

ROB311

Medical diagnosis

A hospital uses a support system for detecting lung problems. The system is designed to help in the diagnosis of **tuberculosis**, **cancer**, and **bronchitis**.

The system will use previous data from the hospital gathered from previous consultations.

At the registration in the hospital, a new patient it is asked to fill in a questionnaire and answer 2 questions: "Have you recently visited Asia?" and "Are you a smoker?".

The data shows that:

$$P(A) = 0.1 \quad P(S) = 0.3$$

- among all the patients, **10% have recently visited Asia**, and **30% are smokers**; $P(T|A)$
- Tuberculosis is present in Asia, and **a patient who recently visited Asia has 10% of having tuberculosis** and **a patient who have not been recently to Asia has only 1% of having tuberculosis**; $P(T|\neg A)$
- (Patients that smoke and complain of lung problems have 20% of having cancer) (against **only 2% for patients that do not smoke**); $P(C|S)$
- Patients that do not smoke are suffering in 80% of cases of only a bronchitis** (against only **60% for people that smoke**). $P(B|\neg S)$

The doctor proposes only 2 tests:

- T_1 : The doctor auscultates the patient's lungs with a stethoscope. **A bronchitis or a lung cancer can be detected in 60% of cases.** When **the patient has none of these two diseases, the doctor will detect it with a probability of 99%**;
- T_2 : The doctor orders an X-Ray. **With the X-Ray the tuberculosis or lung cancer are detected in 70% of cases.** If **the patient has none of these two diseases, nothing will be observed on the X-Ray with a probability of 98%**

$$\begin{aligned} &\rightarrow P(T_1 | B \cup C) \\ &\rightarrow P(T_1 | \neg B \wedge \neg C) \\ &\rightarrow P(T_2 | T \cup C) \\ &\rightarrow P(T_2 | \neg T \wedge \neg C) \end{aligned}$$

Marginal Probabilities

$$P(A) = 0.10, \quad P(S) = 0.3$$

Conditional Probabilities

$$P(T | A) = 0.1, \quad P(T | \neg A) = 0.01$$

$$P(C | S) = 0.2, \quad P(C | \neg S) = 0.02$$

$$P(B | S) = 0.6, \quad P(B | \neg S) = 0.8$$

Test Probabilities

$$P(T_1 | B \cup C) = 0.6$$

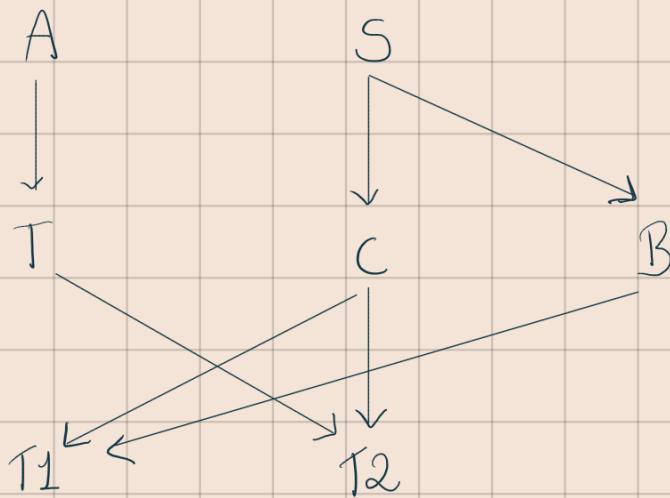
$$P(T_1 | \neg B \wedge \neg C) = 0.01$$

$$P(T_2 | T \cup C) = 0.7$$

$$P(T_2 | \neg T \wedge \neg C) = 0.02$$

Questions:

- Model this problem using a Bayesian network;



- If the patient is not smoking and has not recently visited Asia, can you infer with disease?

~~T~~

$$P(T \wedge \neg A) = P(\neg A) \cdot P(T | \neg A) = 0.9 \cdot 0.01 = 0.009$$

$$P(T \wedge \neg A \wedge \neg S) = P(T \wedge \neg A) \cdot P(\neg S) = 0.009 \cdot 0.7 = 0.0063$$

~~C~~

$$P(C \wedge \neg S) = P(\neg S) \cdot P(C | \neg S) = 0.7 \cdot 0.02 = 0.014$$

$$P(C \wedge \neg A \wedge \neg S) = P(C | \neg S) \cdot P(\neg A) = 0.014 \cdot 0.9 = 0.0126$$

~~B~~

$$P(B \wedge \neg S) = P(\neg S) \cdot P(B | \neg S) = 0.7 \cdot 0.8 = 0.56$$

$$P(B \wedge \neg A \wedge \neg S) = P(B | \neg S) \cdot P(\neg A) = 0.56 \cdot 0.9 = 0.504$$

~~None~~

$$P(N) = 1 - [P(T) + P(B) + P(C)] = 0.4771$$

- According to the disease inferred in Point 2, the doctor decides to auscultate the patient's lungs with a stethoscope? Why?
The stethoscope test is negative. What is the new inferred diagnosis? $P(N) \rightarrow \text{none disease}$

→ Both can be detected in 60% of the cases, plus Bronchitis is the most probable disease

$$\downarrow \left\{ \begin{array}{l} P(T | \neg A \wedge \neg S) \cdot P(\neg T_1 | T) = 0.0063 \cdot 1 = 0.0063 \\ P(C | \neg A \wedge \neg S) \cdot P(\neg T_1 | C) = 0.0126 \cdot 0.4 = 0.00504 \\ P(B | \neg A \wedge \neg S) \cdot P(\neg T_1 | B) = 0.504 \cdot 0.4 = 0.2016 \\ P(N | \neg A \wedge \neg S) \cdot P(\neg T_1 | N) = 0.4991 \cdot 0.99 = 0.49283 \end{array} \right.$$

$$\sum P() = 0.68527$$

• Normalizing:

$$P(T | \neg T_1) = 0.0063 / 0.68527 \approx 0.0092$$

$$P(B | \neg T_1) = 0.00504 / 0.68527 \approx 0.0074$$

$$P(C | \neg T_1) = 0.2016 / 0.68527 \approx 0.2943$$

$$P(N | \neg T_1) = 0.49283 / 0.68527 \approx 0.6891$$

- The doctor orders an X-Ray. The X-Ray test is positive. What is the new inferred diagnosis?

$$I) P(T_2 | T) = 0.7$$

$$P(T_2 | C) = 0.7$$

$$P(T_2 | \neg T \wedge \neg C) = 0.02$$

$$II) \left\{ \begin{array}{l} P(T \wedge T_2) = P(T | \neg T_1) \cdot P(T_2 | T) = 0.0092 \cdot 0.7 = 0.00644 \\ P(C \wedge T_2) = P(C | \neg T_1) \cdot P(T_2 | C) = 0.0074 \cdot 0.7 = 0.00518 \\ \sum P(B \wedge T_2) = P(B | \neg T_1) \cdot P(T_2 | B) = 0.2943 \cdot 0 = 0 \\ P(N \wedge T_2) = P(N | \neg T_1) \cdot P(T_2 | N) = 0.6891 \cdot 0.02 = 0.013782 \end{array} \right.$$

$$\sum_{\text{Sum}} = 0.025402$$

III) Normalizing

$$P(T | T_2) = \frac{0.00644}{\text{Sum}} = 0.2536$$

$$P(B | T_2) = \frac{0}{\text{Sum}} = 0$$

$$P(C | T_2) = \frac{0.00518}{\text{Sum}} = 0.2040$$

$$P(N | T_2) = \frac{0.013782}{\text{Sum}} = 0.5424$$

- Was the X-Ray needed?

The X-Ray was needed to ensure a thorough and accurate diagnosis, as it significantly increased the probabilities of detecting critical conditions like tuberculosis and cancer.