Heuristic Techniques for the Vehicle Routing Problem with Time Windows

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Heuristic Techniques for Solving the Vehicle Routing Problem with Time Windows

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Abstract. Solving the Vehicle Routing Problem (VRP) and its related variants is at the heart of scientific research for optimizing logistic planning. One important variant of the VRP is the Vehicle Routing Problem with Time Windows (VRPTW), where it is assumed that a number of clients must be served by a fleet of vehicles operating from a central depot. Solving this problem requires finding a set of minimum cost routes for the vehicles, while observing a number of pre-defined problem constraints. Since the VRPTW is *NP-hard*, solving it using exact methods is very time consuming. Heuristic and meta-heuristic approaches are usually the preferred option for solving practical size problems. In this paper, we describe the problem and explain some of the most popular heuristic techniques that have been applied for the solution construction and the solution improvement phases of the problem.

Keywords: Vehicle routing with time windows, heuristics, meta-heuristics, combinatorial optimization.

1. Introduction

Solving the Vehicle Routing Problem (VRP) and its related variants is an important field in the area of Operations Research (OR), which has attracted a growing interest in recent years, due to expected benefits of substantial cost reduction and efficient resource consumption. This has indeed encouraged researchers to experiment with diverse algorithms and address different applications, in order to meet the increase in demand for effective vehicle routing decision support tools. In this paper we give an idea about the VRP in general and provide a classification of the different variants of the problem. We then briefly explain the Vehicle Routing Problem with Time Windows (VRPTW), since it is one of the well-studied problems in the vehicle routing literature. We discuss in this paper some of the most popular heuristic techniques that are frequently used for solving this problem.

2. Vehicle Routing Problems (VRPs)

VRPs generally involve problems that deal with the transportation of goods or people between depots and customers, where the objective is to design an optimum schedule for one or more vehicles to service the clients with minimum possible operational cost and maximum customer satisfaction.

The VRP is a generalization of the famous Traveling Salesman Problem (TSP). A general classification of the basic routing and scheduling problems and their interconnections is shown in Fig. 1. Two main problem categories are distinguished in this figure. The first category is called *node routing problems*, in which the service demand is associated with nodes (locations). The second category is called *arc routing problems*, in which the service demand is associated with the arc connecting two nodes. Arc routing problems involve applications like refuse collection and winter gritting, and are often referred to as the Chinese Postman Problem (CPP). As Fig. 1 indicates, the simplest type of node routing problems is the TSP, in which a single vehicle is required to visit a number of nodes, with no restriction on the capacity of the vehicle. It is assumed in the TSP that the trip starts and ends at the same node, without identifying a particular depot point. The multiple-vehicle variant of the TSP is called the Multiple Traveling Salesman Problem (MTSP). If the demands of locations are added to the problem, the capacity of the vehicle is restricted, and a depot point is identified, it is usually called the Capacitated Vehicle Routing Problem (CVRP). Requesting two different service types (pickup and delivery) in the problem instance, transforms it to the Pickup and Delivery Problem (PDP) category. In addition, a service time window (TW) may be imposed on any of the above variants to allow for more realistic applications of the problem, in which the visiting time of each node is restricted between certain pre-defined bounds.

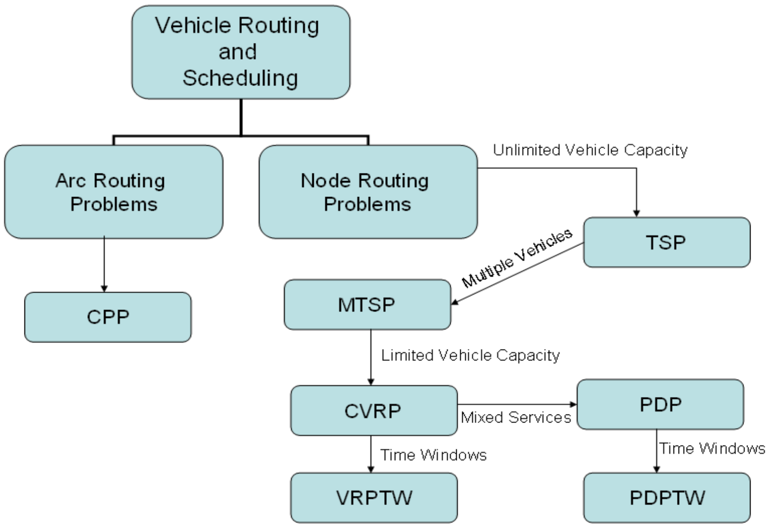

Figure 1: Vehicle routing and scheduling problems

It is known that exact algorithms have their limitations in solving large scale combinatorial optimization problems. In the vehicle routing domain, [1] indicates that today's exact methods cannot consistently solve VRP instances having more than 50-100 customers, which is generally small for most realistic applications. Moreover, given the complexity of the problem, it is very unlikely that an algorithm that solves the problem to optimality in a reasonable time will be developed in the near future. Hence, this paper mainly investigates heuristic approaches as possible solution methods to the problem of interest, which is the VRPTW, discussed in the following sections.

3. The Vehicle Routing Problem with Time Windows (VRPTW)

The VRPTW is an important problem occurring frequently in transportation systems. The problem deals with a number of customer requests that must be dealt with by a fleet of vehicles. All customers are assumed to require the same type of service (either pickup or delivery but not both). Each vehicle route must start and end at a central depot, and each customer must be visited exactly once. Two main constraints are strictly enforced in this problem: the capacity constraint requires that the vehicle capacity should not be exceeded at any time, while the time window (TW) constraint requires that each customer must be visited during a pre-specified time window interval, i.e., if the vehicle reaches the customer before the beginning of its TW, it should wait until the allowed service time begins. Similarly, arrival after the deadline of the TW means a violation of problem constraints. The solution objective is usually hierarchical, such that the minimization of the number of vehicles used is a primary objective, followed by minimizing the total travel distance of the operating fleet, or the total schedule duration.

Solving the VRPTW generally requires two types of decisions: 1) the *assignment or the grouping* decision, which means to assign to each vehicle a subset of nodes from the customers set, and 2) the *routing and* scheduling decision which involves generating a minimum cost route for each vehicle to visit its assigned requests, such that the generated route respects the capacity and TW constraints. In addition, the solution process often consists of two phases: *solution construction*, in which one or more solutions for the problem is generated, and *solution improvement*, in which the initial solution is improved using a heuristic or a meta-heuristic approach [2]. Some heuristic approaches for solving these two phases are discussed in the following two sections.

4. Solution Construction for the VRPTW

Solution construction refers to the creation of a set of routes for the vehicles by selecting nodes (customers) and inserting them in one of the partial routes already created, or in a new route. The decision to select a particular node for insertion is usually based on a cost-minimization criterion, and requires that the insertion of the node in a selected route does not cause violations of problem constraints. Two main types of solution construction exist in the VRPTW literature: *sequential construction*, which builds routes one after the other and parallel construction which builds several routes at the same time. Sequential construction does not attempt to allocate an additional vehicle unless no more requests can be 'feasibly' added to the current vehicle. A parallel construction, on the other hand, initially pre-specifies the number of vehicles that could be used, but more vehicles can be added as needed if the initial estimate of the number of vehicles does not serve all requests without violating the constraints of the problem. Several famous construction heuristic are described by Solomon in [3]. The most important of which are the insertion heuristics, which are briefly explained below.

Insertion Heuristics. This is a class of sequential construction heuristics described by Solomon in [3], and is based on expanding the current route by inserting one un-routed customer at each iteration. The general idea is demonstrated in Fig. 2, where the un-routed customer *k* (left) was inserted between customers *i* and *j* (right) in the progressing route.

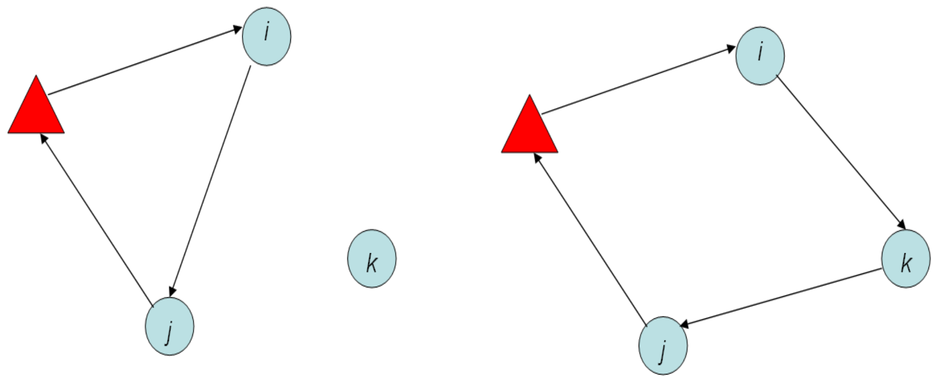

Figure2:The insertion heuristic

More specifically, the insertion heuristic starts by initializing the route under consideration with a seed customer. Other customers are then selected for addition to the current route, based on two cost measures to be defined shortly. When no more un-routed customers can be 'feasibly' inserted in the current route, the process is repeated for a new route. The seed customer could be selected as the customer farthest from the depot, or as the customer having the earliest allowed starting service time.

Three insertion heuristics have been defined by Solomon, based on the criteria used for selecting an un-routed customer to be inserted next in the current route. The first and most successful insertion heuristic (called I1 insertion) proceeds as follows, after initializing the route with a seed customer:

1. Assume the current partial route created is $(i_0, i_1, i_2, \ldots i_m)$, where i_0 and i_m represent the depot;
2. For each un-routed customer u, find its best feasible insertion position in the route, between two customers i and j. The best insertion position is the one that minimizes the first cost measure, which we will call $c1(u)$. $c1(u)$ is a measure that is calculated based on both the extra travel distance and time delay that happens in the route, due to the insertion of u;
3. Find the best customer u^* to be inserted in the route, where u^* is the customer having the maximum value of a second measure, which we will call $c2(u)$. $c2(u)$ is calculated based on both $c1(u)$ (the first measure) and the distance from the depot.

The second insertion heuristic (I2) differs from I1 in the definition of the measure $c2(u)$, since it is now a combined measure of the total route distance and time resulting after the insertion of u (rather than the extra distance and time, as in I1). On the other hand, the third sequential insertion heuristic (I3) differs from I1 in the definition of *c1(u)*, since the urgency of servicing the new customer *u* is now also considered in this measure.

A *parallel construction heuristic* for the VRPTW was proposed by Potvin and Rousseau [4], based on the I1 heuristic described above. In this heuristic several routes are first initialized with seed customers. To determine the number of initial routes, the authors first ran the I1 sequential construction algorithm. Further routes are later added as needed if the initial number of routes does not yield a feasible solution. After determining the best (feasible and least cost) insertion position for each un-routed customer, the customer whose insertion in the solution causes the least increase in the overall cost is selected next for insertion. Besides a weighted sum of the extra travel distance and time delay used by Solomon to determine the cost of the insertion, this parallel heuristic adds a *regret measure* over all routes. The regret measure is a 'lookahead' estimate of the cost of not inserting the customer immediately in its current best route, i.e., if its insertion was postponed to be done later in a different route. More details about different construction techniques for the VRPTW can be found in [2].

5. Solution Improvement for the VRPTW

Solution improvement within the context of the VRPTW refers to the gradual and repeated modification of the initial solution until a certain stopping condition is satisfied. The initial solution is usually a feasible solution, obtained using a construction algorithm, such as the ones described in the previous section. One must then define a *neighborhood move* that can be applied to the initial/current solution to obtain a new solution within its neighborhood, i.e., the new solution only differs in a few attributes from the current solution. For example, the new solution could be obtained by modifying some edges connecting customers in the current solution. Afterwards, the generated solution is evaluated based on the objective function, and may replace the previous solution if it is smaller in cost.

Edge exchanges are the most popular solution improvement method, first described by Lin [5] for the TSP. Edge exchanges are applied to one route in the current solution, and depend on removing a number *k* of edges from the current route, and replacing them with another set of *k* edges. Thus, the process is often called *k-exchange*. A route that cannot be further improved by a *k-exchange* is called a *k-optimal*. Performing all possible *k-exchanges* on a route requires $O(n^k)$ time. Thus, moves beyond 2 or 3 exchanges are very time consuming. The *2-exchange* move is illustrated in Fig. 3.

Several other improvement methods have been described for the VRPTW, a few of them are modifications to the basic edge-exchange moves. For example, Potvin and Rousseau [6] introduce a modification to the *2-Opt* heuristic of [5], called the *2-Opt** exchange heuristic. *2-Opt** works on two different routes (unlike *2-Opt* which exchanges edges belonging to the same route). *2-Opt** tries to combine the two routes without changing the orientation of the tours. For further details about the different improvement heuristics the reader is referred to [2].

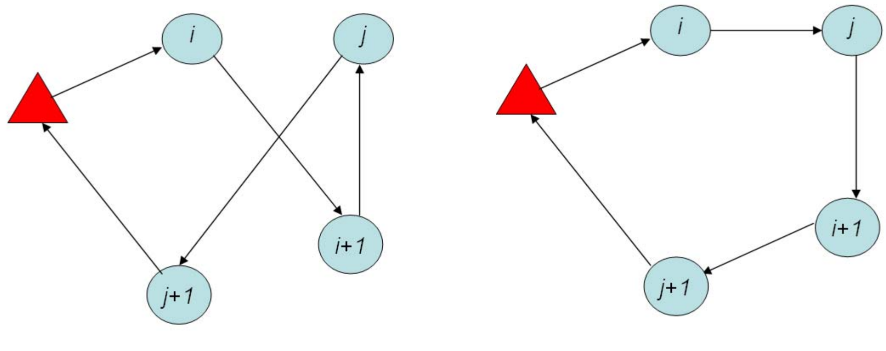

Figure 3: 2-exchange move

6. Summary and Conclusions

Vehicle routing problems have gained considerable attention in the last few decades. This is mainly due to the increase in complexity in transportation and logistics demands, and the urgent need for optimizing client services, reducing operational costs, and limiting the negative environmental impacts that may result from the non-optimal planning of vehicles and their routes. This paper provided a general classification of vehicle routing problems existing in the literature and emphasized one particular important variant, the Vehicle Routing Problem with Time Windows (VRPTW). We presented in this paper a summary of some popular heuristic algorithms that have been applied for the solution construction and the solution improvement phases of the VRPTW.

7. Acknowledgment

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