DOC 221 Dinámica orbital y control de actitud

Solutions to Problems Lecture ADCS - III

Problem 1:

(a) The sun emits photons which contain momentum. This momentum produces a pressure on spacecraft due to momentum exchange.

(b) With $c = 3 \cdot 10^8$ m/s and $q = 0$

$\Rightarrow F_{rad} = m_{Sat} a_{rad} = \frac{\Phi_{rad}(r)}{c}(1+q)A_{\perp} = \frac{1}{3} \cdot 10^{-5}$ N

(c) For reflecting material the momentum exchange doubles and → q = 1

(d) $\Phi(r) = \frac{L_{\odot}}{4\pi r^2} \Leftrightarrow r = \sqrt{\frac{L_{\odot}}{4\pi\Phi(r)}} = 1.76 \cdot 10^{11}$ m where $r_{Sun-Earth} = 1.5 \cdot 10^{11}$ m

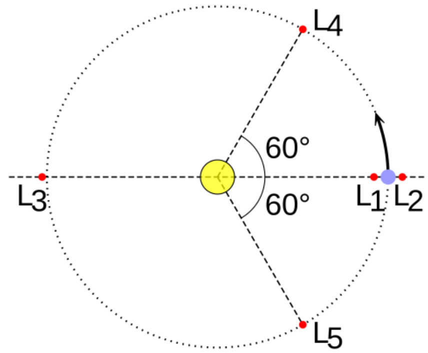
Problem 2:

A satellite in low altitude interacts with the upper atmosphere. The atmosphere produces a drag force that retards the satellite motion and changes the shape of the orbit. The atmospheric drag effect on the satellite orbit takes place through energy dissipation, which means that when an orbit loses energy, then the semi-major axis decreases. This is similar to the Hohmann transfer orbit, where satellite engine fires in the opposite direction to its current path, slowing the satellite and causing it to drop into the lower-energy elliptical/circular orbit.

For an orbit with large eccentricity, the atmospheric drag effect would first make the orbit more circular by gradually lowering its apogee. Later the radius of the circular orbit would continue to decrease until the satellite crashes into the Earth´s surface.

Problem 3:

L_1, L_2, L_3 points are unstable and L_4, L_5 points are stable. However L_4 and L_5 are only stable if the mass ratios of the planets fulfill some conditions. The L_4 and L_5 points of the Sun-Jupiter system contain the Trojan asteroids.



Problem 4:

(a) In a circular orbit, the centripetal force equals to the gravitational force

$$\Rightarrow F_{centripetal} = F_{gravitaion} \Leftrightarrow m\omega^2 r = G\frac{mM}{r^2} \Leftrightarrow r = \sqrt[3]{\frac{GM}{\omega^2}}$$

With $\omega = \frac{2\pi}{T} = \frac{2\pi}{86164}\frac{\text{rad}}{\text{s}} = 7.292\cdot 10^{-5}\ \frac{\text{rad}}{\text{s}}$

$$\Rightarrow r_{Geo} = \sqrt[3]{\frac{\mu}{\omega^2}} = 42164\text{ km}$$

(b) $a_\lambda = -\frac{1}{r\cos\phi}\frac{\partial U}{\partial \lambda}$

(c) The only differences between the two expressions are the

$J_{nm}\cos m(\lambda - \lambda_{nm})$ and $C_{nm}\cos m\lambda + S_{nm}\sin m\lambda$ terms

Using the following trigonometric identity $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

we have

$$\begin{aligned} J_{nm}\cos m(\lambda-\lambda_{nm}) &= J_{nm}(\cos m\lambda \cos m\lambda_{nm} + \sin m\lambda \sin m\lambda_{nm}) \\ &= J_{nm}(\cos m\lambda \tfrac{C_{nm}}{J_{nm}} + \sin m\lambda \tfrac{S_{nm}}{J_{nm}}) \\ &= C_{nm}\cos m\lambda + S_{nm}\sin m\lambda \end{aligned}$$

(d) With the definition of the Legendre polynomials we have

$$P_2(x) = \frac{1}{(-2)^2 2!}\frac{\partial^2}{\partial x^2}(1-x^2)^2 = \frac{1}{4\cdot 2}\frac{\partial^2}{\partial x^2}(1-2x^2+x^4) = -\frac{1}{2}+\frac{3}{2}x^2$$

$$P_{22}(x) = (1-x^2)^{2/2}\frac{\partial^2 P_2(x)}{\partial x^2} = (1-x^2)^{2/2}\frac{\partial^2}{\partial x^2}(-\frac{1}{2}+\frac{3}{2}x^2) = 3(1-x^2)$$

$$P_{22}(\sin\phi) = 3(1-\sin\phi^2) = 3\cos^2\phi$$

U_{22} potential is given by

$$U_{22} = \frac{GM}{r}\left(\frac{R_e}{r}\right)^2 J_{nm}P_{nm}\left(\sin\phi\right)\cos m(\lambda-\lambda_{nm})$$

$$= \frac{GM}{r}\left(\frac{R_e}{r}\right)^2 J_{22}P_{22}\left(\sin\phi\right)\cos 2(\lambda-\lambda_{22})$$

$$= \frac{GM}{r^3}R_e^2 3\cos^2\phi\cos 2(\lambda-\lambda_{22})$$

The East-West acceleration is given by

$$a_{\lambda,22} = -\frac{1}{r\cos\phi}\frac{\partial U_{22}}{\partial\lambda} = -6\frac{GM}{r^4}R_e^2\cos\phi\sin 2(\lambda-\lambda_{22})$$

(e) Using $J_{22} = \sqrt{C_{22}^2+S_{22}^2} = 1.81\cdot 10^{-6}$, $\lambda_{22} = -14.9^o$, $\phi = 0$ $\rightarrow$

$$a_{\lambda,22} = -5.6\cdot 10^{-11}\cos\phi\cdot\sin 2(\lambda-14.9^o)\,\frac{\text{m}}{\text{s}^2}$$

$$= -5.6\cdot 10^{-11}\sin 2(\lambda-14.9^o)\,\frac{\text{m}}{\text{s}^2}$$

(f) The equilibrium points are at about 75^{o} E, 255^{o} E, 162^{o} E and 348^{o} E.

(g) Points are stable at about 75^{o} E and 255^{o} E and unstable at about 162^{o} E and 348^{o} E.

Problem 5 (optional):

Do the exercise self-control proposed in page 66 of lecture ADCS - III.