DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - X

Problem 1:

Consider the spacecraft attitude control problem for a single axis. Neglecting the disturbance torque, the attitude dynamics are given by

$$I\ddot{\theta} = u\ ,$$

where *u* is the control torque. The moment of inertia is I = 1 kg · m^2. The following transient specifications are given for the closed-loop step response:

• Maximum overshoot requirement, M_p = 20 %.

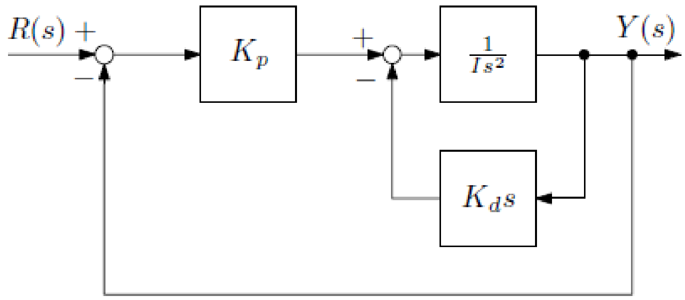
• Settling time requirement, t_s = 60 seconds.

Design a modified proportional-derivative attitude control law such that the transient specifications are satisfied.

Problem 2:

Consider the modified proportional-derivative attitude control about a single spacecraft axis as shown in the figure below, where I = 10 kg · m^2 is the corresponding moment of inertia, y = θ is the corresponding attitude angle, r = θ_d is the desired attitude angle, K_p = 0.1 is the proportional gain and K_d = 0.5 is the derivative gain. With regards to a unit step input:

- **(a)** Determine the settling time.
- **(b)** Determine the percent overshoot.
- **(c)** Determine the rise time.
- **(d)** Is it possible to reduce both the settling time and percent overshoot without changing the proportional gain? Illustrate your reasoning with a diagram.



Problem 3:

Consider the spacecraft attitude control problem for a single axis. The attitude dynamics are given by

$$I\ddot{\theta} = u + T_d$$

where *I* is the related moment of inertia, θ is the related attitude angle, *u* is the control torque applied about the axis by an actuator, and T_d is a disturbance torque. The actuator has dynamics

$$\dot{u} = -\tfrac{1}{T_a}(u - u_c)$$

where *u* is the control torque applied by the actuator, T_a is the actuator time constant, and u_c is the control torque commanded by the control law.

(a) Find the actuator transfer function.

(b) The control law is chosen to be a modified proportional-derivative attitude control law

$$u_c(t) = K_p e(t) - K_d \dot{y}(t)$$

where the plant output is the attitude angle ($y(t) = \theta(t)$), the reference signal is the desired attitude angle ($r(t) = \theta_d$), and $e(t) = r(t) - y(t)$ is the attitude error. Draw a block diagram representation of the closed-loop system.

Problem 4:

Consider a spin-stabilized spacecraft nominally spinning about the principal z-axis with spin-rate ν. The desired angular velocities are therefore $\omega_x = 0$, $\omega_y = 0$ and $\omega_z = \nu$. It may be desirable to control the spin-rate. The equations of motion are (as usual)

$$
\begin{aligned}
I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z &= T_{xc} + T_{xd} \\
I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z &= T_{yc} + T_{yd} \\
I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y &= T_{zc} + T_{zd}
\end{aligned}
$$

where $T_{\cdot c}$ are control torques, and $T_{\cdot d}$ are disturbance torques. The principal moments of inertia are $I_x = 10\ \mathrm{kg \cdot m^2}$, $I_y = 12\ \mathrm{kg \cdot m^2}$, $I_z = 8\ \mathrm{kg \cdot m^2}$.

(a) Assuming small angular velocities $\omega_x = \varepsilon_x$, $\omega_y = \varepsilon_y$, obtain the linearized equation for the spin-rate ω_z.

(b) Given that the output of interest is $y = \omega_z$, and the control input is $u = T_{cz}$, find the plant transfer function $G_p(s)$ such that

$$Y(s) = G_p(s)\left[U(s) + \hat{T}_{dz}(s)\right]$$

(c) The reference signal is the desired spin-rate r = ν. Therefore, the spin-rate error is $e = \nu - \omega = r - y$. Assuming proportional control

$$u(t) = K_p e(t)$$

Draw a block-diagram for the closed-loop system.

(d) Find the closed-loop transfer function relationships from the reference signal R(s) to the error E(s) and from the disturbance $\hat{T}_{\mathrm{dz}}(\mathrm{s})$ to the error E(s). What restriction must be placed on the proportional gain K_p to ensure asymptotic stability?

(e) Find the response e(t) to a step disturbance $T_{\mathrm{dz}}(\mathrm{t}) = \bar{T}_{\mathrm{dz}}$. What restriction must be placed on the proportional gain K_p if the steady-state error to a disturbance of magnitude $\bar{T}_{\mathrm{dz}} = 10^{-5}\,\mathrm{Nm}$ is to be kept below 1 deg/s?

(f) Find the response e(t) to a step reference signal r(t) = $\bar{\nu}$. What restriction must be placed on the proportional gain K_p if the spin-rate y(t) is to be within 2% of the desired spin-rate $\bar{\nu}$ within 10 seconds?

Problem 5 (optional):

Do the exercise self-control proposed in page 102 of lecture ADCS - X.