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Scheduling dial-a-ride paratransit under time-varying, stochastic congestion

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Abstract

This paper discusses a study on the dial-a-ride paratransit scheduling problems arising in paratransit service systems that are subject to tight service time constraints and time-varying, stochastic traffic congestion. Different from existing methodologies, we explicitly incorporate a time-dependent, stochastic travel time model in the problem formulation. A set of recursive relations is first identified to approximate the distribution parameters of arrival times at individual stops of a given route which, coupled with a first-in-first-out (FIFO) assumption, allows us to extend the conventional heuristic algorithms for solving the proposed problem with only marginal increase in computational complexity. Results from a series of numerical experiments on a set of hypothetical problems are described, aiming to illustrate the computational efficiency of the proposed algorithm and the sensitivity of solutions to various model parameters.

Keywords: Vehicle routing and scheduling; Paratransit; Travel time

1. Introduction

Travel times between individual locations in urban traffic environments are often subject to time-varying, stochastic variations due to factors such as random fluctuations in traffic volumes, frequent interruptions of signal controls and unpredictable occurrences of traffic incidents. These variations are commonly one of the major contributing factors that cause dial-a-ride paratransit systems to experience difficulty in adhering their service to pre-established schedules (Fu, 1999).

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This is especially true when schedules are calculated on the basis of models that assume time-unvarying and/or deterministic travel times. It can be expected that in situations of high uncertainty the service vehicles may not be able to follow the schedules generated from these models and thus a reliable service may not be guaranteed. For example, based on the assumption of deterministic travel time it would be feasible to schedule a vehicle to drop off a passenger at his/her destination at his/her appointment time. However, there is a certain amount of uncertainty that the passenger may be dropped off after his appointment time because of the randomness of travel time. The drawback associated with the assumption of static travel time is more straightforward in the sense that the use of a constant travel speed may result in schedules that are sub-optimal and have violated time windows. The objectives of this study are to develop a dial-a-ride vehicle routing and scheduling model which explicitly considers the time-varying, stochastic attribute of travel times, and to develop an algorithm that is efficient enough for solving large sized problems of this type.

The dial-a-ride scheduling problem (DARP) has been the focal point of research since the inception of the demand-responsive paratransit service concept. Wilson et al. (1971), Wilson and Weissberg (1976) and Wilson and Colvin (1977) were among the first to study the DARP with specific interest in developing real-time algorithms for several technology-pioneering paratransit systems. Approximate models were proposed to facilitate economic planning and efficient operations of such systems (Daganzo, 1978; Stein, 1978). Due to high operating costs, most of the dial-a-ride systems turned into reservation-based operations after late 1970s. Research on the DARP also shifted to developing efficient heuristic algorithms for solving the static version of the problem with most requests known in advance. Several efficient heuristics were developed, including the popular parallel insertion algorithm (Jaw, 1984; Roy et al., 1984; Toth and Vigo, 1997), optimization-based method (Sexton and Bodin, 1985a,b), and clustering and mini-clustering-based algorithm (Bodin et al., 1983; Ioachim et al., 1995). Recent surveys on the DARP and more generally the vehicle routing and scheduling problems can be found in Bodin et al. (1983), Desrosiers et al. (1993) and Savelsbergh and Sol (1995). While significant progress has been made in terms of model realism and solution efficiency, most existing studies on the DARP have assumed deterministic and time-unvarying travel times, ignoring any travel time variations caused by traffic congestion which is common in urban routing environments.

In literature related to the DARP, traffic congestion has been considered in some studies to a varied extent. Alfa (1987) examined the travelling salesman problem (TSP) with time-varying (but not stochastic) travel time. More recently, Ahn and Shin (1991) and Malandraki and Daskin (1992) dealt with the vehicle routing problem with time windows and time-varying congestion, and identified the important first-in-first-out (FIFO) property. With the FIFO assumption, Ahn and Shin (1991) found efficient extensions of existing routing heuristics. However, by assuming a known feasible vehicle start time they avoided having to deal with the scheduling problem, which is often critically relevant in a time-dependent context. The vehicle routing problem with stochastic travel time was recently studied by Laporte et al. (1992), in which the objective is to formulate the problem and find an exact algorithm for small sized problems. To the author's knowledge, the DARP with time-varying, stochastic travel times has never been treated in the past in the transportation research literature.

This paper is organized as follows. In Section 2, we introduce a formal definition of the DARP and a model for the time-varying stochastic travel times. Sections 3 and 4 focus on a given route with the objective to identify methods and properties for estimating arrival time distribution parameters at individual stops and validating time window constraints. Section 5 discusses how the DARP can be modeled with respect to the routing objectives of the service provider and passengers when the travel times are modeled as random variables. In Section 6, the problem of finding an optimal schedule for a given route is formulated and solved. Lastly, a computational study is conducted to illustrate the performance sensitivity of the proposed model and algorithm. The conclusions follow in Section 7.

2. Problem definition and travel time model

The static dial-a-ride problem (DARP) is defined to construct a set of feasible and efficient routes and schedules to satisfy transportation requests (trips) made by the system clients. A trip specifies the number of persons to be transported, seating requirement, a pickup location and a drop-off location, and the desired pickup and/or drop-off time. Two types of clients are often involved: wheel-chair passengers who must remain seated in wheel chairs during their travel and those who can use regular seats (also called ambulatories). A mixed fleet of vehicles that can accommodate these seating requirements is available to operate the routes.

The DARP is commonly formulated to minimize a general objective function (or cost function) with a set of service quality constraints (Wilson and Colvin, 1977; Bodin et al., 1983; Jaw et al., 1986; Savelsbergh and Sol, 1995). The cost function is usually defined as a weighted sum of the total client inconvenience, as measured in terms of *excess ride time* (the difference between the scheduled ride time and the ride time without diversions for other passengers) and *service time deviation* (the difference between the scheduled pickup/drop-off times and their most desired pickup/drop-off times), and the cost to the service providers, as often measured in terms of total vehicle travel time and the number of vehicles needed.

The service quality constraints specify that the ride time of each client must be less than a maximum allowable ride time and that all clients must be picked up (dropped off) after (before) their most desired pickup (drop-off) times with service time deviations less than a maximum allowable value. Note that the latter defines a time interval, or *service time window*, during which the service must take place.

When travel times are modeled as random variables, most of the performance measures and schedule variables become random variables and consequently the cost function and service constraints described above need to be redefined. In this paper, we consider travel times as both time-varying and stochastic, and the resulting DARP is called the dial-a-ride problem with time-varying (or dynamic), stochastic travel times (DARP_DS). A more detailed definition follows.

Consider a set of known trip requests. For each trip request, a set of passengers (of different seating requirements) must be transported from an origin location to a destination location. Each trip specifies either a desired pickup time and/or a desired drop-off time. It is assumed that there is a service policy or constraint that designates a maximum allowable deviation from the desired time and a maximum ride time. Note that the later could be a function of the direct ride time of the trip. With the desired service time and service constraints, we can determine a service time window for both stops of a trip (Jaw, 1984). Without loss of generality and describing how the service time windows can be determined, we assume that, for each stop i, it is always possible to identify a service time window $[e_i, l_i]$ for the stop, where e_i and l_i denotes the earliest and latest feasible service start times at the stop, respectively.

For each O–D pair (i, j), let $\{X_{ij}(t),\ t \in T\}$ denote a stochastic process representing the travel time that a vehicle may experience when traveling from stop (location) i to stop j, departing at time t, where T is the time domain. For each time instance t, the mean and standard deviation of $X_a(t)$, denoted as $\mu_{i,j}(t)$ and $\sigma_{i,j}(t)$, respectively, are assumed to be given as part of the information available for the scheduling function. Information on $\mu_{i,j}(t)$ and $\sigma_{i,j}(t)$ may be given as a set of discrete values over a sequence of time intervals, or need to be calculated using a shortest path algorithm or other methods (Fu and Rilett, 2000). Regardless of the way in which the information is provided, we assume that, over the feasible departure time window at stop $i[e_i, l_i]$, the mean travel time $(\mu_{i,j}(t))$ can be approximated as a linear function of departure time (t) at stop i, and that the standard deviation $(\sigma_{i,j}(t))$ can be modeled as a constant (Fig. 1), i.e.

$$\mu_{i,j}(t) = a_{i,j} + b_{i,j} \cdot t, \tag{1}$$

$$\sigma_{i,j}(t) = c_{i,j}, \tag{2}$$

$$e_i \leqslant t \leqslant l_i,$$

where $a_{i,j}$, $b_{i,j}$ and $c_{i,j}$ are parameters that can be determined from the original travel time information and the service time window at stop i. The simplest way to obtain these parameters is to use the boundary values at departure times e_i and l_i. We note that such a model should be sufficiently accurate for representing the travel time variations under typical traffic conditions (not

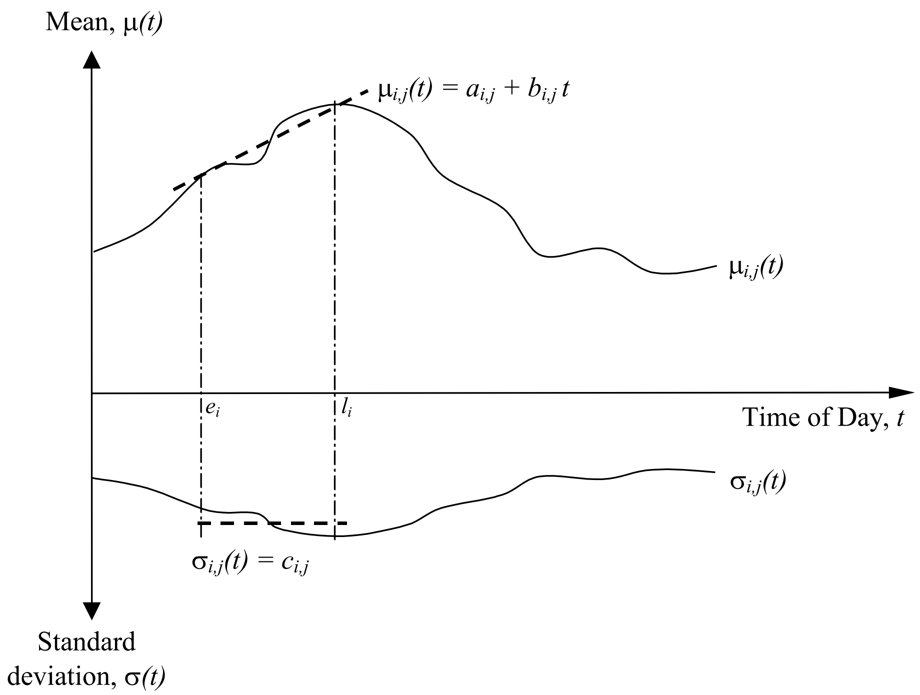

Fig. 1. Time-varying, stochastic travel time model.

vary abruptly and irregularly) with tight trip service time windows (e.g. $l_i - e_i$ is in the order of 30 minutes).

Furthermore, we assume that travel times on individual origin–destination (O–D) pairs at a particular point in time are statistically independent. This assumption is necessary owing to the fact that direct correlation information is difficult to obtain in practice and the consideration of statistical dependency would significantly increase the complexity of the problem.

The objective of the DARP_DS is to determine a set of routes and schedules such that all the stops are made within their service time windows with a a pre-specified minimum probability and the expected total cost including client inconvenience and service hours is minimized. The following sections discuss the modifications needed for realistically formulating and efficiently solving the DARP_DS.

3. Arrival time functions and expected FIFO

In the process of searching for solutions to the DARP, it is necessary to update the arrival times at individual stops of a given route based on the given O–D travel times. This could be accomplished easily if travel times are deterministic or stochastic but not time varying. It is, however, not a trivial task when the travel times are both time-dependent and stochastic, as discussed in Fu and Rilett (1998). It becomes even more complicated when waiting at a stop is allowed or even required as in the case of the scheduling problems. This section describes the assumptions that are introduced to simplify the derivation of the arrival time functions of a given route.

Consider a particular route including N stops and M schedule blocks as shown in Fig. 2. Schedule blocks (SBs) are defined by the deadheading movements; each SB represents a sequence of pickups and deliveries with a vehicle starting empty to pick up a client and ending empty.

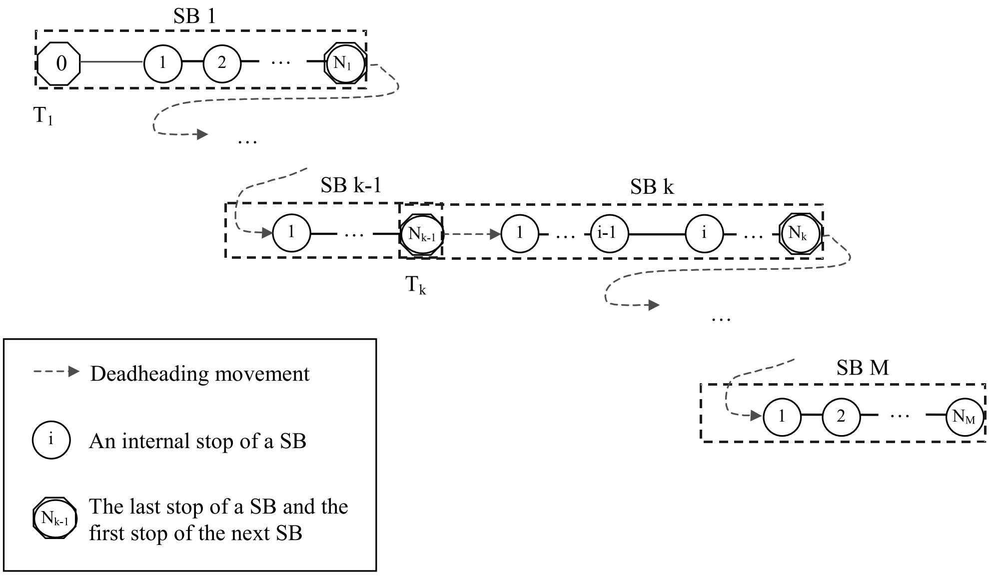

Fig. 2. SBs of a given route.

Within a given SB k, stops are labelled as $s_0^k, s_1^k, s_1^k, \ldots, s_{N_k}^k$ $(N_k \geqslant 2,\ k = 1, 2, \ldots, M)$. For convenience, we drop the reference to the SB and label the stops within each SB sequentially from 0 to N_k. Note that the first stop ($i = 0$) of SB k represents the vehicle depot for the first SB (i.e. $k = 1$) and the last stop of the preceding SB (N_{k-1}) when $k > 1$. Without further clarification, all of notations that are associated with a stop (i) should be considered to have a reference to a SB (k).

Based on common service policy, it is assumed that idle time is not to be scheduled within each SB, that is, vehicle should not be scheduled to wait at a stop when there is one or more passengers onboard. As a result, the schedule for the given route is completely defined by the departure times at all SBs. Denote T_k $(k = 1, 2, \ldots, M)$ as the departure time for SB k. Note that T_1 is the departure time from the vehicle depot and T_k for $k > 1$ is the departure time from stop N_{k-1}.

It should be noted that, in practice (when a schedule is actually implemented), a vehicle may have to wait (or idle) even when there are passengers onboard. For example, a vehicle would have to idle if it arrives at a pickup stop too early according to the earliest ready time for the client.

When the given schedule is followed in operation, the arrival time and departure time at stop i of BS k can be determined by

$$A_i = D_{i-1} + X_{i-1,i}(D_{i-1}), \tag{3}$$

$$D_i = \begin{cases} e_i, & \text{stop } i \text{ is a pickup stop and } A_i < e_i, \\ A_i & \text{otherwise,} \end{cases} \tag{4}$$

$$D_0 = \begin{cases} T_k, & k = 1 \ \text{ or } \ D_{N_{k-1}} \leqslant T_k, \\ D_{N_{k-1}} & \text{otherwise,} \end{cases} \tag{5}$$

$$i = 1, 2, \ldots, N_k, \quad k = 1, 2, \ldots, M,$$

where random variables A_i and D_i denotes the arrival time (or service start time) and the departure time at stop i of SB k, respectively. Eq. (3) relates the arrival time at stop i (A_i) to the departure time at stop $i - 1$ (D_{i-1}) while Eq. (4) represents the service operating rule at each stop. Note that, without loss of generality, we have excluded the dwell time at each stop. Eq. (5) specifies the boundary condition linking service start time (D_0) to the scheduled service start time (T_k) for each SB.

Due to the time-varying stochastic property of travel times, both the arrival time and departure time at each stop are random variables and their distributions depend on the distributions of O–D travel times $(X_{i-1,i}(t))$, the time windows at individual stops (e_i, l_i), and the given schedule (T_k). It has been shown in Fu and Rilett (1998) that it is mathematically intractable to derive such distribution functions. The following assumptions are therefore made to simplify the problem.

Assumption 1. The arrival time and departure time at each stop of any given route is normally distributed with their means and standard deviations depending only on the distribution of the departure time at its preceding stop and the travel time from the preceding stop. This assumption is supported by the Central Limit Theorem and has also been empirically validated by our simulation study.

Assumption 2. At the end of each SB (i.e. vehicle is empty), the variation in departure time for the next SB, which depends on the scheduled departure time (known) and arrival time (random), can be 'absorbed' with driver's behavior response to any deviation from the given schedule (e.g., speed up when the vehicle is behind its schedule).

Based on Assumption 1, the probability distributions of arrival time (A_i) and departure time (D_i) can be completely specified if we can determine their corresponding means and variances. Let $g_{i-1,i}(t)$ be the expected arrival time at location i provided that a vehicle travels directly from location $i-1$ to location i with departure time t, i.e.

$$g_{i-1,i}(t) = t + \mu_{i-1,i}(t) = (1 + b_{i-1,i}) \cdot t + a_{i-1,i}. \tag{6}$$

Denote the inverse function of $g_{i-1,j}(t)$ as $h_{i-1,j}(t)$, i.e.

$$h_{i-1,i}(t) = g^{-1}_{i-1,i}(t) = \frac{1}{1+b_{i-1,i}}t - \frac{a_{i-1,i}}{1+b_{i-1,i}}. \tag{7}$$

Based on the proposed travel time model Eqs. (1) and (2), we now apply the results from Fu and Rilett (1998) to convert Eq. (3) into the following recursive relations in terms of the means and variances of the arrival time and departure time (Eqs. (8) and (9)):

$$\begin{aligned} E[A_i] &= E[D_{i-1}] + E[X_{i-1,i}(D_{i-1,i})] \\ &= E[D_{i-1}] + \mu_{i-1,i}(E[D_{i-1,i}]) \\ &= g_{i-1,i}(E[D_{i-1}]) \end{aligned} \tag{8}$$

$$\begin{aligned} \mathrm{Var}[A_i] &= \{1 + \mu'_{i-1,i}(E[D_{i-1,i}])\} \cdot \mathrm{Var}[D_{i-1}] + \sigma^2_{i-1,i}(E[D_{i-1,i}]) \\ &= \{1 + b_{i-1,i}\} \cdot \mathrm{Var}[D_{i-1}] + c^2_{i-1,i} \\ &\quad i = 1,2,\ldots,N_k, \quad k = 1,2,\ldots,M, \end{aligned} \tag{9}$$

where $\mu'_{i,j}(t)$ is the first-order derivative of the mean travel time from stop i to stop j.

With Assumptions 1 and 2, and the assumption that the schedule of interest is always feasible in an expected sense, i.e. $E[D_{N_{k-1}}] \leqslant T_k$ and $e_i \geqslant E[A_i] \geqslant l_i$ for $i = 1,2,\ldots,N_k$, $k = 1,2,\ldots,M$, Eqs. (4) and (5) can then be converted into the following approximate relations:

$$E[D_i] = E[A_i], \tag{10}$$

$$E[D_0] = T_k, \tag{11}$$

$$\mathrm{Var}[D_i] = \mathrm{Var}[A_i], \tag{12}$$

$$\mathrm{Var}[D_0] = 0, \tag{13}$$

$$i = 1,2,\ldots,N_k, \quad k = 1,2,\ldots,M.$$

With Eq. (6), the expected arrival time at stop i Eq. (8) can also be expressed explicitly as a linear function of T_k, as shown in Eq. (14). This expression will be used in the formulation of the schedule optimization problem discussed in Section 5.

$$E[A_i] = a'_i + b'_i \cdot T_k, \tag{14}$$

where

$$a'_i = \sum_{j=2}^{i} \left\{ a_{i-j,i-j+1} \cdot \prod_{n=1}^{j-1} (1 + b_{i-n-1,i-n}) \right\} + a_{i-1,i},$$

$$b'_i = \sum_{j=1}^{i} (1 + b_{j-1,j}).$$

Note that the equations for arrival time variance (Eqs. 9, 12 and 13), as resulted from Assumptions 1 and 2, have the important property of being independent of the actual vehicle schedule (T_k) while still taking into account the time dependency of travel time variance. This property facilitates efficient updating of arrival times and checking of time feasibility, as discussed in Section 4.

Assumption 3 (*expected FIFO*). For any given O–D pair $(i-1, i)$, the expected FIFO is defined as

$$g_{i-1,i}(t_1) < g_{i-1,i}(t_2) \quad \text{for all } t_1 < t_2$$

or

$$\frac{\mathrm{d}(\mu_{i-1,i}(t))}{\mathrm{d}t} > -1.$$

It means that, on average, earlier departure from location $i-1$ would result in earlier arrival at location i. For the proposed travel time model as given in Eq. (1), it requires $b_{i-1,i}$ be greater than -1. We observe that the expected FIFO assumption is weaker than the strict FIFO assumption used in the deterministic model (Ahn and Shin, 1991) and the stochastic FIFO presented by Laporte et al. (1992) and Wellman et al. (1995). This assumption contributes to the computational efficiency of the screening algorithm discussed in Section 4.

4. Probabilistic time windows and feasibility conditions

The randomness of arrival times at individual stops implies that conventional deterministic time constraints must be modified to consider the uncertainty. We introduce a probabilistic model in which the service time constraint specifies that the probability that a service vehicle arrives at a stop within its service time window must be greater than or equal to a pre-specified threshold value called *minimum reliability*. For example, if a minimum reliability of 90% is required in scheduling, all clients must be scheduled for pickup or drop-off during their service time windows with a probability of 0.90 or over. For a given stop i, the probabilistic time window is therefore

$$P(e_i \leqslant A_i \leqslant l_i) \geqslant \beta, \tag{15}$$

where β is a pre-specified constant representing the minimum required reliability.

Based on Assumption 1, Eq. (15) can be rewritten as

$$P(e_i \leqslant A_i \leqslant l_i) = \Phi\left(\frac{l_i - E[A_i]}{\sqrt{\mathrm{Var}[A_i]}}\right) - \Phi\left(\frac{e_i - E[A_i]}{\sqrt{\mathrm{Var}[A_i]}}\right) \geqslant \beta, \tag{16}$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution of A_i, and $E[A_i]$ and $\mathrm{Var}[A_i]$ are the mean and the variance of A_i, respectively.

To make the computation more efficient, we use the formula shown in Eq. (17) to approximate the distribution function of a standard normal distribution (Karian and Dudewicz, 1991).

$$\Phi(x) = \frac{x(4.4 - x)}{10} + 0.5 \qquad \text{for } 0 \leqslant x \leqslant 2.2. \tag{17}$$

With Eqs. (16) and (17), the feasibility condition of Eq. (15) can therefore be transformed into

$$e'_i \leqslant E[A_i] \leqslant l'_i, \tag{18}$$

where

$$e'_i = e_i + b\sqrt{\mathrm{Var}[A_i]},$$
$$l'_i = l_i - b\sqrt{\mathrm{Var}[A_i]},$$
$$b = 2.2 - \sqrt{9.84 - 10\beta},$$

where e'_i is called the *expected* earliest service start time and l'_i the *expected* latest service start time.

Eq. (18) is essentially a deterministic equivalent to the original probabilistic time window Eq. (15). This modified time window (e'_i, l'_i), which stipulates the feasible time interval for the expected arrival time, depends not only the original time window (e_i, l_i), but also the variation of the arrival time ($\mathrm{Var}[A_i]$) and the required reliability level (β). While it is easy to verify the feasibility of the time window constraint at a stop if the mean and variance of arrival time at the stop are known, it is time-consuming to update the means and variances of arrival times at individual stops during a scheduling process as any minor modification in route or schedule would call for such updating. The following section shows that, with the conditions introduced in Section 3, the need for such updating can be minimized.

4.1. Effective time windows for the stops of a given route

We consider a provisional route and are interested in determining the time window of a stop i (of SB k) on the route that would maintain not only the service time feasibility of its own but also the feasibility of other stops on the route. This new time window, denoted as $[e''_i, l''_i]$, is called the *effective* time window, where e''_i and l''_i denotes the effective earliest and latest service start times, respectively.

The *effective* earliest service start time at stop i (e''_i) is defined as the expected earliest service start time at stop i that is feasible for the stop itself and all of its preceding stops. With the expected FIFO property, we can show that the following forward recursive relation holds:

$$e''_i = \max\{e'_i, g_{i-1,i}(e''_{i-1})\}, \tag{19}$$

$$e''_0 = \begin{cases} \text{vehicle's earliest service start time}, & k = 1, \\ e''_{N_{k-1}}, & k > 1, \end{cases} \tag{20}$$

$$i = 1, 2, \ldots, N_k, \quad k = 1, 2, \ldots, M.$$

Conversely, the effective *latest* service start time at stop i (l''_i) is the expected latest service start time at stop i that is feasible for itself and is not too late for all of its following stops. Again, with the expected FIFO assumption, we can obtain the following backward recursive relation:

$$l''_i = \min\{l'_i, h_{i,i+1}(l''_{i+1})\}, \tag{21}$$

$$l''_{N_k} = \begin{cases} \text{vehicle's latest service end time}, & k = M, \\ l''_{0\,(\text{of } SB\ k+1)}, & k < M, \end{cases} \tag{22}$$

$$i = N_k - 1, \ldots, 1, 0, \quad k = M, M-1, \ldots, 1.$$

For each SB, we can also obtain its feasible earliest and latest expected service *start* times at stop $i = 0$ and the feasible earliest and latest expected service *end* times at stop $i = N_k$. Denote ES_k and LS_k as effective service start times for SB k, and EE_k and LE_k as effective service end times, we have

$$ES_k = H_k(e''_{N_k}), \tag{23}$$

$$LS_k = l''_0, \tag{24}$$

$$EE_k = e''_{N_k}, \tag{25}$$

$$LE_k = G_k(l''_0), \tag{26}$$

$$k = 1, 2, \ldots, M,$$

where $G_k(t)$ is a function specifying the expected arrival time function at stop $i = N_k$ given that the departure time at stop $i = 0$ is t, and $H_k(t)$ is the inverse function of $G_k(t)$, representing the expected arrival (service start) time at stop $i = 0$ given that the arrival time at stop $i = N_k$ is t. Based on the proposed travel time model, both functions are linear in form and can be completely specified by Eq. (14).

We note that the effective time window defined above depends only on the original time window (e_i, l_i) and the variance of arrival time and is independent of the route schedule. This property plays a critical role in simplifying the procedures for feasibility verification (Section 4.2) and schedule optimization (Section 5).

4.2. Necessary conditions for feasible insertion

Consider a potential insertion of stop s into SB k of a given route after stop i ($i = 0, 1, 2, \ldots, N$), the necessary condition for the new route remaining feasible is

$$e''_s \leqslant l''_s, \tag{27}$$

$$g_{i,s}(l''_i) \geqslant e''_s \quad \text{if } s \text{ is a pickup stop and link } (i,s) \text{ is not adeadheading movement}, \tag{28}$$

where the effective time window for stop s, $[e''_s, l''_s]$, can be determined on the basis of Eqs. (19)–(22), i.e.

$$e''_s = \max\{e'_s, g_{i,s}(e''_i)\},$$
$$l''_s = \min\{l'_s, h_{s,i+1}(l''_{i+1})\}.$$

Eq. (27) ensures that a feasible effective time window exists at stop s while Eq. (28) imposes the requirement that no idling at a stop is allowed when a vehicle has one or more passengers onboard. The major feature of this condition is that it does not require the arrival times of the original route be updated each time an insertion of a new stop is tried. As a result, if we maintain the time windows information for a given route, we can use that information to determine the feasibility of inserting any stop at any position without having to update the travel times.

The condition is however not sufficient to guarantee the feasibility of the schedule for the following reasons. First, schedule feasibility with respect to other service constraints such as capacity restriction and ride time requirement needs to be satisfied. Second, while it may be feasible to insert the pickup and drop-off stops of a trip separately, it may not be so after both stops of the trip are inserted.

The third reason is due to the possible change in the variance of arrival times at those stops that follow the inserted stop, which may in turn change the time windows of these stops. Fortunately, it can be reasonably expected that addition of new trips would likely not decrease the variances of arrival times at the existing stops and therefore would not widen the time windows of those stops. Consequently, an insertion that is not feasible under the above condition (without considering increase in variance) would be surely not feasible if increased variance is considered.

As a result, the feasibility of the resulting route still needs to be verified even after the above condition is passed. For the ride time constraint, we assume that the expected ride time must be less than or equal to a maximum allowable ride time which is represented as a linear function of the expected direct ride time (refer to Eq. (33) in Section 5).

5. The cost model

As discussed previously, the DARP is often formulated to minimize the costs to both service provider and passengers. Due to the nature of multiple objectives and the randomness of the system state variables, the utility concept is introduced to combine the objectives of the service provider and the passengers, and to resolve the randomness of the operational measures (Keeney and Raiffa, 1976). Specifically, a set of utility functions, which represents the relationship between the degree of dissatisfaction of the service provider and passengers associated with the routing and routing results, are used to represent the scheduling costs. Here again, we consider a specific route of M SBs, with SB k including N_k stops and $N_k/2$ trips. For each trip j in SB k, its pickup and drop-off stops are labeled as j^+ and j^-. The total cost, C, is defined as

$$C = C_0 + \sum_{k=1}^{M} \left(\sum_{i=1}^{N_k} (\delta_i \cdot C_i^{\mathrm{d}}) + \sum_{j=1}^{N_k/2} C_j^{\mathrm{r}}, \right) \tag{29}$$

where C_0 is the expected cost to the service provider. C_i^{d} the cost to a passenger due to deviation from his/her desired time at stop i of SB k. δ_i a variable indicating whether or not stop i specifies a desired service time; $\delta_i = 1$ if stop i specifies a desired service time; otherwise, $\delta_i = 0$. C_j^{r} is the cost to a passenger (trip) j of SB k due to ride deviation.

The operating cost is commonly assumed to be proportional to the total travel time and the number of vehicles used (Savelsbergh and Sol, 1995). This paper assumes that the operator is risk neutral in terms of travel time and their objective function can therefore be modeled as minimizing the expectation of the total travel time, i.e.

$$C_0 = E[L], \tag{30}$$

where L is a random variable representing the total route travel time; and $E[L]$ is the expectation of the route travel time L.

The expected cost to a passenger due to the deviation from his/her most desired pickup and drop-off time, C_i^{d}, is defined as follows:

$$C_i^{\mathrm{d}} = \int_{-\infty}^{\infty} f_{A_i}(x) \cdot U(x - \tau_i)\,\mathrm{d}x, \tag{31}$$

where $f_{Ai}(x)$ is the probability density function of the arrival time A_i; τ_i the passenger's desired service (pickup/drop-off) time at stop i; and $U(x)$ is a utility function representing a passenger's dissatisfaction as a function of the deviation from his/her desired time (x). A general form of the utility function that is commonly used to model passengers' risk-aversion attitude under uncertainty is the quadratic function (Wilson and Colvin, 1977; Jaw et al., 1986). This study uses the utility function $U(x) = c_{\mathrm{d}}x^2$, where c_{d} is an externally specified constant indicating how much weight is allocated in the general objective function.

Based on the given form of the utility function, Eq. (31) can be transformed as a function of the mean and variance of the arrival time at the stop, as shown in Eq. (32):

$$C_i^{\mathrm{d}} = c_{\mathrm{d}} \cdot \{(E[A_i] - \tau_i)^2 + \mathrm{Var}[A_i]\}. \tag{32}$$

From Eq. (32), it can be seen that the inconvenience due to time deviation depends not only on the expected deviation from the most desired time ($E[A_i] - \tau_i$), but also on the variance of the arrival time ($\mathrm{Var}[A_i]$). This makes intuitive sense in that the large arrival time variation would also cause inconvenience to a passenger even if the expected arrival time deviation is 0.

The expected cost to a passenger due to ride deviation, C_i^{r}, is defined as

$$C_j^{\mathrm{r}} = c_{\mathrm{r}} y_j^2, \tag{33}$$

where c_{r} is an externally set constant representing the weight allocated to the ride diversion in the general objective function; y_j is the relative deviation of ride time for trip j, defined as

$$y_i = \frac{E[\tilde{R}_j] - E[\bar{R}_j]}{E[\bar{R}_j]}, \tag{34}$$

$\bar{R}_j$ is the direct ride time for trip j with its expected direct ride time denoted as $E[\bar{R}_j]$. We assume that

$$E[\bar{R}_j] = \mu_{j^+,j^-}\left(\frac{e_{j^+} + l_{j^+}}{2}\right) = a_{j^+,j^-} + b_{j^+,j^-} \cdot \frac{e_{j^+} + l_{j^+}}{2}. \tag{35}$$

$\tilde{R}_j$ is the scheduled ride time for trip j with its expected direct ride time, $E[\tilde{R}_j]$, defined as

$$E[\tilde{R}_j] = E[A_{j^-}] - E[A_{j^+}]. \tag{36}$$

6. Optimizing the schedule for a given route

As a generalization of the deterministic DARP, the DARP_DS is NP-complete, suggesting that any optimal solution method to this problem may be viable only for small sized problems. Since our study concerns itself mainly with the development of methods to solve problems of practical size, it is therefore focused on efficient heuristic approaches. One of the most effective heuristics that have been successfully applied to solve the DARP, as well as the general vehicle routing problems with time constraints, is the parallel insertion algorithm (Bodin et al., 1983; Roy et al., 1984; Jaw et al., 1986; Solomon, 1987; Toth and Vigo, 1997). This algorithm was extended to solve the DARP_DS with the new objective functions and constraints discussed above. The modifications that are made to the original algorithm include the step to test the feasibility of inserting a trip into a provisional route and the *optimization* procedure to identify the best feasible schedule for a given route. Section 4 has detailed the necessary condition for the feasibility test. In this section, we focus on the step for optimizing the schedule for a given route. Appendix provides the pseudocode of the complete algorithm.

The optimization step is to minimize the total additional cost due to inserting an unscheduled trip into a provisional route. The cost function is defined in Eq. (29) and the additional cost is the difference between the cost before and after inserting a trip. As discussed in the preceding section, the cost function is primarily related to three measures, including the total travel time for each vehicle, the arrival time at each stop and the ride time of each passengers. Once a vehicle's visiting sequence is determined, an optimal schedule for this vehicle needs to be obtained by minimizing the total cost of the schedule based on the travel time information. In deterministic cases, it has been shown that the problem can be formulated as one or more unidimensional optimization problems and solved efficiently (Jaw, 1984; Sexton and Bodin, 1985b; Dumas et al., 1990). The following section discusses the extension of this modeling and optimization methodology to handle the time-varying, stochastic case.

Here again, we consider a given route of M SBs. As discussed above, because it is not allowed to schedule any idle time within a SB, the decision variables for the given route involve M variables representing the departure times for the M SBs ($T_k,\ k = 1, 2, \dots, M$). For each SB, its effective service start and end times ($ES_k,\ LS_k,\ EE_k$ and LE_k) can be obtained using Eqs. 23 and 24. For a specific SB k ($k > 1$), the following three cases exist that define its relationship with its preceding SB ($k - 1$), as shown in Fig. 3:

- *Case* I: $EE_k > LS_k$ that is, it would still be too late for servicing the SB k even if a vehicle is able to visit all stops of SB $k - 1$ at the earliest feasible time. Under this case, no feasible schedule exists.
- *Case* II: $LE_k > ES_k$ that is, it would still be too early for servicing the SB k even if a vehicle completes SB $k - 1$ at the latest feasible time. Under this case, idle time is required no matter what feasible service start times (T_{k-1} and T_k) are used, which implies that schedules for SB $k - 1$ and k can just be optimized independently.
- *Case* III: otherwise. In this case, there is an overlap between the feasible service start time window of SB k and the feasible end time window of SB $k - 1$. It implies that, in determining the optimal departure times for these two SBs, we have to consider the potential interaction between them. While it is feasible to design a sequential optimization procedure to solve this

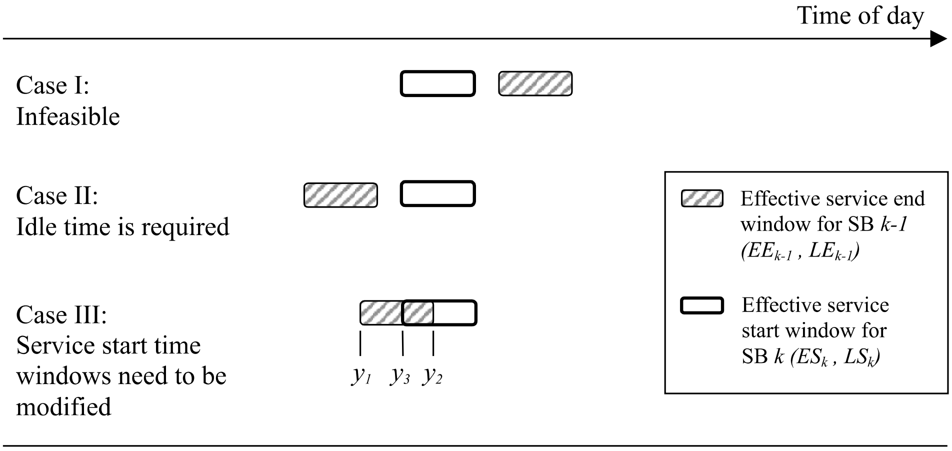

Fig. 3. Effective service start/end time windows for two consecutive SBs.

problem (Dumas et al., 1990), we introduce a sub-optimal, but more efficient treatment to resolve this problem, as discussed as follows.

Under Case III, the feasible service start times of SB $k-1$ and SB k are modified so that the optimal departure times can be decided separately as in Case II. We introduce the following modifications (refer to Fig. 3).

$$ES_{k-1} = H_{k-1}(y_1), \tag{37}$$

$$LS_{k-1} = H_{k-1}(y_3), \tag{38}$$

$$ES_k = y_3, \tag{39}$$

$$LS_k = y_2, \tag{40}$$

where

$$y_1 = EE_{k-1}$$

$$y_2 = \min\{LS_k, LE_{k-1}\}$$

$$y_3 = \begin{cases} ES_k, & ES_k > EE_{k-1}, \\ \frac{1}{2}(y_1 + y_2) & \text{otherwise.} \end{cases}$$

The above treatment can be sequentially applied to the SBs forward from $k = 2$ to $k = M$, which would result in a set of feasible windows for the expected service start times of all SBs. The problem of determining the optimal schedule for the given route becomes solving M independent, unidimensional quadratic optimization problems. For SB k, the model is then

$$\min_{T_k} C(T_k) = \{E[A_{N_k}] - T_k\} + \sum_{i=1}^{N_k} C_i^{\mathrm{d}} + \sum_{j=1}^{N_k/2} C_j^{\mathrm{r}}$$

s.t.

- C_i^{d} and C_j^{r} are quadratic functions of $E[A_i]$ defined by Eqs. (32) and (33).
- $E[A_i]$ is a linear function of T_k defined by Eq. (14).
- $ES_k \leqslant T_k \leqslant LS_k$ (service start time window for SB k).

The optimal service start time can be easily found by setting the derivative of $C(T_k)$ with respect to T_k equal to 0. The solution is

$$T_k^* = \begin{cases} ES_k & \text{if } T_k' < ES_k, \\ LS_k & \text{if } T_k' > LS_k, \\ T_k' & \text{otherwise,} \end{cases} \tag{41}$$

where

$$T_k' = \frac{1 - b'_{N_k} - c_{\mathrm{d}} \sum_{i=1}^{N_k} \delta_i b_i'(a_i' - \tau_i) - c_{\mathrm{r}} \sum_{j=1}^{N_k/2} \frac{(b'_{j-} - b'_{j+})(a'_{j-} - a'_{j+} - \bar{R}_j)}{\bar{R}_j^2}}{c_{\mathrm{d}} \sum_{i=1}^{N_k} \delta_i b_i'^2 + c_{\mathrm{r}} \sum_{j=1}^{N_k/2} \left(\frac{b'_{j-} - b'_{j+}}{\bar{R}_j} \right)^2}, \quad c_{\mathrm{d}} > 0 \ \text{or} \ c_{\mathrm{r}} > 0. \tag{42}$$

Note that if $c_{\mathrm{d}} = c_{\mathrm{r}} = 0$, the solution is either ES_k or LS_k depending on the value of the first component of the optimization function (i.e. $E[A_{N_k}] - T_k$).

7. Computational analysis

The objective of this section is to examine the computational requirements of the proposed algorithm and the sensitivity of the solutions to model parameters through a set of hypothetical problems. The scheduling algorithm described in this paper was coded in Visual C++ as a Windows application program and all the runs presented in this section were executed on a Pentium II 300 IBM-compatible with 256 MB RAM with Windows NT operating system. The system performance is measured using the statistics listed in Table 1. The test problems were created with the following settings:

Table 1
Notations and description of the schedule statistics

Notation	Description
NVS	Total number of vehicles required to service the transportation requests
NTS	Total number of trips scheduled
TT (h)	The expected total travel time of all routes, excluding idle time
VP (trips/vehicle/h)	The ratio of total number of passengers (or trips) to the expected total vehicle time, = NTS/TT
RT (min)	The expected total ride time of all trips divided by total number of trips
TD (min)	The expected total service time deviation divided by total number of trips
CPU (s)	computing time (Pentium II 300 MHz IBM-compatible with 256 MB RAM)

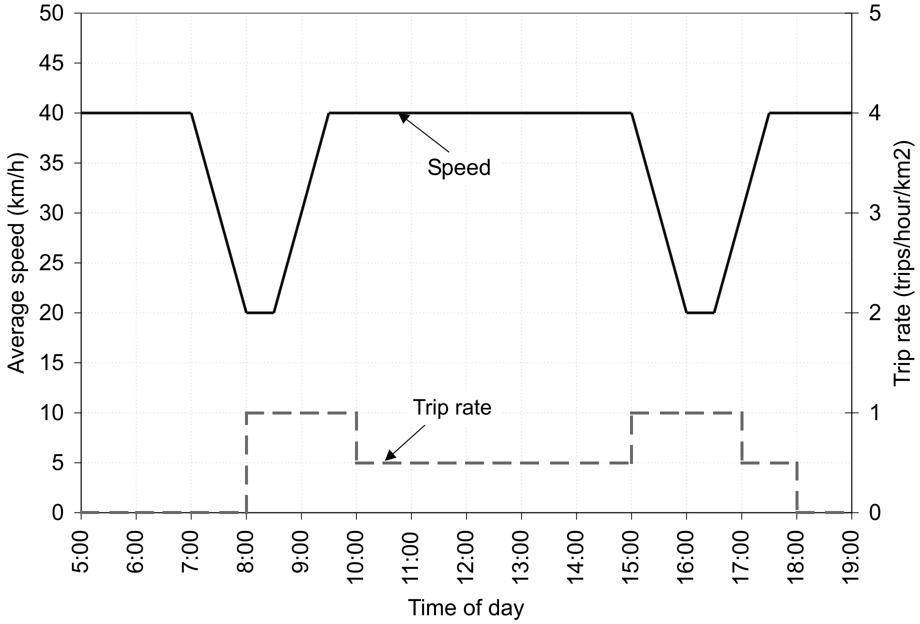

Fig. 4. Time-dependent demand and travel speed.

- The service area is a square of 20×20 sq. km.
- The travel time between any two locations (or O–D travel time) is assumed to be normally distributed with its mean equal to the rectangular distance between the two locations divided by an average travel speed. The average travel speed is time-dependent as defined in Fig. 4, which would thus make the travel time time-dependent. To model the variance of the travel time, the variance-to-mean ratio of travel time [1] (denoted as V/M) is assumed to be the same for all O–D pairs and has a value equal to 10 or 20 s.
- There are a total of 120 vehicles available and all of them are located at the center of the service area. All vehicles have the same seating capacity of 8 seats/vehicle. Each vehicle's available service time is from 6:00 AM to 18:00 PM.
- Passenger trips (pickup and drop-off stops) are assumed to be uniformly distributed over the service area. Each trip has either a desired pickup time or a desired drop-off time (50% each in this study) with a service time between 8:00 AM and 18:00 PM. A total of 2800 trips were generated according the time-dependent trip rate given in Fig. 4. The trips were partitioned to yield five problem sets: P500 (the first 500 trips from 8:00 AM), P1000 (the first 1000 trips from 8:00 AM), P1500 (the first 1500 trips from 8:00 AM), P2000 (the first 2000 trips from 8:00 AM), and P2800 (all of the trips).

[1] It should be noted that the variance-to-mean ratio is different from the more commonly used term coefficient of variation (defined as the ratio of standard deviation to mean). Past research has indicated that the variance-to-mean appeared to be more likely to be constant than the coefficient of variation in a given road network and therefore has been selected for use (Mullen and Fu, 1996; Mohammadi, 1997). Note that, corresponding to variance-to-mean ratios of 10 and 20 s, the coefficients of variation are 0.07 and 0.11, respectively, for a trip of 30 min.

Table 2
Computational efficiency of the solution algorithm

Problem	NT	NTS	NVS	TT	VP	RT	TD	CPU
P500	500	All	72	186.5	2.69	36.6	12.3	91
P1000	1000	All	95	333.6	2.97	32.3	10.9	352
P1500	1500	All	99	497.2	2.95	29.8	9.6	803
P2000	2000	All	107	672.7	2.91	28.6	8.7	1157
P2800	2800	All	119	1068.1	2.62	28.8	8.7	2042

- The maximum service time deviation from each passenger's most desired time is set to 30 min and the maximum ride time is set equal to $20 + 2.0 \times$ expected direct ride time.

The first objective of our numerical experiment was to find out whether or not the proposed heuristic for the DARP_DS is efficient enough for solving large sized problems. The five sub-problems described previously were scheduled under a set of fixed parameter values: $c_d = 0.1$, $c_r = 0.1$, $\beta = 90\%$, $V/M = 10$ s, initial number of vehicles = 70, and the results are summarized in Table 2. It can be seen that, while the required computational time increases at a rate faster than the increase in problem size, the algorithm is shown to be efficient enough for solving problems of practical size on a PC.

We next examine how the objective function parameters c_d and c_r, which are associated with the service time deviation and the ride deviation, influences the routing and scheduling results under a given pattern of O–D travel times. The problem set P1000 was selected for this analysis and the results under various combinations of model parameters are summarized in Table 3. The following observations can be made from these results:

- There is an expected relationship between the cost to the service provider as measured by the number of vehicles required (NVS) and the total travel time (TT) or vehicle productivity (VP) and the parameters c_d, c_r and β which represents the weight placed on the quality of service to passengers. As the values of c_d and c_r increase, the passengers' satisfaction is weighted higher in the routing objective and as a result, more vehicles are required to service the same number of trips.
- The average ride time correlates fairly strongly with the parameter c_r, which is expected as this parameter determines the optimization weight placed on minimizing the passengers' ride deviation. It is also interesting to observe that this performance measure is fairly insensitive to other parameters such as c_r, β, and variation in travel time (V/M). Similarly, the average deviation from passenger's desired service seems to depend only on the parameter associated with it, i.e. c_d.
- The required reliability level (β), which represents the on-time performance that is to be maintained, has a critical impact on the cost to the service provider (NVS, TT and VP). The higher the required reliability level becomes, the more vehicles are needed in order to service the same number of trips, and the longer the total travel time is. A more detailed analysis on how travel time variations influence the expected performance of paratransit service is provided in Fu (1999).

Table 3
Summary of scheduling statistics for P1000 under various combinations of modeling parameters

Sensitivity to the service time deviation parameter ($c_r = 0$, $\beta = 90\%$, $V/M = 10$ s)							
c_d	INV	NTS	NVS	TT	VP	RT	TD
0.0	90	All	91	300.4	3.34	42.4	14.2
0.1	90	All	109	394.3	2.52	42.0	8.1
0.2	90	All	111	393.5	2.53	42.0	7.2
Sensitivity to the ride deviation parameter ($c_d = 0$, $\beta = 90\%$, $V/M = 10$ s)							
c_r	INV	NTS	NVS	TT	VP	RT	TD
0.0	90	All	91	300.4	3.34	42.4	14.2
0.1	90	All	107	365.3	2.69	30.1	16.1
0.2	90	All	112	373.8	2.62	29.9	16.1
Sensitivity to the required reliability level ($c_d = 0 = c_r = 0$, $V/M = 10$ s)							
β	INV	NTS	NVS	TT	VP	RT	TD
50%	90	All	90	260.9	3.86	42.0	13.3
70%	90	All	90	280.7	3.58	42.5	14.2
90%	90	All	91	300.4	3.34	42.4	14.2
Sensitivity to the travel time variation ($c_d = 0 = c_r = 0$, $\beta = 90\%$)							
V/M	INV	NTS	NVS	TT	VP	RT	TD
0	90	All	90	261.0	3.86	42.0	13.3
10	90	All	91	300.4	3.34	42.4	14.2
20	90	All	100	316.6	3.10	41.5	14.0

8. Conclusions

In this paper, we argued that travel time in urban routing environments is inherently time-varying and stochastic due to temporal and spatial variation of traffic congestion and therefore should be explicitly modeled as such in formulating the dial-a-ride scheduling problem (DARP) in order to produce more reliable and efficient schedules. The paper initiated an effort in this direction by introducing the dial-a-ride problem with dynamic and stochastic travel times (DARP_DS), and has made several potentially important contributions. The first was to introduce a unique travel time model to represent the time-dependency and randomness of travel times, taking into account model realism and simplicity, as well as information availability. The second principle contribution was from identifying the assumptions that lead to efficient methods for determining arrival time distribution parameters and probabilistic service time windows. With these assumptions, we were able to extend the screening strategy, which was originally proposed for quickly verifying the feasibility of an insertion step for the deterministic DARP, for solving the DARP_DS. The travel time model and associated methodologies may also be applied to other routing and scheduling problems where traffic congestion needs to be considered. The third contribution was to introduce a set of cost models that consider multiple routing objectives and the randomness of routing cost, and to formulate and solve the problem of optimizing the schedule for a given route under the new travel time model. Finally, the model and algorithm has been successfully integrated into an interactive scheduling tool (Fu, 2000) and was able to solve problems of realistic size on personal computers.

Currently, we are attempting to extend the proposed methodology to the dynamic dial-a-ride scheduling problems where real-time information on vehicle service status and traffic congestion may be available via advanced information technologies and randomness in system states resolves gradually as time unfolds. We will report our results in future publication.

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Appendix A. Parallel insertion algorithm

This appendix details the parallel insertion algorithm that has been used to solve the DARP_DS. The algorithm includes four functions as described as follows.

Notations

U	the set of trips to be scheduled, including $\lvert U\rvert$ trips
IU	the set of trips taken from U to be considered for immediate insertion
V	the set of fleet vehicles available for service, including $\lvert V\rvert$ vehicles
IV	the set of vehicles initially taken from V to be considered in scheduling, including $\lvert IV\rvert$ vehicles ($\lvert IV\rvert \leqslant \lvert V\rvert$)
S	vehicle schedule created during the scheduling process with S^*, S', S'' and $S^\wedge$ representing the best schedules identified within different search scopes
C	additional cost to insert a trip to an existing route with C^*, C', C'' and $C^\wedge$ representing the insertion cost associated with the best schedules identified within different search scopes

Algorithm

Function 1: *main*( )

1. *Initialization*:

- Determine service time windows (e_i, l_i) for each stop (i) of each trip in the (unscheduled) trip list U.
- Sort U according to the earliest pickup time (e_i) of each trip.
- Set the initial set of active vehicles (IV), and optionally initiate each active vehicle with a seed trip.
- Set the size of candidate trip set ($\lvert IU\rvert$).

2. Consider the top $n = \min\{|IU|, |U|)$ trips of U as the set of candidate trips (IU). If $n = 0$, *terminate*, otherwise, set $j = 1$ and $C^* = $ INFINIT.
3. Select jth trip from IU, do the following:
   3.1. Call function *Find_Best_Vehicle_To_Insert*(j), which returns $\{k', S', C'\}$:
      If $C' < C^*$, then
         $j^* = j;\ k^* = k';\ C^* = C';\ S^* = S'$
   3.2. $j = j + 1,$ if $j \leqslant n_0$ then go to Step 3.
4. Assign trip j^* to vehicle k^*; update the schedule and the effective time windows of the schedule (Eqs. (19)–(22)); remove trip j^* from U, and go to Step 2.

Function 2: *Find_Best_Vehicle_To_Insert*(*Trip i*)

1. Set $k = 1$ and $C' = $ INFINIT.
2. Select kth vehicle in the active vehicle set IV, and do the following:
   2.1. Call the function *Find_Best_Positions_To_Insert*(k, i), which returns $\{S'', C''\}$:
      If $C'' < C'$, then
         $k' = k;\ C' = C'';\ S' = S''$
   2.2. $k = k + 1$, if $k \leqslant |IV|$ then go to Step 2.
3. if $C' = $ INFINIT and $|IV| < |V|$ then
      Move one vehicle from V to IV and set $k = |IV|$, go to Step 2
   else if $C' = $ INFINIT and $|IV| = |V|$ then
      Set trip i as rejected.
4. Return $\{k', S', C'\}$.

Function 3: *Find_Best_Positions_To_Insert*(*Vehicle k*, *Trip j*)

1. Set $n = 1$ and $C'' = $ INFINIT.
2. For the nth option of inserting trip j into the provisional route of vehicle k, do the following:
   2.1. Check the conditions: capacity constraint and the necessary condition Eqs. 27 and 28. If any one of the conditions is violated, then go to 2.3.
   2.2. Call function *Optimize_Schedule*(k), *which returns* $\{S^\wedge, C^\wedge\}$
      If $C^\wedge < C''$, then
         $C'' = C^\wedge;\ S'\prime = S^\wedge$
   2.3. $n = n + 1$, if $n \leqslant$ total no. of options, then go to Step 2.
3. Return $\{S'', C''\}$.

Function 4: *Optimize_Schedule*(*Vehicle k*)

1. Identify the SBs in the given route and determine and record service time windows for all SB based on Eqs. (23)–(26). Set $k = 1$.
2. For the kth SB, do the following:
   2.1. Compare the service start and end time windows:
      Case I: Set $C^\wedge = $ INFINIT, and go to Step 4.

Case II: Go to Step 2.2.

Case III: Modify the service time windows for both SB k and $k+1$.

2.2. Determine the optimal departure time for SB k based on Eq. (41).

3.3. $k = k+1$ and go to Step 2.

3. Update the schedule based on the optimal departure times ($S^\wedge$) and calculate the additional insertion cost ($C^\wedge$).
4. Return $\{S^\wedge, C^\wedge\}$.

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