Obligatorio Series Temporales

Ventas semanales del agua / temperatura media semanal en Uruguay

Fuentes de datos

La temperatura se descargó de la web https://en.tutiempo.net/) y los datos son de la estación meteorológica cercana al aeropuerto de Carrasco. La bajada fue por día desde el 01/01/2017 y se pasó a semanal haciendo el promedio de los valores de la semana.

La venta de agua está en litros, también es semanal y del mismo período pero en este caso es la suma de los valores de la semana. Los datos se obtuvieron de una empresa que hace venta minorista, por lo que el factor stock no tiene incidencia en el dataset. Se excluyó aguas saborizadas del análisis y solo se tomó en cuenta agua mineral tanto con como sin gas y en todos sus calibres.

```
In [1]: #install.packages("xlsx")
    #install.packages("astsa")

library(xlsx)
    library(lubridate)

options(repr.plot.width=10, repr.plot.height=5) #ajusta tamaño de graficas

Attaching package: 'lubridate'

The following object is masked from 'package:base':
    date
```

Adicionalmente a los criterios dados en clase para identificar la mejor predicción de la serie (AIC, BIC, ruido blanco Gaussiano, varianza) utilizaremos la función mape definida a continuación.

```
In [2]: mape <- function(actual,pred){
    mape <- mean(abs((actual - pred)/actual))*100
    return (mape)
}</pre>
```

```
In [3]: base <- read.xlsx("Base time series.xlsx", sheetIndex = 1)

base_train <- base[base$Class == 'Train',]
base_test <- base[base$Class == 'Test',]

Litros=round(base_train[4] / 10000, 2)
# Normalizamos los datos de venta para pasarlos a la misma escala que la tempe ratura

Temp = round(base_train[5], 2)
#Temp</pre>
```

In [4]: head(base) head(Litros) head(Temp)

A data.frame: 6 x 6

Year	Weeknum	Yearweek	Sales_Liters	Temp_Avg	Class
<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<fct></fct>
2017	1	1	263778.2	22.88571	Train
2017	2	2	249777.5	23.24286	Train
2017	3	3	266308.4	23.52857	Train
2017	4	4	261133.6	21.95714	Train
2017	5	5	292004.5	24.58571	Train
2017	6	6	236130.0	21.11429	Train

A data.frame: 6

x 1

Sales_Liters

<dbl></dbl>
26.38
24.98
26.63
26.11
29.20
23.61

A data.frame:

6 x 1

Temp_Avg

<dbl></dbl>
22.89
23.24
23.53
21.96
24.59
21.11

Metodología

Según lo visto en el curso utilizaremos la receta de Box-Jenkins para trabajar con nuestra serie temporal, es decir vamos a:

Graficar los datos

Trasnformar los datos (por ejemplo, transformación logarítmica, o detrend o ambos, diferenciación).

Identificar los órdenes de dependencia (acf, pacf).

Estimación de parámetros (fit, básicamente mínimos cuadrados o máxima verosimilitud).

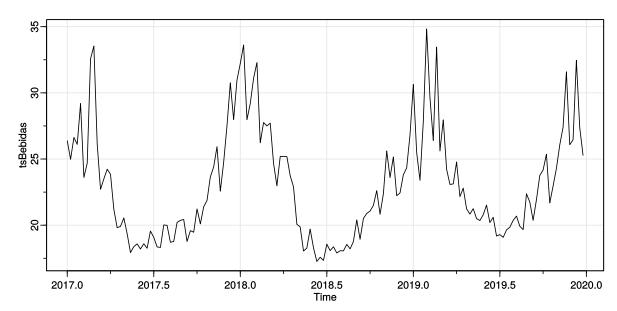
Diagnóstico (análisis de residuos por ejemplo).

Elección del modelo (criterios de información tipo AIC, evitar overfitting, etc.)

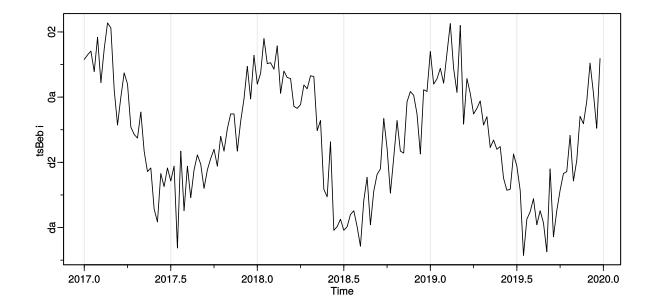
Predicción en base a estimadores lineales calculados recursivamente e intervalos de confianza.

Graficamos los datos

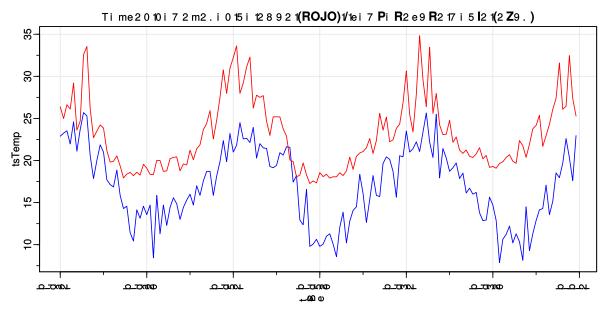
```
In [5]: tsBebidas = ts(Litros, start=c(2017,1,1), freq = 52)
In [6]: tsTemp = ts(Temp, start=c(2017,1,1), freq = 52)
In [7]: tsplot(tsBebidas)
```



In [8]: tsplot(tsTemp)



tsplot(tsTemp, ylim=c(min(tsTemp),max(tsBebidas)), col = "blue", main = "VENTA In [9]: S SEMANALES DE AGUA (ROJO) / TEMPERATURA MEDIA (AZUL)") lines(tsBebidas, col = "red")



Prueba de algunos modelos

Se procede a ajustar algunos modelos que representen la serie de Venta de Agua a partir de la temperatura y de sus valores historicos:

1.
$$A_t=eta_0+eta_1 t+w_t$$
 .

2.
$$A_t=eta_0+eta_1 t+eta_2 (T_t-ar{T})+w_t.$$

$$egin{align} 2. \ A_t &= eta_0 + eta_1 t + eta_2 (T_t - ar{T}) + w_t. \ 3. \ A_t &= eta_0 + eta_1 t + eta_2 (T_t - ar{T}) + eta_3 (T_t - ar{T})^2 + w_t. \end{array}$$

Modelo 1 : regresión lineal ventas vs trend

```
In [10]: A=tsBebidas
         T=tsTemp
In [11]: # Modelo 1
         trend = time(A)
                              # time
         fit = lm(A \sim trend, na.action=NULL)
                      # regression results
         summary(fit)
         num = length(A)
                                                                # sample size
         AIC(fit)/num - log(2*pi)
                                                               # AIC
         BIC(fit)/num - log(2*pi)
                                                                # BIC
         Call:
         lm(formula = A ~ trend, na.action = NULL)
         Residuals:
                     1Q Median
                                   3Q
         -5.7745 -3.2703 -0.9166 2.3601 11.6573
         Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
         (Intercept) -416.3563 774.8193 -0.537 0.592
                       0.2177
                                 0.3839
                                           0.567
                                                    0.571
         Residual standard error: 4.152 on 154 degrees of freedom
         Multiple R-squared: 0.002084, Adjusted R-squared: -0.004396
         F-statistic: 0.3216 on 1 and 154 DF, p-value: 0.5715
         3.87274004614084
         3.93139112320333
```

Modelo 2 : regresión lineal ventas vs trend + centered temp

```
In [12]: # Modelo 2
        temp = T-mean(T) # center temperature
        trend = time(A) # time
        fit = lm(A \sim trend + temp , na.action=NULL)
        summary(fit)
                    # regression results
        num = length(A)
                                                         # sample size
        AIC(fit)/num - log(2*pi)
                                                         # AIC
        BIC(fit)/num - log(2*pi)
                                                         # BIC
        Call:
        lm(formula = A ~ trend + temp, na.action = NULL)
        Residuals:
           Min
                   1Q Median
                                3Q
                                       Max
        -6.1859 -1.7303 -0.3024 1.2555 8.2124
        Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
        trend
        temp
                     0.7466
                               0.0483 15.458 < 2e-16 ***
        ---
        Signif. codes: 0 '***, 0.001 '**, 0.01 '*, 0.05 '.', 0.1 ', 1
        Residual standard error: 2.603 on 153 degrees of freedom
        Multiple R-squared: 0.6104, Adjusted R-squared: 0.6054
        F-statistic: 119.9 on 2 and 153 DF, p-value: < 2.2e-16
        2.9449006406558
        3.02310207673912
```

Modelo 3 : regresión lineal ventas vs trend + centered temp + temp^2

```
In [13]: # Modelo 3
         Tmed=mean(T)
         temp = T-Tmed # center temperature
         temp2 = temp^2
                                   # square it
         trend = time(A) # time
         fit = lm(A \sim trend + temp + temp2, na.action=NULL)
         summary(fit)
                      # regression results
         num = length(A)
                                                                # sample size
         AIC(fit)/num - log(2*pi)
                                                                # AIC
         BIC(fit)/num - log(2*pi)
                                                                # BIC
         Call:
         lm(formula = A ~ trend + temp + temp2, na.action = NULL)
         Residuals:
            Min
                     1Q Median
                                     30
                                            Max
         -6.2873 -1.4381 -0.2104 1.2219 8.2644
         Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
         (Intercept) -1.998e+03  4.756e+02  -4.201  4.51e-05 ***
                     1.001e+00 2.356e-01 4.248 3.74e-05 ***
                     7.615e-01 4.648e-02 16.381 < 2e-16 ***
         temp
                    3.790e-02 9.996e-03 3.791 0.000216 ***
         temp2
         ---
         Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
         Residual standard error: 2.496 on 152 degrees of freedom
         Multiple R-squared: 0.6441, Adjusted R-squared: 0.6371
```

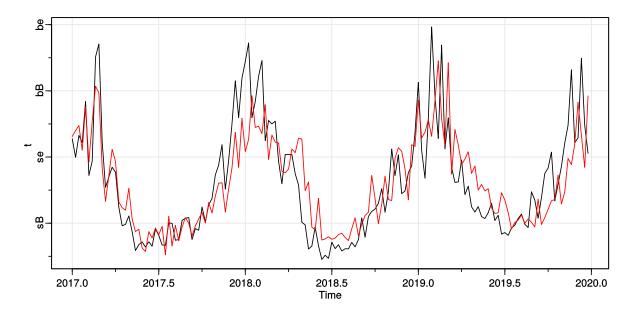
F-statistic: 91.69 on 3 and 152 DF, p-value: < 2.2e-16

2.86737058328647

2.96512237839062

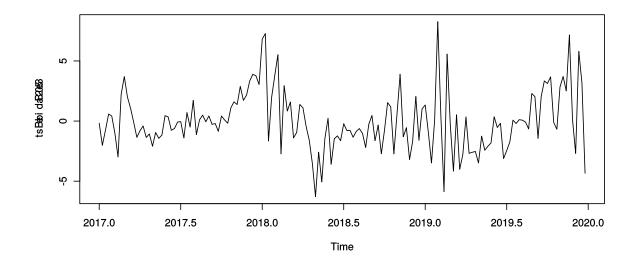
In [14]: # El modelo 3 es por ahora la mejor aproximacion de acuerdo a los indices de A
IC y BIC

predictions = ts(fitted(fit), start = 2017,frequency=52) #Creo una serie tempo
ral con los valores ajustados.
tsplot(A)
lines(predictions,col=2)



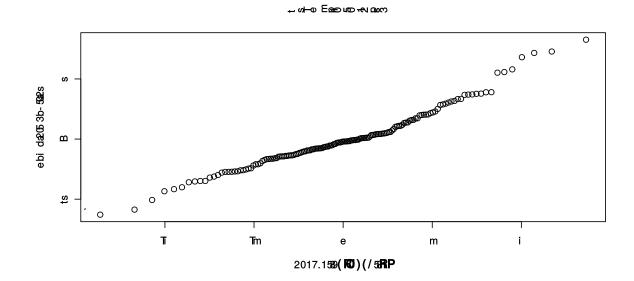
Residuos modelo 3

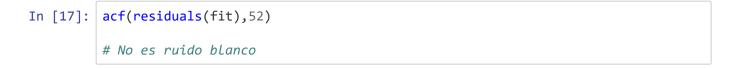
In [15]: plot(residuals(fit))

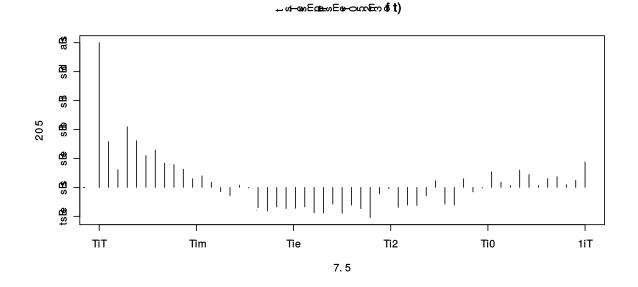


```
In [16]: qqnorm(residuals(fit))
qqline(residuals(fit))

# No es Gaussiano
```







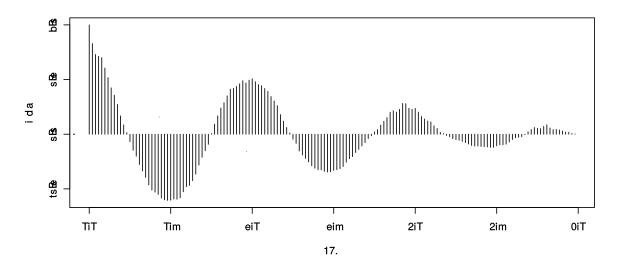
Como se puede observar todavia quedan componentes de autocorrelacion

Procederemos con análisis de componentes estacionarias tanto en la serie de Agua como en la de Temperaturas

Análisis de componentes estacionarias

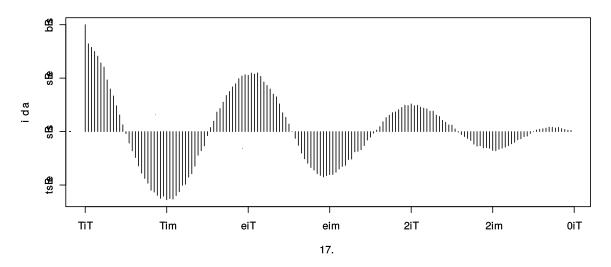
In [18]: acf(tsBebidas,181,main="Venta Semanal de Agua")

- ഗ⊢ഘടേഹാ⊟– 5 വേഗമാ **ft**

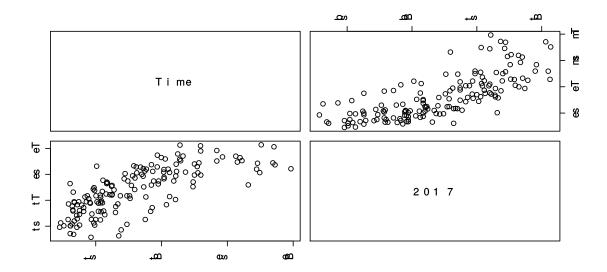


In [19]: acf(tsTemp,181,main="Temperatura promedio Semanal")

+ on the ongo to the or both

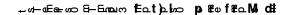


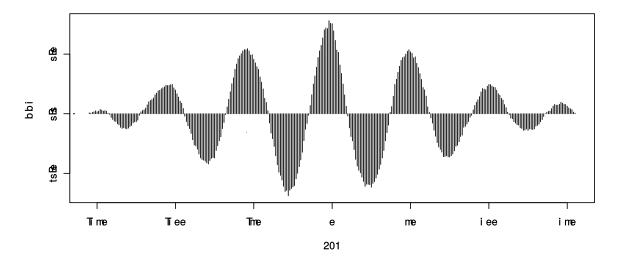
```
In [20]: pairs(cbind(Agua=tsBebidas, Temp=tsTemp))
# Observamos una alta correlación
```

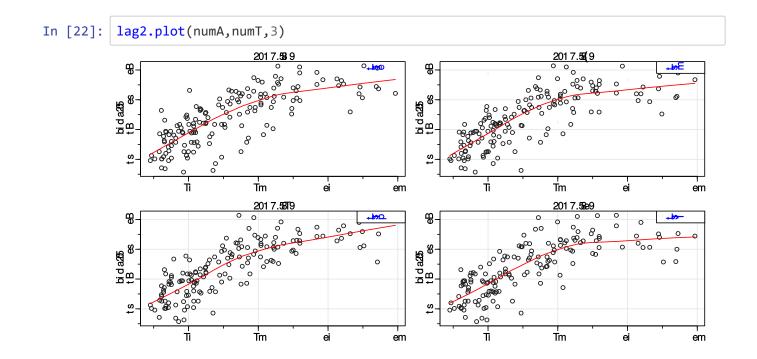


In [21]: numA=as.numeric(A)
 numT=as.numeric(T)
 ccf(numA,numT, 157, main="Venta Semanal Agua vs Temperatura Media", ylab="CCF"
)

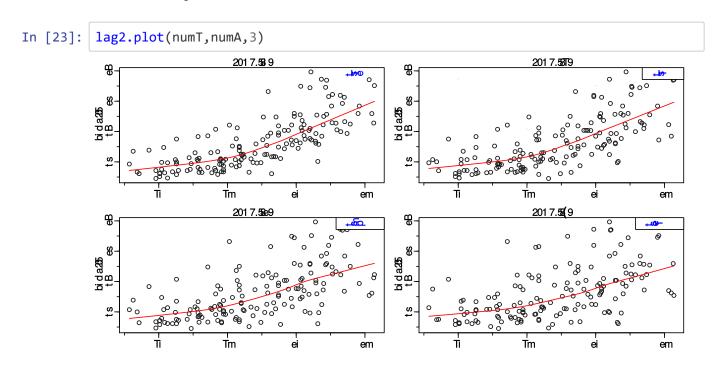
#Vemos una muy fuerte correlación cruzada en el mismo lag, además se notan cor relaciones negativas muy altas a los 6 meses y positivas al año, tanto para ad elante como para atrás







La serie de Ventas esta correlacionada fuertemente con la temperatura de la semana actual pero tambien se observa correlacion con las siguientes



En sentido inverso tambien se observa que el pronostico de temperatura para una semana dos y hasta tres tienen gran correlacion con la venta actual de Agua

Observaciones:

- La serie de Venta de Bebidas al igual que la Temperatura muestra correlaciones fuertes cada 6 y cada 12 meses.
- Cada 6 meses se obtiene el pico de correlacion negativa debido a estar en la estación opuesta.
- Cada 12 meses se da el pico de correlacion positivo porque tiene una fuerte estacionalidad anual.
- Se observa cierta correlacion en las ventas de acuerdo a las temperaturas experimentadas en las semanas anteriores y tambien pronosticos.

Se propone un modelo:

$$A_t = eta_0 + eta_1 T_t + eta_2 T_{t-1} + eta_3 T_{t+1} + w_t.$$

Es decir, estimar la venta de agua actual por una media más algo que depende de la temperatura actual, la previa y el pronostico de la semana siguiente.

Problema: como alineamos las series A y T_t , T_{t-1} , T_{t+1} .

```
In [24]: temp_ant=lag(tsTemp,-1)
temp_post=lag(tsTemp,1)
```

In [25]: agua = ts.intersect(A,tsTemp,temp_ant,temp_post, dframe=TRUE)
agua

A data.frame: 154 x 4

Sales_Liters	Temp_Avg	Temp_Avg.1	Temp_Avg.2
<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
24.98	23.24	22.89	23.53
26.63	23.53	23.24	21.96
26.11	21.96	23.53	24.59
29.20	24.59	21.96	21.11
23.61	21.11	24.59	23.70
24.68	23.70	21.11	25.69
32.57	25.69	23.70	25.31
33.54	25.31	25.69	20.57
26.14	20.57	25.31	17.86
22.71	17.86	20.57	20.00
23.50	20.00	17.86	21.86
24.23	21.86	20.00	21.00
23.85	21.00	21.86	17.71
21.20	17.71	21.00	17.14
19.81	17.14	17.71	16.86
19.91	16.86	17.14	18.86
20.55	18.86	16.86	15.86
19.36	15.86	18.86	14.29
17.93	14.29	15.86	14.57
18.38	14.57	14.29	11.43
18.59	11.43	14.57	10.43
18.20	10.43	11.43	14.14
18.60	14.14	10.43	13.14
18.26	13.14	14.14	14.57
19.56	14.57	13.14	13.57
19.08	13.57	14.57	14.71
18.36	14.71	13.57	8.43
18.32	8.43	14.71	15.86
20.02	15.86	8.43	11.29
19.99	11.29	15.86	14.71
		•••	
20.79	16.21	16.00	13.79
21.52	13.79	16.21	12.86
20.20	12.86	13.79	12.93

Sales_Liters	Temp_Avg	Temp_Avg.1	Temp_Avg.2
<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
20.60	12.93	12.86	15.64
19.18	15.64	12.93	14.71
19.29	14.71	15.64	12.86
19.08	12.86	14.71	7.86
19.64	7.86	12.86	10.64
19.85	10.64	7.86	11.21
20.36	11.21	10.64	12.21
20.69	12.21	11.21	10.21
19.92	10.21	12.21	11.29
19.67	11.29	10.21	10.36
22.37	10.36	11.29	8.14
21.75	8.14	10.36	14.50
20.37	14.50	8.14	9.29
21.93	9.29	14.50	11.29
23.76	11.29	9.29	12.86
24.17	12.86	11.29	14.14
25.37	14.14	12.86	14.29
21.68	14.29	14.14	17.07
22.96	17.07	14.29	13.57
24.29	13.57	17.07	15.14
26.06	15.14	13.57	18.53
27.40	18.53	15.14	17.96
31.59	17.96	18.53	19.67
26.08	19.67	17.96	22.61
26.45	22.61	19.67	20.33
32.46	20.33	22.61	17.61
27.39	17.61	20.33	22.96

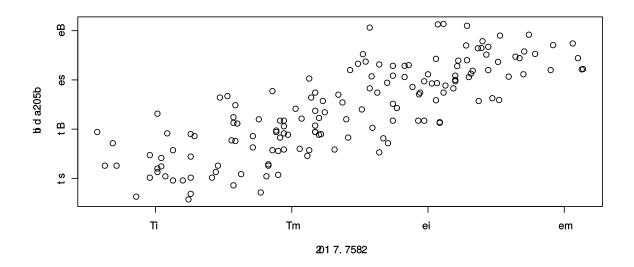
Modelo 4: Regresión ventas t vs temperatura t + temperatura t+1 + temperatura t-1

```
In [26]:
         summary(fit1 <- lm(A ~ tsTemp + +temp ant + temp post , data=agua, na.action=N</pre>
         ULL))
         Call:
         lm(formula = A ~ tsTemp + +temp_ant + temp_post, data = agua,
             na.action = NULL)
         Residuals:
             Min
                      1Q Median
                                       3Q
                                              Max
         -6.1714 -1.9582 -0.4968 1.2242 8.9426
         Coefficients: (2 not defined because of singularities)
                     Estimate Std. Error t value Pr(>|t|)
                                                    <2e-16 ***
         (Intercept) 10.97180
                                  0.87288
                                            12.57
                      0.70791
                                  0.04956
                                            14.29
                                                    <2e-16 ***
         tsTemp
         temp_ant
                           NA
                                       NA
                                               NA
                                                        NA
         temp_post
                           NA
                                       NA
                                               NA
                                                        NA
         Signif. codes: 0 (***, 0.001 (**, 0.01 (*, 0.05 (., 0.1 (), 1
         Residual standard error: 2.726 on 154 degrees of freedom
         Multiple R-squared: 0.5699,
                                         Adjusted R-squared: 0.5671
         F-statistic: 204.1 on 1 and 154 DF, p-value: < 2.2e-16
         plot(temp_post,temp_ant)
In [27]:
         cor(temp_post,temp_ant)
```

A matrix: 1 x 1 of type dbl

 Temp_Avg

 Temp_Avg
 1

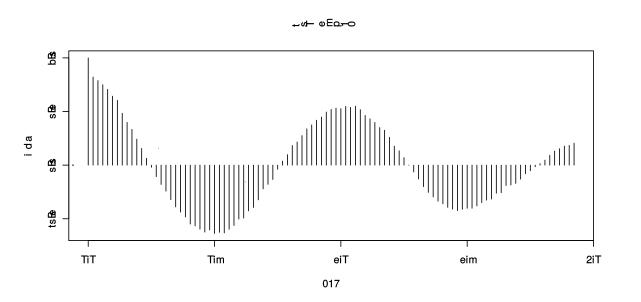


Los Coeficientes correspondientes a la temperatura actual se obtienen, pero para la anterior y para el pronostico futuro no ya que existe una fuerte correlacion entre ambos ; *Nota error: Error "not defined because of singularities" will occur due to strong correlation between your independent variables*

En conclusión: obligatoriamente tenemos que sacar la periodicidad de la serie

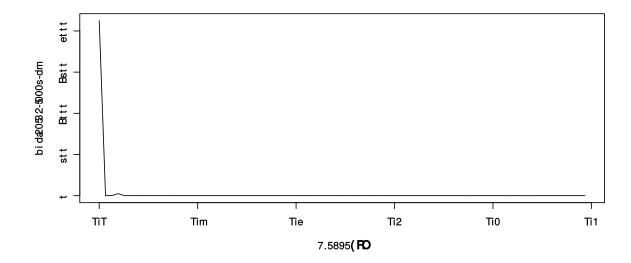
Componentes Estacionales y Periodograma de las ventas

In [28]: plot(acf(T,lag.max=100))



```
In [29]: # Periodograma

n=length(A)
    I = abs(fft(A))^2/n # the periodogram
    P = (4/n)*I[1:(n/2)]
    # the scaled periodogram
    f = (0:(n/2-1))/n
    # frequencies
    plot(f, P, type="l", xlab="Frequency", ylab="Scaled Periodogram")
```

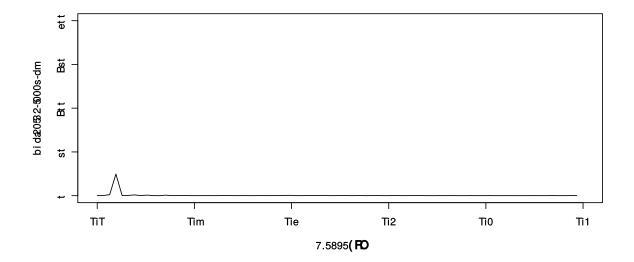


El pico se debe a la componente de continua que tiene la serie, es decir que la transformada de fourier grafica todas las componentes , y la continua es una delta (salto infinito) en el origen

```
In [30]: # Para sacar La componente de continua se resta La media nada mas
Amed=mean(A)
A=A-Amed
```

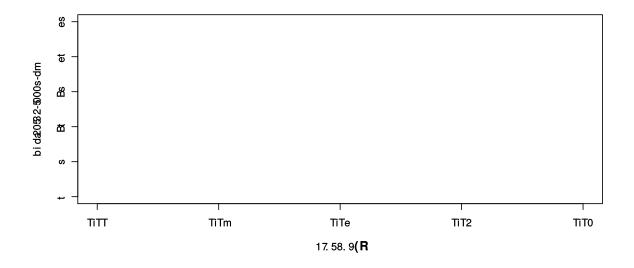
```
In [31]: # Periodograma
    n=length(A)
    I = abs(fft(A))^2/n # the periodogram
    P = (4/n)*I[1:(n/2)]
    # the scaled periodogram
    f = (0:(n/2-1))/n
    # frequencies
    plot(f, P, type="l", xlab="Frequency", ylab="Scaled Periodogram",xlim=c(0,0.5)),ylim=c(0,200))

## Ahi si graficando en todo el rango se visualiza mejor
```



```
In [32]: # Periodograma
    n=length(A)
    I = abs(fft(A))^2/n # the periodogram
    P = (4/n)*I[1:(n/2)]
    # the scaled periodogram
    f = (0:(n/2-1))/n
    # frequencies
    plot(f, P, type="l", xlab="Frequency", ylab="Scaled Periodogram",xlim=c(0,0.08)
    ),ylim=c(0,25))

## Ahi si graficando en todo el rango se visualiza mejor
```



In [33]: which.max(P) # hay otra compnente en frecuencia 0.04 (periodo de 25 semanas) y posiblement e en 0.05 (periodo 20 semanas) ,mas chiquitas y otra en 0.7, 14 semanas

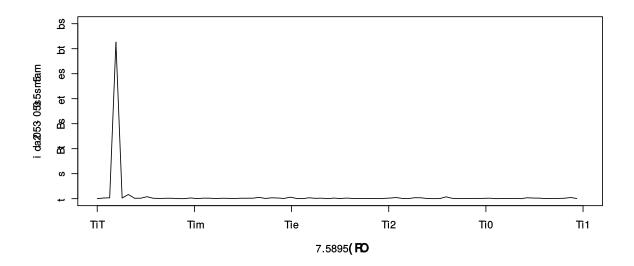
4

```
In [34]:
        0 0.00641025641025641 0.0128205128205128 0.0192307692307692
        0.0256410256410256
                        0.032051282051282 \quad 0.0384615384615385 \quad 0.0448717948717949
        0.0512820512820513
                        0.0576923076923077
                                         0.0641025641025641
                                                         0.0705128205128205
        0.0769230769230769
                        0.0833333333333333
                                         0.0897435897435897
                                                         0.0961538461538462
        0.102564102564103  0.108974358974359
                                       0.115384615384615
                                                       0.121794871794872
        0.128205128205128
                       0.134615384615385
                                       0.141025641025641
                                                       0.147435897435897
        0.153846153846154  0.16025641025641
                                      0.166666666666667
                                                      0.173076923076923
        0.179487179487179  0.185897435897436
                                       0.192307692307692
                                                       0.198717948717949
        0.217948717948718
                                                       0.224358974358974
        0.230769230769231
                       0.237179487179487
                                       0.243589743589744
                                                       0.25
        0.269230769230769
                                                       0.275641025641026
        0.294871794871795
                                                       0.301282051282051
        0.320512820512821
                                                       0.326923076923077
        0.346153846153846
                                                      0.352564102564103
        0.358974358974359  0.365384615384615
                                       0.371794871794872
                                                       0.378205128205128
        0.384615384615385
                       0.391025641025641
                                       0.397435897435897
                                                       0.403846153846154
        0.423076923076923
                                                      0.429487179487179
        0.435897435897436
                       0.442307692307692
                                       0.448717948717949
                                                       0.455128205128205
        0.461538461538462
                       0.467948717948718
                                       0.474358974358974
                                                       0.480769230769231
        In [35]:
        Tmed
        mean(T)
        T=T-Tmed
        17.0547435897436
```

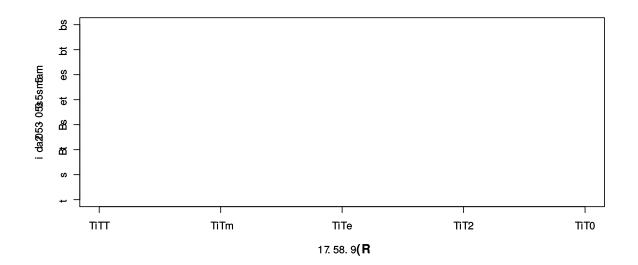
Periodograma de la temperatura

17.0547435897436

```
In [36]: n=length(T)
    I = abs(fft(T))^2/n # the periodogram
    P = (4/n)*I[1:(n/2)]
    # the scaled periodogram
    f = (0:(n/2-1))/n
    # frequencies
    plot(f, P, type="l", xlab="Frequency", ylab="Scaled Periodogram",xlim=c(0,0.5)),ylim=c(0,35))
```



```
In [37]: plot(f, P, type="1", xlab="Frequency", ylab="Scaled Periodogram",xlim=c(0,0.08
),ylim=c(0,35))
```



```
In [38]: max(P)
```

31.3344135457411

```
In [39]: (which.max(P))
In [40]: f1=f[4]
         f2=f[8]
         f3=f[9]
         f4=f[12]
In [41]:
         f1
         1/f1
         f2
         1/f2
         f3
         1/f3
         f4
         1/f4
         0.0192307692307692
         52
         0.0448717948717949
         22.2857142857143
         0.0512820512820513
         19.5
         0.0705128205128205
         14.1818181818182
```

La componente f1 (1/0.19 ~ 52 semanas) y f3 (1/0.050 ~ 20 semanas) se encuentras en ambas series

```
In [42]: periodo_ppal= 1/f1
    periodo_secundario = 1/f3
    periodo_secundario

52
    19.5

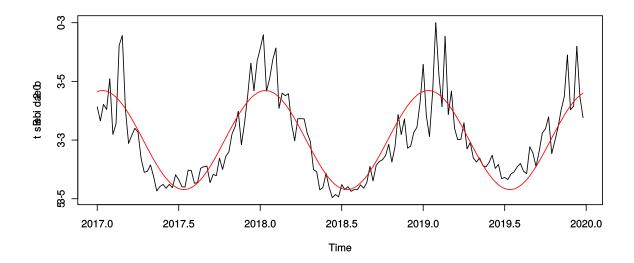
In [43]: # Normalizamos La ts Ventas
    trend=time(A)
    amp_max = max(c( max(A), max(-A)))
    A_norm=A/amp_max
```

Modelo 5 : ajuste de las ventas normalizadas vs sin + cos

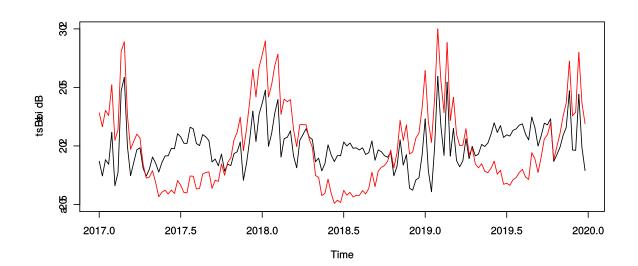
```
In [44]: fit_estacional = lm(A_norm \sim sin(2*pi*trend*f1*52) + cos(2*pi*trend*f1*52), n
         a.action=NULL)
In [45]: | summary(fit_estacional)
         Call:
         lm(formula = A_norm \sim sin(2 * pi * trend * f1 * 52) + cos(2 *
             pi * trend * f1 * 52), na.action = NULL)
         Residuals:
                        1Q Median
                                          3Q
              Min
                                                  Max
         -0.39164 -0.12326 -0.01602 0.09543 0.59454
         Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)
                                       -2.883e-14 1.484e-02 0.000
         (Intercept)
         sin(2 * pi * trend * f1 * 52) 8.453e-02 2.099e-02 4.027 8.87e-05 ***
         cos(2 * pi * trend * f1 * 52) 4.135e-01 2.099e-02 19.702 < 2e-16 ***
         Signif. codes: 0 '***, 0.001 '**, 0.01 '*, 0.05 '.', 0.1 ', 1
         Residual standard error: 0.1854 on 153 degrees of freedom
         Multiple R-squared: 0.7255, Adjusted R-squared: 0.7219
         F-statistic: 202.2 on 2 and 153 DF, p-value: < 2.2e-16
In [46]:
        num = length(A)
                                                                 # sample size
         AIC(fit_estacional)/num - log(2*pi)
                                                                            # AIC
         BIC(fit_estacional)/num - log(2*pi)
                                                                            # BIC
         -2.33880399516711
         -2.26060255908379
```

```
In [47]: #transformo el resultado en una serie
predictions = ts(fitted(fit_estacional),start=2017, frequency=52)

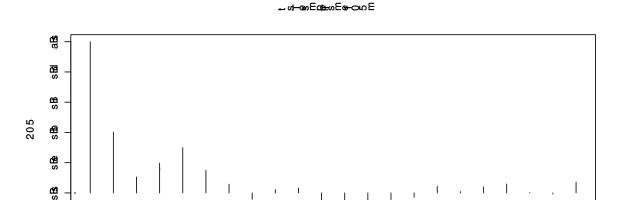
plot(A_norm )
lines(predictions,col=2)
```



```
In [48]: residuos=residuals(fit_estacional)
In [49]: plot(residuos, ylim=c(-0.5,1) )
lines(A_norm,col=2)
```







Tie

17.

Ti0

Mejora el modelo pero existen todavia algunas componentes de correlacion , vamos a probar ajustar un modelo con mas componentes estacionarias

Modelo 6 : modelo no lineal con tres componentes estacionarias

Tim

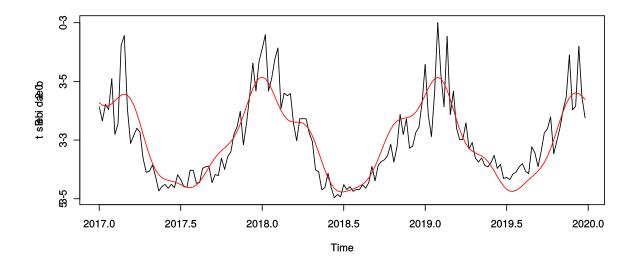
ΤiΤ

```
In [51]: ## haciendo un planteo de modelo no linea7
    fit2_estacional <- nls( A_norm ~ a *(sin(2*pi*trend*f_1*52) + cos(2*pi*trend*f_1*52)) + b*(sin(2*pi*trend*f_2*52) + cos(2*pi*trend*f_2*52)) + c*(sin(2*pi*trend*f_3*52) + cos(2*pi*trend*f_3*52)),start=list(a=0.5,b=0.4,c=0.7,f_1=f1,f_2=f3,f_3=f4))</pre>
```

```
In [52]: | summary(fit2_estacional)
         Formula: A_norm ~ a * (sin(2 * pi * trend * f_1 * 52) + cos(2 * pi * trend *
             f_1 * 52) + b * (\sin(2 * pi * trend * f_2 * 52) + \cos(2 * f_1 * 52))
             pi * trend * f_2 * 52)) + c * (sin(2 * pi * trend * f_3 *
             52) + cos(2 * pi * trend * f_3 * 52))
         Parameters:
              Estimate Std. Error t value Pr(>|t|)
             2.985e-01 1.395e-02 2.139e+01 < 2e-16 ***
            4.916e-02 1.395e-02 3.524e+00 0.000563 ***
         c 4.582e-02 1.395e-02 3.285e+00 0.001270 **
         f_1 1.923e-02 7.087e-08 2.714e+05 < 2e-16 ***
         f 2 5.128e-02 4.302e-07 1.192e+05 < 2e-16 ***
         f 3 7.050e-02 4.617e-07 1.527e+05 < 2e-16 ***
         Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
         Residual standard error: 0.1742 on 150 degrees of freedom
         Number of iterations to convergence: 9
         Achieved convergence tolerance: 2.887e-07
In [53]: coef(fit2_estacional)
         # f_1, f_2 y f_3 dan muy cercano a lo que nos había dado el periodograma
```

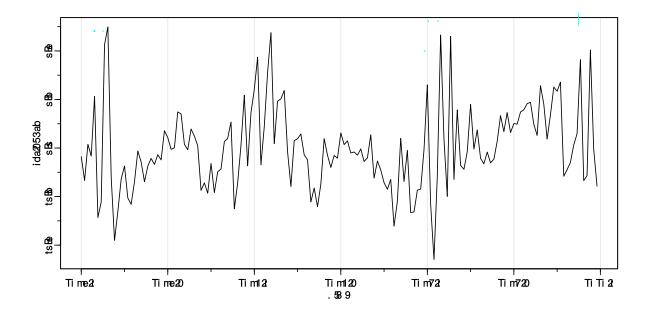
- **a** 0.298453735422649
- **b** 0.049163126437584
- c 0.0458202265832083
- **f_1** 0.0192316544523848
- **f_2** 0.0512809153936186
- **f_3** 0.0705013288438199

```
In [54]: #transformo el resultado en una serie
    predictions2 = ts(fitted(fit2_estacional),start=2017, frequency=52)
    plot(A_norm )
    lines(predictions2,col=2)
```

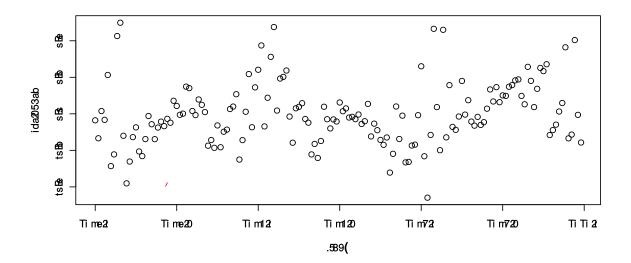


Estudiamos los residuos del modelo 6

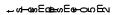
```
In [55]: round(mape(A_norm, predictions2))
```

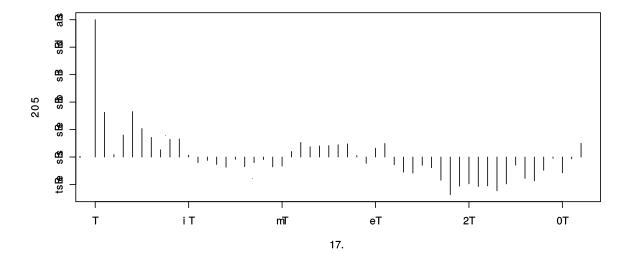


In [57]: plot(trend,residuos2)
 lines(A_norm,col=2)



```
In [58]: acf(residuos2,52)
# No es ruido blanco
```



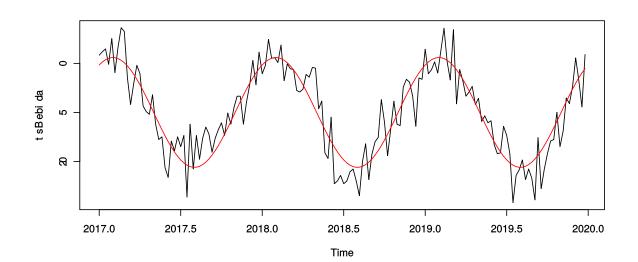


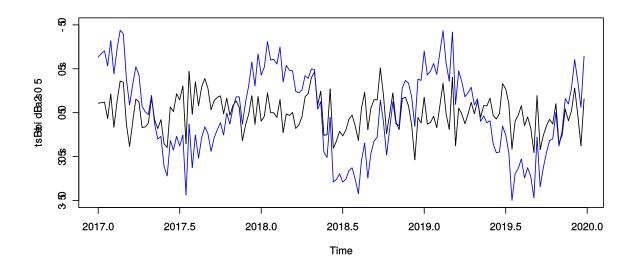
Componentes Estacionales de la temperatura

```
In [61]: | summary(fit_temp_estacional)
         Call:
         lm(formula = T_norm \sim sin(2 * pi * trend * f1 * 52) + cos(2 *
             pi * trend * f1 * 52), na.action = NULL)
         Residuals:
              Min
                        1Q
                             Median
                                                  Max
                                          3Q
         -0.53721 -0.14075 -0.01347 0.15190 0.50941
         Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)
                                       -5.085e-14 1.697e-02
                                                                0.00
         (Intercept)
                                                                        <2e-16 ***
         sin(2 * pi * trend * f1 * 52) 3.063e-01 2.400e-02
                                                               12.76
         cos(2 * pi * trend * f1 * 52) 5.261e-01 2.400e-02
                                                               21.92
                                                                        <2e-16 ***
         Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
         Residual standard error: 0.212 on 153 degrees of freedom
         Multiple R-squared: 0.8078, Adjusted R-squared: 0.8053
         F-statistic: 321.6 on 2 and 153 DF, p-value: < 2.2e-16
In [62]: | num = length(T)
                                                                  # sample size
         AIC(fit_temp_estacional)/num - log(2*pi)
                                                                                  # AIC
         BIC(fit_temp_estacional)/num - log(2*pi)
                                                                                  # BIC
         -2.07041399043761
         -1.99221255435429
In [63]: #transformo el resultado en una serie
         predictions = ts(fitted(fit_temp_estacional),start=2017, frequency=52)
```

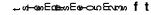
plot(T)

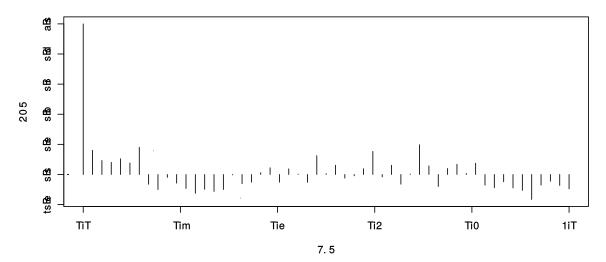
lines(predictions*temp_max,col=2)





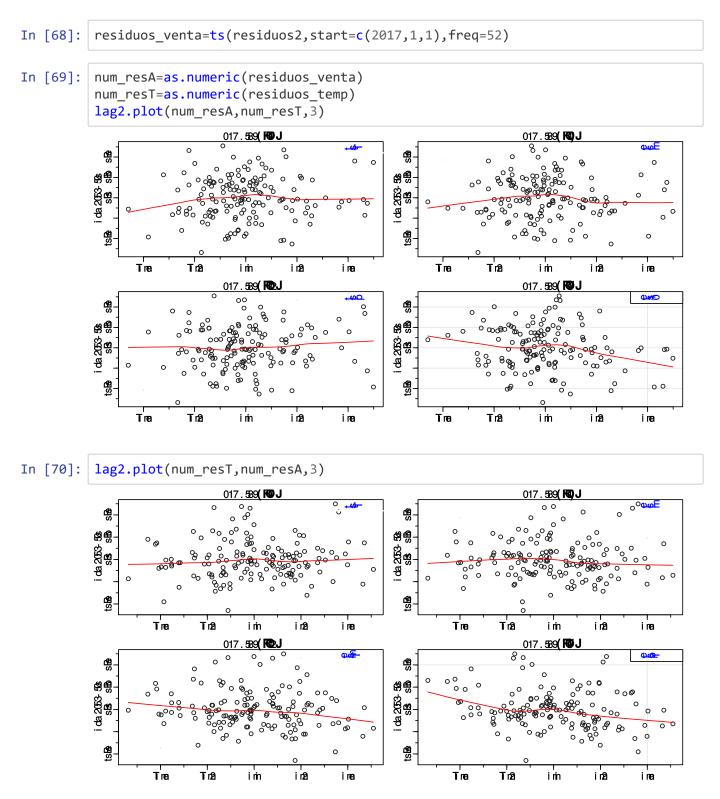




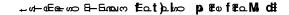


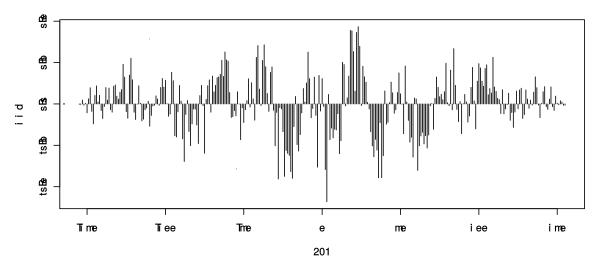
El residuo queda practicamente blanco por lo tanto podemos considerar que conseguimos extraer toda la componente estacionaria con frecuencia f1, periodo 52 Semanas

Buscamos mejorar nuestro Modelo 6 extrayendo las componentes estacionarias para explicar el comportamiento de la serie sin las tres componentes del modelo



```
In [71]: ccf(num_resA,num_resT, 157, main="Venta Semanal Agua vs Temperatura Media", yl
    ab="CCF")
# En los residuos vemos muy poca correlación
```





Modelo 7 : Función de Ventas vs temperatura t + residuos(temperatura t-1) + temperatura t+1

$$resA_{t} = eta_{0} + eta_{1}(resT_{t}) + eta_{2}(resT_{t-1}) + eta_{3}(resT_{t+1}) + w_{t}.$$

Es decir, estimar la venta de agua actual por una media más algo que depende de la temperatura actual, el residuo en el cambio de la semana previa y el pronostico de la semana siguiente.

```
In [72]: restemp_ant=lag(residuos_temp,-1) # past temp
    restemp_post=lag(residuos_temp,1) # forecast tem

    trend = time(residuos_temp) # time

    residuos_temp2 = residuos_temp^2 # square it
    restemp_ant2 = restemp_ant^2
    restemp_post2 = restemp_post^2

# Aplicamos la función sign para tener valores positivos y negativos según si
    crece o decrere la temperatura

residuos_temp_exp = sign(residuos_temp)*exp(residuos_temp) # exponential it
    restemp_ant_exp = sign(restemp_ant)*exp(restemp_ant)
    restemp_post_exp = sign(restemp_post)*exp(restemp_post)
```

	trend	residuos_venta	residuos_temp	residuos_temp2	residuos_temp_exp	restemp_ant
	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>
2	2017.019	-0.133886957	0.11346833	0.0128750619	1.1201564	0.10848384
3	2017.038	0.015265424	0.12008213	0.0144197188	1.1275895	0.11346833
4	2017.058	-0.033013742	-0.06707530	0.0044990953	-0.9351248	0.12008213
5	2017.077	0.212810317	0.21130752	0.0446508662	1.2352922	-0.06707530
6	2017.096	-0.286161760	-0.16595070	0.0275396344	-0.8470880	0.21130752
7	2017.115	-0.222120870	0.12580218	0.0158261883	1.1340578	-0.16595070
8	2017.135	0.425804115	0.36100484	0.1303244931	1.4347704	0.12580218
9	2017.154	0.498890182	0.34688221	0.1203272688	1.4146501	0.36100484
10	2017.173	-0.120112919	-0.13339042	0.0177930038	-0.8751234	0.34688221
11	2017.192	-0.380711640	-0.38536464	0.1485059046	-0.6802026	-0.13339042
12	2017.212	-0.261273537	-0.10296701	0.0106022042	-0.9021567	-0.38536464
13	2017.231	-0.127611140	0.15515092	0.0240718074	1.1678342	-0.10296701
14	2017.250	-0.074074320	0.12280602	0.0150813180	1.1306651	0.15515092
15	2017.269	-0.206086220	-0.16935419	0.0286808411	-0.8442098	0.12280602
16	2017.288	-0.231704479	-0.16218444	0.0263037942	-0.8502844	-0.16935419
17	2017.308	-0.138404346	-0.12097472	0.0146348821	-0.8860564	-0.16218444
18	2017.327	-0.012150767	0.16965801	0.0287838397	1.1848996	-0.12097472
19	2017.346	-0.057064281	-0.08310920	0.0069071392	-0.9202506	0.16965801
20	2017.365	-0.138660899	-0.18103563	0.0327739004	-0.8344056	-0.08310920
21	2017.385	-0.075128616	-0.07950494	0.0063210362	-0.9235735	-0.18103563
22	2017.404	-0.042718704	-0.35270718	0.1244023532	-0.7027830	-0.07950494
23	2017.423	-0.067533800	-0.39694494	0.1575652876	-0.6723710	-0.35270718
24	2017.442	-0.027369437	0.06634844	0.0044021149	1.0685990	-0.39694494
25	2017.462	-0.048859963	0.01180252	0.0001392995	1.0118724	0.06634844
26	2017.481	0.071473500	0.21515740	0.0462927089	1.2400571	0.01180252
27	2017.500	0.043084146	0.14715274	0.0216539293	1.1585309	0.21515740
28	2017.519	-0.005371428	0.30421736	0.0925482022	1.3555637	0.14715274
29	2017.538	0.001043961	-0.35385561	0.1252137931	-0.7019763	0.30421736
30	2017.558	0.148824836	0.47062247	0.2214855121	1.6009905	-0.35385561
31	2017.577	0.140599457	-0.01875045	0.0003515795	-0.9814242	0.47062247
126	2019.404	0.028146434	0.16715506	0.0279408149	1.1819375	0.07601870
127	2019.423	0.133388620	-0.03151877	0.0009934326	-0.9689728	0.16715506
128	2019.442	0.067627578	-0.07286154	0.0053088036	-0.9297296	-0.03151877

	trend	residuos_venta	residuos_temp	residuos_temp2	residuos_temp_exp	restemp_ant
	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>
129	2019.462	0.146069067	-0.01103662	0.0001218069	-0.9890241	-0.07286154
130	2019.481	0.063639526	0.33152824	0.1099109747	1.3930955	-0.01103662
131	2019.500	0.100799654	0.27113662	0.0735150683	1.3114542	0.33152824
132	2019.519	0.098332119	0.10301545	0.0106121823	1.1085085	0.27113662
133	2019.538	0.148367550	-0.41584755	0.1729291859	-0.6597808	0.10301545
134	2019.558	0.157302675	-0.09709320	0.0094270885	-0.9074714	-0.41584755
135	2019.577	0.183059809	-0.02745108	0.0007535616	-0.9729223	-0.09709320
136	2019.596	0.188287946	0.08008782	0.0064140585	1.0833822	-0.02745108
137	2019.615	0.098083216	-0.14749796	0.0217556494	-0.8628642	0.08008782
138	2019.635	0.052099668	-0.04881421	0.0023828266	-0.9523581	-0.14749796
139	2019.654	0.257238969	-0.17716428	0.0313871838	-0.8376422	-0.04881421
140	2019.673	0.180532063	-0.45384588	0.2059760813	-0.6351806	-0.17716428
141	2019.692	0.036855974	0.19509428	0.0380617772	1.2154256	-0.45384588
142	2019.712	0.136992506	-0.42118977	0.1774008251	-0.6562655	0.19509428
143	2019.731	0.251193676	-0.25950262	0.0673416119	-0.7714352	-0.42118977
144	2019.750	0.233736197	-0.14993969	0.0224819113	-0.8607599	-0.25950262
145	2019.769	0.271451616	-0.07638265	0.0058343085	-0.9264616	-0.14993969
146	2019.788	-0.116230400	-0.12923066	0.0167005638	-0.8787712	-0.07638265
147	2019.808	-0.089097273	0.10145409	0.0102929319	1.1067791	-0.12923066
148	2019.827	-0.059289658	-0.35231532	0.1241260870	-0.7030584	0.10145409
149	2019.846	0.012647132	-0.25507176	0.0650616005	-0.7748609	-0.35231532
150	2019.865	0.059258076	0.04079386	0.0016641387	1.0416374	-0.25507176
151	2019.885	0.364149900	-0.09227659	0.0085149694	-0.9118529	0.04079386
152	2019.904	-0.134334896	0.02540200	0.0006452616	1.0257274	-0.09227659
153	2019.923	-0.113580138	0.28062988	0.0787531295	1.3239635	0.02540200
154	2019.942	0.403813280	-0.02713986	0.0007365719	-0.9732251	0.28062988
155	2019.962	-0.005647531	-0.37717292	0.1422594141	-0.6857975	-0.02713986
4						•

```
In [74]: | num = length(res agua)
                                                                    # sample size
         summary(fit_res <- lm(residuos_venta ~ residuos_temp + residuos_temp2 + resi</pre>
         duos temp exp + restemp ant + restemp ant2 +restemp ant exp + restemp post +
         restemp_post2 + restemp_post_exp , data=res_agua, na.action=NULL))
         AIC(fit res)/num - log(2*pi)
                                                                 # AIC
         BIC(fit res)/num - log(2*pi)
                                                                 # BIC
        Call:
        lm(formula = residuos_venta ~ residuos_temp + residuos_temp2 +
            residuos temp exp + restemp ant + restemp ant2 + restemp ant exp +
            restemp_post + restemp_post2 + restemp_post_exp, data = res_agua,
            na.action = NULL)
        Residuals:
             Min
                       1Q
                           Median
                                        3Q
                                               Max
         -0.50487 -0.10989 -0.01250 0.09026 0.45414
        Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
                                                        0.9458
        (Intercept)
                         -0.0015059 0.0221173 -0.068
                          0.3081217 0.1161844 2.652
        residuos_temp
                                                        0.0089 **
        residuos_temp2
                          0.1493599 0.2562121 0.583 0.5608
        residuos_temp_exp -0.0615818  0.0236611 -2.603
                                                       0.0102 *
        restemp ant
                          0.1819881 0.1169788 1.556 0.1220
        0.7148
        restemp_ant_exp -0.0515322 0.0235756 -2.186
                                                        0.0304 *
        restemp_post
                        -0.0235971 0.1161218 -0.203
                                                        0.8393
        restemp_post2
                         0.0467675 0.2578199 0.181
                                                        0.8563
        restemp_post_exp -0.0006471 0.0237585 -0.027
                                                        0.9783
        Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
        Residual standard error: 0.1698 on 144 degrees of freedom
        Multiple R-squared: 0.08345, Adjusted R-squared: 0.02617
        F-statistic: 1.457 on 9 and 144 DF, p-value: 0.1695
        -10.7026413633411
        -7.66568876092749
In [75]: | fit_nls <- nls(residuos_venta ~ g*residuos_temp + h*residuos_temp2 + i*restemp</pre>
```

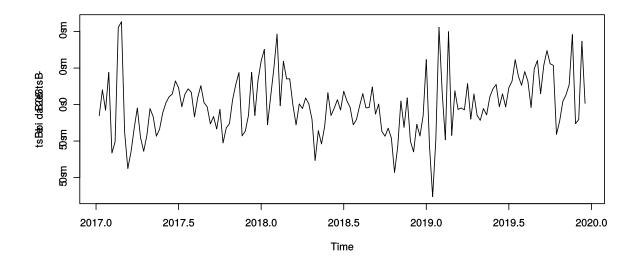
_ant + j*restemp_post+ k*restemp_post_exp, start=list(g=0.5,h=0.5,i=1.1,j=0.9,

k=0.1), data=res_agua, na.action=**NULL**)

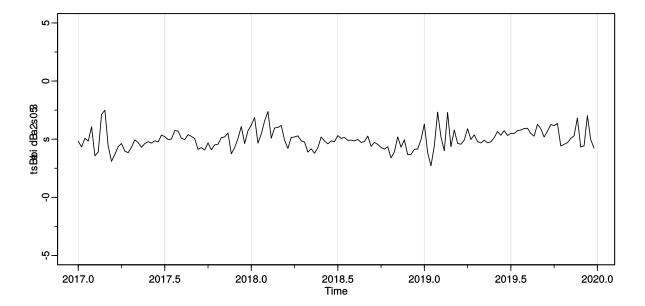
```
summary(fit_nls)
In [76]:
         AIC(fit_res)/num - log(2*pi)
                                                                       # AIC
         BIC(fit_res)/num - log(2*pi)
                                                                       # BIC
         Formula: residuos_venta ~ g * residuos_temp + h * residuos_temp2 + i *
             restemp_ant + j * restemp_post + k * restemp_post_exp
         Parameters:
             Estimate Std. Error t value Pr(>|t|)
           0.0605975 0.0682539
                                    0.888
                                             0.376
                       0.1942592
         h -0.0551075
                                  -0.284
                                             0.777
         i -0.0210050
                       0.0687997
                                   -0.305
                                             0.761
                                   -0.179
                                             0.858
         j -0.0201838
                       0.1124583
         k -0.0003184 0.0227548
                                   -0.014
                                             0.989
         Residual standard error: 0.1738 on 149 degrees of freedom
         Number of iterations to convergence: 1
         Achieved convergence tolerance: 1.034e-08
         -10.7026413633411
         -7.66568876092749
```

Análisis de residuos Modelo 7

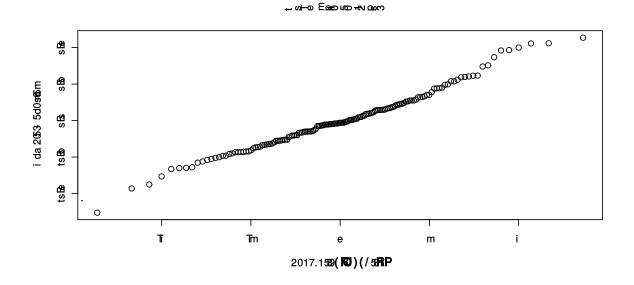
```
In [77]: plot(residuals(fit_res))
```



In [78]: # hasta aqui se obtiene la mejor aproximacion de acuerdo a los indices de AIC
 y BIC
 predict_nls = ts(fitted(fit_nls), start = 2017,frequency=52) #Creo una serie t
 emporal con los valores ajustados.
 tsplot(residuos_venta,ylim=c(-2,2))

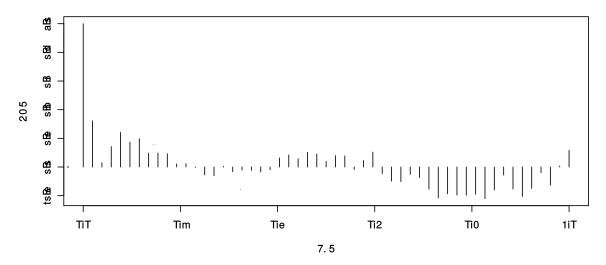


In [79]: qqnorm(residuals(fit_res))
qqline(residuals(fit_res))









Hacemos predicción combinando lo aprendido en el Modelo 6 y el Modelo 7 y comparamos con los datos de test, los cuáles nuestro modelo no ha visto cuando se entrenó

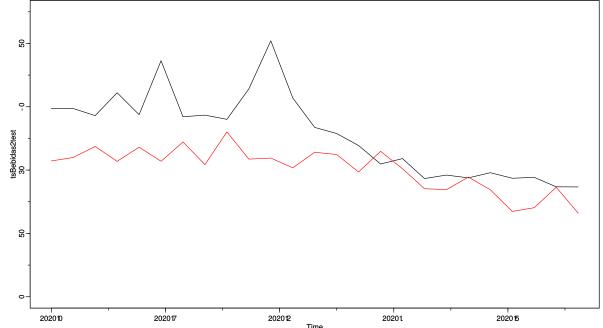
```
In [81]: head(base_test)
```

A data.frame: 6 x 6

	Year	Weeknum	Yearweek	Sales_Liters	Temp_Avg	Class
	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<fct></fct>
157	2020	157	1	297444.3	21.44286	Test
158	2020	158	2	297221.2	21.98571	Test
159	2020	159	3	285992.0	23.73333	Test
160	2020	160	4	322162.2	21.37500	Test
161	2020	161	5	287530.2	23.61429	Test
162	2020	162	6	372673.3	21.38571	Test

```
In [82]: Litros_test=base_test[4] / 10000
# Normalizamos los datos de venta para pasarlos a la misma escala que la tempe
ratura

Temp_test = round(base_test[5],2)
#Temp
```

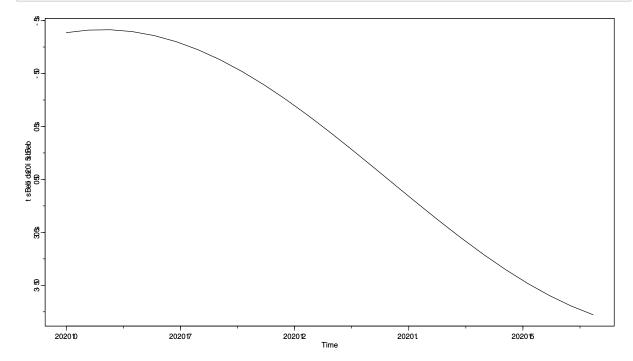


Reconstruyendo el Modelo 7 y agregamos las componentes estacionales del Modelo 6

- **a** 0.298453735422649
- **b** 0.049163126437584
- **c** 0.0458202265832083
- **f_1** 0.0192316544523848
- **f_2** 0.0512809153936186
- **f_3** 0.0705013288438199

```
In [88]: # Modelo 7 (nls de los residuos)
         coef(fit_nls)
                             g
                                 0.0605974974459932
                                 -0.0551074739958787
                             h
                                -0.0210049526591887
                              i
                                 -0.0201838025907531
                              j
                                 -0.000318394006200873
In [89]:
         coefs_estacional=as.numeric(coef(fit2_estacional))
         a=coefs estacional[1]
         b=coefs estacional[2]
         c=coefs_estacional[3]
         f1=coefs estacional[4]
         f2=coefs_estacional[5]
         f3=coefs_estacional[6]
In [90]:
         coefs_nls=as.numeric(coef(fit_nls))
         g=coefs_nls[1]
         h=coefs nls[2]
         i=coefs_nls[3]
         j=coefs_nls[4]
         k=coefs nls[5]
         print(trend_test)
         Time Series:
         Start = c(2020, 1)
         End = c(2020, 25)
         Frequency = 52
          [1] 2020.000 2020.019 2020.038 2020.058 2020.077 2020.096 2020.115 2020.135
          [9] 2020.154 2020.173 2020.192 2020.212 2020.231 2020.250 2020.269 2020.288
         [17] 2020.308 2020.327 2020.346 2020.365 2020.385 2020.404 2020.423 2020.442
         [25] 2020.462
```

```
In [91]: T_estacional_test = (sin(2*pi*trend_test*f1*52) + cos(2*pi*trend_test*f1*52))
tsplot(T_estacional_test)
```



```
In [92]: residuos_temp_test=as.numeric((tsTemp_test-Tmed)/temp_max) - (T_estacional_tes
t)
```

```
In [93]:    restemp_ant_test=lag(residuos_temp_test,-1)  # past temp
    restemp_post_test=lag(residuos_temp_test,1)  # forecast tem

#trend_test= time(residuos_temp_test)  # time

residuos_temp_test2 = residuos_temp_test^2  # square it
    restemp_ant_test2 = restemp_ant_test^2
    restemp_post_test2 = restemp_post_test^2

residuos_temp_test_exp = sign(residuos_temp_test)*exp(residuos_temp_test)  #
    exponential it
    restemp_ant_test_exp = sign(restemp_ant_test)*exp(restemp_ant_test)
    restemp_post_test_exp = sign(restemp_post_test)*exp(restemp_post_test)
```

In [94]: res_agua_test = ts.intersect(trend_test,residuos_temp_test, residuos_temp_test
2, residuos_temp_test_exp, restemp_ant_test,restemp_ant_test2,restemp_post_te
st,restemp_post_test2,restemp_ant_test_exp,restemp_post_test_exp, dframe=TRUE)
res_agua_test

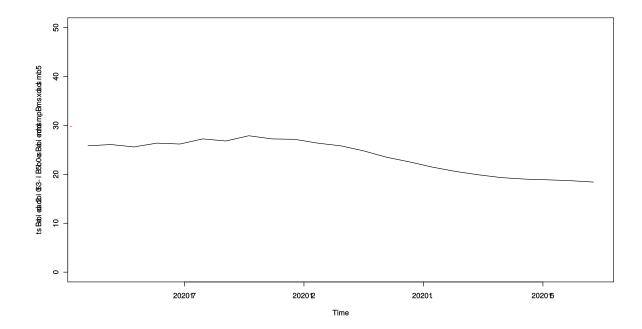
A data.frame: 23 x 10

trend_test	residuos_temp_test	residuos_temp_test2	residuos_temp_test_exp	restemp_ant_test ı
<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>	<ts></ts>
2020.019	-0.8729057	0.761964385	-0.4177360	-0.9087645
2020.038	-0.6870673	0.472061492	-0.5030492	-0.8729057
2020.058	-0.9254409	0.856440822	-0.3963566	-0.6870673
2020.077	-0.6453473	0.416473154	-0.5244804	-0.9254409
2020.096	-0.8294173	0.687933018	-0.4363035	-0.6453473
2020.115	-0.4213617	0.177545713	-0.6561527	-0.8294173
2020.135	-0.7165118	0.513389132	-0.4884531	-0.4213617
2020.154	-0.0435464	0.001896289	-0.9573881	-0.7165118
2020.173	-0.3856468	0.148723430	-0.6800107	-0.0435464
2020.192	-0.2285604	0.052239841	-0.7956783	-0.3856468
2020.212	-0.2464316	0.060728532	-0.7815848	-0.2285604
2020.231	0.1784770	0.031854045	1.1953954	-0.2464316
2020.250	0.3053893	0.093262608	1.3571532	0.1784770
2020.269	0.1743147	0.030385630	1.1904302	0.3053893
2020.288	0.6996156	0.489461945	2.0129787	0.1743147
2020.308	0.5704454	0.325407917	1.7690548	0.6996156
2020.327	0.3944574	0.155596635	1.4835790	0.5704454
2020.346	0.5389985	0.290519408	1.7142892	0.3944574
2020.365	0.9071036	0.822837009	2.4771374	0.5389985
2020.385	0.8320348	0.692281869	2.2979899	0.9071036
2020.404	0.5901345	0.348258728	1.8042311	0.8320348
2020.423	0.7700610	0.592993937	2.1598980	0.5901345
2020.442	1.2211279	1.491153354	3.3910103	0.7700610
4				

In [95]: A_test = a *($\sin(2*pi*trend_test*f1*52)$ + $\cos(2*pi*trend_test*f1*52)$) +b*($\sin(2*pi*trend_test*f2*52)$) + c*($\sin(2*pi*trend_test*f3*52)$) + cos($2*pi*trend_test*f3*52$))

```
In [97]: plot((A_test + residuos_venta_test)*amp_max+Amed,ylim=c(0,50))
    lines(tsBebidas_test,col=2)
    pred_test=(A_test + residuos_venta_test)*amp_max+Amed

# Predicción vs datos reales
```



La predicción da muy bien. No toma los picos de ventas que se deben a compras de pánico por el Covid 19. En nuestro dataset no habíamos tenido un comportamiento similar por lo que nos parece razonable y esperable que el modelo no captara estos picos.

```
length(pred test)
In [98]:
         length(tsBebidas_test)
         23
         25
In [99]:
         pred_agua_test = ts.intersect(pred_test, tsBebidas_test, dframe=TRUE)
         print(pred agua test$pred test)
         print(pred_agua_test$Sum.Litros)
         Time Series:
         Start = c(2020, 2)
         End = c(2020, 24)
         Frequency = 52
          [1] 25.83519 26.09151 25.60390 26.38106 26.19529 27.24741 26.81400 27.89104
          [9] 27.24611 27.15471 26.37487 25.82787 24.79523 23.48804 22.53880 21.46317
         [17] 20.59290 19.89757 19.32672 19.02654 18.88048 18.70279 18.41705
         NULL
```

```
In [100]:
            head(pred_agua_test)
            A data.frame: 6 x 2
             pred_test Sales_Liters
                <dbl>
                             <dbl>
             25.83519
                          29.72212
             26.09151
                          28.59920
             25.60390
                          32.21622
             26.38106
                          28.75303
             26.19529
                          37.26733
             27.24741
                          28.42089
In [101]:
            round(mape(pred_agua_test$pred_test,pred_agua_test$Sales_Liters),2)
            10.68
```

Fue posible predecir la venta de de agua para el perido de 2020 con una exactitud del 89,3%

Modelo 8: ARMAX - Multivariable AR

Finalmente realizaremos un analisis utilizando una libreria que permite aplicar modelos Autoregresivos (AR) y de media móvil (MA) para procesos que utilizan varias variables o sea ARIMA multivariable.

Este análisis se realizó basandose en el Capítulo 5.6 de Libro del Curso

Loading required package: urca Loading required package: lmtest

```
In [104]: tsBebidas2=tsBebidas^2
    tsTemp2=tsTemp^2
    x = cbind(tsBebidas, tsTemp,tsTemp2)
    summary(VAR(x, p=1, type='both')) # Primer aporxmacion para un corrimiento
#lag=1,orden de p=1 incluyendo const + trend
```

```
VAR Estimation Results:
Endogenous variables: tsBebidas, tsTemp, tsTemp2
Deterministic variables: both
Sample size: 155
Log Likelihood: -1347.9
Roots of the characteristic polynomial:
0.8876 0.4186 0.2243
Call:
VAR(y = x, p = 1, type = "both")
Estimation results for equation tsBebidas:
tsBebidas = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + const + trend
            Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1 0.639741 0.074095 8.634 8.05e-15 ***
tsTemp.l1
          -0.177046 0.310845 -0.570 0.5698
tsTemp2.l1
           0.011818 0.009517 1.242 0.2163
           7.046036 2.896846 2.432 0.0162 *
const
         0.007677 0.004425 1.735 0.0848 .
trend
---
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.256 on 150 degrees of freedom
Multiple R-Squared: 0.7118, Adjusted R-squared: 0.7041
F-statistic: 92.63 on 4 and 150 DF, p-value: < 2.2e-16
Estimation results for equation tsTemp:
_____
tsTemp = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + const + trend
            Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1 0.407587 0.074357 5.482 1.74e-07 ***
tsTemp.l1 0.940211 0.311941 3.014 0.00303 **
tsTemp2.l1 -0.012433 0.009551 -1.302 0.19502
         -3.814365 2.907064 -1.312 0.19149
const
          -0.009049 0.004441 -2.038 0.04334 *
trend
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.264 on 150 degrees of freedom
Multiple R-Squared: 0.743,
                        Adjusted R-squared: 0.7361
F-statistic: 108.4 on 4 and 150 DF, p-value: < 2.2e-16
Estimation results for equation tsTemp2:
_____
tsTemp2 = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + const + trend
             Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1 14.17696 2.52823 5.607 9.6e-08 ***
             19.02666 10.60644 1.794 0.07485 .
tsTemp.l1
```

```
tsTemp2.l1 -0.04947 0.32475 -0.152 0.87913 const -301.45909 98.84425 -3.050 0.00271 ** trend -0.31209 0.15099 -2.067 0.04046 *
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 76.98 on 150 degrees of freedom Multiple R-Squared: 0.7431, Adjusted R-squared: 0.7363 F-statistic: 108.5 on 4 and 150 DF, p-value: < 2.2e-16

Covariance matrix of residuals:

tsBebidas tsTemp tsTemp2 tsBebidas 5.089 1.578 59.58 tsTemp 1.578 5.125 169.07 tsTemp2 59.581 169.075 5925.37

Correlation matrix of residuals:

tsBebidas tsTemp tsTemp2 tsBebidas 1.0000 0.3090 0.3431 tsTemp 0.3090 1.0000 0.9702 tsTemp2 0.3431 0.9702 1.0000

· Obtuvimos una aproximacion:

 $A_t = 7.046036 - 0.007677t + 0.639741A_{t-1} - 0.177046T_{t-1} + 0.011818T_{t-1}^2$

\$selection

AIC(n) 5 HQ(n) 3 SC(n) 2 FPE(n) 5

\$criteria

A matrix: 4 x 10 of type dbl

	1	2	3	4	5	6	
AIC(n)	8.835624	8.573480	8.459981	8.423318	8.410002	8.432541	8.477
HQ(n)	8.960176	8.772764	8.733996	8.772064	8.833480	8.930750	9.050
SC(n)	9.142159	9.063936	9.134358	9.281615	9.452221	9.658681	9.887
FPE(n)	6875.393682	5291.249477	4726.024502	4560.058462	4506.092848	4618.111350	4842.589

←

rden. Según el li ncia a BIC, por l		s BIC para mu	Itivariate time se	eries. En este caso

```
In [106]: summary(fit <- VAR(x, p=2, type="both"))</pre>
```

```
VAR Estimation Results:
Endogenous variables: tsBebidas, tsTemp, tsTemp2
Deterministic variables: both
Sample size: 154
Log Likelihood: -1315.41
Roots of the characteristic polynomial:
0.9144 0.6659 0.4576 0.4311 0.3714 0.3714
VAR(y = x, p = 2, type = "both")
Estimation results for equation tsBebidas:
tsBebidas = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + tsBebidas.l2 + tsTemp.l2
+ tsTemp2.12 + const + trend
            Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1 0.619942 0.088101 7.037 7.09e-11 ***
          -0.366504
                      0.339318 -1.080 0.2819
tsTemp.l1
                      0.010165 1.716 0.0883 .
tsTemp2.l1
            0.017443
tsBebidas.l2 0.102528
                      0.092728 1.106 0.2707
           0.510119
                      0.343256 1.486 0.1394
tsTemp.12
tsTemp2.12 -0.017253
                      0.010309 -1.673 0.0964 .
         3.459272
                      3.551789 0.974 0.3317
const
           0.005516
trend
                      0.004643 1.188 0.2367
_ _ _
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.258 on 146 degrees of freedom
Multiple R-Squared: 0.7187, Adjusted R-squared: 0.7052
F-statistic: 53.28 on 7 and 146 DF, p-value: < 2.2e-16
Estimation results for equation tsTemp:
tsTemp = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + tsBebidas.l2 + tsTemp.l2 + t
sTemp2.12 + const + trend
            Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1 0.218250 0.080841 2.700 0.007759 **
           0.485003
tsTemp.l1
                      0.311358 1.558 0.121469
tsTemp2.ll -0.006142
                      0.009327 -0.659 0.511249
                      0.085087 3.789 0.000221 ***
tsBebidas.l2 0.322386
tsTemp.12
           0.938867
                      0.314972 2.981 0.003369 **
tsTemp2.12 -0.022754
                      0.009460 -2.405 0.017408 *
         -9.952794
                      3.259121 -3.054 0.002686 **
const
trend
          -0.010050
                      0.004260 -2.359 0.019651 *
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.072 on 146 degrees of freedom
Multiple R-Squared: 0.7878, Adjusted R-squared: 0.7776
```

F-statistic: 77.41 on 7 and 146 DF, p-value: < 2.2e-16

Estimation results for equation tsTemp2:

tsTemp2 = tsBebidas.l1 + tsTemp.l1 + tsTemp2.l1 + tsBebidas.l2 + tsTemp.l2 +
tsTemp2.l2 + const + trend

```
Estimate Std. Error t value Pr(>|t|)
tsBebidas.l1
             7.01441
                       2.75062 2.550
                                        0.0118 *
              6.59962
                      10.59395 0.623
tsTemp.l1
                                        0.5343
tsTemp2.l1
              0.08114 0.31737 0.256
                                        0.7986
tsBebidas.12 12.18446 2.89509 4.209 4.47e-05 ***
tsTemp.12
             24.19451 10.71691 2.258
                                        0.0255 *
            -0.56831 0.32187 -1.766
                                        0.0795 .
tsTemp2.12
const
           -479.23740 110.89154 -4.322 2.85e-05 ***
trend
             -0.35398
                       0.14496 -2.442 0.0158 *
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

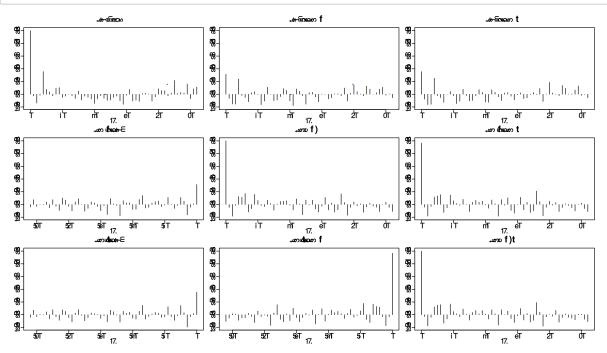
Residual standard error: 70.49 on 146 degrees of freedom Multiple R-Squared: 0.7871, Adjusted R-squared: 0.7768 F-statistic: 77.09 on 7 and 146 DF, p-value: < 2.2e-16

Covariance matrix of residuals:

tsBebidas tsTemp tsTemp2 tsBebidas 5.097 1.466 56.61 tsTemp 1.466 4.292 141.11 tsTemp2 56.609 141.113 4968.62

Correlation matrix of residuals:

tsBebidas tsTemp tsTemp2 tsBebidas 1.0000 0.3133 0.3557 tsTemp 0.3133 1.0000 0.9663 tsTemp2 0.3557 0.9663 1.0000



• Este grafico muestra la autocorrelacion ACF sobre la diagonal y la CCF sobre la diagonal inversa

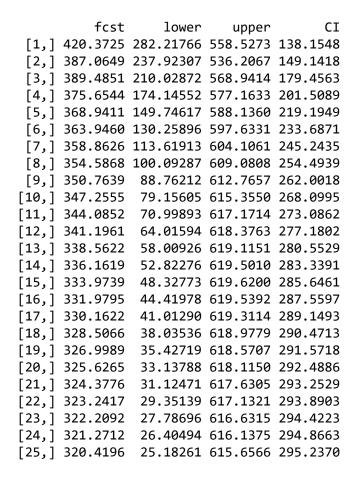
In [108]: (fit.pr = predict(fit, n.ahead = 25, ci = 0.95)) # 25 weeks ahead
fanchart(fit.pr) # plot prediction + error

\$tsBebidas

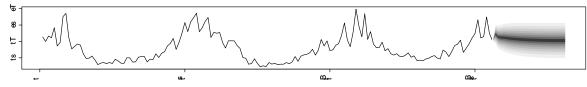
fcst lower upper CI[1,] 27.21901 22.79399 31.64402 4.425015 [2,] 26.34663 20.83829 31.85497 5.508341 [3,] 26.17810 20.11304 32.24315 6.065052 [4,] 26.14887 19.59543 32.70230 6.553435 [5,] 25.99588 19.09121 32.90054 6.904664 [6,] 25.89958 18.72442 33.07474 7.175157 [7,] 25.81573 18.42397 33.20748 7.391757 [8,] 25.73522 18.17040 33.30004 7.564820 [9,] 25.66600 17.96096 33.37104 7.705041 [10,] 25.60448 17.78493 33.42403 7.819554 [11,] 25.54982 17.63634 33.46330 7.913482 [12,] 25.50180 17.51094 33.49267 7.990865 [13,] 25.45969 17.40487 33.51451 8.054815 [14,] 25.42298 17.31519 33.53076 8.107788 [15,] 25.39121 17.23946 33.54296 8.151750 [16,] 25.36396 17.17567 33.55224 8.188287 [17,] 25.34083 17.12214 33.55952 8.218689 [18,] 25.32147 17.07746 33.56548 8.244008 [19,] 25.30556 17.04045 33.57067 8.265110 [20,] 25.29281 17.01010 33.57552 8.282708 [21,] 25.28294 16.98555 33.58033 8.297390 [22,] 25.27570 16.96606 33.58534 8.309644 [23,] 25.27087 16.95100 33.59075 8.319876 [24,] 25.26825 16.93983 33.59667 8.328420 [25,] 25.26764 16.93208 33.60320 8.335558

\$tsTemp

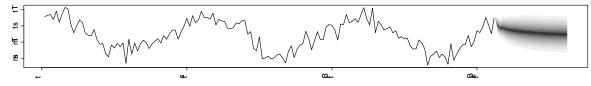
fcst lower upper CI[1,] 20.19165 16.131255 24.25204 4.060393 [2,] 19.32192 14.917727 23.72610 4.404189 [3,] 19.36015 14.060786 24.65950 5.299359 [4,] 18.97708 13.050859 24.90330 5.926220 [5,] 18.78630 12.341232 25.23137 6.445069 [6,] 18.63721 11.769555 25.50486 6.867651 [7,] 18.48883 11.281150 25.69650 7.207677 [8,] 18.36237 10.881547 25.84319 7.480822 [9,] 18.24901 10.545743 25.95227 7.703264 [10,] 18.14496 10.260623 26.02930 7.884339 [11,] 18.05081 10.018115 26.08351 8.032700 [12,] 17.96496 9.810317 26.11960 8.154643 [13,] 17.88666 9.631463 26.14186 8.255198 [14,] 17.81527 9.476956 26.15359 8.338318 [15,] 17.75018 9.343004 26.15735 8.407173 [16,] 17.69082 9.226515 26.15512 8.464303 [17,] 17.63671 9.124939 26.14848 8.511771 [18,] 17.58740 9.036143 26.13865 8.551254 [19,] 17.54247 8.958344 26.12659 8.584125 [20,] 17.50155 8.890038 26.11306 8.611512 [21,] 17.46430 8.829954 26.09864 8.634344 [22,] 17.43040 8.777009 26.08378 8.653387 [23,] 17.39956 8.730282 26.06884 8.669278 [24,] 17.37153 8.688985 26.05407 8.682542 [25,] 17.34606 8.652439 26.03967 8.693617



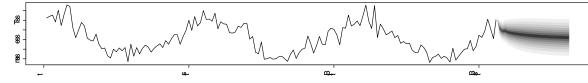
Time2i0175078i09 (1801.) Qi9fiJ



Ti me2i 017.5078i 09 (1801JPO Z



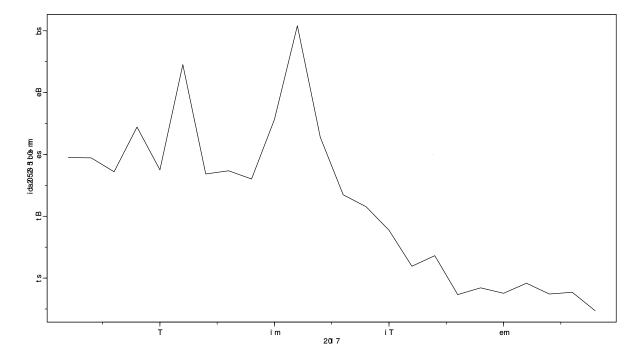
Ti me2i 017.5078i 09 (RDUPO) Z2



```
In [109]: | x test = cbind(tsBebidas test, tsTemp test)
          print(x_test)
          Time Series:
          Start = c(2020, 1)
          End = c(2020, 25)
          Frequency = 52
                    tsBebidas_test tsTemp_test
          2020.000
                          29.74443
                                         21.44
          2020.019
                          29.72212
                                         21.99
          2020.038
                          28.59920
                                         23.73
                          32.21622
                                         21.38
          2020.058
          2020.077
                          28.75303
                                         23.61
                                         21.39
          2020.096
                          37.26733
          2020.115
                          28.42089
                                         24.44
          2020.135
                          28.67805
                                         20.86
          2020.154
                          28.01215
                                         26.03
          2020.173
                          32.82848
                                         21.73
          2020.192
                          40.40622
                                         21.90
          2020.212
                          31.39477
                                         20.36
          2020.231
                          26.73832
                                         22.81
                          25.77284
                                         22.46
          2020.250
          2020.269
                          23.87235
                                         19.70
          2020.288
                          20.95774
                                         22.96
          2020.308
                          21.81312
                                         20.21
          2020.327
                          18.66503
                                         17.06
          2020.346
                                         16.91
                          19.21226
          2020.365
                          18.77107
                                         18.89
          2020.385
                          19.58575
                                         16.89
          2020.404
                          18.71079
                                         13.47
          2020.423
                          18.85455
                                         14.06
                          17.35902
          2020.442
                                         17.29
          2020.462
                          17.34106
                                         13.21
In [110]: | print(x_test[1:25,1])
           [1] 29.74443 29.72212 28.59920 32.21622 28.75303 37.26733 28.42089 28.67805
           [9] 28.01215 32.82848 40.40622 31.39477 26.73832 25.77284 23.87235 20.95774
          [17] 21.81312 18.66503 19.21226 18.77107 19.58575 18.71079 18.85455 17.35902
          [25] 17.34106
In [111]: | print(fit.pr\fcst\sebidas[,1])
           [1] 27.21901 26.34663 26.17810 26.14887 25.99588 25.89958 25.81573 25.73522
           [9] 25.66600 25.60448 25.54982 25.50180 25.45969 25.42298 25.39121 25.36396
          [17] 25.34083 25.32147 25.30556 25.29281 25.28294 25.27570 25.27087 25.26825
          [25] 25.26764
In [112]:
          mape(x test[,1],fit.pr$fcst$tsBebidas[,1])
```

21.3553427928945

```
In [113]: tsplot((x_test[1:24,1]))
    lines(fit.pr$fcst$tsBebidas[,1],col=4)
```



Como se puede observar el ajuste utilizando el paquete de **ARMAX**, *vars* no logra ajustar correctamente al final del semiciclo de los datos de test. Si bien este modelo es posible implementarlo rapidamente para tener una idea de valores de prediccion aproximados, lo mas recomendale es realizar antes todo el proceso de analisis de componentes estacionarias y luego aplicar un modelo predictivo.

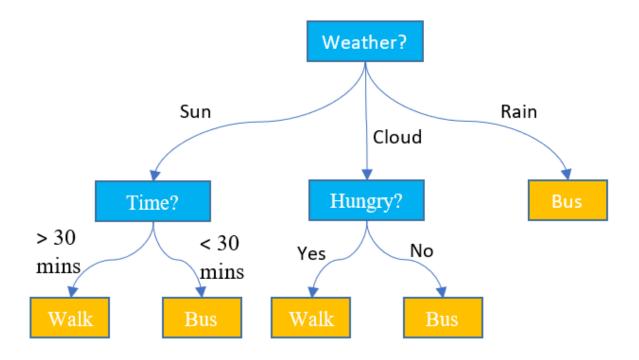
Modelo 9: Random forest

El modelo Random Forest es un bagging aplicado en árboles de decisión. Bagging es crear diferentes modelos usando muestras aleatorias con reemplazo y luego combinar o ensamblar los resultados.

Random Forest consiste en la creación de varios Decision Tree independientes, cada uno de los Tree genera su propia predicción y las mismas son promediadas para llegar al resultado final. Esto permite reducir la varianza de dicho modelo.

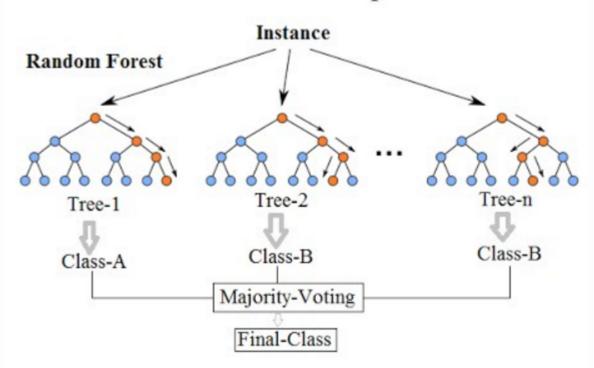
Los árboles son los candidatos ideales para el bagging, dado que ellos pueden registrar estructuras de interacción compleja en los datos, y si crecen suficientemente profundo, tienen relativamente baja parcialidad.

DECISION TREE



RANDOM FOREST: conjunto de decision tree promediados

Random Forest Simplified



```
In [114]: library(dplyr)
    library(randomForest)
    library(tidyverse)
    library(forecast)
```

```
Attaching package: 'dplyr'
The following object is masked from 'package:MASS':
     select
The following objects are masked from 'package:lubridate':
     intersect, setdiff, union
The following objects are masked from 'package:stats':
     filter, lag
The following objects are masked from 'package:base':
     intersect, setdiff, setequal, union
randomForest 4.6-14
Type rfNews() to see new features/changes/bug fixes.
Attaching package: 'randomForest'
The following object is masked from 'package:dplyr':
     combine
— Attaching packages ————
                                                            ----- tidyverse 1.2.1
— Conflicts —
                                                            — tidyverse_conflicts()
X lubridate::as.difftime() masks base::as.difftime()
x stringr::boundary() masks strucchange::boundary()
x randomForest::combine() masks dplyr::combine()
X lubridate::date()
X dplyr::filter()
masks base::date()
masks stats::filte
                               masks stats::filter()
X lubridate::intersect() masks base::intersect()
X dplyr::lag()
X ggplot2::margin()
X dplyr::select()
X lubridate::setdiff()
X lubridate::union()
masks stats::lag()
masks randomForest::margin()
masks MASS::select()
masks base::setdiff()
masks base::union()
Attaching package: 'forecast'
The following object is masked from 'package:astsa':
     gas
```

In [115]: head(base_train) head(base_test)

A data.frame: 6 x 6

Year	Weeknum	Yearweek	Sales_Liters	Temp_Avg	Class
<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<fct></fct>
2017	1	1	263778.2	22.88571	Train
2017	2	2	249777.5	23.24286	Train
2017	3	3	266308.4	23.52857	Train
2017	4	4	261133.6	21.95714	Train
2017	5	5	292004.5	24.58571	Train
2017	6	6	236130.0	21.11429	Train

A data.frame: 6 x 6

	Year	Weeknum	Yearweek	Sales_Liters	Temp_Avg	Class
	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<fct></fct>
157	2020	157	1	297444.3	21.44286	Test
158	2020	158	2	297221.2	21.98571	Test
159	2020	159	3	285992.0	23.73333	Test
160	2020	160	4	322162.2	21.37500	Test
161	2020	161	5	287530.2	23.61429	Test
162	2020	162	6	372673.3	21.38571	Test

```
In [116]: rf = randomForest(Sales_Liters ~ Year + Weeknum + Yearweek + Temp_Avg , data = base_train)
```

In [117]: print(rf)

Call:

randomForest(formula = Sales_Liters ~ Year + Weeknum + Yearweek + Temp_
Avg, data = base_train)

Type of random forest: regression Number of trees: 500

No. of variables tried at each split: 1

Mean of squared residuals: 399772516 % Var explained: 76.56

```
In [118]: predictions_train = predict(rf, newdata = base_train)
    round(mape(base_train$Sales_Liters, predictions_train),2)

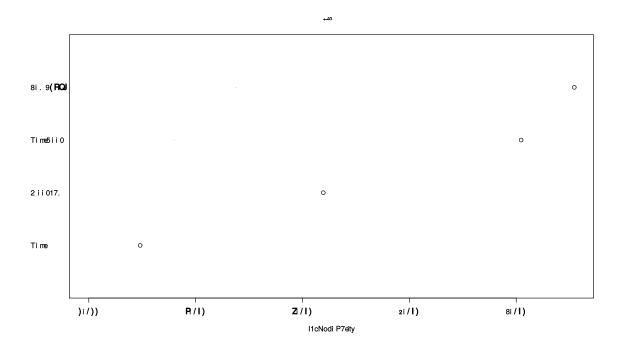
    predictions_test = predict(rf, newdata = base_test)
    round(mape(base_test$Sales_Liters, predictions_test),2)

3.84

16.2
```

MAPE es 4% en train y 16% en test, es un claro caso de overfitting. Vamos a probar utilizando menos variables

```
In [119]: varImpPlot(rf)
# Para ver que variables impactan más en el modelo
```



Yearweek y Temp_Avg son las variables que más influyen

```
In [121]: predictions_train = predict(rf_revised, newdata = base_train)
    round(mape(base_train$Sales_Liters, predictions_train),2)

predictions_test = predict(rf_revised, newdata = base_test)
    round(mape(base_test$Sales_Liters, predictions_test),2)

3.87

9.91
```

El modelo mejora desde 16% en test a 10%

Probamos utilizar solo la temperatura

```
In [122]: rf_revised2 = randomForest(Sales_Liters ~ Temp_Avg , data = base_train)
          print(rf_revised2)
          Call:
           randomForest(formula = Sales_Liters ~ Temp_Avg, data = base_train)
                         Type of random forest: regression
                               Number of trees: 500
          No. of variables tried at each split: 1
                    Mean of squared residuals: 944755305
                              % Var explained: 44.6
In [123]:
          predictions train2 = predict(rf revised2, newdata = base train)
          round(mape(base train$Sales Liters, predictions train2),2)
          predictions_test2 = predict(rf_revised2, newdata = base_test)
          round(mape(base test$Sales Liters, predictions test2),2)
          5.56
          13.93
```

Sube el error en train y test, es razonable porque nos quedamos sin variable de tiempo

'matrix.unlist.predictions_test...nrow...25..byrow...T.'

```
In [126]: names(RF_Pred)[names(RF_Pred) == "matrix.unlist.predictions_test...nrow...25..
byrow...T."] <- "Sales_Liters"
head(RF_Pred)</pre>
```

A data.frame: 6

x 1

Sales_Liters

<dbl></dbl>
283048.7
285721.4
271756.5
263674.6
288575.1
266826.7

A data.frame: 6 x 2

Weeknum Sales_Liters

<int></int>	<dbl></dbl>
157	283048.7
158	285721.4
159	271756.5
160	263674.6
161	288575.1
162	266826.7

In [128]: train1 <- base_train %>% select(c(Weeknum, Sales_Liters)) head(train1)

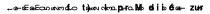
A data.frame: 6 x 2

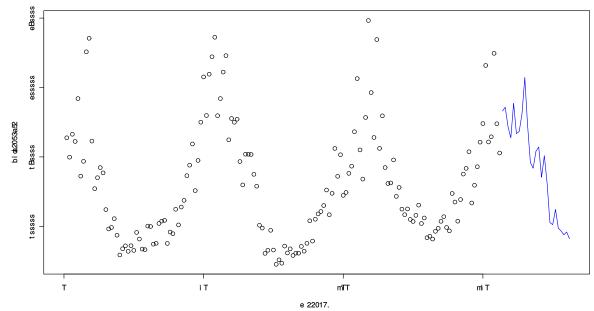
Weeknum	Sales_Liters
<dbl></dbl>	<dbl></dbl>
1	263778.2
2	249777.5
3	266308.4
4	261133.6
5	292004.5
6	236130.0

```
In [129]: test <- base_test %>% select(c(Weeknum, Sales_Liters))
head(test)
```

A data.frame: 6 x 2

	Weeknum	Sales_Liters
	<dbl></dbl>	<dbl></dbl>
157	157	297444.3
158	158	297221.2
159	159	285992.0
160	160	322162.2
161	161	287530.2
162	162	372673.3





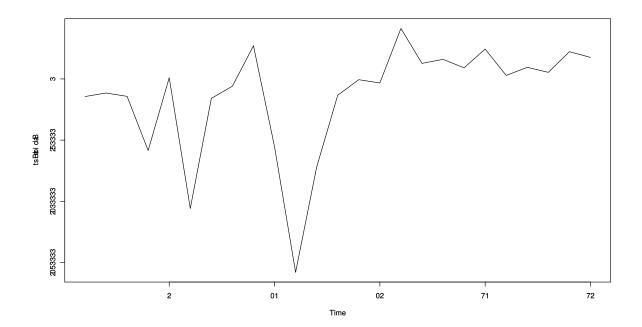
Vemos si los residuos del modelo son ruido blanco Gaussiano

```
In [134]: residuals <- round(RF_Pred$Sales_Liters - test$Sales_Liters)</pre>
```

```
In [135]: residuals_ts <- ts(residuals, start=157, end=181, frequency=1)
    residuals_ts
    ts.plot(residuals)</pre>
```

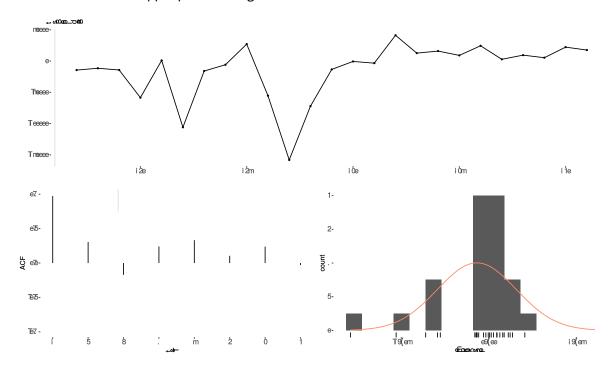
A Time Series:

-14396 -11500 -14236 -58488 1045 -105847 -15884 -5897 27106 -55359 -158054 -72069 -13305 -607 -3319 41252 12716 15997 9094 24431 2838 9494 5383 22283 17617



In [136]: checkresiduals(residuals_ts)

Warning message in modeldf.default(object): "Could not find appropriate degrees of freedom for this model."



El ACF da muy bien en una serie que ha demostrado tener muy alta autocorrelación.

El ruido es blanco pero no llega a ser gaussiano. Entendemos que esto se da porque los grandes desaciertos son las compras de pánico en las que el modelo se queda corto, y no tiene grandes errores donde se pase. Como no tuvimos compras de pánico en los datos de entrenamiento es entendible que esto suceda y no lo vemos como una falla del modelo.

Para mejorar las predicciones de cara a futuro podemos crear una nueva variable dummy que sea situación de emergencia y llenar las semanas de marzo y abril donde esto sucedió así el modelo puede aprender de ello de cara a futuro y no nos altera a las demás variables.

CONCLUSIONES

Utilizamos datos reales de venta de agua y de la temperatura, para los cuáles no había un estudio previo adaptado a la realidad de nuestro país. Se trabajó mucho en la calidad de los datos previo a los análisis de la serie para poder llegar a un modelo que performe bien en la realidad y no sea sesgado por errores en los datos.

Realizamos 9 modelos en total (más varios que fueron a modo borrador) en los que incluimos diversas técnicas aprendidas en clase así como algunas otras que investigamos, ya sea con el libro o con papers que encontramos en la web.

Estas técnicas incluyeron:

- Análisis de componentes estacionarias
- Ajustes lineales y no lineales
- · Análisis de residuos
- · Correlaciones cruzadas
- Manejo de herramientas para series multivariables
- · Técnicas de machine learning

Para validar los modelos utilizamos la metodología vista en clase de analizar el ruido y entender si es ruido blanco gaussiano con la menor varianza posible. Además dividimos nuestro dataset en train y test y comparamos nuestras predicciones con los datos de test que el modelo no utilizó para entrenar, como herramienta para evitar overfitting y tener otro indicador de la efectividad de predicción del modelo a efectos de la elección del modelo.

Nuestros modelos que realizaron mejores predicciones fueron el Random forest así como el Modelo 7 (basado en el análisis de residuos y agregando las componentes estacionarias). Para hacer la elección entendemos que en accuracy están muy parejos, por lo que elegiríamos el Modelo 7 por su mayor interpretabilidad ya que tiene una base estadística con una clara explicación y no es tan randómica como es un random forest. Igualmente nos quedamos con el random forest como modelo challenger para tener un respaldo y por si en los eventos futuros se adapta mejor a la dinámica de la realidad.