DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - I

Problem 1:

Consider the Earth.

What is the radial acceleration due to gravitational force?

(a) At sea surface.
(b) For an Earth observation satellite at 800 km altitude.
(c) For a GPS satellite at 20200 km altitude.
(d) For a geostationary satellite at 35800 km altitude.

Problem 2:

In lecture I we have seen that the energy of a photon is given by following formula $E = hc/\lambda$.

(a) Please write the energy of the photon as a function of frequency ν.
(b) Calculate the energy in joules for a green (λ = 525 nm) light photon as well as for gamma ray ($\lambda = 10^{-12}$ m) and for radio wave (λ = 1 m).

Problem 3:

In lecture I page 34 we have seen that Planck's distribution describes the variation of the power per unit area (intensity, I) emitted from a thermal source (black body) as a function of the emission wavelength λ and the temperature of the source T.

(a) What is the physical meaning of the integral of the Planck distribution? Please write the formula for the integral.
(b) Wien's law describes the dependence of the wavelength corresponding to maximum emission intensity (λ_{max}) as a function of the source temperature T. Show that Planck´s law

$$I(T,\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Can be used to derive Wien´s law $\lambda_{\max} = \frac{b}{T}$, with $b = 2.898 \times 10^{-3} \mathrm{m\,K}$.

Hint: Use product and chain rule. The calculation gets horrible messy, however you can obtain an expression of the form 5 = (5-x)e^x where x is a trivial function of λ and T. The numerical solution of this equation is $x \approx 4.96511$ (or x = 0 not physical).

Note: Planck's law originates from first principles derivation and agrees very well with observations, whereas Wien's law is empirical (means it originates from observations and not from first principles).

Problem 4:

Explain qualitatively the atmospheric loss.

Assume a volume of gas at the top of a planet’s atmosphere. The gas will have a range of velocities described by the Maxwell-Boltzmann distribution. Please write the Maxwell-Boltzmann distribution formula and plot the function for different temperatures. Why do particles/molecules escape particles into space? Which particle/molecules escape first into space? Please compare temperature, escape velocity and atmosphere composition of e.g. Earth, Mars and Jupiter.

Problem 5:

Consider a star with mass M, radius R and uniform density $\rho_0 = \frac{3}{4\pi}\frac{M}{R^3}$. Use the equation of hydrostatic equilibrium ($\frac{\mathrm{d}p}{\mathrm{d}r} = -g(r)\rho(r)$) and show that the pressure as a function of radius in the star is given by $p(r) = \frac{2\pi}{3} G \rho_0{}^2 (R^2 - r^2)$.