DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - VIII

Problem 1:

Consider rigid body with following inertia matrix

$$\mathbf{J} = \begin{bmatrix} 1500 & 0 & -1000 \\ 0 & 2700 & 0 \\ -1000 & 0 & 3000 \end{bmatrix} \text{ kg m}^2$$

about a body-fixed frame with its origin at center of mass.
Find eigenvalues, eigenvectors and rotation matrix.

Problem 2:

Assume the rotational kinetic energy has the form:

$$2T_r = 20\omega_x^2 + 30\omega_y^2 + 15\omega_z^2 - 20\omega_x\omega_y - 30\omega_x\omega_z$$

Please write the rotational kinetic energy in principle axis.

Hint:

The above rotational kinetic energy has the form $2T_r = \boldsymbol{\omega}^T \mathbf{J} \boldsymbol{\omega}$. Find the appropriate transformation so that the rotational kinetic energy in principle axis has the form $2T_r = \boldsymbol{\omega}^T \mathbf{I} \boldsymbol{\omega}$, where **I** is now a diagonal matrix.

Problem 3:

Assume to have a solid sphere with a non-uniform mass distribution. The mass density varies with the radius according to $\sigma = kr^3$ (which means the outer portion of the sphere is much denser than the inner portion). Calculate the moment of inertia about a line which passes through the center of mass and express it in terms of radius a and mass M of the sphere.

Problem 4:

Consider the moment of inertia matrix

$$\mathbf{J} = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix}$$

Show that for any three-dimensional body

$$J_{xx} - J_{yy} + J_{zz} > 0$$