DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS – IXB

Problem 1:

Given a homogenous cylinder of radius R and height h, what is the smallest possible cylinder height such that this object would still be gravity gradient stabilized in attitude (assume height is aligned with $\hat{o}_3$)?

Problem 2:

A spacecraft with a principal axes body-fixed frame b, has corresponding principal moments of inertia $I_x = 100\ \mathrm{kg\cdot m^2}$, $I_y = 120\ \mathrm{kg\cdot m^2}$ and $I_z = 80\ \mathrm{kg\cdot m^2}$. The spacecraft attitude relative to the Earth centered inertial frame G is described by a yaw-pitch-roll (3-2-1) Euler sequence, represented by the rotation matrix

$$\mathbf{C}_{bG}(\phi,\theta,\psi) = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix}$$

Where s_b = sin b and c_b = cos b. Where ϕ, Θ and ψ are the roll, pitch and yaw angles, respectively. Currently, the attitude is represented by
ϕ = Θ = ψ = π/4 rad, and the spacecraft orbital position (in ECI) coordinates is

$$\vec{\mathbf{R}}_o = \begin{bmatrix} 0 \\ 0 \\ R_o \end{bmatrix} \mathrm{km},$$

with R_o = 7000 km.

Determine the gravity-gradient torque acting on the spacecraft. Express the result in spacecraft body coordinates. Note that μ = 3.986 · 10^5 $\mathrm{km^3/s^2}$ is the Earth´s gravitational constant.

Problem 3:

A spacecraft with a principal axes body-fixed frame b, has corresponding principal moments of inertia $I_x = 8\ \mathrm{kg \cdot m^2}, I_y = 12\ \mathrm{kg \cdot m^2}, I_z = 10\ \mathrm{kg \cdot m^2}$.

It is desired to spin the spacecraft about the principal z-axis with angular velocity ω_z = 0.1 rad/s. Assuming that the spacecraft has the momentum wheel with spin-axis aligned with the principal z-axis, determine the required relative wheel angular momentum h_s, to make the desired attitude motion passively stable under torque-free conditions and without energy dissipation.

Problem 4:

Consider a nominally non-spinning dual-spin spacecraft with principal inertias I_1, I_2 and I_3. The wheel axis coincides with the 3-axis of a body-fixed principal axes frame. The wheel relative angular momentum is given by $h_s > 0$.

a) Show that this situation corresponds to an equilibrium for torque-free motion
b) Show that small perturbations to the spacecraft angular velocity lead to purely oscillatory behavior in ω_1 and ω_2 with frequency

$$\Omega_p = \frac{h_s}{\sqrt{I_1 I_2}}.$$