentum storage in reaction wheel

One approach to estimate wheel momentum (H) is to integrate worst-case disturbance torque (T_D) over a full orbit. If disturbance is gravity-gradient, maximum disturbance accumulates in ¼ of an orbit. Simplified expression for sinusoidal disturbance is:

$$H = T_D P$$

$$H = \left(\frac{2}{\pi} T_D\right) \frac{P}{4}$$

Orbital period P

$$P = 5544\,\mathrm{s} = 92.4\,\mathrm{min}$$

$$\boxed{T_g = 1.2 \times 10^{-5}\ \mathrm{N\,m}}$$ Constant

$$\boxed{T_m = 3.1 \times 10^{-5}\ \mathrm{Nm}}$$ Cyclic

$$\boxed{T_{sp} = 2.4 \times 10^{-7}\ \mathrm{Nm}}$$ Cyclic

$$\boxed{T_a = 1.7 \times 10^{-6}\ \mathrm{Nm}}$$ Constant

$$\boxed{H = T_g P = 0.064\,\mathrm{N\,m\,s}}$$

$$H = \frac{2}{\pi} T_g \frac{P}{4} = 0.027\,\mathrm{N\,m\,s}$$

Select and size hardware

4

Reaction wheel: *Hamster*

H_{max}	0.1 Nms @ 15000 rpm
Dimensions	100x100x150 mm
Mass	0.315 kg
Power	1 W
Reaction torque	30 mNm

Moment of inertia of reaction wheel (I_{RW})

$$I_{RW} = \frac{H_{\max}}{\omega_{\max}} = \frac{0.1\,\mathrm{N\,m\,s}}{15000\,\frac{2\pi}{60}\,\frac{1}{\mathrm{s}}} = 6.37\times10^{-5}\,\mathrm{kg\,m^2}$$

Saturation time (t_{sat}) calculated only for constant disturbances

$$T_D = I_{RW}\,\dot{\omega}(t)$$

$$\dot{\omega}(t) = \frac{T_D}{I_{RW}} \xrightarrow{\omega(0)=0} \omega(t) = \frac{T_D}{I_{RW}} t_{sat} \longrightarrow t_{\text{sat}} = \frac{I_{RW}}{T_D}\omega_{\max}$$

$$T_D = T_a = 1.7\times10^{-6}\ \mathrm{Nm} \longrightarrow t_{\text{sat}} = 981\,\mathrm{min}$$

$$T_D = T_g = 1.2\times10^{-5}\ \mathrm{Nm} \longrightarrow t_{\text{sat}} = 145\,\mathrm{min}$$

Select and size hardware

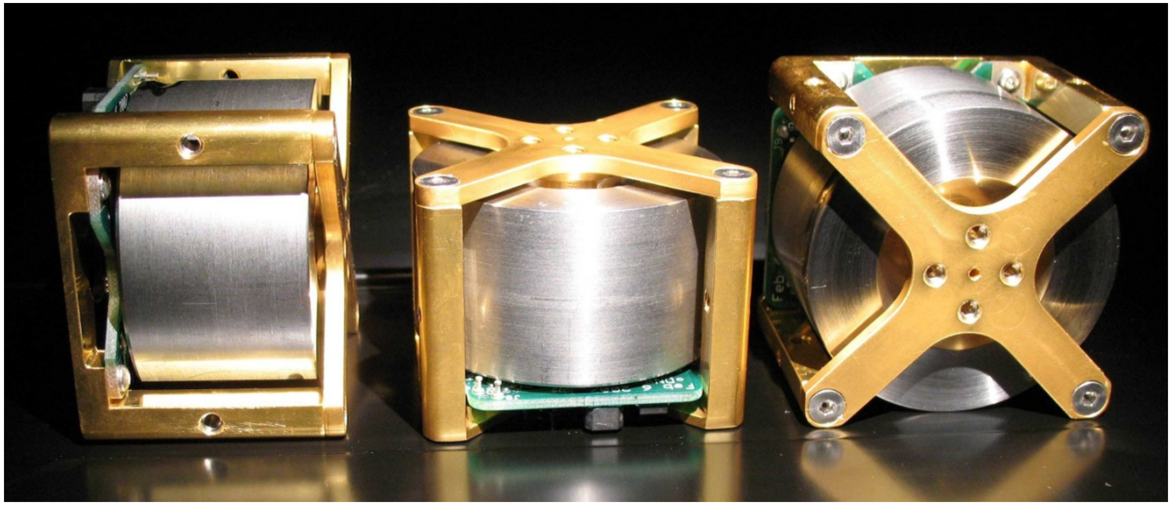
4

- Saturation time

Only two 2% of lifespan
in 20^o reorientation mode

$$t_{sat} = 981\,\text{min}\,0.98 + 145\,\text{min}\,0.02 = 964\,\text{min}$$

Finally…
- 3 reaction wheels *Hamster*, one on each axes and redundant one
- Mass of reaction wheel: 0.945 kg (mass budget for ADCS subsystem is 3 kg)



Select and size hardware

- Mini thruster

  Objectives:

  - Maneuver spacecraft over large angles (initial acquisition and emergency)
  - Dump extra momentum from a reaction wheels
  - Control attitude as redundant system

- Estimate force to compensate disturbances

  Thruster force must at least equalize maximal external disturbance
  Assume thruster moment arm L = 0.3 m

$$T_D = T_m = 3.1 \times 10^{-5}\ \mathrm{Nm}$$

$$F = \frac{T_D}{L} = 1 \times 10^{-4}\ \mathrm{N}$$

It is a very small value
for rocket engine market

Select and size hardware

4

- Mini thruster
  - Estimate force to reorient satellite

Maximal angular velocity 0.3º/s

Assume that thruster burns during 3 s

Take into account mayor moment of inertia (I_z)

$$\left.\begin{aligned}\ddot{\theta} &= \frac{\dot{\theta}}{\Delta t} = 0.1^{\circ}/\mathrm{s}^2 = 1.7\times 10^{-3}\,\mathrm{rad/s}^2 \\ I_Z &= 32.8\,\mathrm{kg\,m}^2\end{aligned}\right\} \quad T = FL = I_Z\ddot{\theta} \quad \Rightarrow \quad F = \frac{I_z\ddot{\theta}}{L} = 0.19\,\mathrm{N}$$

This is small but feasible

Select and size hardware

4

- Mini thruster
  - Estimate force for momentum dumping of reaction wheel

Maximal stored momentum of wheel H = 0.1 Nms

Assume that thruster burns during Δt = 5 s and L = 0.3 m

$$F\,L\,\Delta t = H$$

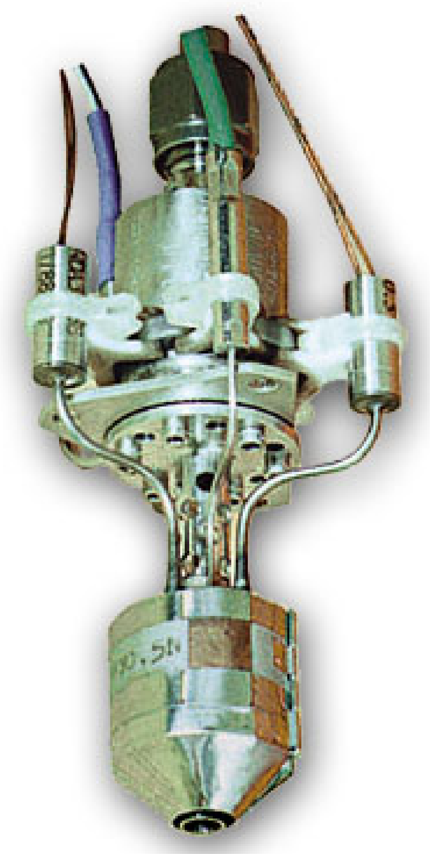
$$F = \frac{H}{L\,\Delta t} = 0.07\,\mathrm{N}$$

Select and size hardware

4

- Mini thruster

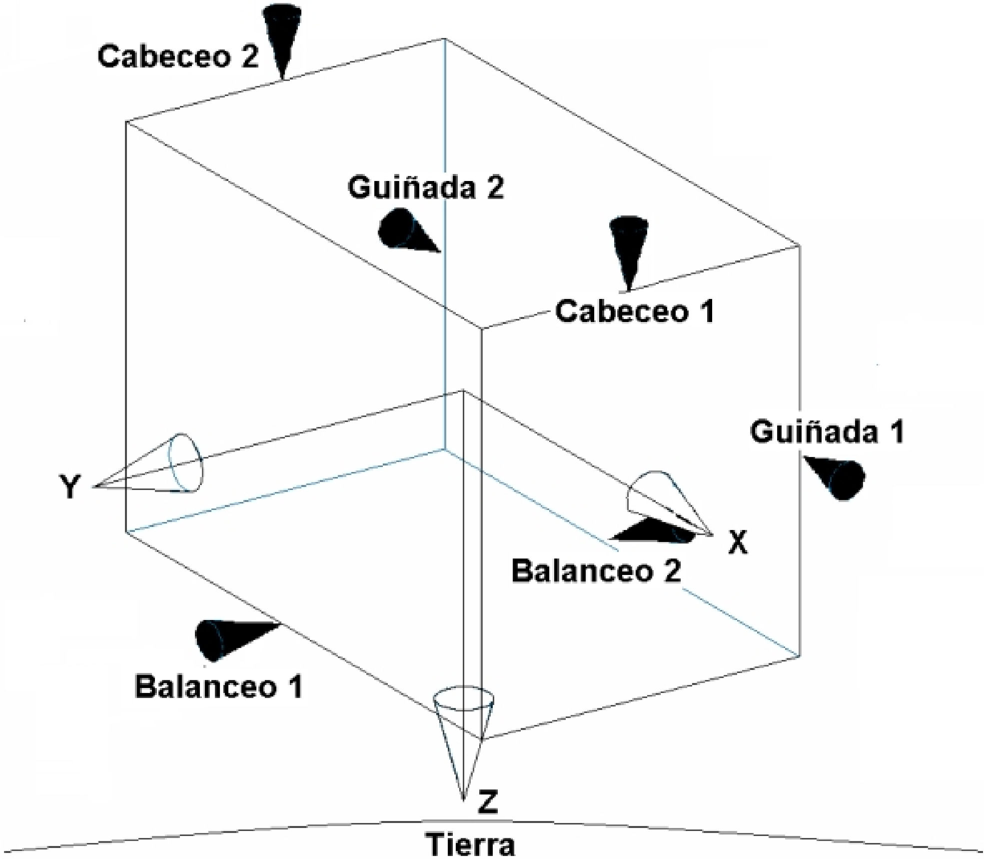
EADS Astrium
0.5 N HYDRAZINE THRUSTER Model CHT 0.5

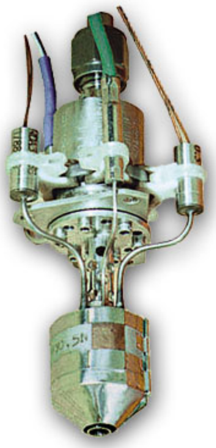


• Fuel / Propellent	Hydrazine
• Thrust in vacuum	0.5 N
• I_{sp} in vacuum	227.3 s
• Chamber pressure	22 bar
• Length	113 mm
• Diameter of nozzle	4.8 mm
• Mass	0.195 kg

Select and size hardware

- Mini thruster





Six thruster: two for each axis

Select and size hardware

4

- Mini thruster
  - Estimation of propellant mass (m_p)

Mission of 2 years

Momentum dumping of reaction wheels each 964 min ➔ 1091 pulses

Reorientation 4 pulses each month ➔ 96 pulses

Total impulse $$I_T = Ft_p = 1091 \cdot 5\,\mathrm{s} \cdot 0.07\,\mathrm{N} + 96 \cdot 3\,\mathrm{s} \cdot 0.19\,\mathrm{N} = 436.6\,\mathrm{Ns}$$

Definition of specific impulse, I_{sp}:

$$I_{sp} = \frac{\int_0^{t_p} F(t)\,\mathrm{d}t}{g_0 \int_0^{t_p} G(t)\,\mathrm{d}t} = \frac{Ft_p}{g_0 m_p} \qquad m_p = \frac{Ft_p}{g_0 I_{sp}} = 0.20\,\mathrm{kg}$$

Select and size hardware

4

- Sensors

Selection influenced by:

- Required orientation of spacecraft and its accuracy
- Redundancy
- Fault tolerance
- Field of view requirements
- Available data rates

Full 3-axis knowledge requires at least two external vector measurements

Select and size hardware

4

- Sensors

Sensor	Typical precision	Mass (kg)	Power (W)
• Sun *1	0.005º to 3º	0.5 to 7	0 to 3
• Stars	0.0003º to 0.01º	3 to 7	5 to 20
• Horizon	0.1º to 1º	2 to 5	5 to 10
• Magnetometer	0.5º to 3º	0.6 to 1.2	<1

*1 take into account eclipse

Horizon sensor is added for initial acquisition modes and emergency modes

Select and size hardware

4

Characteristics of star sensor

Mechanical	
Dimensions (with baffle)	80mm x 100mm x 180mm
Mass	**1.1 kg**

Electrical	
Power consumption, max.	**2.5 W**
Input voltage range	9 VDC to 18 VDC

Performance	
Accuracy (x,y / z axis)	**0.005º / 0.0338º**
Acquisition probability	>99.7%
Update period	4 Hz to 8 Hz
Field of view	14° x 14°
Time to first acquisition	max. 900 ms

Using at least two vectors separated by some angular distance between stars
→ Have full 3-axes determination

Select and size hardware

4

Characteristics of Sun sensor

Accuracy (º)	**0,0167**
Field of view (º)	100 x 50
Number of pixels on CCD	1,728
Frecuency measurement	10 Hz
Power consumed (W)	**2,5**
Mass	**0,35 kg**

Sun sensor needs to be replaced during eclipse

Select and size hardware

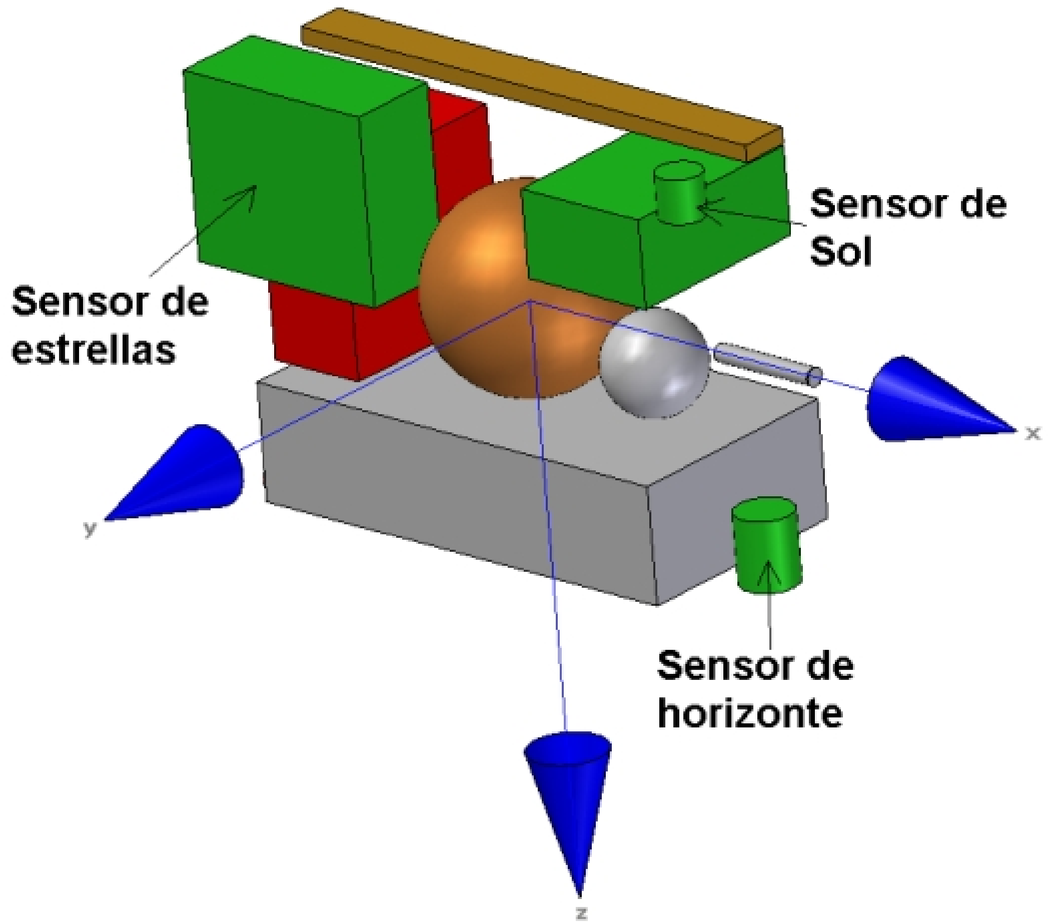
Characteristics of Earth horizon sensor

Accuracy (º)	**±0,2 (3σ)**
Range of operation around nadir (º)	±10
Field of view of each telescope (º)	20 (high) x 14,8 (wide)
Field of view of combined telescopes (º)	120º
Operational temperature (ºC)	45º C (stabilized by heaters)
Range of operation in orbit (ºC)	-40 a 40
Spectral band (μm)	14 a 16
Overall mass (kg)	**1**
Overall dimension (mm)	**190 (diameter), 120 (depth)**
Power consumed by telescopes and heaters (W)	**5**

Earth sensors are infrared systems to detect contrast between cold deep space and heat of atmospheric ground

Select and size hardware

4



Select and size hardware

- Summary

	Nr	Unit mass [kg]	Total [kg]		Power [W]
Reaction wheels	3	0.32	0.95		1.0
Thrusters	6	0.20	1.17		
Fuel	1	0.20	0.20		
Star sensor	1	1.10	1.10		2.5
Sun sensor	1	0.35	0.35		2.5
Horizon sensor	1	1.00	1.00		5.0
			4.77	**> 3 kg**	**11.0**

- Pass data to other subsystems
- Review / discuss with other system designers
- **ITERATE**

Steps of ADCS design process

1 Define control modes and requirements of ADCS

2 Select type of attitude control

3 Quantify external and internal disturbance environment

4 Select and size ADCS hardware

5 Define ADCS algorithm

Iterate and document process

Dynamics and control

Dynamic equation

$$\mathbf{T} = \dot{\mathbf{h}}_I = \dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h}$$

$$\mathbf{h} = \mathbf{h}_S + \mathbf{h}_W$$

Angular momentum of satellite

Angular momentum of reaction wheel

$$\mathbf{T}_C + \mathbf{T}_D = \dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h}$$

Control torque

Disturbance torque

$$T_{Cx} + T_{Dx} = \dot{h}_{Sx} + \dot{h}_{wx} + \left(\omega_y h_{Sz} - \omega_z h_{Sy}\right) + \left(\omega_y h_{Wz} - \omega_z h_{Wy}\right)$$

$$T_{Cy} + T_{Dy} = \dot{h}_{Sy} + \dot{h}_{wy} + \left(\omega_z h_{Sx} - \omega_x h_{Sz}\right) + \left(\omega_z h_{Wx} - \omega_x h_{Wz}\right)$$

$$T_{Cz} + T_{Dz} = \dot{h}_{Sz} + \dot{h}_{wz} + \left(\omega_x h_{Sy} - \omega_y h_{Sx}\right) + \left(\omega_x h_{Wy} - \omega_y h_{Wx}\right)$$

Complicated nonlinear equations

Control theory does not provide exact analytical solutions

Make assumptions and linearize equation to use control technique

Attitude control discussed in detail in Lecture ADCS - X

Dynamics and control

- **Dynamic equation** (Restrict to control design of linear system)

  Assume small angles and rates → Kinematic equation $\omega_i = \dot{\theta}_i \qquad i = x, y, z$

  Neglect second order terms of dynamic equation $\omega_i \omega_j \approx 0$

  Reaction wheel axis coincides with principle axis of inertia of satellite

- **Linearized dynamic equations**

$$T_{Cx} + T_{Dx} = \dot{h}_{Sx} + \dot{h}_{Wx}$$
$$T_{Cy} + T_{Dy} = \dot{h}_{Sy} + \dot{h}_{Wy}$$
$$T_{Cz} + T_{Dz} = \dot{h}_{Sz} + \dot{h}_{Wz}$$

Note:
If $T_C = T_D = 0$→
Angular momentum conservation

Dynamic equations are decoupled and have same form for each axis
Consider only one axis at a time and use $h_i = I_i \omega_i \qquad i = x, y, z$

$$T_{Cx} + T_{Dx} = \dot{h}_{Sx} + \dot{h}_{Wx} \Leftrightarrow T_{Cx} + T_{Dx} = I_{Sx}\ddot{\theta}_{Sx} + \dot{h}_{Wx}$$

Write as $$T_C + T_D = I_S \ddot{\theta} + \dot{h}_W$$

Assume reference angle is zero, then measured angle is directly error in orientation

Dynamics and control

5

Control laws

Proportional control torque: → $T_C = -K\theta$

Control is proportional to error angle

(K is proportional gain)

Proportional derivative control torque *: → $T_C = -K_1\dot{\theta} - K_2\theta$

Add damping to control

(K_1 is derivative gain)

* The reaction wheels provide this control law (Sidi, 1997)

Dynamics and control

Dynamic equation for reaction wheel can be written as

$$T_C = -\dot{h}_W = -K_1\dot{\theta} - K_2\theta = -K\left(\tau\dot{\theta} + \theta\right)$$

With

- ***t***: time constant
- ***K***: gain

They depend on electromechanical design of reaction wheels

Replacing in equation of motion:

$$T_D = I_S\ddot{\theta} + K\tau\dot{\theta} + K\theta$$

It has been considered that exterior torque is due to perturbation, T_D

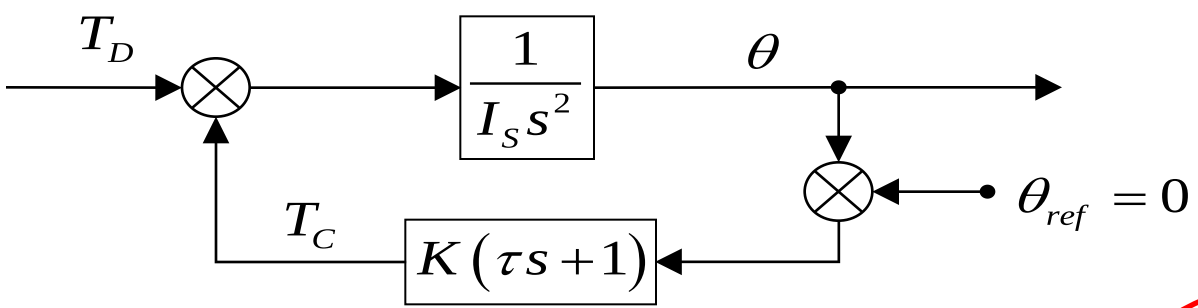
For analysis apply Laplace transformation

$$L\left(T_D\right) = \left[I_S s^2 + K\tau s + K\right] L\left(\theta\right)$$

Dynamics and control

This is a closed-loop system whose second order transfer function is:

$$G(s)=\frac{L(\theta)}{L(T_D)}=\frac{1}{I_S s^2+K\tau s+K}$$



Or:

$$G(s)=\frac{L(\theta)}{L(T_D)}=\frac{1}{I_S}\frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_n=\sqrt{\frac{K}{I_S}}$$

$$\xi=\frac{\tau}{2}\sqrt{\frac{K}{I_S}}$$

ω_n→undamped natural frequency
(is frequency of oscillation of closed-loop system without damping)
ξ→ damping ratio
(is measure of resistance to change system output)

Dynamics and control

5

The system response to a step function of magnitude T_D is evaluated using Laplace transformation (Laplace transformation of unit step function is 1/s)

$$L(T_D) = \frac{T_D}{s} \Rightarrow L(\theta) = \frac{T_D}{I_S} \frac{1}{s\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)}$$

With $0 < \xi < 1$ and $\omega_n > 0$ the inverse Laplace transformation of $L(\Theta)$ is given by:

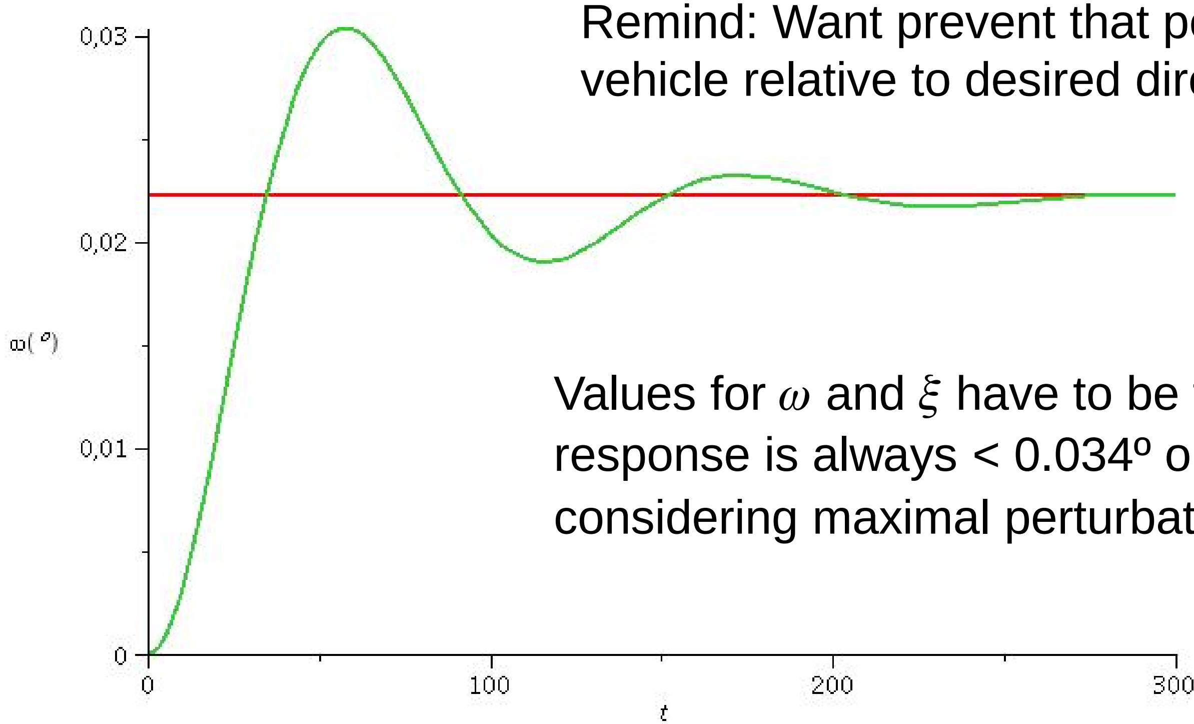
$$\theta(t) = \frac{T_D}{I_S \omega_n^{\ 2}} \left[1 - e^{-\xi\omega_n t} \left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin\omega_d t \right) \right]$$

with

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

Dynamics and control

$$\theta(t)=\frac{T_D}{I_S\omega_n^{\;2}}\left[1-e^{-\xi\omega_n t}\left(\cos\omega_d t+\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)\right]$$

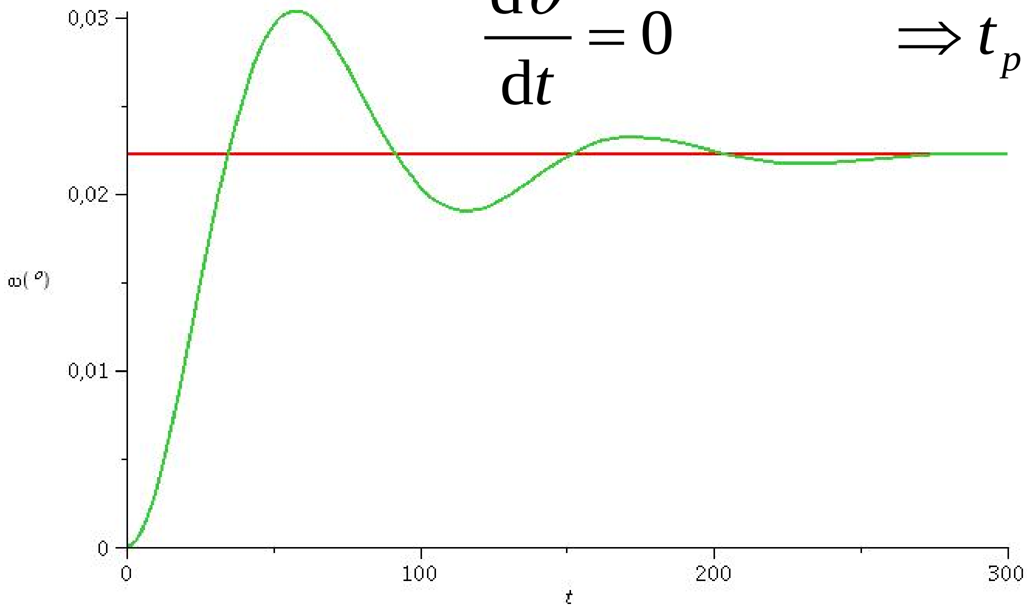


Remind: Want prevent that pointing angle of vehicle relative to desired direction is < 0.034º

Values for ω and ξ have to be found so that the response is always < 0.034º on all axes, when considering maximal perturbation

Dynamics and control

5



$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0 \qquad \Rightarrow t_p = k\frac{\pi}{\omega_d} \qquad k = 0, 1, 2\ldots$$

With k =1 replace in θ(t):

$$\theta_{\max} = \frac{T_D}{\omega_n{}^2 I_S}\left(1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\right) \quad \Rightarrow \quad \omega_n{}^2 = \frac{T_D}{\theta_{\max} I_S}\left(1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\right)$$

Dynamics and control

5

$$\omega_n^{\ 2} = \frac{T_D}{\theta_{max} I_S}\left(1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\right)$$

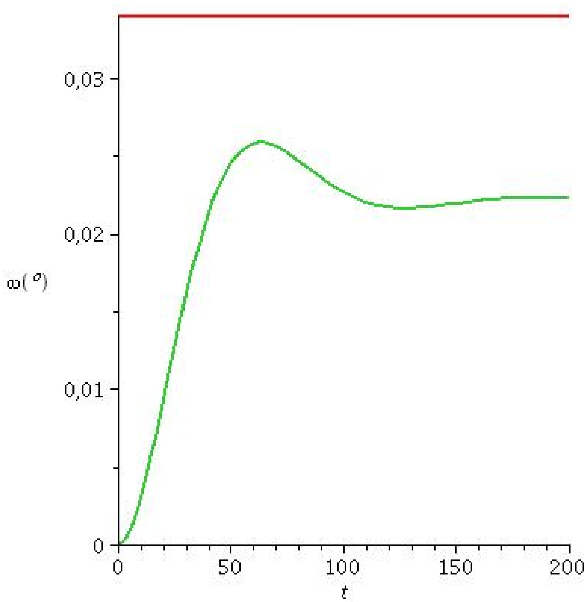
$$\boxed{\omega = 0.057\,\text{rad/s}}$$

Fix damping at 0.5

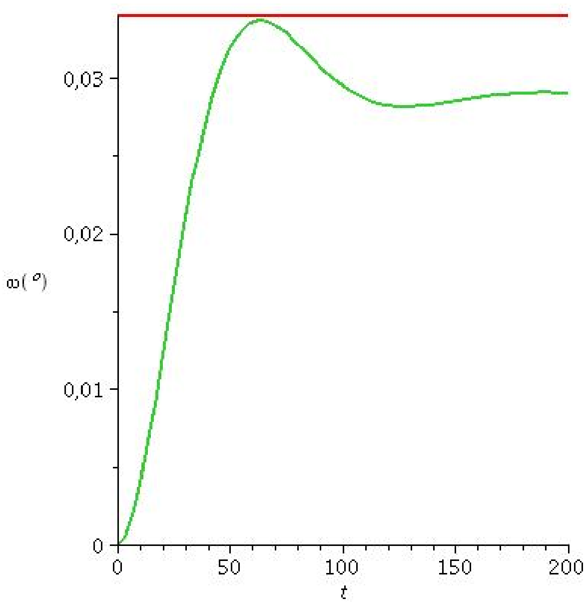
θ_{max} = 0.034º

$I_s = I_Y$ (smallest moment of inertia)

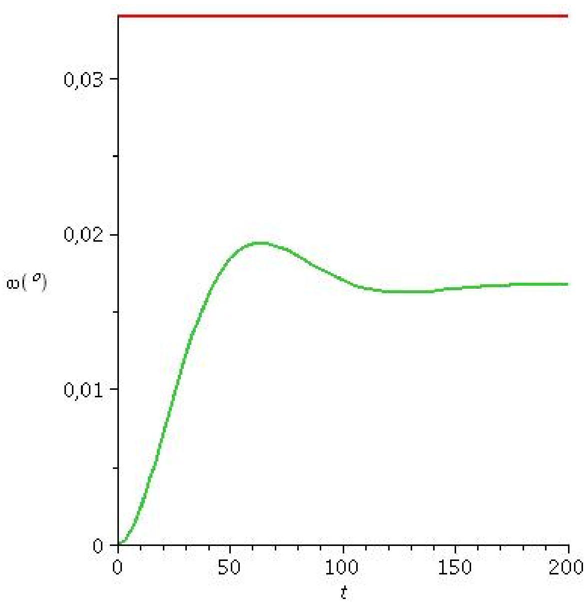
Take into account maximal environmental disturbance (magnetic field)



$I_S=I_X$



$I_S=I_Y$



$I_S=I_Z$

Dynamics and control

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Finally…

$$\xi = 0.5$$
$$\omega = 0.057\,\text{rad/s}$$

$$\omega_n = \sqrt{\frac{K}{I_S}}$$
$$\xi = \frac{\tau}{2}\sqrt{\frac{K}{I_S}}$$

$$K = 0.0614\,\frac{\text{N m}}{\text{rad}}$$
$$\tau = 17.5\,\text{s}$$

Give these values to reaction wheel manufacturer to tune reaction wheel

Conclusion: Hardware design

Summary

	Nr	Unit mass [kg]	Total [kg]		Power [W]
Reaction wheels	3	0.32	0.95		1.0
Thrusters	6	0.20	1.17		
Fuel	1	0.20	0.20		
Star sensor	1	1.10	1.10		2.5
Sun sensor	1	0.35	0.35		2.5
Horizon sensor	1	1.00	1.00		5.0
			4.77	**> 3 kg**	**11.0**

- Pass data to other subsystems
- Review / discuss with other system designers
- **ITERATE**

Conclusion: ADCS design

1 Define control modes and requirements of ADCS

↓

2 Select type of attitude control

↓

3 Quantify external and internal disturbance environment

↓

4 Select and size ADCS hardware

↓

5 Define ADCS algorithm

↓

Iterate and document process