DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - II

Problem 1:

A satellite is in a Keplerian orbit around the Earth with perigee height above the Earth of h_p = 300 km and an apogee height above the Earth of h_a = 10000 km. Please calculate

(a) the semi-major axis *a.*
(b) the eccentricity *e.*
(c) the velocity at perigee $v_{p.}$
(d) the velocity at apogee $v_{a.}$
(e) the orbital period *T*.

Problem 2:

Consider a satellite at *perigee height* h_p = 800 km altitude above the Earth's surface, and velocity (perpendicular to the radius) of v = 8 km/s.

(a) Compute the semi-major axis *a.*
(b) Compute the eccentricity *e.*
(c) Compute the maximum radius of this orbit $r_{a.}$
(d) Compute the maximum altitude $h_{max.}$

Problem 3:

(a) What is the definition for the radial acceleration, the North-South acceleration and the East-West acceleration based on the potential formulation for the gravity field of the Earth?

(b) What is the general equation for the radial acceleration due to the $J_{2,0}$ term as well as due to the $J_{3,0}$ term for a satellite?
To derive the acceleration calculate first Legendre polynomial

$$P_n(x) = \frac{1}{(-2)^n n!} \frac{\partial^n}{\partial x^n}(1-x^2)^n \quad \text{and set} \quad P_{20}(x)|_{x=\sin\phi} = P_{20}\left(\sin\phi\right)$$

and later the potential.

(c) What is the general equation for the North-South acceleration due to the $J_{2,0}$ term as well as due to the $J_{3,0}$ term for a satellite?

(d) What is the general equation for the East-West acceleration due to the $J_{2,0}$ term as well as due to the $J_{3,0}$ term for a satellite?

(e) What is the numerical expression for radial acceleration due to $J_{2,0}$ term for a satellite at 400 km altitude?

Problem 4:

(a) What is the main characteristic of sun-synchronous orbit?

(b) Show that for sun-synchronous orbit, which is also constant in the argument of the perigee (frozen orbit), with semi-major axis *a*, the eccentricity *e* is given by

$$e = \left[1 - \left(\frac{3J_2R_e^2}{2\dot{\Omega}_{avg}}\sqrt{\frac{\mu}{5a^7}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

where $\dot{\Omega}_{avg} = \dfrac{360^o}{year} = \dfrac{2\pi}{year}$.

(c) Compute the eccentricity and radii of perigee for a geocentric sun-synchronous frozen orbits with semi-major axes *a* = 10000 km, *a* = 15000 km and *a* = 20000 km. Conclude that there is a range of semi-major axes for which a geocentric sun-synchronous frozen orbit is possible.

(d) Sketch a plot of semi-major axis versus radius of perigee for a geocentric sun-synchronous frozen orbit.

(e) Find the minimum value of *a* for which a geocentric sun-synchronous frozen orbit is possible.

(f) Using an iterative procedure, determine the maximum value of *a* for which a geocentric frozen orbit is possible, given that the orbit should stay at least 200 km above the Earth.