DOC 221 Dinámica orbital y control de actitud

Solutions to Problems Lecture ADCS - IV

Problem 1:

Solar sail problem:

The solar radiation pressure is $p_s = 2\frac{\Phi}{c} \approx 2\frac{1362}{3\cdot 10^8} = 9\cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{m}^2}$.

The force is given by $F = p_s A = 9\cdot 10^{-6}\cdot 10^4 = 9\cdot 10^{-2}\,\mathrm{N}$.

The acceleration is $a = \frac{F}{m} = \frac{9\cdot 10^{-2}\,\mathrm{N}}{1000\,\mathrm{kg}} = 9\cdot 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}^2}$.

The time needed is $s = \frac{1}{2}at^2 \Leftrightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\cdot 4\cdot 10^8}{9\cdot 10^{-5}}} \approx 3\cdot 10^6\,\mathrm{s{=}34.5\ days}$.

To provide a suitable large momentum transfer the sail is required to have a large surface area, while maintaining as low as a mass as possible. The sail is slowly but continuously accelerated.

The main problem in solar sail design is the packing and subsequent deployment of large area of thin film.

Problem 2: Please see lecture.

Problem 3:

Problem 4:

The Euler equation, which describes the rotation of an object about its symmetry axis, say the 1-axis, is $I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$ where $T_1 = -b\omega_1$ is the component of torque along 1-axis. Because the object is symmetric about the 1-axis, we have $I_3 = I_2$ and the above equation becomes

$$I_1\dot{\omega}_1 = -b\omega_1 \quad \Rightarrow \quad \omega_1 = \tilde{\omega}\mathrm{e}^{-\frac{b}{I_1}t}$$