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Taller 1 Calculo Integral

1. Evaluar los limites

a.) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n^3}(i-2)^2$

$$\lim_{n\to\infty} \frac{1}{n^3}\sum_{i=1}^{n}(i^2-4i+4) = \lim_{n\to\infty} \frac{1}{n^3}\sum_{i=1}^{n} i^2 - 4\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} 1$$

$$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - 4\left(\frac{n(n+1)}{2}\right) + 4n\right]$$

$$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3+3n^2+n}{6} - 2n^2-2n + 4n\right]$$

$$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3+3n^2+n}{6} - 2n^2+2n\right]$$

$$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3+3n^2+n-12n^2+12n}{6}\right] = \lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3-9n^2+13n}{6}\right]$$

$$\lim_{n\to\infty} \frac{2n^3-9n^2+13n}{6n^3}$$ Aplicando la regla de L'Hopital resolvemos el limite

$$\lim_{n\to\infty} \frac{6n^2-18n}{18n^2} = \lim_{n\to\infty} \frac{12n-18}{36n} = \lim_{n\to\infty} \frac{12}{36} = \frac{1}{3}$$

R/ $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n^3}(i-2)^2 = \frac{1}{3}$

$\longrightarrow$

b.) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(1+\frac{3i}{n}\right)^2$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\frac{(3i+n)^2}{n^2}\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(3i+n)^2}{n^3}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(9i^2+6in+n^2)}{n^3} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{18i^2+12in+2n^2}{n^3}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{18i^2}{n^3} + \sum_{i=1}^{n} \frac{12in}{n^3} + \sum_{i=1}^{n} \frac{2n^2}{n^3}$$

$$\lim_{n\to\infty} \frac{18}{n^3}\sum_{i=1}^{n} i^2 + \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{2}{n}\sum_{i=1}^{n} 1$$

$$\lim_{n\to\infty} \frac{18}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{12}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{2}{\cancel{n}}\cdot \cancel{n}$$

$$\lim_{n\to\infty} \frac{36n^3+54n^2+18n}{6n^3} + \frac{6n^2+n}{n^2} + 2$$

$$\lim_{n\to\infty} \frac{36n^3+54n^2+18n}{6n^3} + \lim_{n\to\infty} \frac{6n^2+n}{n^2} + \lim_{n\to\infty} 2$$

Aplicando la regla de L'Hopital resolvemos los limites

$$\lim_{n\to\infty} \frac{108n^2+108n+18}{18n^2} + \lim_{n\to\infty} \frac{12n+1}{2n}\left[+\ 2\right]$$

$$\lim_{n\to\infty} \frac{216n+108}{36n} + \lim_{n\to\infty} \frac{12}{2}\left[+\ 2\right]$$

$$\lim_{n\to\infty} \frac{216}{36} + \left[6+2\right] = 6+6+2 = 14$$

R/ $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(1+\frac{3i}{n}\right)^2 = 14$

2. Evaluar por el metodo de sumas de Riemann el area bajo la curva de:

$y = -\frac{x^2}{2} + 4x + 2$ en el eje x entre las rectas $x=1$ y $x=6$

- $\int_1^6 -\frac{x^2}{2} + 4x + 2\,dx$ $\quad a=1 \quad b=6 \quad \Delta x = \frac{6-1}{n} = \frac{5}{n}$

$$f\left(1+\frac{5K}{n}\right) = -\frac{\left(1+\frac{5K}{n}\right)^2}{2} + 4\left(1+\frac{5K}{n}\right) + 2$$

$$= -\frac{(5K+n)^2}{2n^2} + 4 + \frac{20K}{n} + 2$$

$$= \frac{-(5K+n)^2}{2n^2} + \frac{20K}{n} + 6$$

$$= \frac{-25K^2+30Kn-n^2}{2n^2} + 6$$

- $$\int_1^6 -\frac{x^2}{2} + 4x + 2\,dx = \lim_{n\to\infty} \sum_{K=1}^{n} \left(\frac{-25K^2+30Kn-n^2}{2n^2} + 6\right)\left(\frac{5}{n}\right)$$

$$\lim_{n\to\infty} \sum_{K=1}^{n} \frac{-125K^2+150Kn-5n^2}{2n^3} + \frac{30}{n}$$

$$\lim_{n\to\infty} \sum_{K=1}^{n} \frac{-125K^2}{2n^3} + \sum_{K=1}^{n} \frac{150Kn}{2n^3} - \sum_{K=1}^{n} \frac{5n^2}{2n^3} + \sum_{K=1}^{n} \frac{30}{n}$$

$$\lim_{n\to\infty} \frac{-125}{2n^3} \sum_{K=1}^{n} K^2 + \frac{75}{n^2} \sum_{K=1}^{n} K - \frac{5}{2n} \sum_{K=1}^{n} 1 + \frac{30}{n} \sum_{K=1}^{n} 1$$

$$\lim_{n\to\infty} \frac{-125}{2n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{75}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{5}{2} + 30$$

$$\lim_{n\to\infty} \frac{-250n^3-375n^2-125n}{12n^3} + \lim_{n\to\infty} \frac{75n^2+75n}{2n^2} + \lim_{n\to\infty} \frac{55}{2}$$

Aplicando la regla de L'Hopital resolvemos los limites :

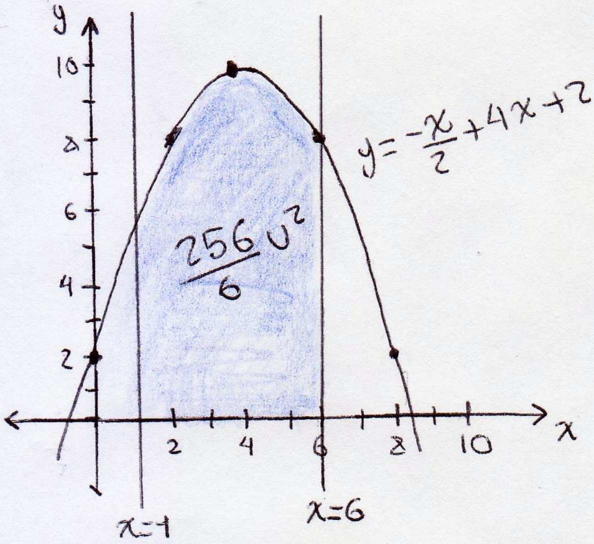
$$\lim_{n\to\infty}\left(\frac{-750n^2-750n-125}{36n^2}\right)+\lim_{n\to\infty}\left(\frac{150n+75}{4n}\right)+\frac{55}{2}$$

$$\lim_{n\to\infty}\left(\frac{-1500n-750}{72n}\right)+\lim_{n\to\infty}\left(\frac{150}{4}\right)+\frac{55}{2}$$

$$\lim_{n\to\infty}\left(-\frac{1500}{72}\right)+\frac{150}{4}+\frac{55}{2}=-\frac{1500}{72}+\frac{150}{4}+\frac{55}{2}=\frac{256}{6}u^2$$

R/ $\int_1^6 -\frac{x}{2}+4x+2\,dx=\frac{256}{6}u^2$

Grafica :



x	y
0	2
2	8
4	10
6	8
8	2

3. probar la desigualdad $-\sqrt{2}\leq\int_5^9\left(\frac{1}{x-1}\right)dx\leq 2$

- $\frac{1}{4}\int_5^9\left(\frac{1}{x-1}\right)dx$ $\quad u=x-1$, $du=1$

$\frac{1}{4}\int_5^9\frac{du}{u}=\frac{1}{4}\left[\ln u\Big|_5^9 \ln(x-1)-\ln(x-1)\right]$

$\frac{1}{4}\left[\ln(8)-\ln(4)\right]=\frac{1}{4}\left[0,6931\right]=0,1732$

R/ Se demuestra la desigualdad ya que $-\sqrt{2}\leq 0,1732\leq 2$

4. a) Para $f(x) = \operatorname{sen} x$, determine una antiderivada que satisfaga $F(0) = 3$

$$\int \operatorname{sen} x \, dx = -\cos x + C \qquad \begin{array}{l} y = 3 \\ x = 0 \end{array}$$

$$3 = -\cos(0) + C \qquad R/ \; y = -\cos x + 4$$

$$3 = -1 + C$$

$$C = 4$$

b.) Para $f(x) = x^3 - 20$, determine una antiderivada que satisfaga $F(2) = 5$

$$\int x^3 - 20 \, dx = \int x^3 \, dx - \int 20 \, dx = \frac{x^4}{4} - 20x + C$$

$$\begin{array}{l} y = 5 \\ x = 2 \end{array} \qquad 5 = \frac{16}{4} - 40 + C$$

$$5 = -36 + C$$

$$C = 41$$

$$R/ \; y = \frac{x^4}{4} - 20x + 41$$

5. Resolver las integrales:

a.) $\int_1^2 \left(\frac{x^2 + 3x + 4}{\sqrt[5]{x}} \right) dx$

$$\int_1^2 \frac{x^2}{x^{1/5}} dx + \int_1^2 \frac{3x}{x^{1/5}} dx + \int_1^2 \frac{4}{x^{1/5}} dx$$

$$\int_1^2 x^{9/5} \, dx + 3\int_1^2 x^{4/5} \, dx + 4\int_1^2 \frac{1}{x^{1/5}} dx$$

$$\int_1^2 x^{9/5} dx + 3\int_1^2 x^{4/5} dx + 4\int_1^2 x^{-1/5} dx$$

$$\frac{1}{14/5} x^{14/5} + \frac{3}{9/5} x^{9/5} + \frac{4}{4/5} x^{4/5} + C$$

$$\frac{5}{14} x^{14/5} + \frac{5}{3} x^{9/5} + 5x^{4/5} \Big|_1^2 \quad F(2) - F(1) = 9{,}972$$

$$R/ \int_1^2 \frac{x^2 + 3x + 4}{\sqrt[5]{x}} dx = 9{,}972$$

b. $\int_0^{\sqrt{3}} \left(\frac{dx}{\sqrt{12-x^4}} \right)$

- $\int_a^b f(x)\,dx \simeq \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right)$

- $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{12-x^4}} \simeq \frac{\sqrt{3}}{6}\left(\frac{\sqrt{3}}{6} + 4\left(0{,}2957\right) + \frac{\sqrt{3}}{3} \right)$

$$\simeq \frac{\sqrt{3}}{6}\left(\frac{\sqrt{3}}{6} + \frac{2957}{2500} + \frac{\sqrt{3}}{3} \right)$$

$$\simeq \frac{\sqrt{3}}{6}\left(2{,}049\right) \simeq 0{,}5914$$

R/ $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{12-x^4}} \simeq 0{,}5914$