DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS – III

Problem 1:

(a) What is solar radiation pressure?
(b) What is the force of a flux of energy of 1000 W/m^2 normal to the plane of 1 m^2 of total absorbent material for a satellite?
(c) What would happen if the material would be a mirror (total reflecting)?
(d) Assume the solar luminosity is given by $L_{\odot} = 3.9 \times 10^{26}$ W. At what distance from the Sun is the satellite? Compare the result to the Sun-Earth distance.

Problem 2:

Explain qualitatively what is the effect of atmospheric drag on a satellite in an orbit with large eccentricity?

Problem 3:

In Lecture ADCS – III we have seen the restricted three body problem. Please make a figure showing the five Lagrange equilibrium points for the Sun-Jupiter configuration. Can you say something about the stability? Are there objects in these stability points?

Problem 4:

(a) Calculate the orbit radius of a geostationary satellite? Use sidereal day T = 86164 s.

(b) What is the definition for the East-West acceleration based on the potential formulation for the gravity field of the Earth?

(c) We have seen that the perturbation of the non-spherical part of the Earth´s gravitational potential can be written as

$$U_{pert}(r,\lambda,\phi) = \frac{GM}{r}\left(\sum_{n=2}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_{n0}(\sin\phi) + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n \left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right] P_{nm}(\sin\phi)\right) \qquad (1)$$

Show that the perturbation of the non-spherical part of the Earth´s gravitational potential can be also written as

$$U_{pert}(r,\lambda,\phi) = \frac{GM}{r}\left(\sum_{n=2}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_{n0}(\sin\phi) + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n J_{nm} P_{nm}(\sin\phi)\cos m(\lambda-\lambda_{nm})\right) \qquad (2)$$

when the following definitions are used

$$J_{nm} = \sqrt{C_{nm}^2 + S_{nm}^2} \quad , \quad \cos(m\lambda_{nm}) = \frac{C_{nm}}{J_{nm}} \quad , \quad \sin(m\lambda_{nm}) = \frac{S_{nm}}{J_{nm}}$$

(d) What is the general equation for the East-West acceleration due to the J_{22} term for a satellite using e.g. the equation (2)?

The Legendre polynomials are given by

$$P_n(x) = \frac{1}{(-2)^n n!}\frac{\partial^n}{\partial x^n}(1-x^2)^n$$

$$P_{nm}(x) = (1-x^2)^{m/2}\frac{\partial^m P_n(x)}{\partial x^m}$$

Calculate first the U_{22} potential and later the East-West acceleration.

(e) What is the numerical equation for the East-West acceleration due to J_{22} term for a geostationary satellite?

$$C_{22} = 1.57\cdot 10^{-6} \quad , \quad S_{22} = -0.90\cdot 10^{-6} \quad , \quad \lambda_{22} = -14.9^o$$

(f) What are the locations of equilibrium points?

(g) Which points are stable or unstable?

Problem 5 (optional):

Do the exercise proposed in page 63 of Lecture ADCS - III.