DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - IXA

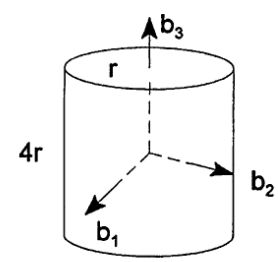
Problem 1:

Consider a cylinder of mass *m*, radius *r*, length 4*r* (assume uniform mass distribution).

The body is spinning with initial conditions

$$\vec{\boldsymbol{\omega}}(t=0)=\Omega_0\left(\frac{3}{5}\vec{\mathbf{b}}_1+\frac{4}{5}\vec{\mathbf{b}}_3\right).$$

Solve for **ω**(t).



Problem 2:

A spacecraft is launched into a low Earth orbit. The spacecraft principal moments of inertia are $I_x = 98\ \mathrm{kg\cdot m^2}$, $I_y = 102\ \mathrm{kg\cdot m^2}$ and $I_z = 150\ \mathrm{kg\cdot m^2}$. For stability, the launch vehicle deploys the spacecraft such that it is in a major axis spin when released, with ω_z=0.5 rad/s. Because no deployment is prefect, the spacecraft also has some angular velocity about the other two principle axes, given by ω_x = 0.1 rad/s and ω_y = 0.02 rad/s. Making appropriate approximations:

a) Describe the resulting spacecraft attitude motion if there are no disturbance torques.
b) Determine the nutation angle.
c) Determine the precession rate.

Problem 3:

Explore the polhode trajectory (intersection of kinetic energy ellipsoid and angular momentum ellipsoid) for the torque-free rotation for the two particular body geometries where the body is either axisymmetric with $I_1 = I_2$, or the body principal inertias are all equal with $I_1 = I_2 = I_3$. This last condition occurs if the body is a homogenous cube and the body frame B is a principal coordinate frame, or the body is a homogenous sphere with any body fixed frame B.

Problem 4:

The principal inertias of a rigid satellite are given by

$$I_1 = 210 \text{ kg·m}^2, \; I_2 = 200 \text{ kg·m}^2 \text{ and } I_1 = 118 \text{ kg·m}^2 .$$

At time t_0 the body angular velocity vector is $\omega=(0.05, 0.02, -0.02)^T$ rad/s. Numerically solve the resulting torque-free motion for 100 s and plot the resulting attitude in terms of the 3-2-1 Euler angles. Assume the initial Euler angles to be zeros. The kinematic equation for the 3-2-1 Euler angles is given by

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_2} \begin{bmatrix} \cos\theta_2 & \sin\theta_1 \sin\theta_2 & \cos\theta_1 \sin\theta_2 \\ 0 & \cos\theta_1 \cos\theta_2 & -\sin\theta_1 \cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$