DOC
Dinámica Orbital
y
Control de Actitud

Salvatore Mangano

MÁSTER UNIVERSITARIO EN SISTEMAS ESPACIALES
POR LA UNIVERSIDAD POLITÉCNICA DE MADRID

ADCS
Attitude Determination & Control System
and
Orbital Mechanics

Salvatore Mangano

MÁSTER UNIVERSITARIO EN SISTEMAS ESPACIALES
POR LA UNIVERSIDAD POLITÉCNICA DE MADRID

My profile

PhD in physics at ETH Zurich

18 years of experience in quantitative analytical research in five European institutes

Participation in more than 80 relevant publications specialized in particle and astroparticle physics

E-mail: salvatore.mangano@upm.es
salvatore.mangano@ciemat.es

URL: https://moodle.upm.es

Course overview

Lecture 1: **Introduction and Spacecraft environment**

Orbital Mechanics

Lecture 2: **Orbital Mechanics and Perturbations**

Lecture 3: **Perturbations and Coordinate Systems**

ADCS

Lecture 4: **Attitude Hardware and Attitude Concepts**

Lecture 5: **Space Project**

Lecture 6: **Attitude Kinematics**

Lecture 7: **Attitude Determination**

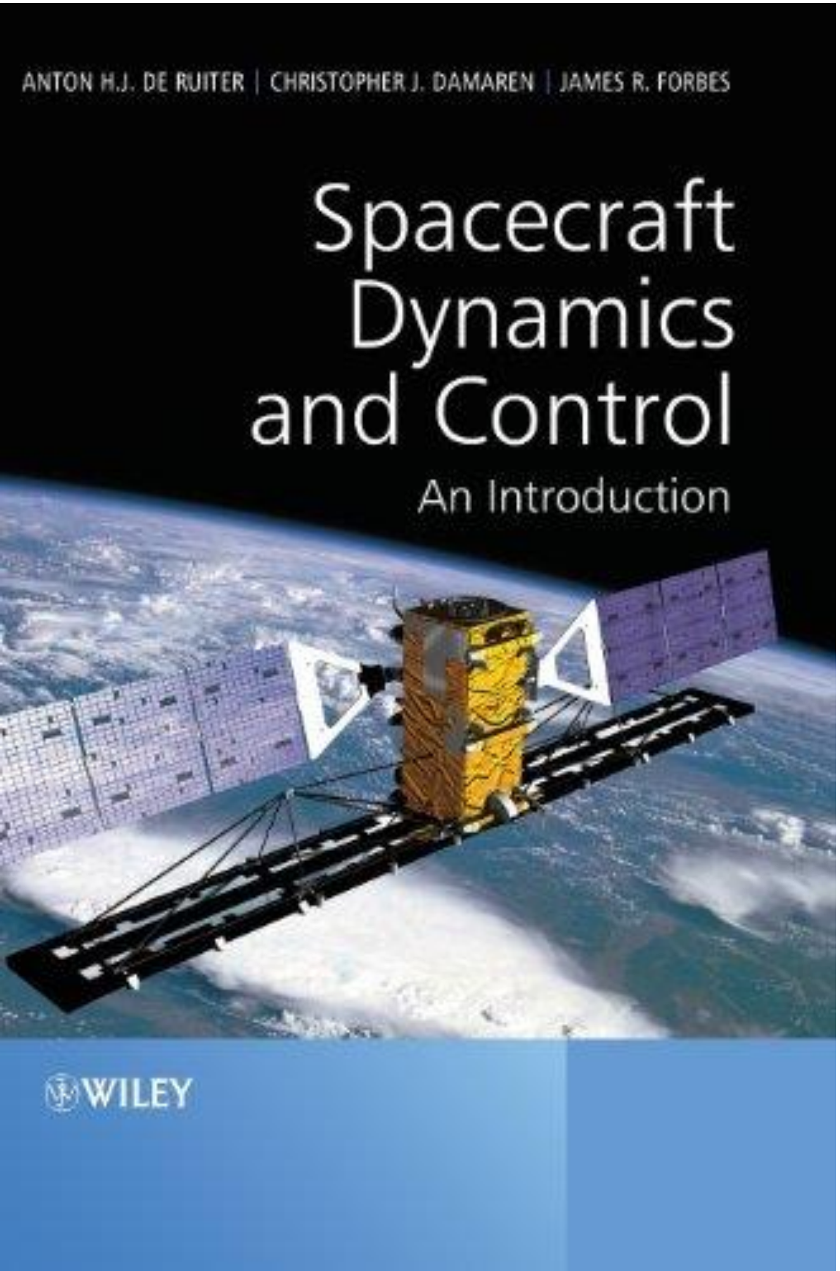
Lecture 8: **Rigid Body Dynamics**

Lecture 9A: **Attitude Dynamics**

Lecture 9B: **Attitude Dynamics**

Lecture 10: **Attitude Control**

Bibliography I



Part of:		Part of:
Chapter 1	→	Lecture 6
Chapter 2	→	Lecture 8
Chapter 3	→	Lecture 2 (short)
Chapter 7	→	Lecture 2
Chapter 10	→	Lecture 3
Chapter 11	→	Lecture 4
Chapter 12	→	Lecture 4
Chapter 13-16	→	Lecture 9
Chapter 17	→	Lecture 10
Chapter 25	→	Lecture 7

Bibliography II

- James R. Wertz (1978)

  "*Spacecraft attitude determination and contro*l"
- James R. Wertz and Wiley Larson (1999)

  "*Space mission analysis and design*"
- B. Wie (2008)

  "*Space vehicle dynamics and control*"
- D. A. Vallado (2007)

  "Fundamentals of astrodynamics and applications"
- Peter C. Hughes (1986)

  "*Spacecraft attitude dynamics*"
- William T. Thomson (1986)

  "*Introduction to space dynamics*"

Bibliography III

- M. J. Sidi (1997)

  “*Spacecraft dynamics and control*”
- T. Logsdon (1998)

  “*Orbital mechanics*”
- P. Fortescue and J. Stark (1991)

  “*Spacecraft systems engineering*”
- H. Schaub and J. L. Junkins (2009)

  “*Analytical mechanics of space systems* ”
- V. L. Pisacane and R. C. Moore (1994)

  “*Fundamentals of space systems*”
- Malcolm Macdonald et al. (2014)

  “*International handbook of space technology*”
- Charles D. Brown (2002)

  “*Elements of spacecraft design*”

Useful information

- Homework **50 %**

  - Written reports

  - (Solve problems and oral presentation)
- Exam (written) **50 %**

  - Final exam

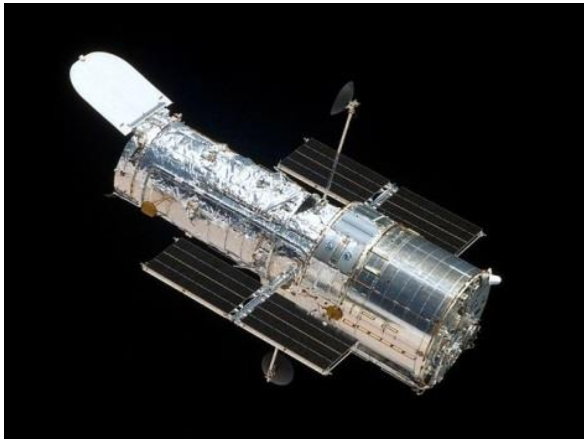
  - Tuesday 14/01/2020   10:00-13:00
- Number of credit points **4.5**

Introduction to Lecture: Basic Ideas

What is spacecraft attitude?

Most spacecraft have instruments or antennas that must be pointed in specific directions

- Hubble must point its main telescope
- Communications satellite must point their antennas



Orientation of spacecraft in space is called its **attitude**

To **control attitude**, spacecraft operator (or an algorithm) must have ability to:

- Determine current attitude
- Determine error between current and desired attitude
- Apply torques to remove errors

Spacecraft attitude determination and control

Principle requirement is to point spacecraft payload (instrument, antenna, solar array), therefore spacecraft needs an **attitude determination and control system** (ADCS)

Attitude **determination**: use of **sensors** to estimate attitude

Knowledge of **kinematics** needed to do determination

Attitude **control:** maintain specific attitude with **actuators**

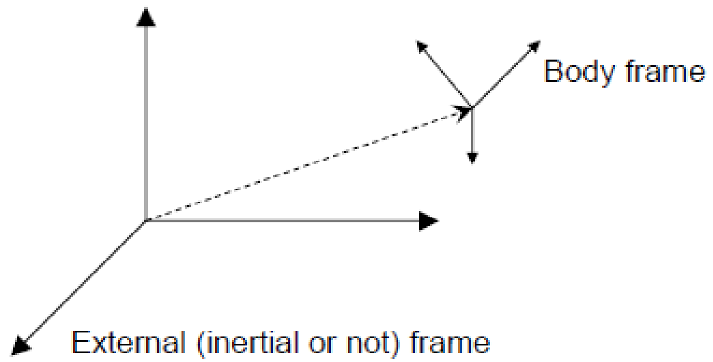
Knowledge of **dynamics** needed to do control

Mission requirements dictate spacecraft sensor configuration

Attitude determination

Determine attitude or angular orientation of a body-fixed coordinate frame with respect to an external known (inertial or not) frame

Generally involves finding a rotation matrix



Requires two or more attitude sensors:

- Sun sensor (Direction to Sun)
- Earth horizon sensor (Earth´s shape)
- Magnetometer (Earth´s magnetic field)
- Star tracker (Inertial frame of universe)
- etc.

Orbital motion versus attitude motion

- Spacecraft are free bodies
- Free to translate and rotate
- Mathematically described by six degrees of freedom
- Orbital motion is study of spacecraft translation
- Attitude motion is study of spacecraft rotation
- Orbital and attitude motions are uncoupled and treated separately
- This course covers mainly attitude motion (Lecture 4 to 10)
- Orbital motion mostly in previous course (121 Entorno espacial y análisis de misión) and in this course in Lecture 2 + 3

Orbital dynamics versus attitude dynamics

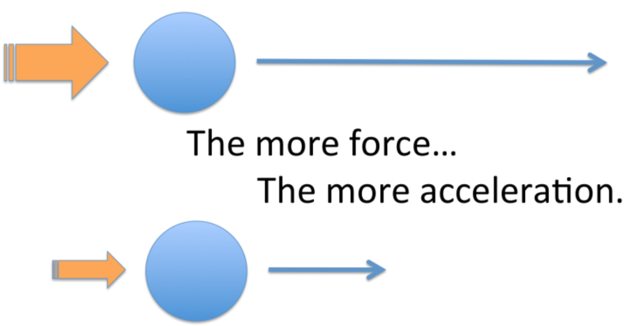
Position and orientation of rigid body in space has **six degrees of freedom** (3 components for translation and 3 components for rotation)

Orbital dynamics and control

Linear momentum

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$$ (Newton 1687)

Motion of center of mass



Attitude dynamics and control

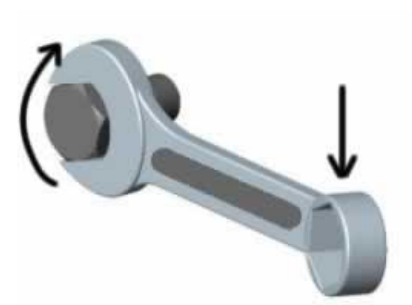
Angular momentum

$$\frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \vec{T}$$ (Euler 1736)

$$\vec{h} \equiv \vec{r} \times \vec{p}$$

$$\vec{T} \equiv \vec{r} \times \vec{F}$$

Motion relative to center of mass



Other possible notation for $\frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \vec{T}$ $\quad \frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \vec{N}$ $\quad \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \vec{M}$

Orbital control versus attitude control

Define nominal orbit for satellite

Satellite should maintain nominal orbit

Orbit is controlled by
applying **forces** to satellite

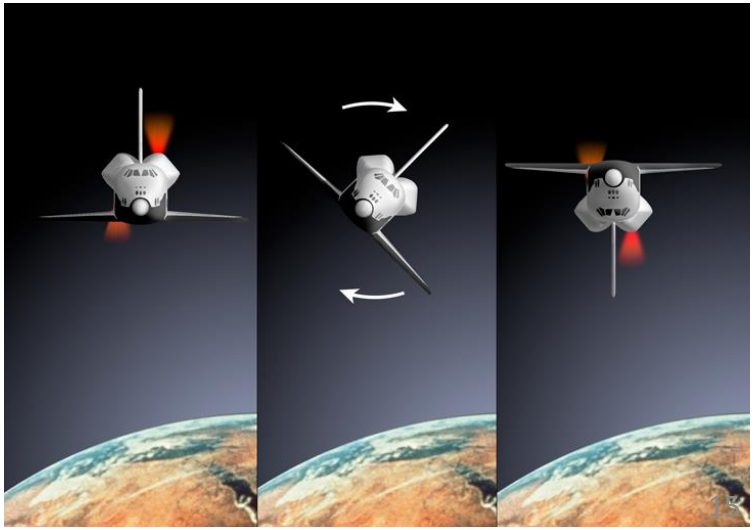
Forces are applied by velocity actuators

Define nominal attitude for satellite

Satellite should maintain nominal attitude

Attitude controlled by
applying **torques** to satellite

Torques are applied by attitude actuators



Attitude motion: Euler´s equation

Many small satellites can be treated as single rigid bodies

Motion described by Euler´s equation

$$\frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \vec{T}$$

Euler´s equations are simply

rate of change of angular momentum = torque

relation expressed **in body coordinates**

Euler equation is a vector equation (3 scalar equations)

$$\mathbf{I}\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} + \vec{\omega}\times\mathbf{I}\cdot\vec{\omega} = \vec{T}$$

Note: reason to use body coordinates is to keep moment of inertia expression simple (constant)

Euler´s equation

Euler´s **vector differential equation**

$$\frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \vec{T}$$

$\vec{h}$ is angular momentum

$\vec{T}$ is torque

Becomes **matrix differential equation** when expressed in body-fixed reference frame

$$\mathbf{I}\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} + \vec{\omega} \times \mathbf{I} \cdot \vec{\omega} = \vec{T}$$

$\mathbf{I}$ is inertia matrix

$\vec{\omega}$ is angular velocity

Becomes **coupled nonlinear differential equation** when expressed in principle reference frame

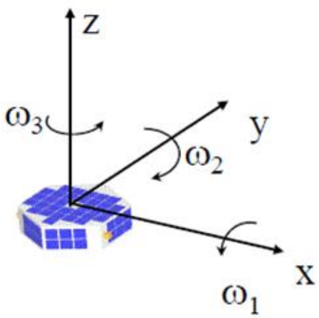
$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$

Attitude dynamics and kinematics

Attitude dynamics of spacecraft

Euler´s equations give evolution of angular velocity (as seen in body coordinates). For some problems this may be all we need.

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$



Attitude kinematics of spacecraft

To find attitude, need to integrate again using equation expressing angular velocity in terms of attitude parameterization

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_2} \begin{bmatrix} \cos\theta_2 & \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 \\ 0 & \cos\theta_1\cos\theta_2 & -\sin\theta_1\cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Orbit and attitude are interdependent

In general orbit and attitude are interdependent:

Example:

1. In low altitude Earth orbit, attitude will affect atmospheric drag which will affect orbit
2. Orbit determines spacecraft position which determines both atmospheric density and magnetic field strength, which will again affect attitude

But this dynamic coupling is normally ignored and time history of spacecraft position is assumed to be known and to be an input for attitude dynamics

What is ADCS?

Orientation of spacecraft axes with respect to inertial reference frame

Attitude Determination:
Knowledge of spacecraft **orientation** with respect to frame of reference

Attitude Control:
Process of achieving and maintaining desired **orientation** or attitude rate

Attitude Determination and Control System
- Provide rates stabilization and pointing
- Controls spacecraft body axes such that errors are within defined limits
- Provides spacecraft attitude knowledge to support mission objectives

ADCS - I
Spacecraft Environment

Outline

Gravity

Sun

- Solar constant
- Solar radiation pressure
- Solar wind

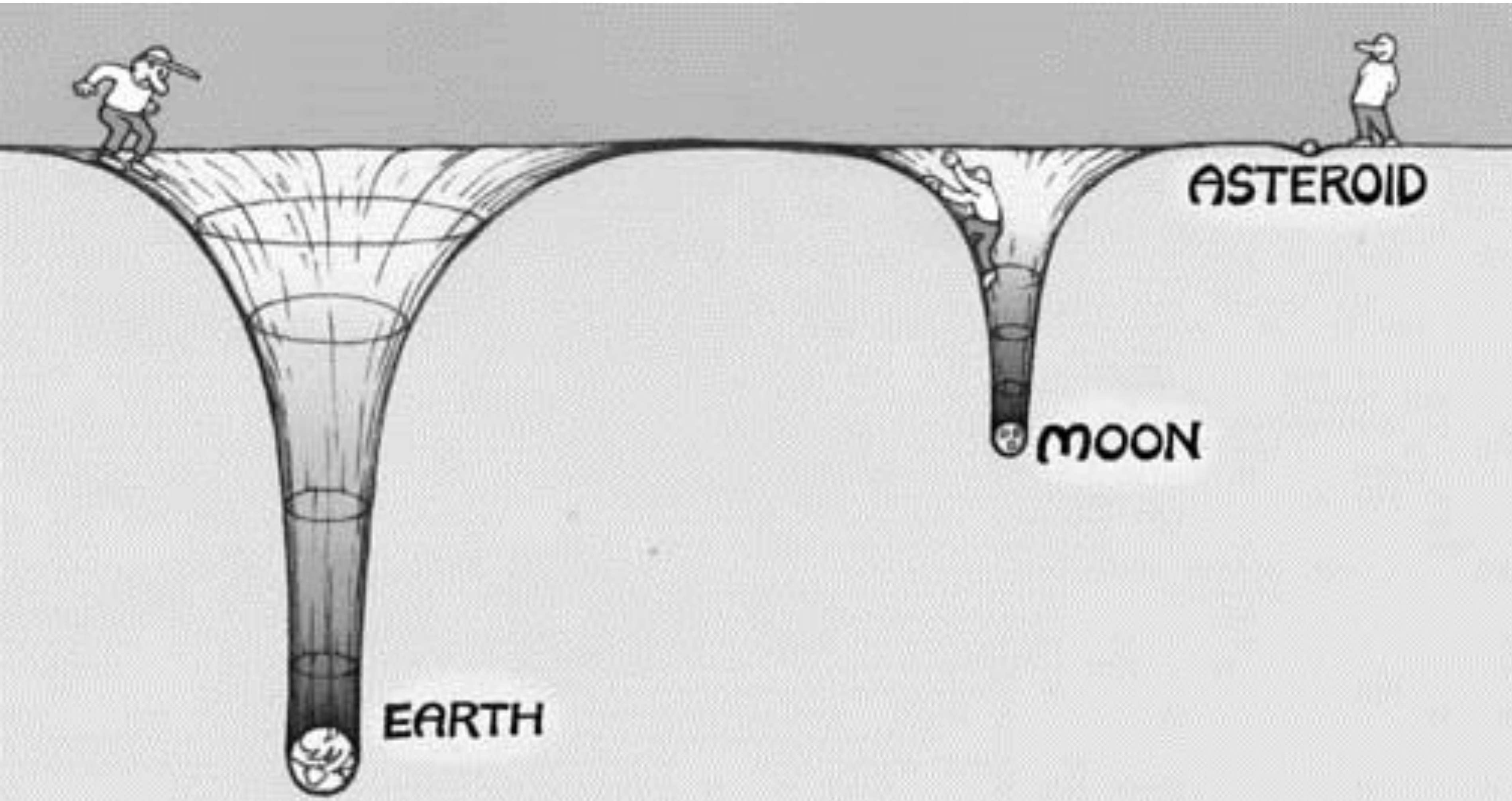
Earth atmosphere

Earth magnetic field

Spacecraft environment relevant to ADCS:

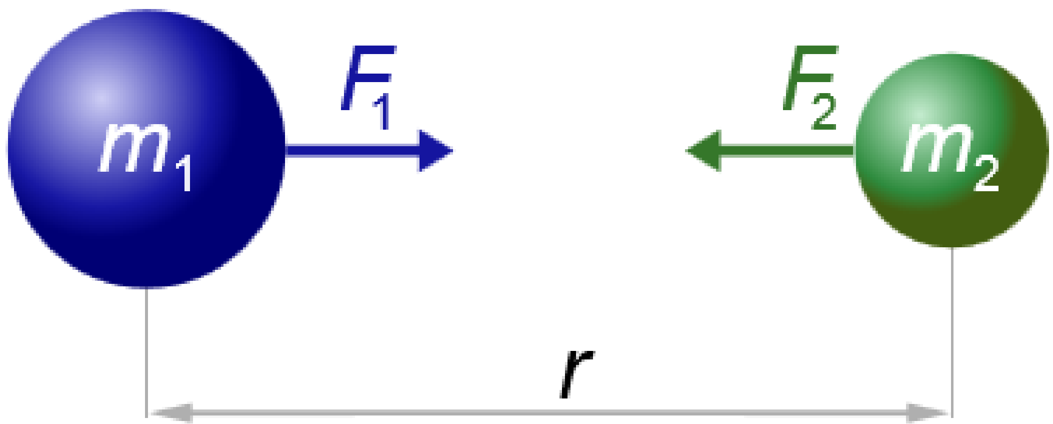
Not detailed description

Gravity



Gravitational force

Every point mass **attracts** every other point mass by a force pointing along the line intersecting both points. The force is proportional to the **product of two masses** and **inversely proportional to square of distance** between the point masses:



$$\boxed{F_1 = F_2 = G\frac{m_1 m_2}{r^2}}$$

Gravity

$$F = G\frac{m_1 m_2}{r^2} \quad \text{with} \quad G = 6.674\cdot 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$$

At surface of Earth

$$F_{Earth} = G\frac{m\,m_{Earth}}{{R_{Earth}}^2} \qquad m_{Earth} = 5.972\cdot 10^{24}\ \mathrm{kg}$$

$$g(R_{Earth}) = G\frac{m_{Earth}}{{R_{Earth}}^2} = 9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \qquad G m_{Earth} = \mu_{Earth} = 398600\ \mathrm{km}^3\ \mathrm{s}^{-2}$$

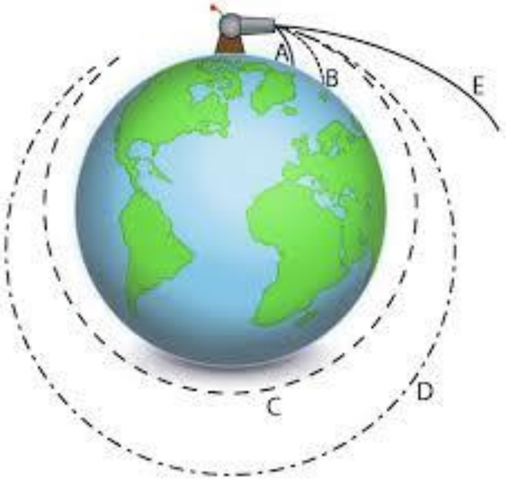
What is acceleration from Earth at given satellite distance?
Why does satellite (or Moon) not fall upon Earth?

Side remark:
Limitations of Newton's gravity → Need of Einstein general relativity
E.g. perihelion precession of Mercury, gravitational lensing,
gravitational time delay, gravitational time dilation

Newton´s cannonball

Earth has curvature with about 8000 m horizontal distance to 5 m height
Assume: Projectile fired horizontally at 8000 m/s

When does projectile hit ground if we assume no air resistance?



Projectile vertical distance traveled in 1 second

$$y = y_0 + v_{0y}t + \tfrac{1}{2}gt^2$$

$$y - y_0 = \tfrac{1}{2}gt^2 = \tfrac{1}{2}9.8(1)^2 = 4.9\ \mathrm{m}$$

Projectile horizontal distance traveled in 1 second

$$x = x_0 + v_{0x}t = 8000\ \mathrm{m}$$

Projectile follows Earth curvature and never hits ground

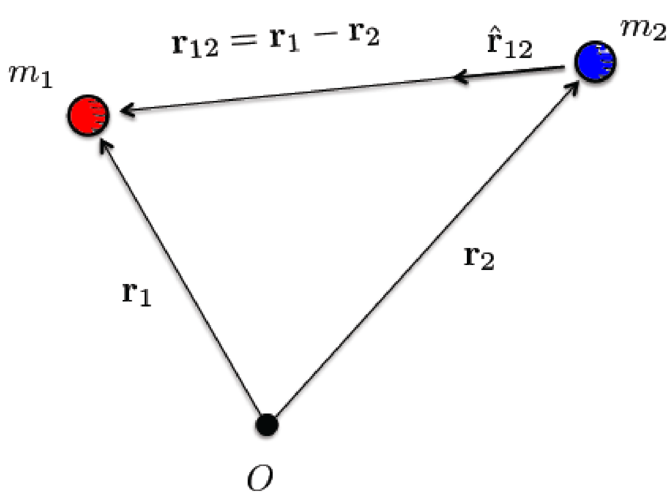
Travels 8000 m horizontal and drops 5 m vertical → It´s a satellite

→ Satellite misses ground all the time

(Assume gravity, otherwise satellite follows a straight line)

Gravitational force in vector form

$$\vec{F}_{12} = -G\frac{m_1 m_2}{|\vec{r}_{12}|^2}\frac{\vec{r}_{12}}{|r_{12}|}$$

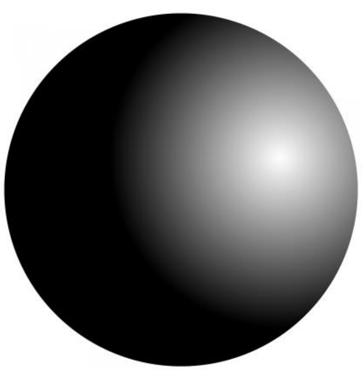


Spherically symmetric mass distribution

An object with **spherically-symmetric** distribution of mass exerts same gravitational attraction on external bodies as if all the object mass were **concentrated at a point** at its center

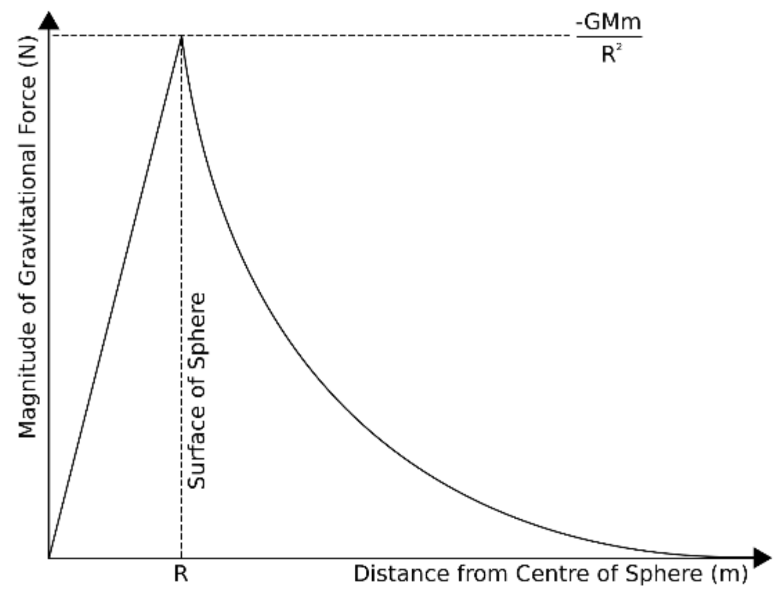
$$F_{R_{\oplus}>r} = \frac{GMm}{R^3_{\oplus}} r \qquad F_{R_{\oplus}<r} = \frac{GMm}{r^2}$$

Sphere of mass M



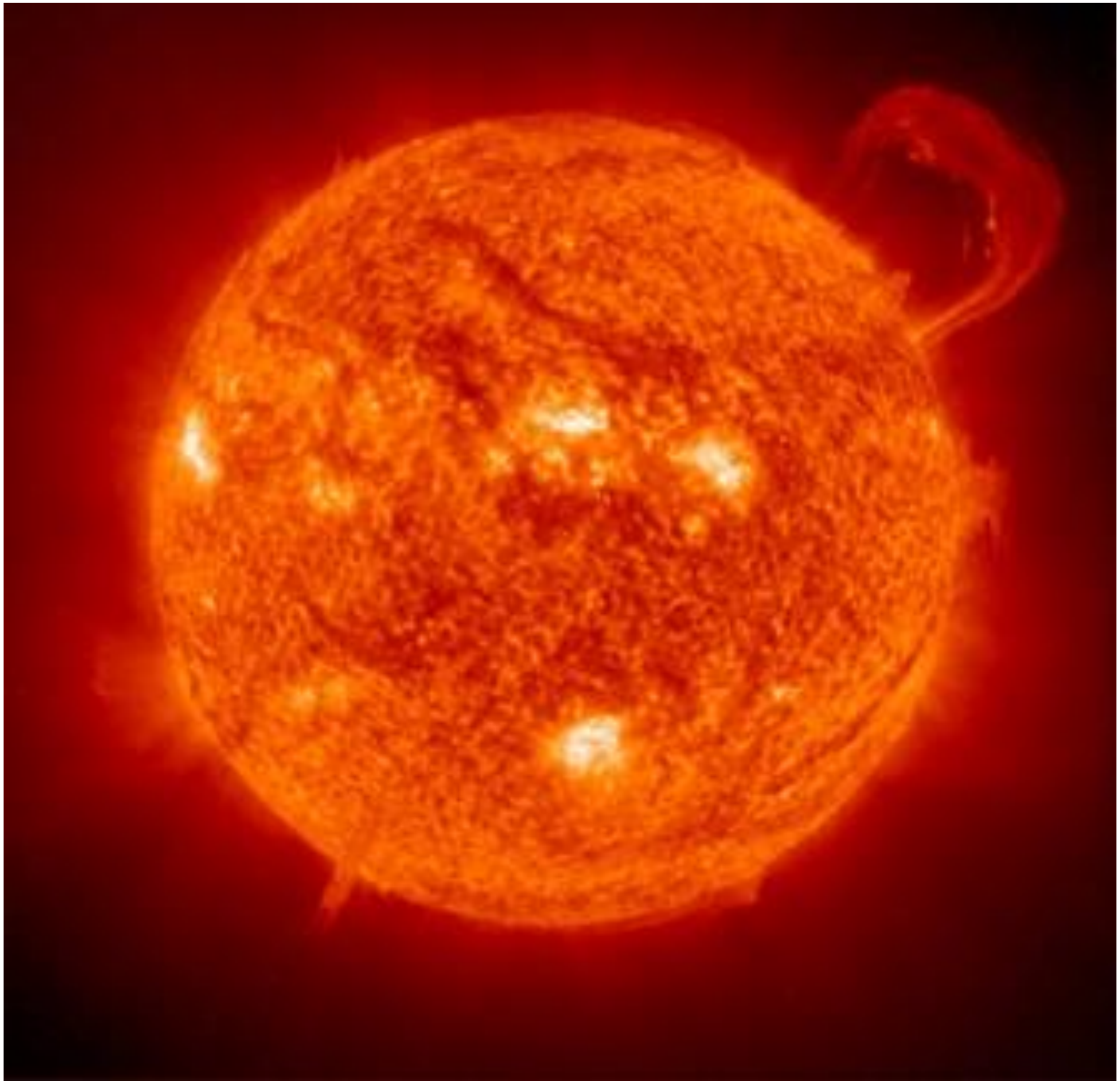
=

Point mass M



$$M(r) = M \frac{r^3}{R^3_{\oplus}}$$

Sun



Sun

All heat of solar system from Sun (excluding planetary radioactive decays)

Sun dominates space environment of whole solar system

Sun mass is 99.9% of the total solar system

Sun is fusion reactor with surface temperature of 5800 K

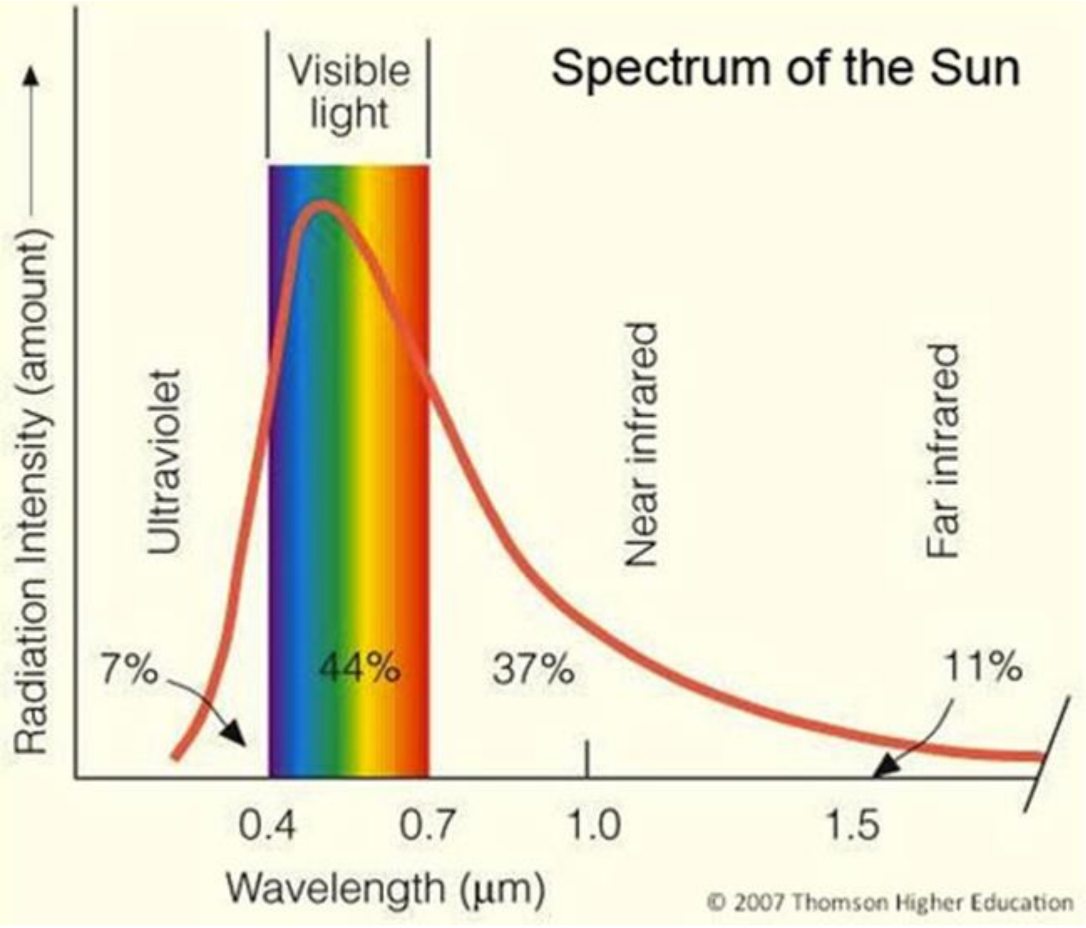
Light spectrum is nearly that of black body

Sun is ordinary star and is one of 10^{11} stars from our galaxy

Next nearest star is at 3.5 light years away (1 light year = 10^{13} km)

Radius = 7×10^5 km and Sun at 150×10^6 km from Earth (500 light seconds)

Solar spectrum



Sun emits electromagnetic radiation with characteristic spectral shape (red curve)

Strongest output in visible light (44 %)

Solar spectrum covers essentially entire electromagnetic spectrum, but mostly concentrated in wavelength interval between 300 nm and 1000 nm

Solar spectrum goes from X-rays to radio waves

Energy of photon (light)

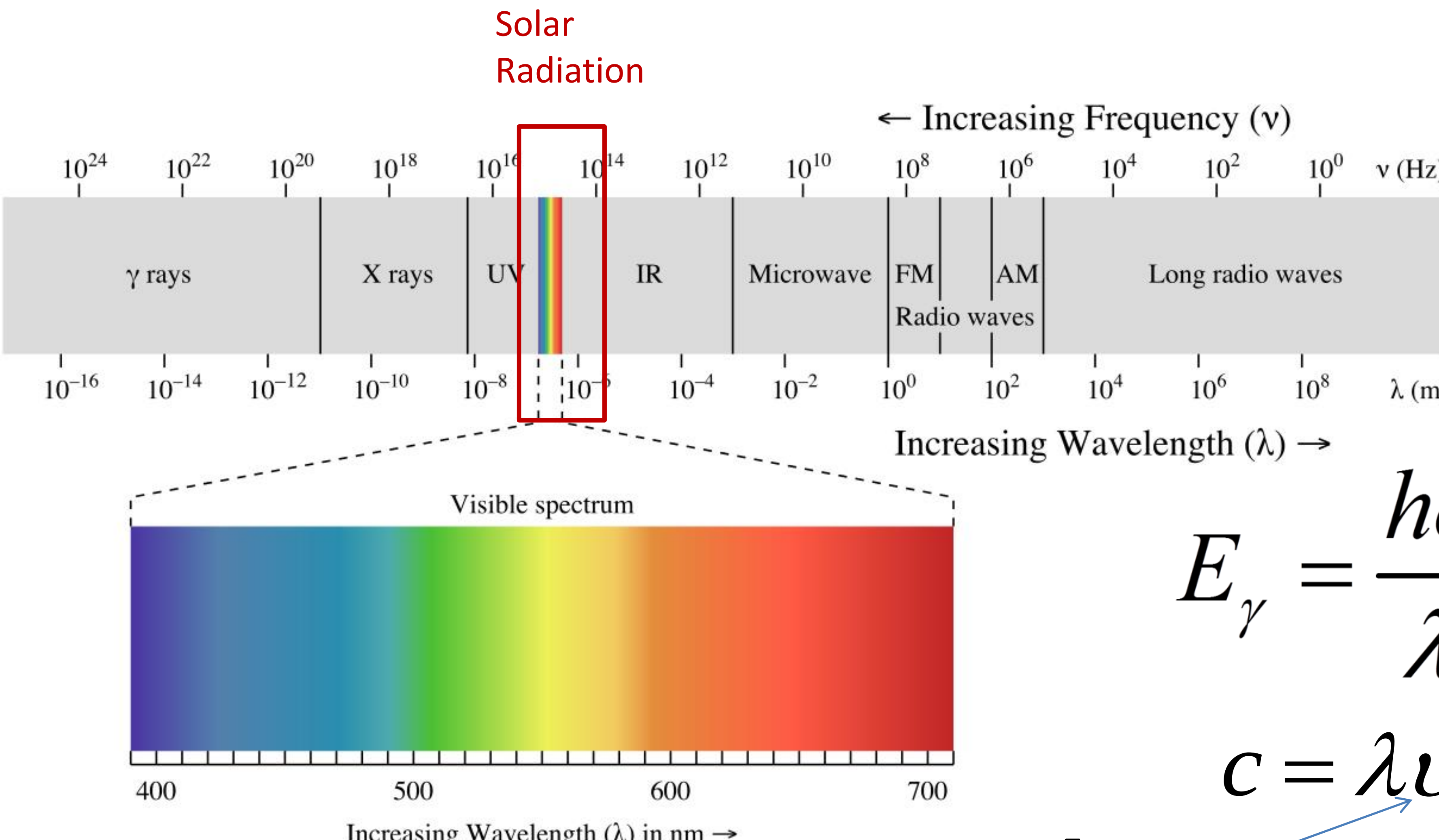
Planck constant **Speed of light**

Energy of photon $$E_\gamma = \frac{hc}{\lambda}$$

Wavelength

$$h = 6.6 \cdot 10^{-34}\ \mathrm{J \cdot s}$$

Electromagnetic spectrum

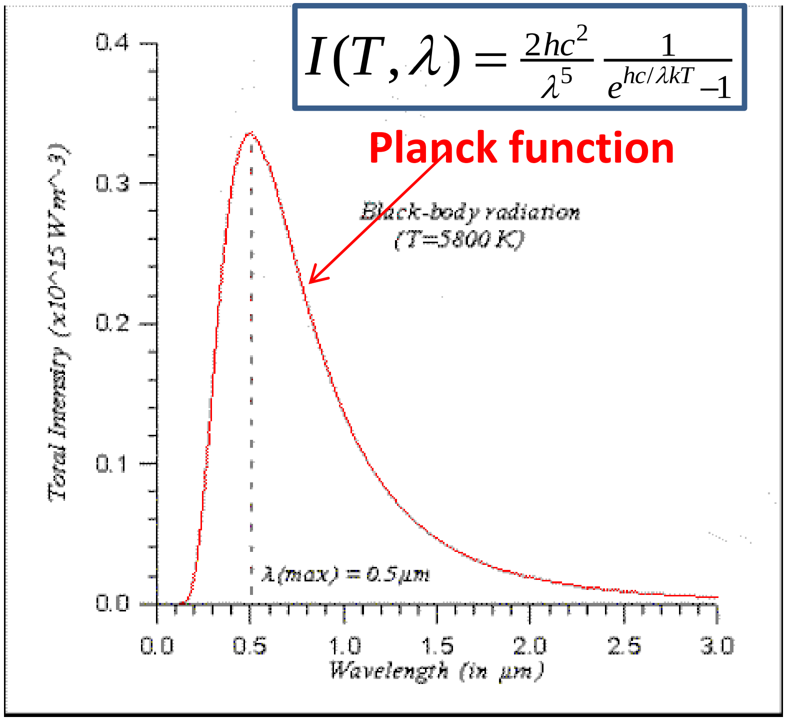


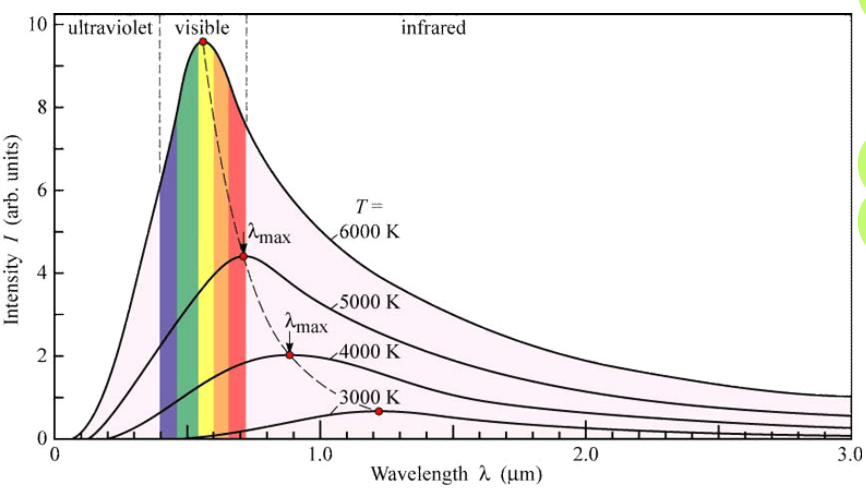
$$E_\gamma = \frac{hc}{\lambda}$$

$$c = \lambda \upsilon$$

Frequency

Black-body radiation





Black-body is object that emits (or absorbs) electromagnetic radiation with 100% efficiency at all wavelengths

Black-body radiation is electromagnetic radiation emitted by a body with definite temperature
Amount of radiation emitted by a black-body depends on temperature of blackbody

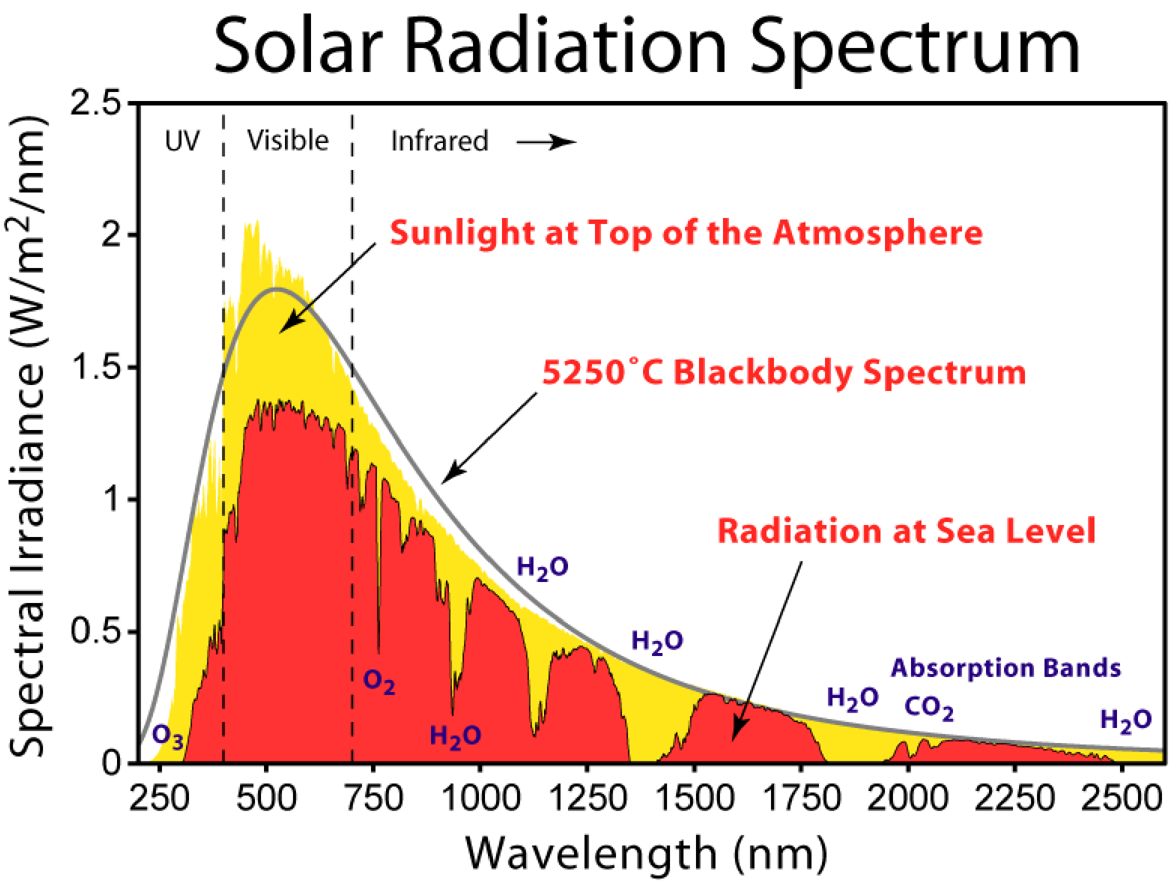
Black-body spectrum given by **Planck function** which describes electromagnetic radiation emitted by black-body in thermal equilibrium at definite temperature

Peak wavelength of blackbody spectrum is inversely proportional to absolute temperature T **(Wien´s law)**

$$\lambda_{\max} = \frac{const}{T}$$

(Derivation: differentiate Planck function with respect to wavelength and set result to zero)

Sun nearly black-body



Solar radiation spectrum above atmosphere and at surface

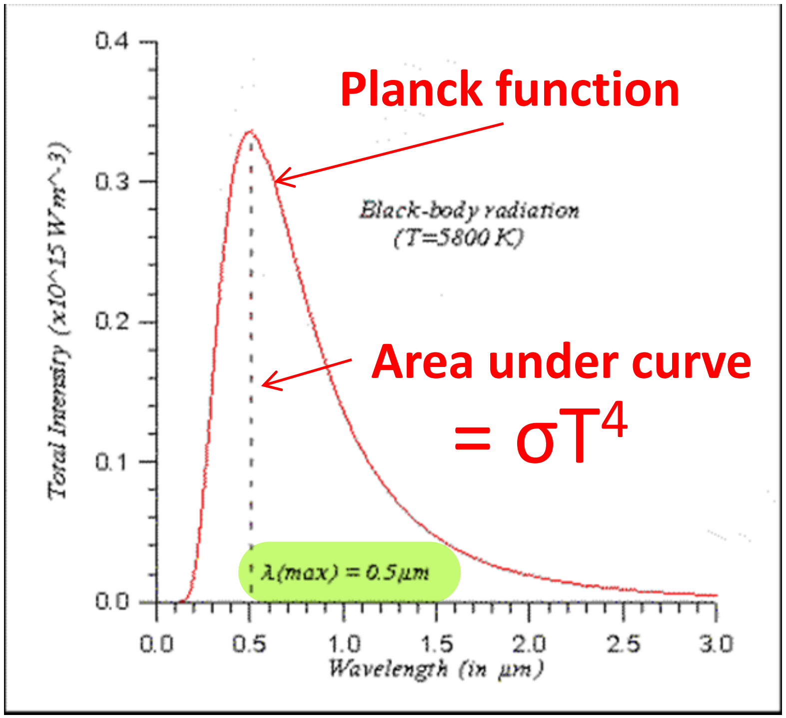
In yellow true extraterrestrial solar radiation spectrum

Black line is black-body spectrum (Planck function) for $T \approx 5525$ K

In red solar radiation spectrum at sea level after absorption by gases in atmosphere

X-rays and gamma-rays are produced but comprise very small amounts of Sun's total output power

Integral of Planck function
Stefan-Boltzmann law



Integration of Planck function over all wavelengths is given by **Stefan-Boltzmann law**

Total energy emitted by blackbody is proportional to **fourth power** of body´s **temperature**

Intensity $$\boxed{I = \sigma T^4}$$

with $$\sigma = 5.67 \cdot 10^{-8} \, \frac{\mathrm{W}}{\mathrm{m^2 K^4}}$$

Energy emitted from Sun

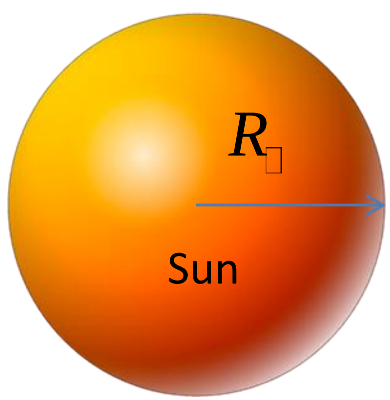
Luminosity = Surface times intensity

$$\boxed{L_{\odot} = AI = 4\pi R_{\odot}^{2}\sigma T^{4}}$$

Luminosity = Total energy per second

Luminosity for Sun

$$L_{\odot} = 4\pi(7\times10^{8})^{2}5.67\cdot10^{-8}\cdot5800^{4} = 3.9\times10^{26}\,\mathrm{W}$$



From Solar luminosity to solar flux density

Solar luminosity [W]

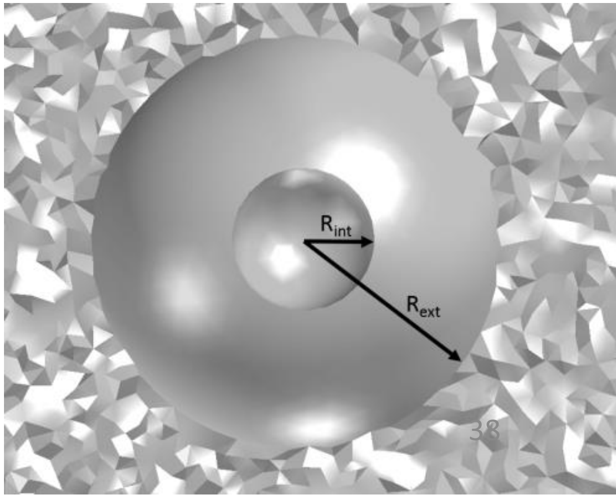
Sun emits constant flux of energy per time

$$L_{\odot} = 3.9 \times 10^{26}\ \mathrm{W}$$

Solar flux density [W/m²]

Solar energy per unit area for a sphere with radius *r*

$$\boxed{\Phi(r) = \frac{L_{\odot}}{4\pi r^2}}$$

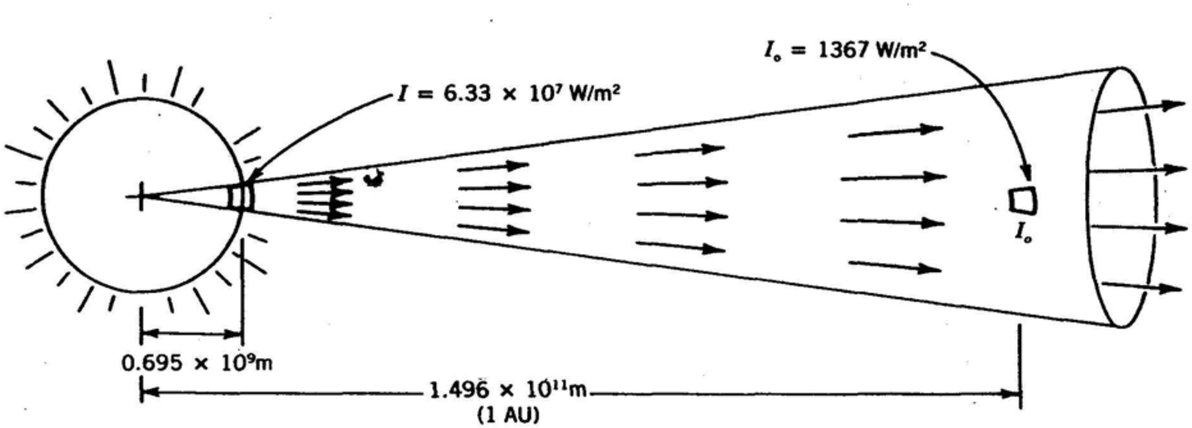


Solar flux density reaching Earth

Solar constant [W/m²]

Solar flux density at mean distance Earth-Sun

$$\Phi = \frac{L_{\odot}}{4\pi r^2} = 1362 \frac{\mathrm{W}}{\mathrm{m}^2}$$



Solar power passing through concentric spherical surfaces with different radius

$$\boxed{\Phi(r) = \Phi_0 \frac{r_0^{\,2}}{r^2}}$$

Solar flux annual variation model due to distance variation

Constant model:

Mean solar flux at 1 AU:

Max solar flux (in early January):

Min solar flux (in early July):

Solar constant:

$$\Phi = 1362 \pm 10 \ \frac{\mathrm{W}}{\mathrm{m}^2}$$

$$\Phi = 1412 \ \frac{\mathrm{W}}{\mathrm{m}^2}$$

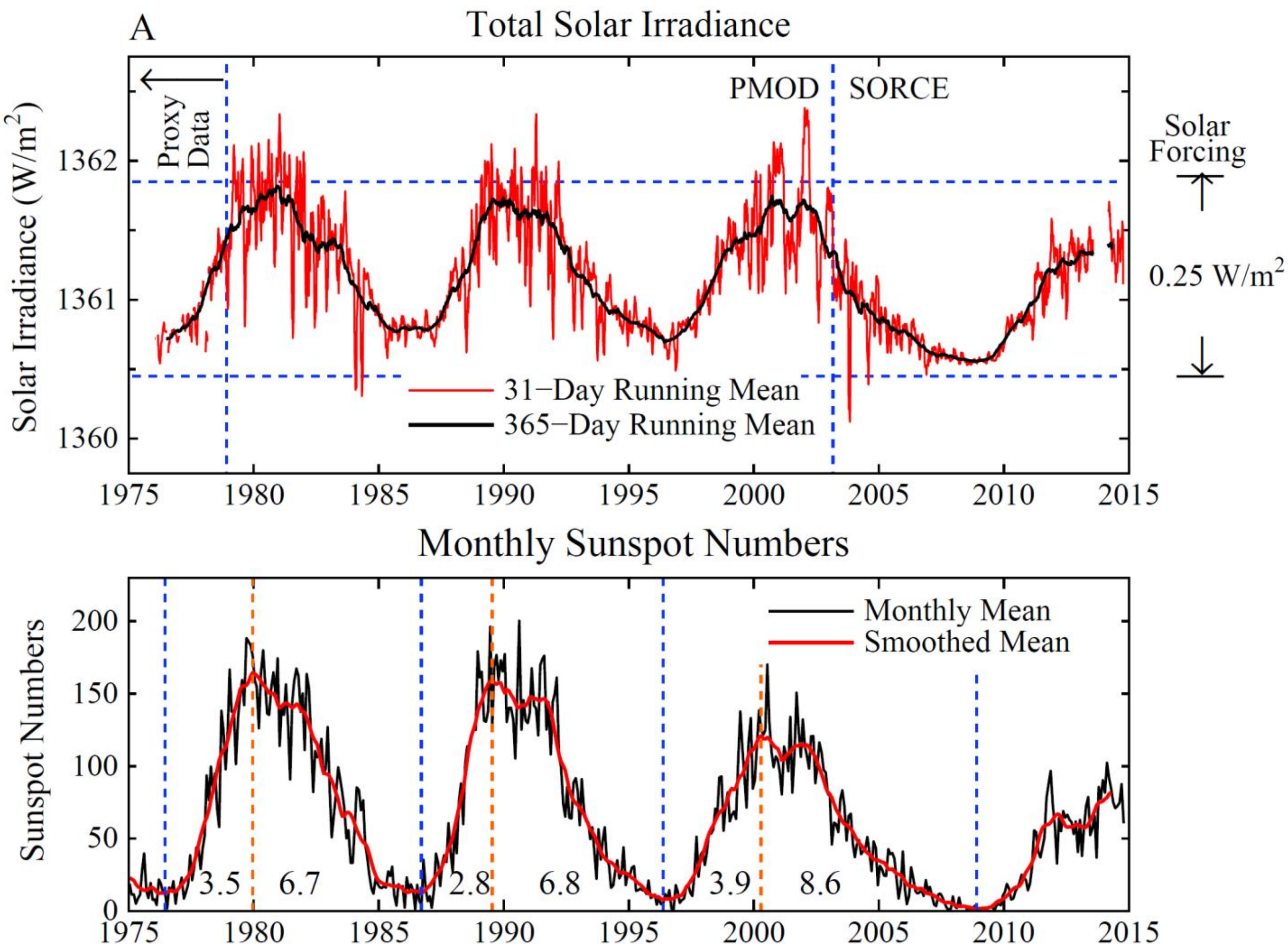
$$\Phi = 1321 \ \frac{\mathrm{W}}{\mathrm{m}^2}$$

Annual variation model:

$$\Phi = \frac{1362}{1.004 + 0.0334 \cos D} \ \frac{\mathrm{W}}{\mathrm{m}^2}$$

D = phase angle of year staring from 4th July (Earth aphelion)

Solar constant nearly constant at fixed distance



Solar energy output varies by about 0.1%

Mostly correlated with number of sunspots

Number of sunspots are responsible for small changes in brightness of Sun

Albedo = [Reflected]/[Incoming] sunlight

Albedo is percentage of sunlight that is reflected to space by planet

Spectral characteristics of reflected radiation similar to incident radiation

Albedo coefficient highly variable over planet surface

Oceans absorb most radiation (values between 0.05 and 0.10)

Ice or fresh snow reflect (values around 0.85)

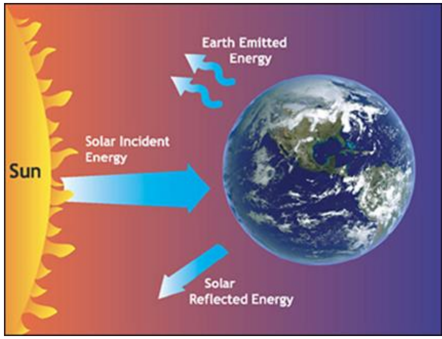
Planetary albedo of Earth is about 0.3

Earth radiation balance

Solar surface: T = 5800 K
thermal radiation mostly visible wavelengths (shortwave radiation)

Earth´s surface: T = 300 K
thermal radiation mostly in infrared wavelength (long wave radiation)

Earth in thermodynamic equilibrium with surrounding
Total energy received from Sun equals to total energy radiated from Earth into space
(Earth reflects (55%) and absorbs (45%) radiation from Sun and emits long wave radiation into space)



Consequence for interpretation for satellite Earth observation:
- Infrared images provide information of temperature distribution
- Visible radiation is reflected sunlight: albedo or surface reflectivity

Radiation pressure

Electromagnetic radiation produces pressure over satellite

Sun:

- Solar radiation: electromagnetic radiation from X-ray to radio wave
- Solar wind: charged particles (mainly protons) and electrons

Earth:

- Albedo: reflection and scattering of incident solar radiation
- Earth´s infrared radiation reemission

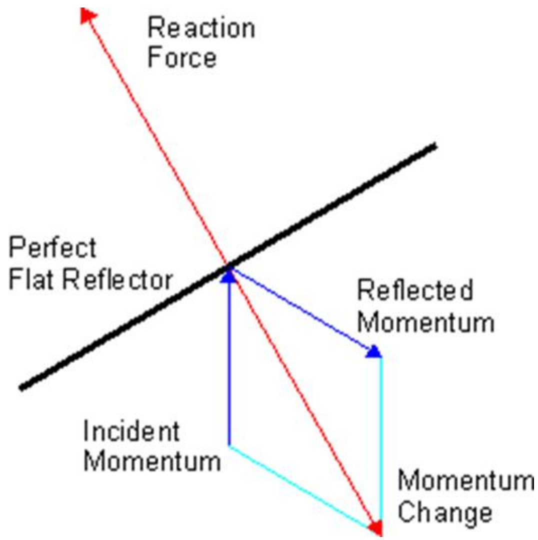
Note: Earth is a minor source of radiation pressure

Cause of radiation pressure

Electromagnetic radiation as well as each photon carries:

Energy $E_\gamma = \frac{hc}{\lambda}$

Linear momentum $p_\gamma = \frac{h}{\lambda}$



Impact of electromagnetic radiation (photons) on surfaces produce radiation pressure

Solar radiation pressure [N/m²]: $$p_s = \frac{\Phi}{c}$$

Φ is solar constant and c is speed of light

Near Earth: Solar radiation pressure $p_s = 4.5 \cdot 10^{-6}\, \frac{\mathrm{N}}{\mathrm{m}^2}$

Solar wind

Particles expelled from Sun:
mainly electrons, protons and alpha particles

Flow approximately radially from Sun

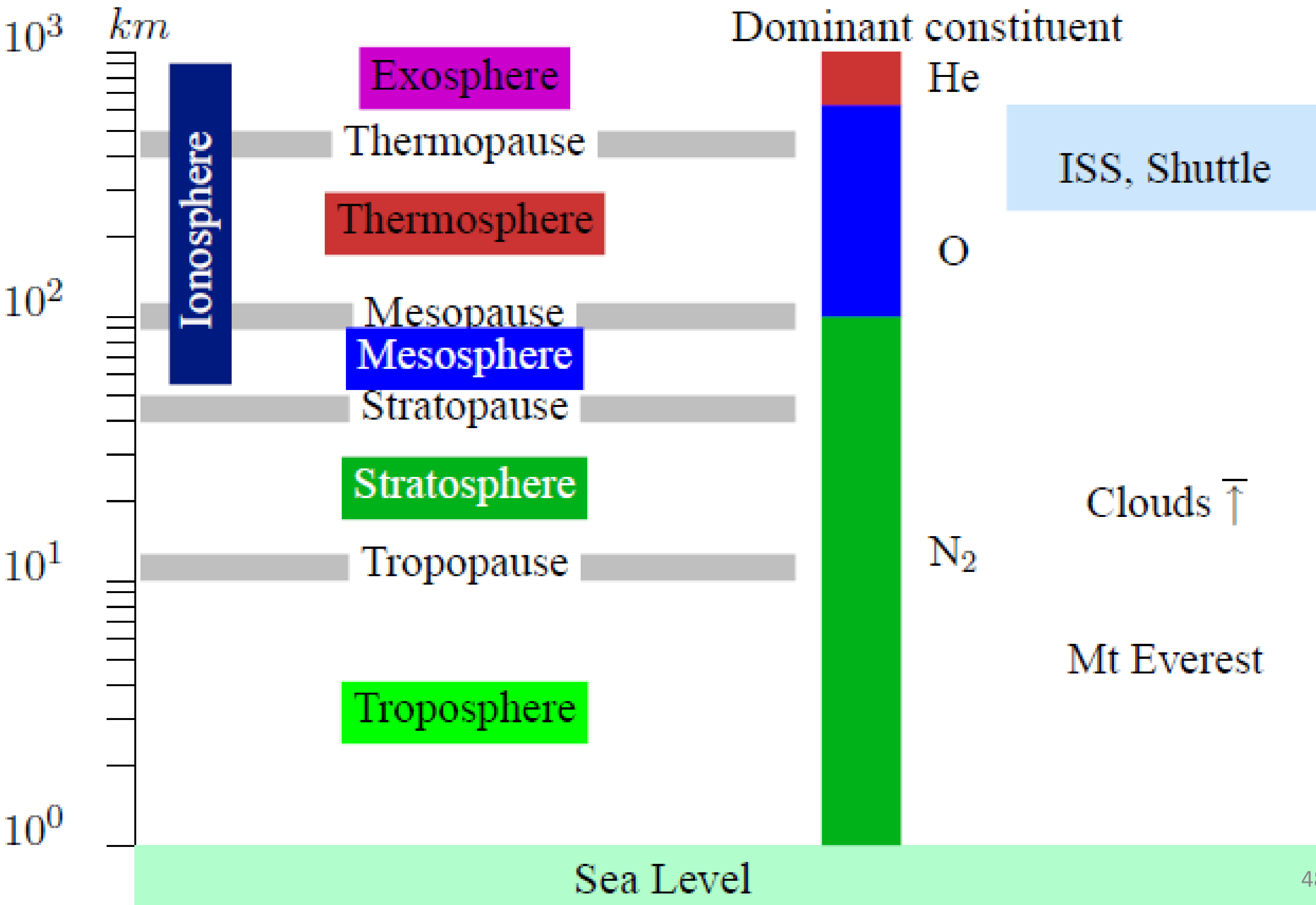
Velocity ranging from 400 km/s to 2500 km/s

Density from around 50 particles cm^{-3} near Mercury
to 0.2 particles cm^{-3} at Jupiter

Solar wind exerts pressure approximately 4 orders magnitude less than solar radiation pressure

Earth´s atmosphere

Structure of Earth´s atmosphere



Structure of atmosphere

- **Troposphere (0 - 10 km)**
  Warmed by Earth and also heated by Sun (weather in this layer)
  Contains 80% of total mass of Earth´s atmosphere
- **Stratosphere (10 - 50 km)**
  Heated from above by absorption of UV light by Ozon (jet aircraft in this layer)
  Contains 19.9% of total mass of Earth´s atmosphere
  Very stable atmospheric conditions
- **Mesosphere (50 - 90 km)**
  Heated by radiation from stratosphere and cooled by radiation into space
  Coldest place with average temperature of around -85^{o} C
  Too high for aircraft and balloons and too low for orbital spacecraft
- **Thermosphere (90 - 700 km)**
  Very sensitive to solar activity (orbital spacecraft in this layer)
  Atoms have large energy (high temperature), but because of low density
  no significant heat transfer possible
- **Exosphere (700 - 10000 km)**
  Composed mainly of very low density of hydrogen and helium (orbital spacecraft)
  Atoms travel hundreds of kilometer without colliding with each other (no gas like behavior)

Atmospheric gases of Earth

- Atmosphere (0-120 km) well mixed (78% N_2, 21% O_2)
- Primarily O from 120 km – 600 km (O_2 breaks down into O by UV)
- Primarily Helium above 600 km
- Particle density for Earth is 10^{10} cm^{-3} at 200 km altitude
- Particle density for Earth is 10^{6} cm^{-3} at 1000 km altitude
- Temperature above 120 km increase with altitude, but because of few molecules not enough for heat transfer

Escape of planetary atmospheres

Atmosphere is gaseous envelope around terrestrial planets

Assume atmosphere is controlled by following factors:

- Gravity (Attracts atmospheric particles)
- Temperature (How fast particles move away)
- Composition (Lighter particles move faster than heavier particles)

Compare gravitational attraction and speed of particles for given temperature (escape particle speed versus thermal particle speed)

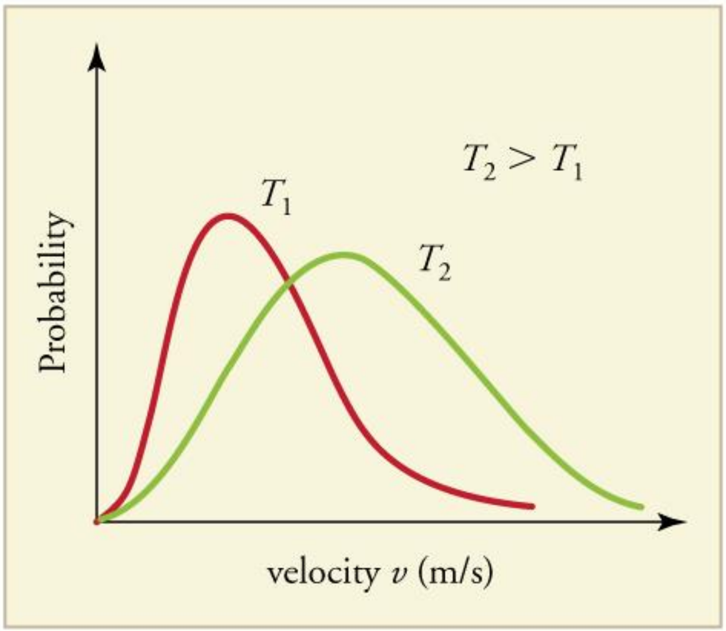
Escape speed given by conservation of energy $\frac{1}{2}mv_e^2 - \frac{GMm}{r} = 0$

What is thermal speed?

Maxwell-Boltzmann distribution

Maxwell-Boltzmann distribution describes speed *v* (or energy) distribution of particles/molecules for given temperature *T* (in thermal equilibrium)

$$\boxed{f(v) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{mv^2}{2kT}}}$$



Speed distribution depends on mass of particle *m* and temperature *T* (*k* is Boltzmann constant)

Most probable speed v_p given by

$$\frac{df(v)}{dv} = 0 \Leftrightarrow v_p = \sqrt{\frac{2kT}{m}}$$

At higher temperature: Maxwell-Boltzmann distribution shifted to higher speed and broadened

Similar to: Planck´s distribution describes wavelength λ (or energy) distribution of electromagnetic radiation (photons) for given temperature *T* (in thermal equilibrium)

Escape speed versus thermal speed

Thermal speed given by Maxwell-Boltzmann distribution with most probable particle speed: $v_p \sim \sqrt{\frac{kT}{m}}$ *m* = mass of particle

Escape speed of particle: $v_e \sim \sqrt{\frac{GM}{r}}$ *M* = mass of planet

In outer region of atmosphere sufficiently fast particle escape into space

To maintain atmosphere thermal speed should be much smaller than escape speed: $\boxed{v_p \ll v_e \quad \Leftrightarrow \quad kT \ll \frac{GMm}{r}}$

- Large terrestrial planets (large *M*) can have atmosphere whereas small planets or asteroids cannot
- Colder atmosphere (small *T*) → particle longer time in atmosphere
- Heavy particles (large *m*) longer time kept than lighter particles → Earth´s atmosphere has little hydrogen or helium

Simple static atmospheric model

In **static** model, properties change only with location

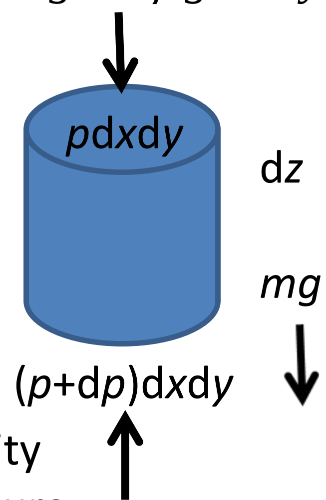
Height: Hydrostatic equilibrium

Assume: ideal gas, constant temperature and atmosphere is spherically symmetric

$$m = \rho \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$$

$$F_{top} = p \, \mathrm{d}x \, \mathrm{d}y$$

$$F_{weight} = -mg = -\rho g \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$$



$$F_{top} + F_{bottom} + F_{weight} = 0$$

Equilibrium in vertical direction (Volume is not moving)

$$p \, \mathrm{d}x \, \mathrm{d}y - (p + \mathrm{d}p) \, \mathrm{d}x \, \mathrm{d}y - g \rho \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 0$$

$$\mathrm{d}p = -g \rho \, \mathrm{d}z \qquad \boxed{\frac{\mathrm{d}p}{\mathrm{d}z} = -g \rho}$$

Hydrostatic equation

$$p_{gas} = \rho R T$$

Ideal gas law

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = -\rho \frac{g}{RT} \qquad \int_{\rho_0}^{\rho} \frac{\mathrm{d}\rho}{\rho} = -\int_{h_0}^{h} \mathrm{d}z \, \frac{g}{RT}$$

ϱ = density

p = pressure

R = gas constant

T = temperature

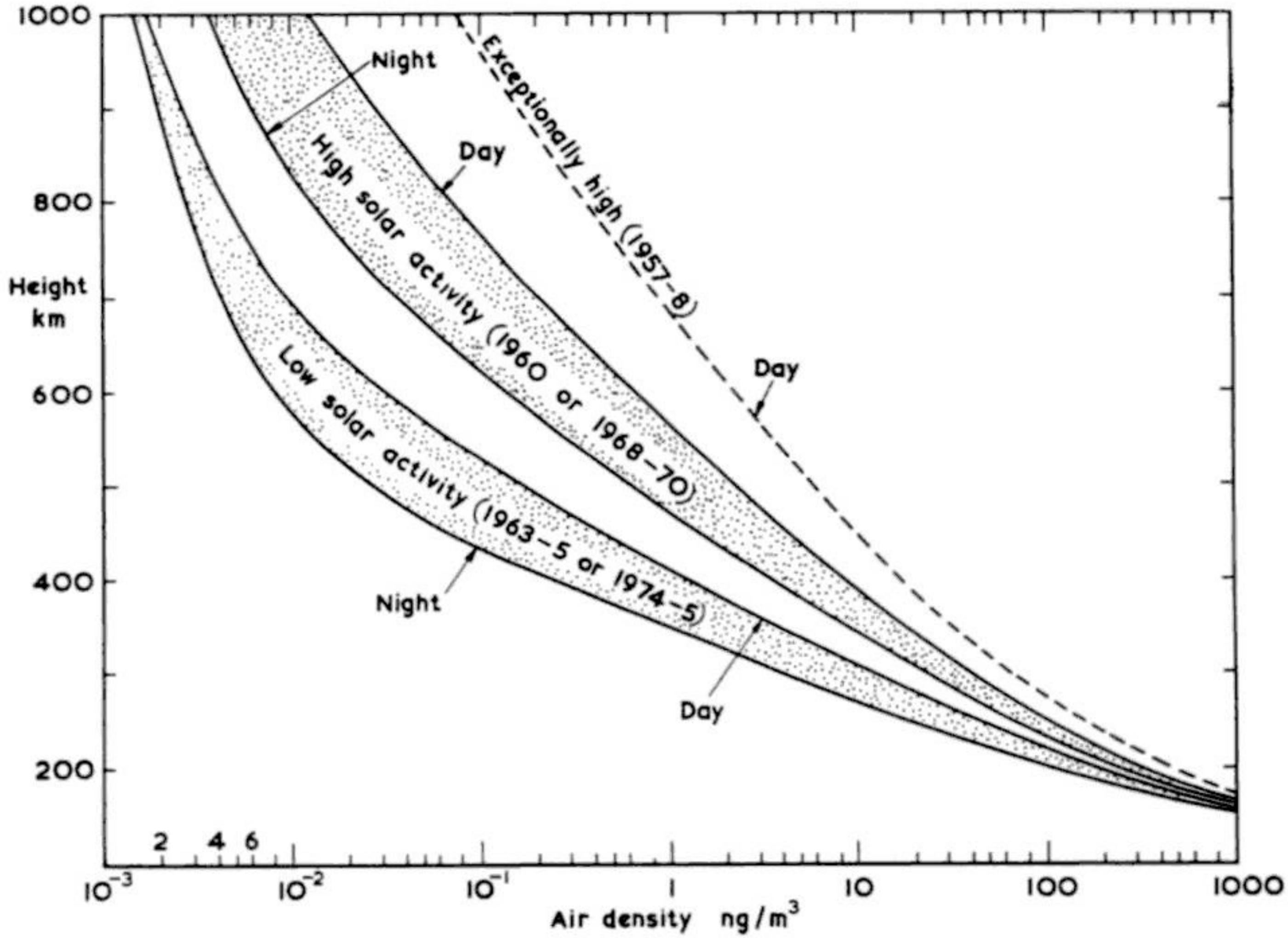
Valid to several 100 km

$$\boxed{\rho = \rho_0 \mathrm{e}^{-h/H}} \qquad H = RT / g$$

Dynamic atmospheric model

- Earth atmosphere is not static → Need of **dynamic** model
- Atmosphere varies with season and time of day
  (affects temperature and density)
- Causes of time related changes:
  - Earth day-night cycle (zenith angle of Sun)
  - Variable solar distance (effects are small)
  - Sun 27 day rotation (rotation of Sun)
  - 11 year sun spot cycle (cycle of solar magnetic activity)
- Very difficult to quantify and model (Jacchia model)
  In time varying atmospheric models, atmospheric properties
  change with location and time
- Principle effect of atmosphere on orbiting body:
  - retarding (drag) force against velocity vector
  - caused by particles which make up atmosphere

Diurnal variations in atmosphere

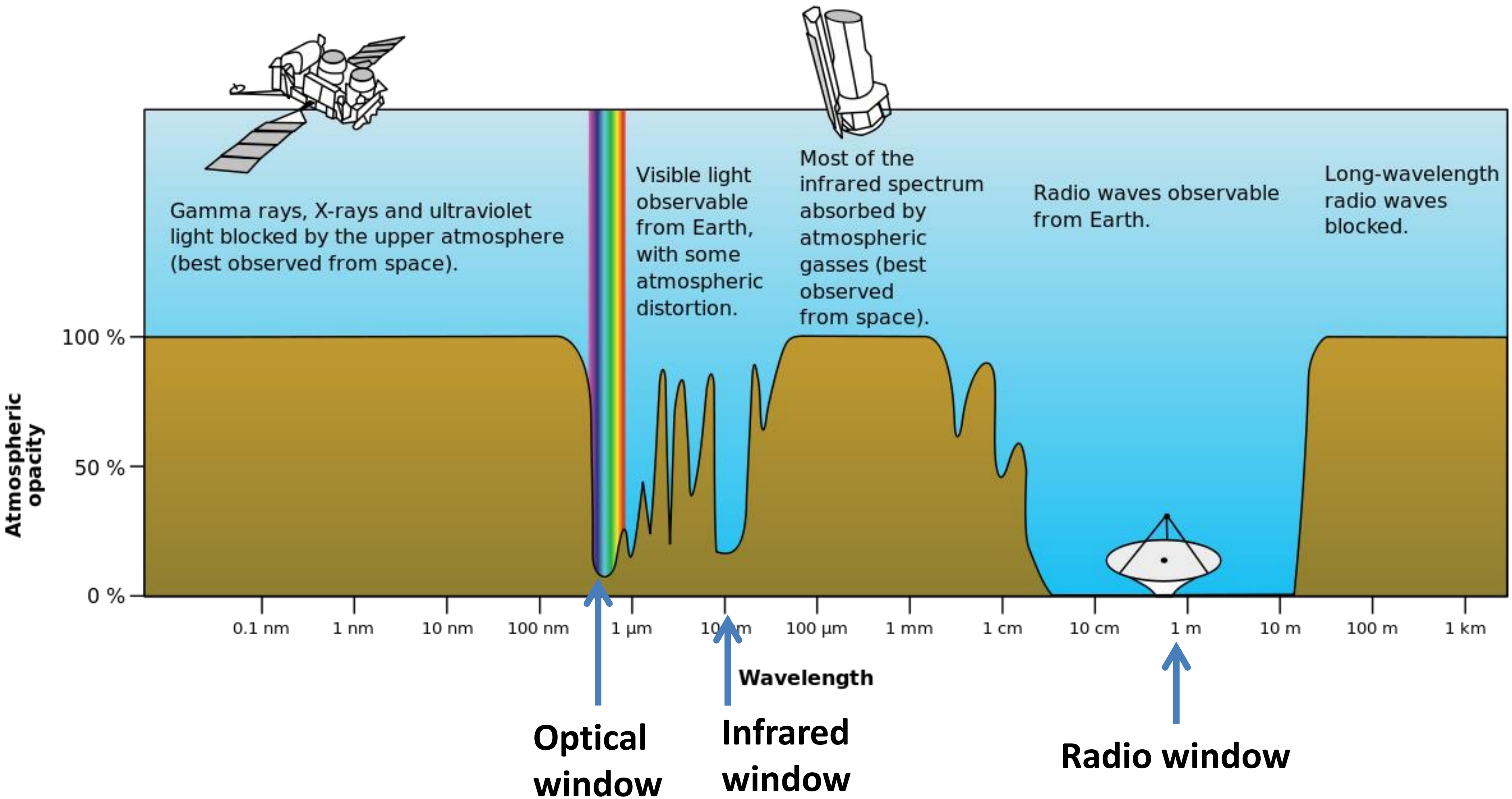


Variations of atmospheric density from 150 – 1000 km for high and low solar activity and for diurnal variations

Factor of 10 change in density for diurnal variation

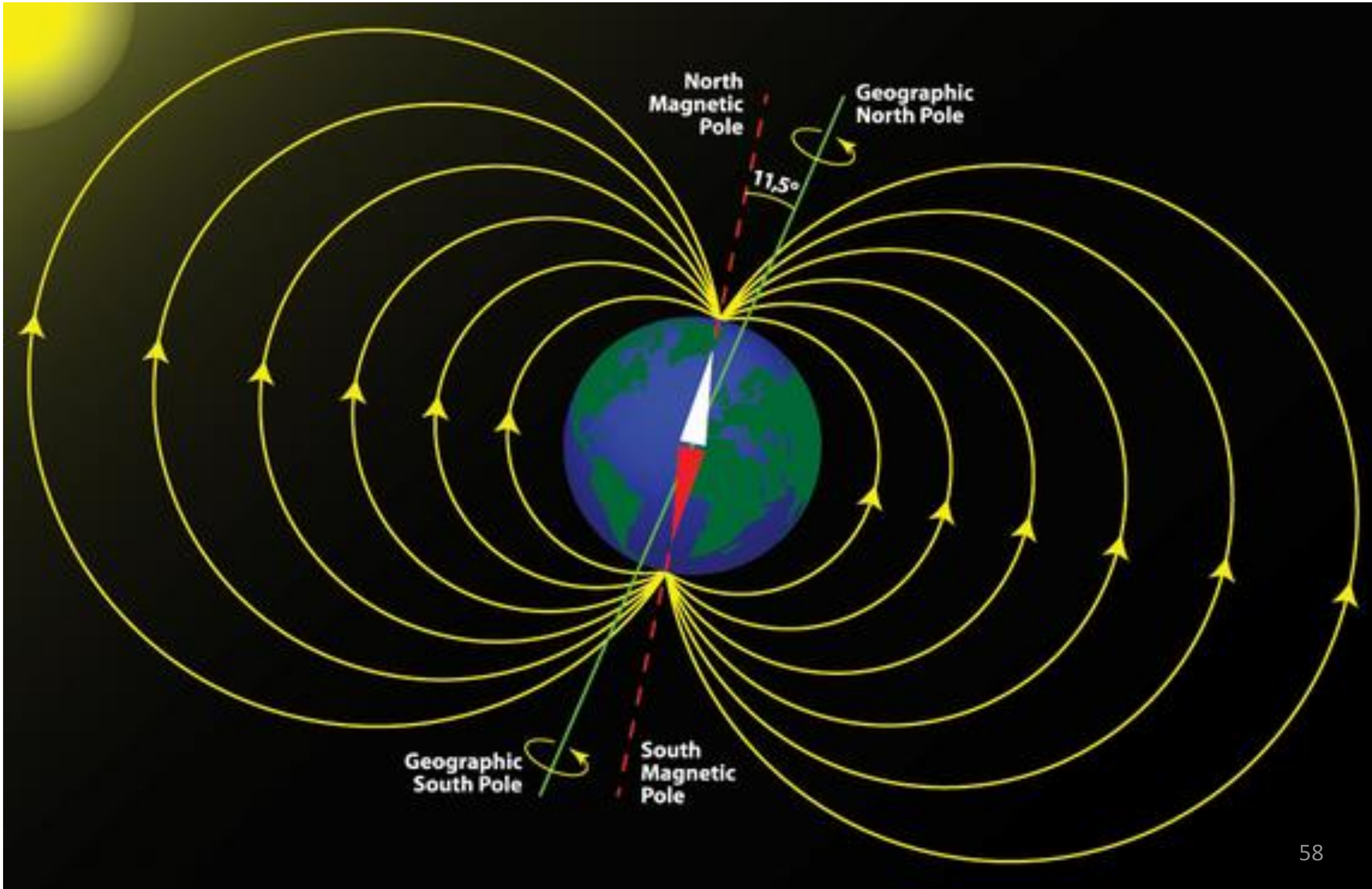
Large effect of solar activity Factor of 100 and more in density

Transmittance of atmosphere at different wavelengths



100% transmittance or opacity means that NO radiation reaches surface of Earth

Earth magnetic field



Earth magnetic field

Earth has solid metal core, mostly iron, surrounded by liquid iron core and again surrounded by mantel

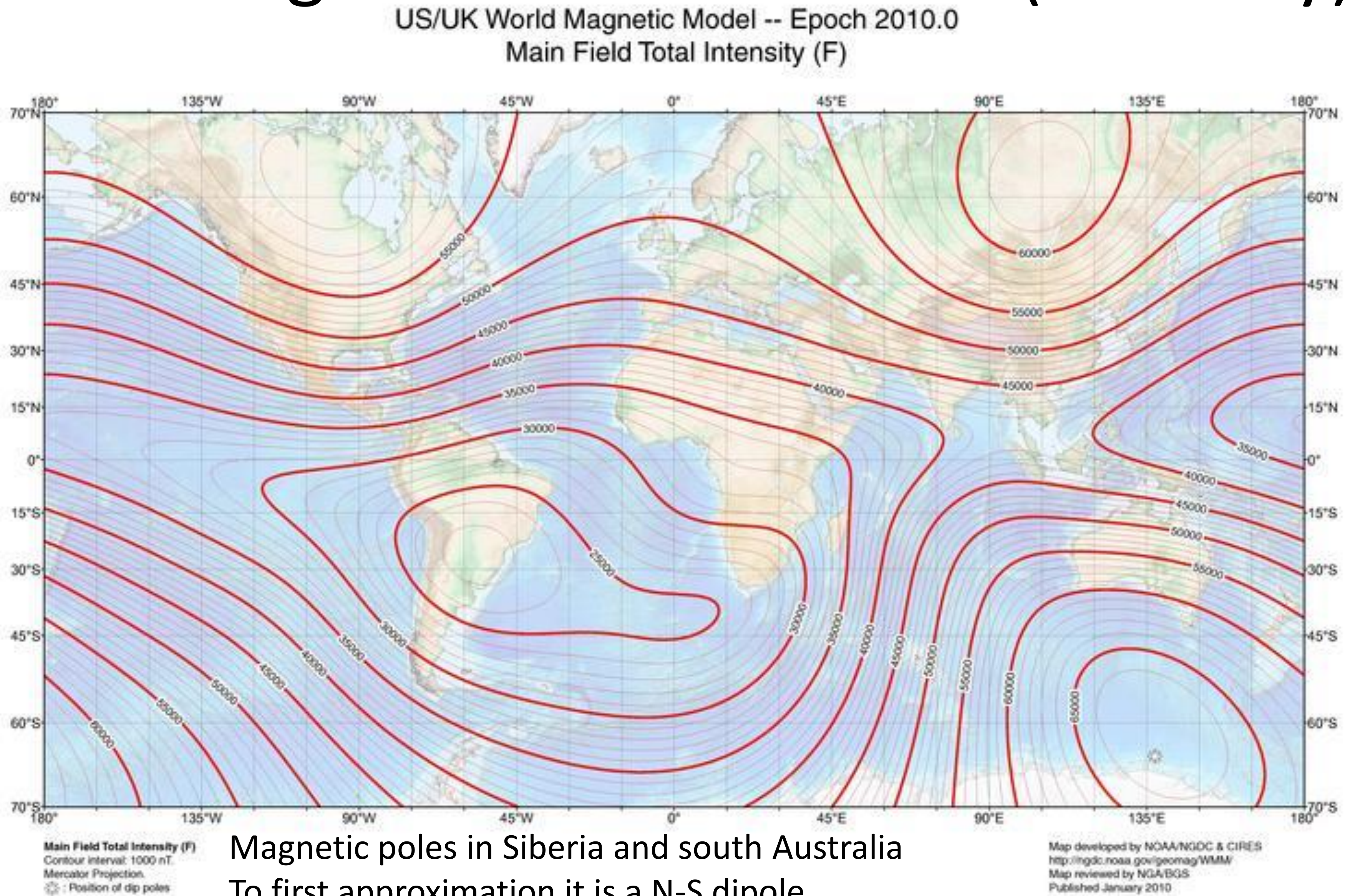
Convective motion in ferromagnetic molten core produces Earth´s magnetic field through dynamo effect

Dynamo effect:
- Weak changing magnetic field generate electric current
- Electric current loops generate magnetic field

In first order magnetic fields are modeled by a **magnetic dipole**

Magnetic field varies with time and has changed polarity in past through complex process
Magnetic field varies slowly:
- dipole strength decreases by $\approx$0.05% /year
- drifts around

Earth magnetic field at surface (intensity)



Magnetic poles in Siberia and south Australia

To first approximation it is a N-S dipole

(about 30000 nT at equator, about 60000 nT at poles, nT = NanoTesla)

Magnetic storm produce small magnetic field variation on Earth surface

Model for Earth magnetic field

No surface electric currents means
magnetic field ***B*** has zero curl
(Static Maxwell equation without currents)

$$\vec{\nabla} \times \vec{B} = 0$$

B field can be gradient of scalar potential *V*
(curl of gradient of scalar field always zero)

$$\vec{B} = -\vec{\nabla} V$$

No magnetic monopoles (Maxwell equation)

$$\vec{\nabla} \cdot \vec{B} = 0$$

Take two equations together follows **Laplace equation**

$$\boxed{\nabla^2 V = 0}$$

Assume: spherical boundary at Earth surface
→Solution of Laplace equation expressed by spherical harmonics

Laplace equation and spherical harmonics

$$\nabla^2 V = 0$$

Write Laplace equation in spherical coordinates $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial V}{\partial r}) + \frac{1}{r^2 \sin\phi}\frac{\partial}{\partial \phi}(\sin\phi \frac{\partial V}{\partial \phi}) + \frac{1}{r^2 \sin^2\phi}\frac{\partial^2 V}{\partial \lambda^2} = 0$

Use separation of variables → **Spherical harmonic functions**

For exterior region of sphere

$$V(r,\lambda,\phi) = R_E \sum_{n=1}^{\infty}\left(\frac{R_E}{r}\right)^{n+1}\sum_{m=0}^{n}[g_{nm}\cos m\lambda + h_{nm}\sin m\lambda]P_{nm}(\phi)$$

Spherical harmonic expansion of magnetic scalar potential

g_{nm} and h_{nm} are time dependent empirically determined coefficients

More on spherical harmonic functions in Lecture 2

Magnetic field is derivative of potential $\vec{B} = -\vec{\nabla} V$

Magnetic field can be now calculated at any point

Comments on magnetic field model

- Dipole model of Earth's magnetic field is first order approximation of rather complex true Earth´s magnetic field
- Dipole model is dominant term of spherical harmonics expansion of Earth´s magnetic field
- Dipole model is slightly tilted (11.5^{o}) with respect to Earth´s spin axis
- Greater precision achieved using spherical harmonics expansion
- Expansion coefficients are published in IGRF (International Geomagnetic Reference Field)
- Expansion coefficients are time dependent (i.e. given table of coefficients will have an epoch plus corrections as functions of time)
- Spherical harmonic expansion well suited for computer code, but for analytical purposes use dipole model
- Few order expansion gives adequate accuracy for most ADCS uses

Magnetic fields of Sun and Earth

Sun has around 100 times stronger magnetic field than Earth and goes beyond all planets (Interplanetary magnetic field)

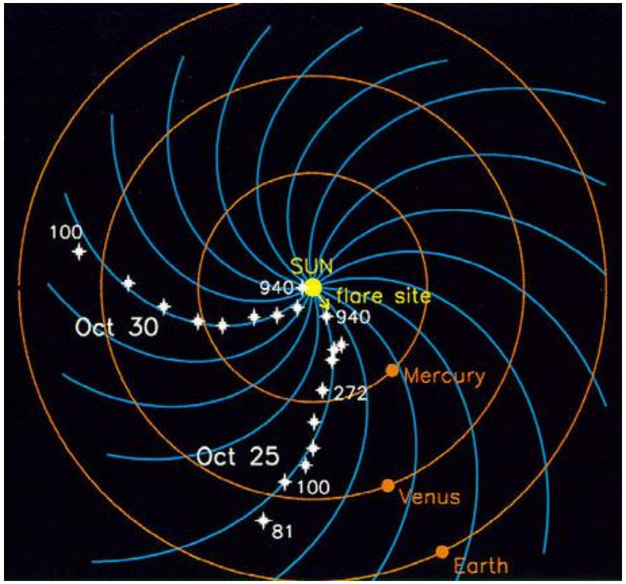
Sun´s magnetic field carry solar wind (charged particles)

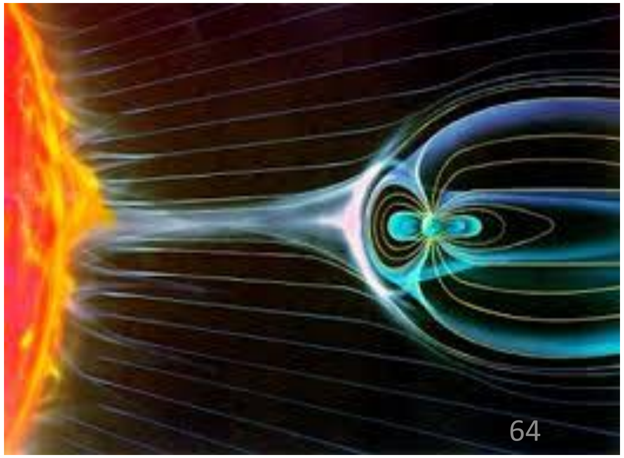
Because Sun rotates (27 days) interplanetary magnetic field has spiral shape

Interaction of geomagnetic field with solar magnetic field produce fast changing structure: Magnetosphere

Magnetosphere deflects solar wind

Contact of Earth´s magnetic field and interplanetary magnetic field is called Magnetopause





Magnetosphere

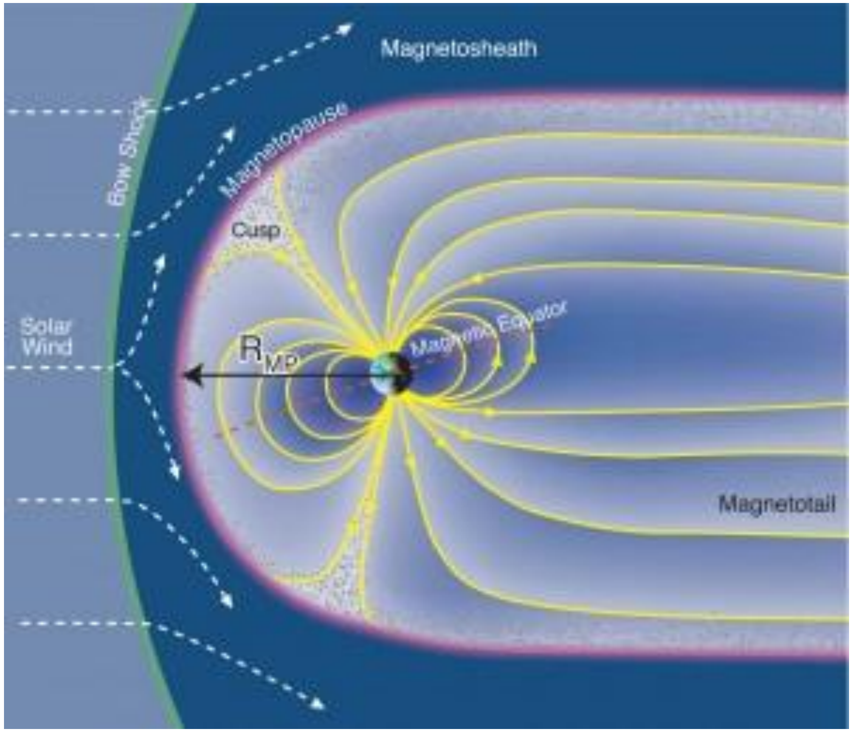
Geomagnetic field perturbation primarily do to Sun by solar winds

Day side of Earth magnetic field is compressed by solar wind

Solar winds distort Earth field at high altitude (10 Earth radii)

(No spherical harmonic solution valid)

Earth field slows and deflects solar wind



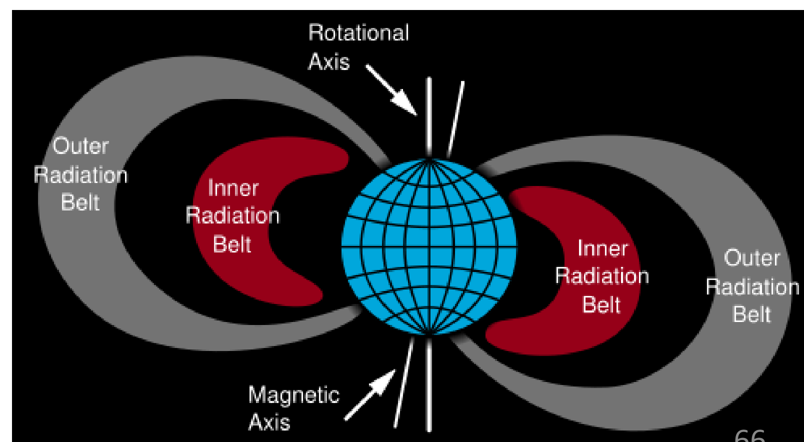
Trapped radiation (Van Allen radiation belts)

Trapped (by magnetic field) energetic electrons and protons

Toroidal belts around Earth (inner region of magnetosphere)
made of electrons and protons
- electron energy between 100 keV to few MeV
- proton energy between 100 keV to 100 MeV

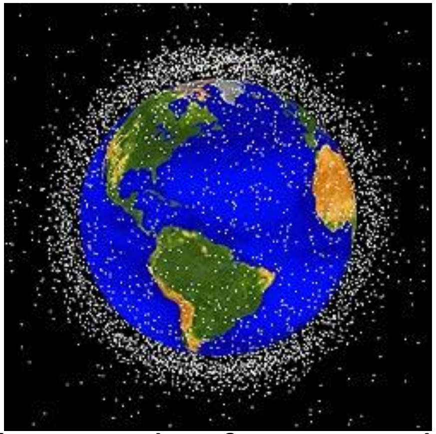
Two big zones centered along Earth´s magnetic equator
- inner belt (1000 km – 6000 km) mainly protons with some electrons
- outer belt (13000 km – 60000 km) mainly electrons
- recently temporary third belt discovered

Sources
- solar wind
- cosmic rays



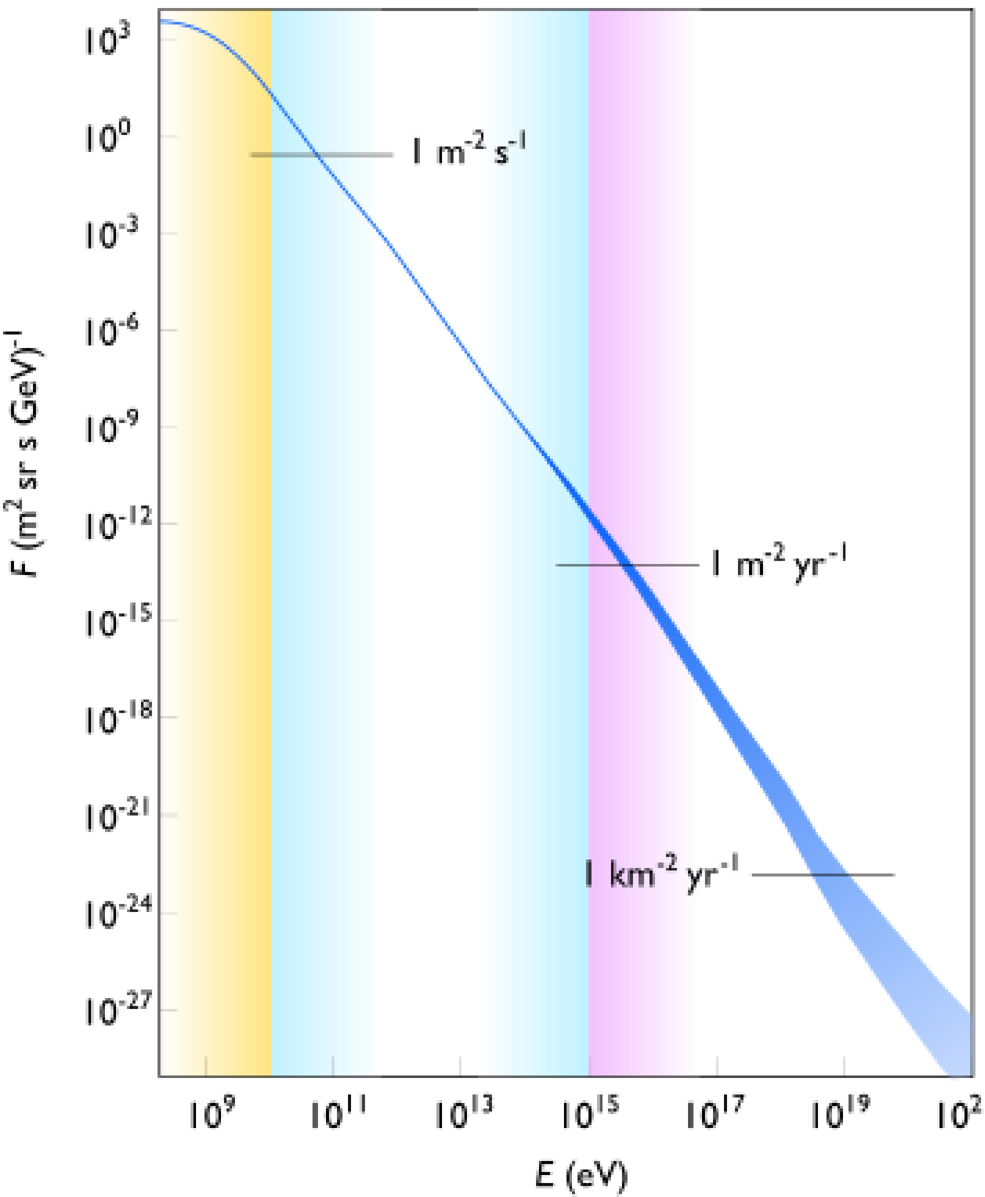
Micrometeoroids and space debris

- Micrometeoroids are stony, metallic objects travelling through space at Earth escape speed and beyond
- Space debris produced by space borne explosions, old inactive satellites
- Most debris in nearly circular orbits barely over atmosphere (but also in geosynchronous altitude, where number of available orbit slots is limited, often clustered over same primary ground target footprint)
- Particle impact may destroy spacecraft or produce torque
- Pieces may be small but mean impact velocity 20 to 30 km/s with enormous kinetic energy
- Large micrometeoroids are tracked individually
- Small micrometeoroids are treated statistically
- Video: [The space debris story](https://www.youtube.com/watch?v=OGfU2u1__OI) (https://www.youtube.com/watch?v=OGfU2u1__OI)



Thousands of man-made fragments in orbit

Cosmic rays



Cosmic rays in interplanetary space are primarily of interplanetary protons and ionized heavy nuclei from about 1 GeV/nucleon

Coming from all directions (isotropic)

Low Energy (yellow):
mainly solar cosmic rays

Intermediate energy (blue):
mainly galactic cosmic rays

Highest energy (purple):
extragalactic cosmic rays

Sources are mostly outside solar system
- Supernova explosion
- Quasars

Summary

Overview of ADCS and lecture

Spacecraft environment:

Gravity

- Newton´s Cannonball

Sun

- Electromagnetic spectrum → Planck´s distribution → Stefan Boltzmann law → Luminosity → Solar flux density
- Solar radiation pressure
- Solar wind

Earth atmosphere

- Maxwell-Boltzmann distribution (thermal vs. escape speed)

Earth magnetic field

- Spherical harmonic expansion of magnetic potential

Bibliography

- J. R. Wertz (1978)

  “Spacecraft attitude determination and control”
- P. Fortescue and J. Stark (1991)

  “Spacecraft systems engineering”
- M. Macdonald et al. (2014)

  “International handbook of space technology”